CoEPP
ARC Centre of Excellence for Particle Physics at the Terascale

Tracking with silicon detector

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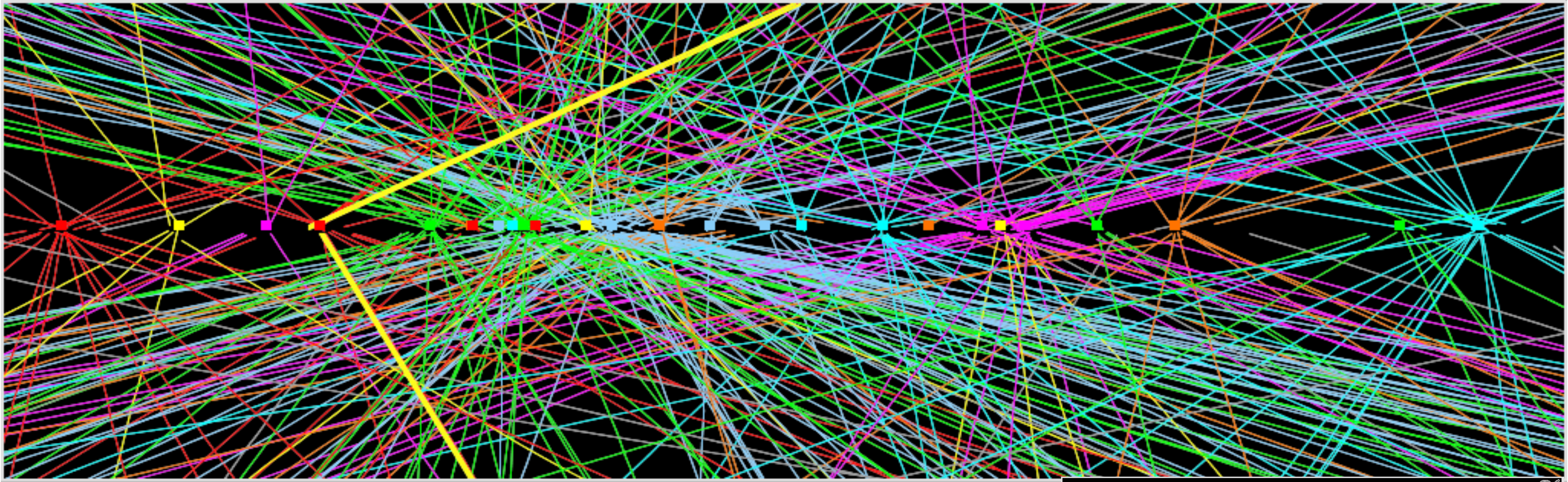
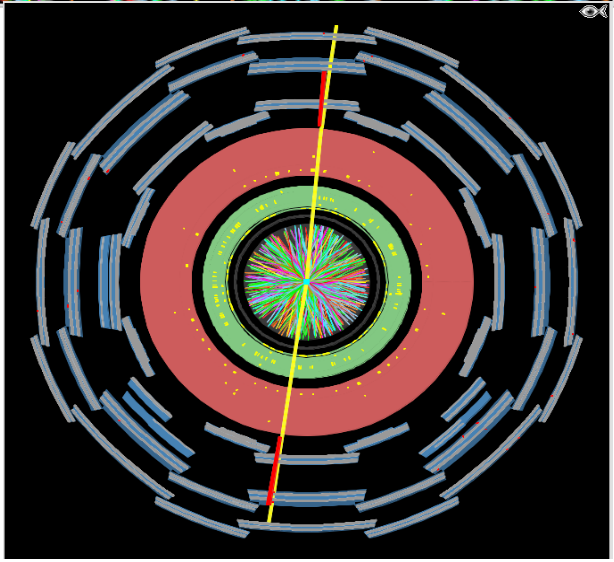
Outline

1. **What is a silicon tracking system**
2. **Track parameters and resolutions**
3. **Silicon detectors**
4. **Tracking systems**
5. Understanding track fitting (optional)
6. **Performance and applications**

WHAT IS
A (SILICON) TRACKING SYSTEM?

ATLAS Event: Z→μμ at high pileup



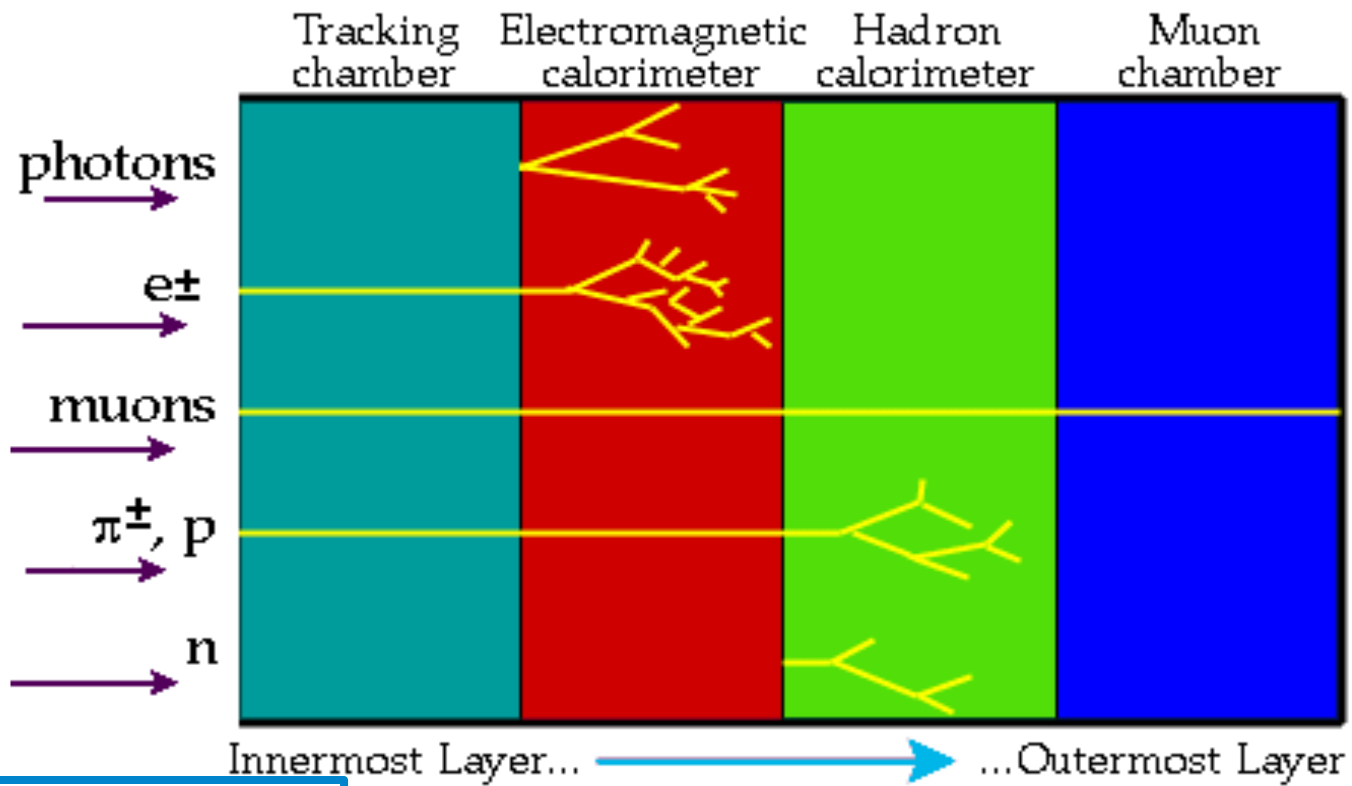


Reconstruction of charged particles produced in particle physics experiments:

- with high granularity
- with high precision:
  - position down to O(10 μm)
  - momentum down to 10^{-3}

Unique way to access the region of the interaction vertex (pileup, short lived particles)

A particle physics detector



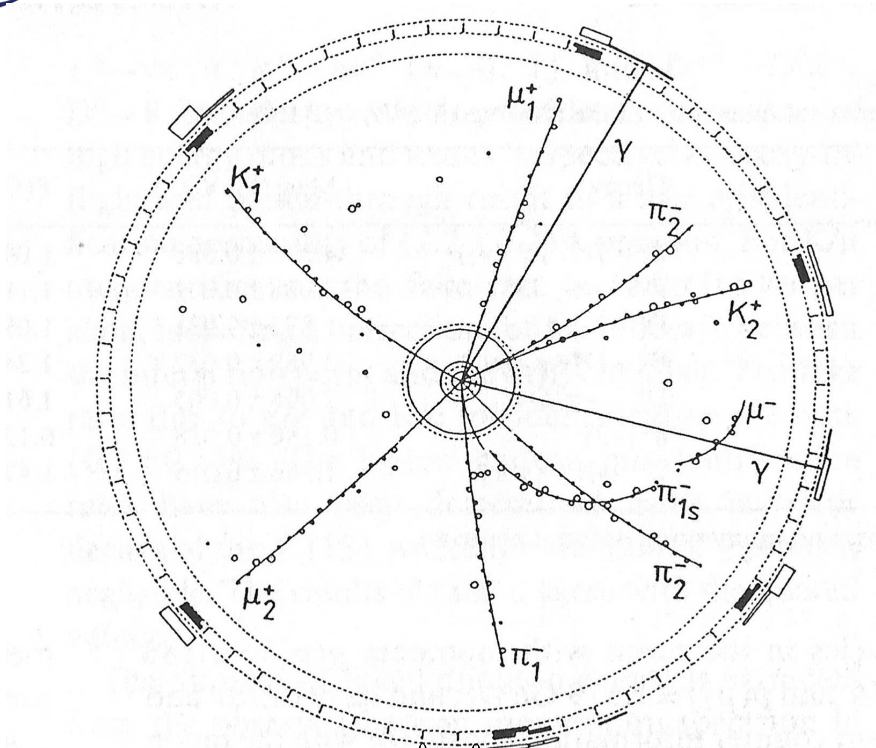
Momentum measurement, Production and decay vertex reconstruction, hadron identification. Light material (~1X_0)

Electron indentification, photon and π^0 measurement. ~20 X_0, 1-2 λ_I

Energy measurement of charged and neutral hadrons. 5-6λ_I

Muon identification, if energettic enough to pass through the calorimeters (tipically >2-3 GeV)

Evolution of tracking systems

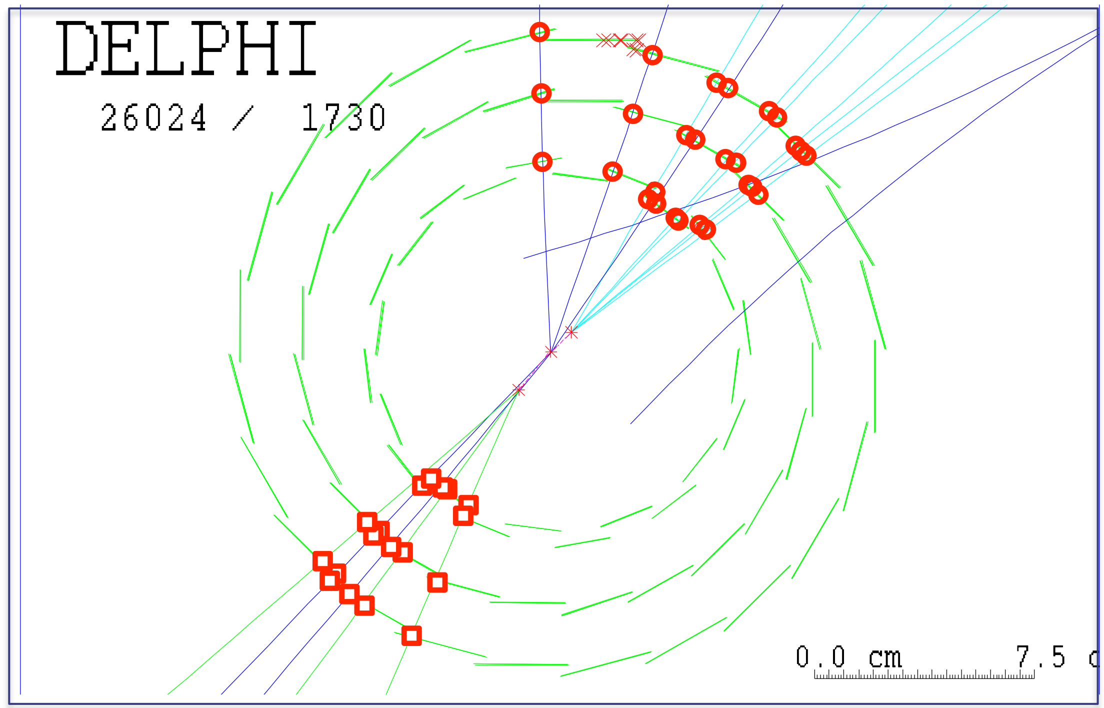


1) $B_1^0 \to D_1^{*-}\mu_1^+\nu_1,\ D_1^{*-} \to \pi_1^-\bar{D}^0,\ \bar{D}^0 \to K_1^+\pi_1^-$

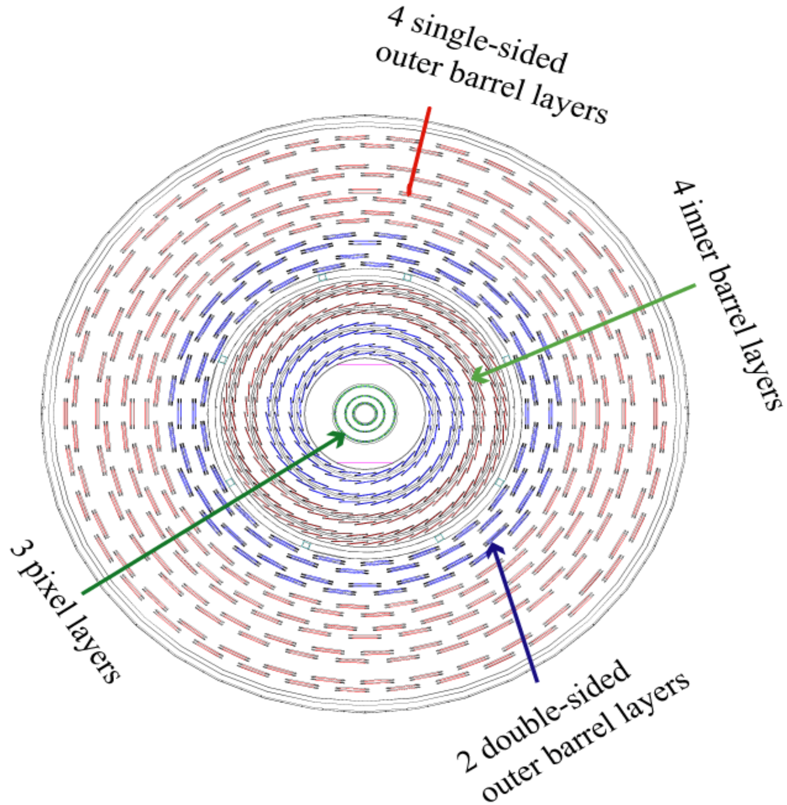
2) $B_2^0 \to D_2^{*-}\mu_2^+\nu_2,\ D_2^{*-} \to \pi^0 D^-,\ D^- \to K_2^+\pi_2^-\pi_2^-$

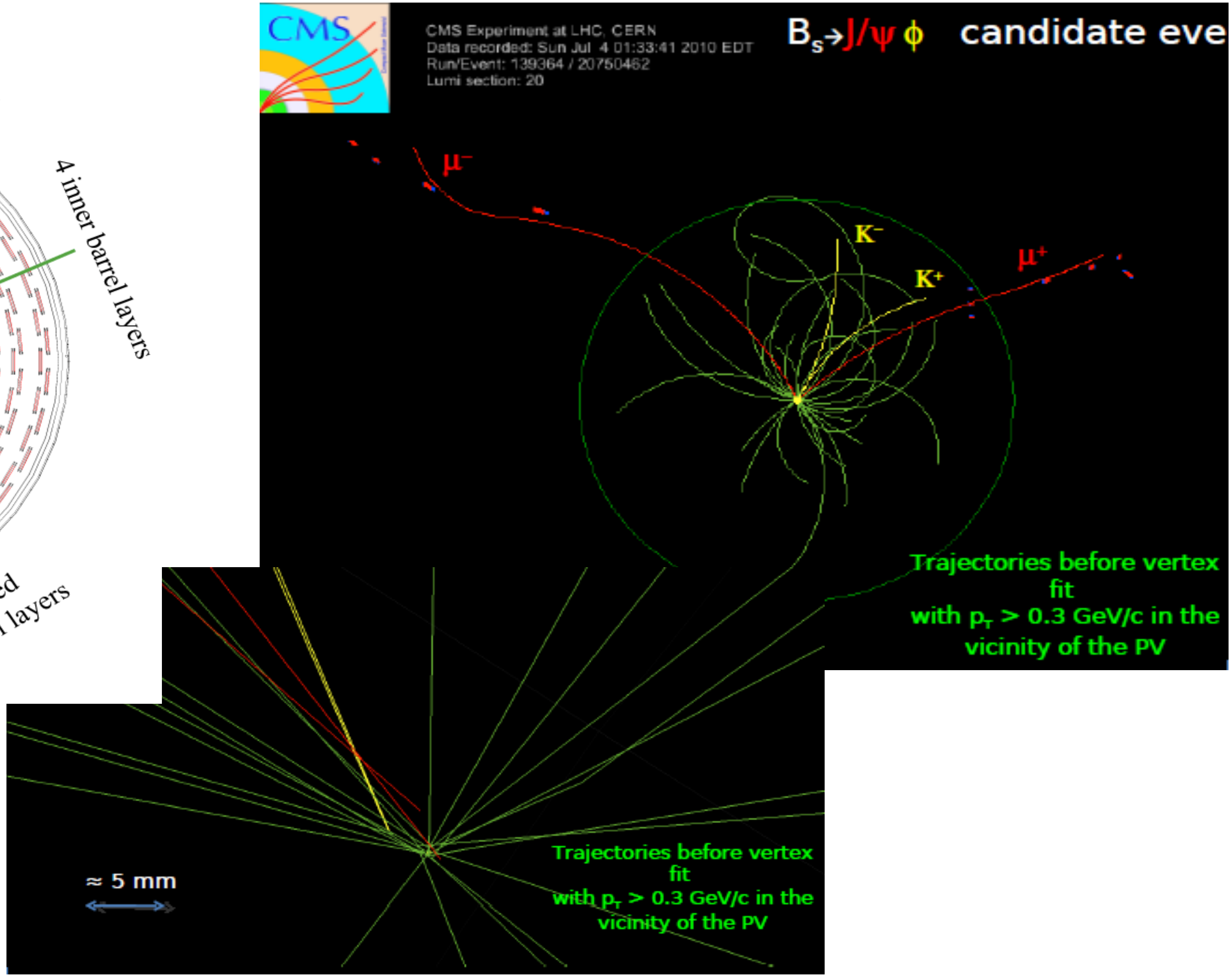
From continuous tracking with gas detector...

...through the addition of few very precise Si-based measurements near to the interaction region...



Evolution of tracking systems





...to all Si trackers.

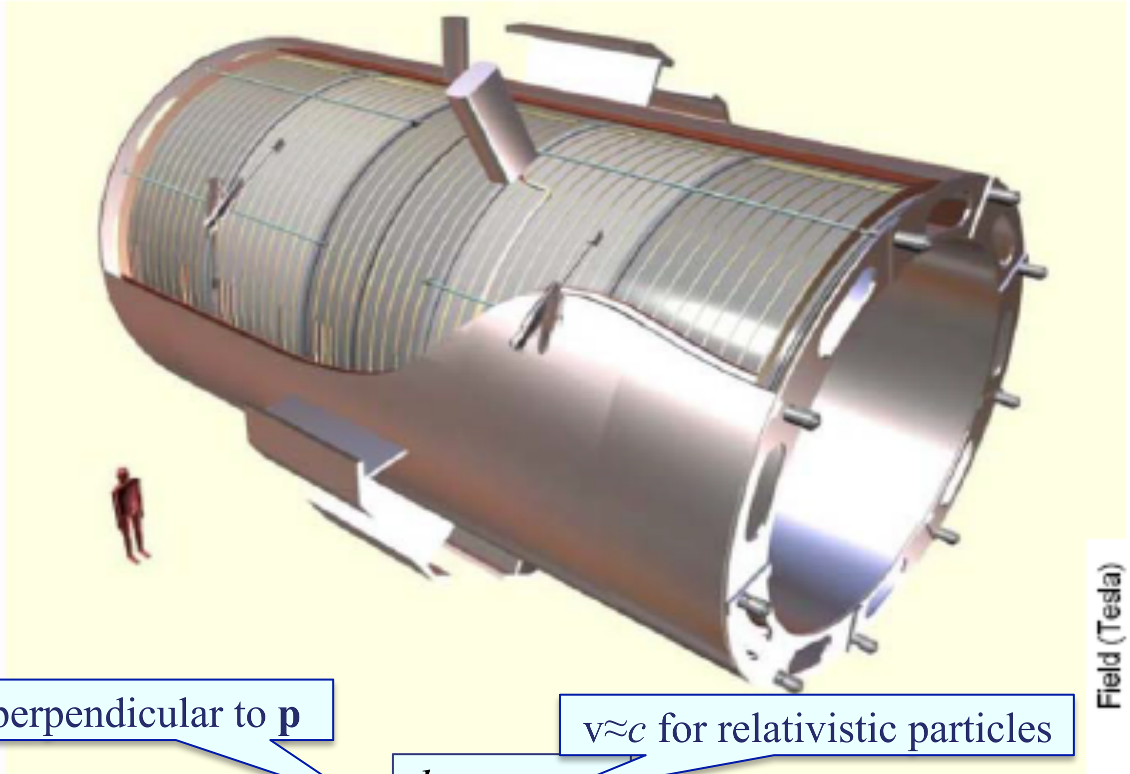
References

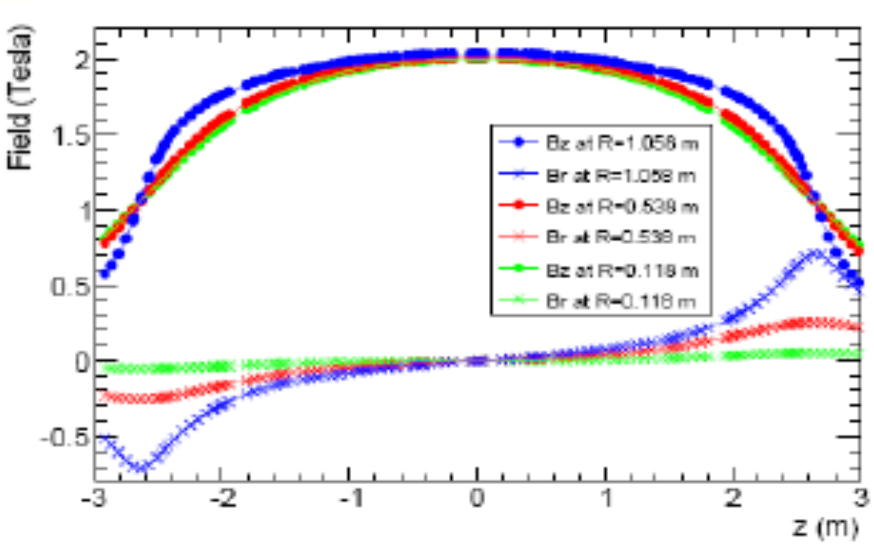
- A lot of the contents of today slides is taken from:
  - F. Ragusa, *New Journal of Physics 9 (2007) 336*
  - The lectures given at the CERN EDIT 2011 School (especially by P. Wells) **http://edit2011.web.cern.ch/edit2011**
  - C. Haber's lectures at the TIPP 2011 conference **http://conferences.fnal.gov/tipp11**
- Books
  - Kleinknecht, *Detectors for Particle Radiation*, Cambridge University Press
  - Fernow, *Introduction to experimental particle physics*, Cambridge University Press
  - Regler, *Data analysis techniques for high-energy physics experiments*, Cambridge University Press

THE TRACK MODEL

Do not try to follow all formulas during the lecture...they are there just for reference

Superconducting solenoids





perpendicular to **p**

v≈*c* for relativistic particles

$$\frac{d\mathbf{p}}{dt} = e\mathbf{v}\times\mathbf{B}$$

- momentum measured from angular deflection $\frac{1}{p}\frac{d\mathbf{p}}{dt}$
- solenoid magnets mostly used at colliders:
  uniform field on large volumes
  ...but dipoles or toroids too

Motion in a magnetic field

- The **Lorentz force** does not change the energy of a particle

$$\frac{d\mathbf{p}}{dt} = e\mathbf{v} \times \mathbf{B}$$

- since we measure a trajectory, we explicit the position vector **r**:

$$m\gamma \frac{d\mathbf{v}}{dt} = e\mathbf{v} \times \mathbf{B}$$

$$m\gamma \frac{d^2\mathbf{r}}{dt^2} = e\frac{d\mathbf{r}}{dt} \times \mathbf{B}$$

- and, since v is constant, can use the path length s=vt:

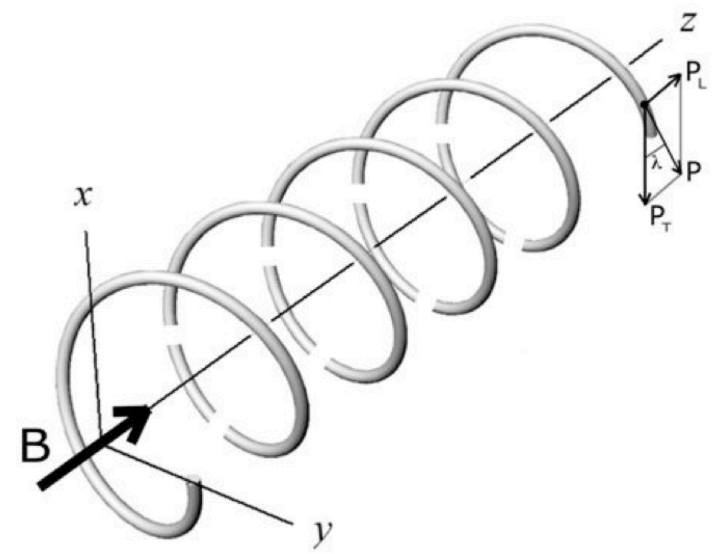
$$ds = \mathrm{v}\,dt$$

$$m\gamma \mathrm{v} \frac{d^2\mathbf{r}}{ds^2} = e\frac{d\mathbf{r}}{ds} \times \mathbf{B}$$

- finally:

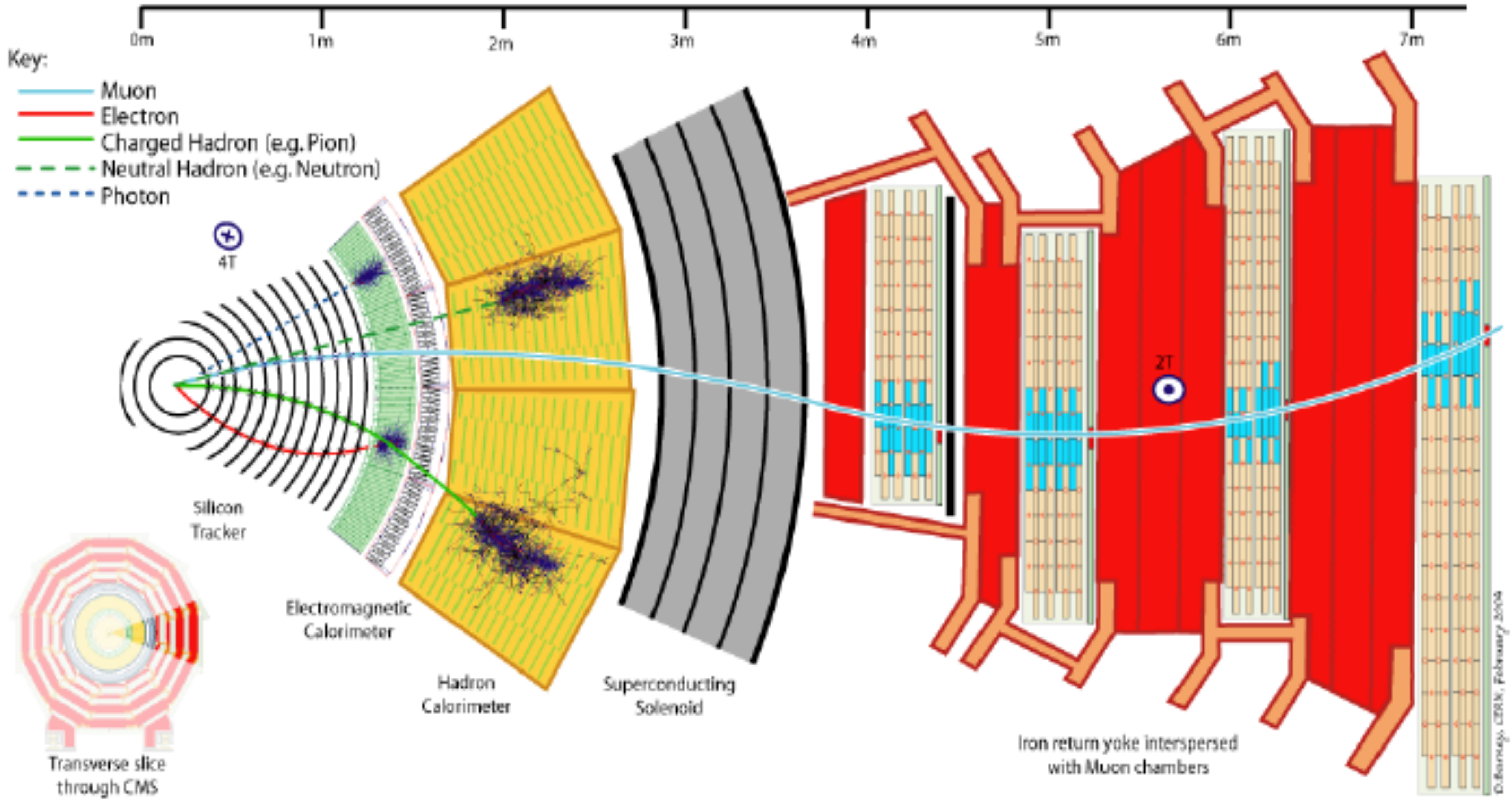
$$\frac{d^2\mathbf{r}}{ds^2} = \frac{e}{p}\frac{d\mathbf{r}}{ds} \times \mathbf{B}$$

- If the **B** field is **homogeneous** the trajectory in a **helix**:



- In the more general case of **inhomogeneous** magnetic field, **B**(s) varies along the trajectory **r**(s), and the differential equation needs to be solved numerically.

A CMS slice


0m
1m
2m
3m
4m
5m
6m
7m
Key:
Muon
Electron
Charged Hadron (e.g. Pion)
Neutral Hadron (e.g. Neutron)
Photon
4T
2T
Silicon
Tracker
Electromagnetic
Calorimeter
Hadron
Calorimeter
Superconducting
Solenoid
Iron return yoke interspersed
with Muon chambers
Transverse slice
through CMS
D.Barney, CERN, February 2004

The helix equation

$$R[\mathrm{m}] = \frac{p_{\mathrm{T}}[\mathrm{GeV}]}{0.3B[\mathrm{T}]}$$

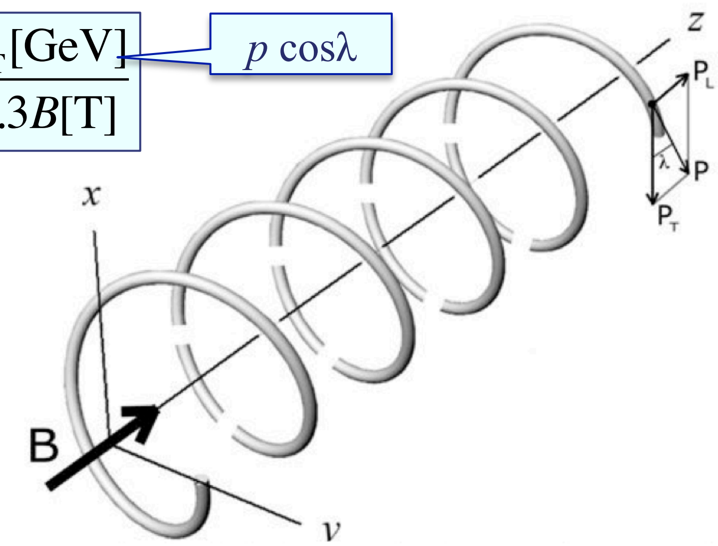
$p \cos\lambda$

- The **helix** is described in **parametric** form:

$$x(s) = x_0 + R\left[\cos\left(\Phi_0 + \frac{hs\cos\lambda}{R}\right) - \cos\Phi_0\right]$$

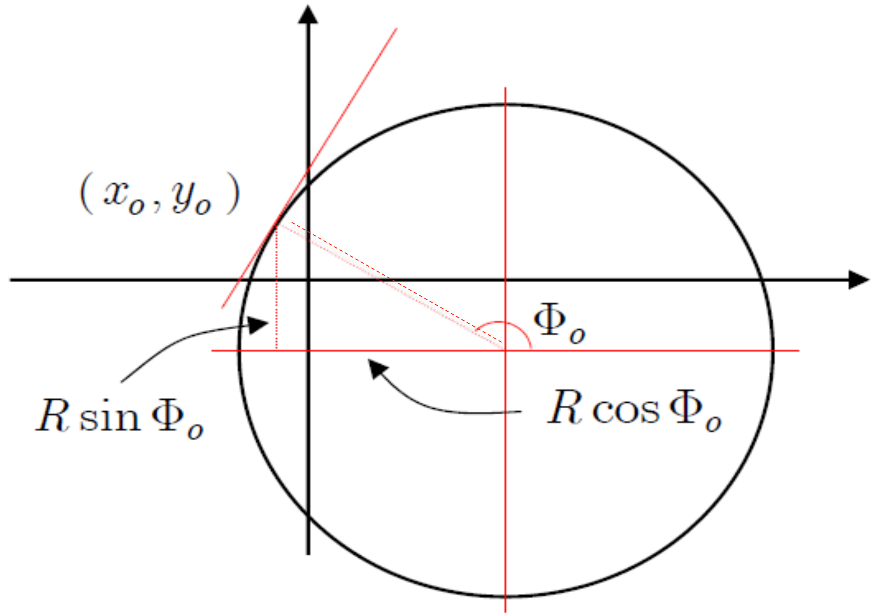
$$y(s) = y_0 + R\left[\sin\left(\Phi_0 + \frac{hs\cos\lambda}{R}\right) - \sin\Phi_0\right]$$

$$z(s) = z_0 + s\sin\lambda$$



- λ is the **dip-angle**
- $h=\pm1$ is the sense of rotation (**sign** of the charge)
- The projection on the x-y plane is a circle:

$$\left(x - x_0 + R\cos\Phi_0\right)^2 + \left(y - y_0 + R\sin\Phi_0\right)^2 = R^2$$

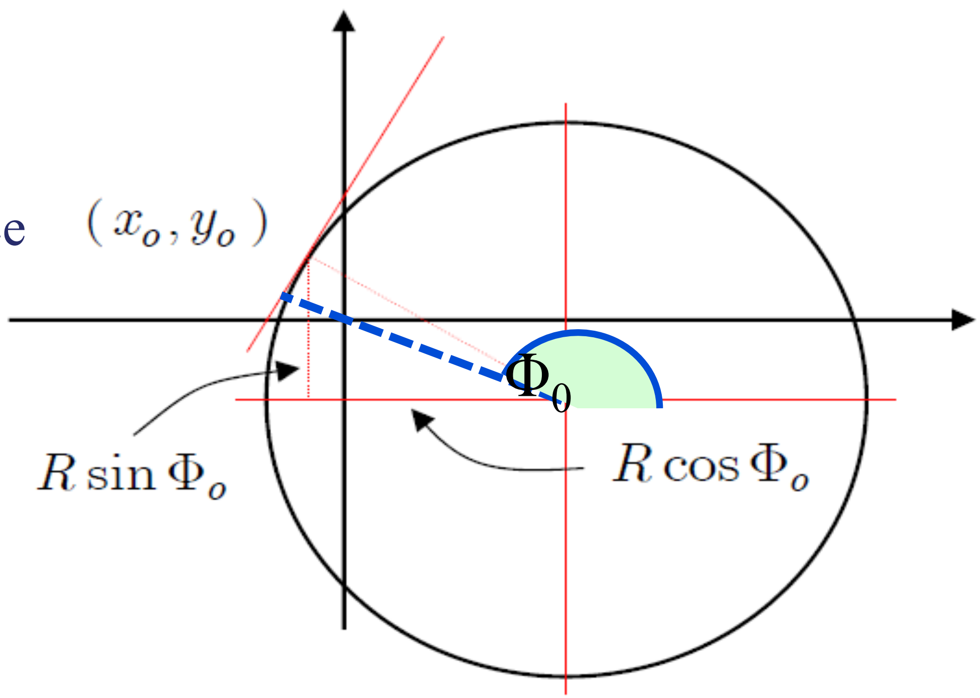


- x_0 and y_0 are the coordinates at $s=0$
- Φ_0 is also related to the slope of the tangent of the circle at $s=0$

Perigee parameters

- In the helix equation:
  - The s=0 point is an arbitrary choice
  - A common use case is when **the track is reconstructed in a region of size ≪ *R***
    - p_T=1 GeV, B=2 T, R=1.7 m
    - radius of ATLAS traking system is 1.05 m
    - …or if interested in the proximity of the interaction region
- Choose as reference point the **perigee**: the closest point to the origin of the reference frame (i.e. detector center)
- Write as a Taylor expansion in s/R
  - this is an approximation!
    - error $O(s^3/R^2)$
  - but it will be very useful for future examples

$$p_T[\text{GeV}] = 0.3B[\text{T}]R[\text{m}]$$



Perigee parameters

- Development in s/R:

$$x(s) = x_0 - hs\cos\lambda\sin\Phi_0 - \frac{1}{2}\frac{s^2\cos^2\lambda}{R}\cos\Phi_0$$
$$y(s) = y_0 + hs\cos\lambda\cos\Phi_0 - \frac{1}{2}\frac{s^2\cos^2\lambda}{R}\sin\Phi_0$$
$$z(s) = z_0 + s\sin\lambda$$

- we can now introduce the **perigee parameters**:
  - **impact parameter** d_0:

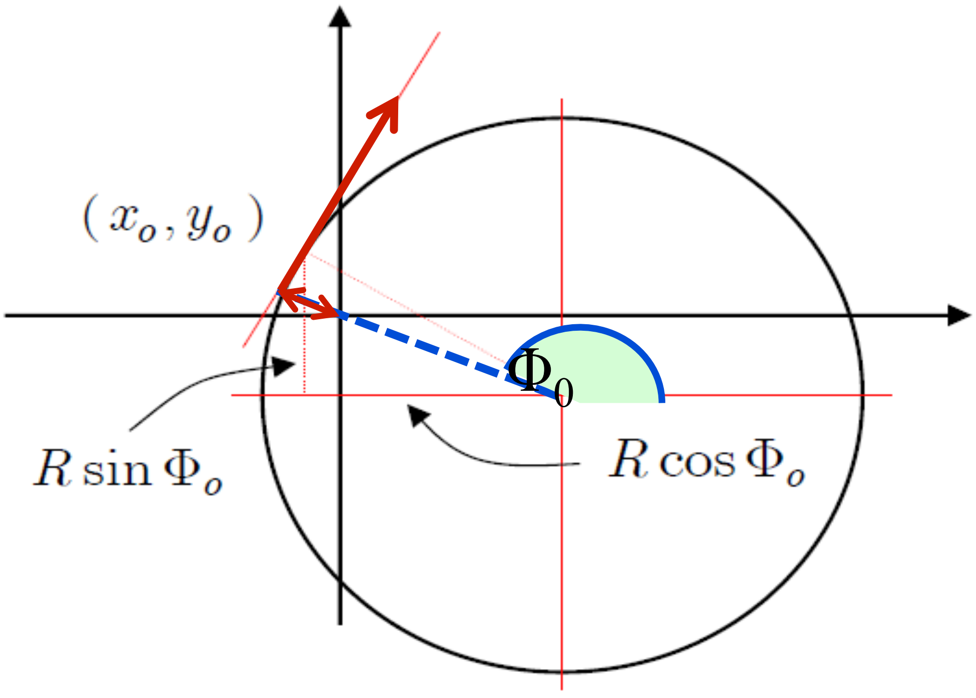
    $$x_0 = d_0 h\cos\Phi_0, \quad y_0 = d_0 h\sin\Phi_0$$

    notice it has a sign!
  - the **direction of the track** at the perigee φ_0:

    $$\cos\varphi_0 = h\sin\Phi_0, \quad \sin\varphi_0 = -h\cos\Phi_0$$
  - the **curvature** $\kappa = \frac{h}{R}$

    which includes the sign of the charge
  - and the **polar angle** $\vartheta = \frac{\pi}{2} - \lambda$

$$x(s) = -d_0\sin\varphi_0 + s\sin\vartheta\cos\varphi_0 + \tfrac{1}{2}\kappa s^2\sin^2\vartheta\sin\varphi_0$$
$$y(s) = d_0\cos\varphi_0 + s\sin\vartheta\sin\varphi_0 - \tfrac{1}{2}\kappa s^2\sin^2\vartheta\cos\varphi_0$$
$$z(s) = z_0 + s\cos\vartheta$$



High-p_T parabolic approximation

- Starting from the parametric trajectory

$$\begin{aligned} x(s) &= -d_0 \sin\varphi_0 + s\sin\vartheta\cos\varphi_0 + \tfrac{1}{2}\kappa s^2 \sin^2\vartheta \sin\varphi_0 \\ y(s) &= d_0\cos\varphi_0 + s\sin\vartheta\sin\varphi_0 - \tfrac{1}{2}\kappa s^2\sin^2\vartheta\cos\varphi_0 \\ z(s) &= z_0 + s\cos\vartheta \end{aligned}$$

- It is now interesting to define a change of coordinates $x,y \rightarrow x',y'$, with the x'-axis directed along the track direction:

$$\begin{aligned} x' &= x\cos\phi_0 + y\sin\phi_0 \\ y' &= -x\sin\phi_0 + y\cos\phi_0 \end{aligned}$$

$$\begin{aligned} x'(s) &= s\sin\vartheta \\ y'(s) &= d_0 - \tfrac{1}{2}\kappa s^2\sin^2\vartheta \\ z(s) &= z_0 + s\cos\vartheta \end{aligned}$$

- In these coordinates the trajectory has a simple expression in the **longitudinal** ρ-z and **transverse** ρ,y' planes:

$$\begin{aligned} z &= z_0 + x'\tan\vartheta \\ y' &= d_0 - \tfrac{1}{2}\kappa x'^2 \end{aligned}$$

  - Sometimes $r=\sqrt{(x^2+y^2)}$ is used instead of x':
    - this is a "double" approximation valid for $r \gg d_0$
  - If rotating to **an axis *near* to the particle direction** (the jet-axis for example)

$$y' = d_0 + x'\tan(\phi_0 - \phi_{jet}) - \tfrac{1}{2}\kappa x'^2$$

leading term in $(\varphi_0$-$\varphi_{jet})$
This equation will be our *workhorse*

Impact parameter resolution

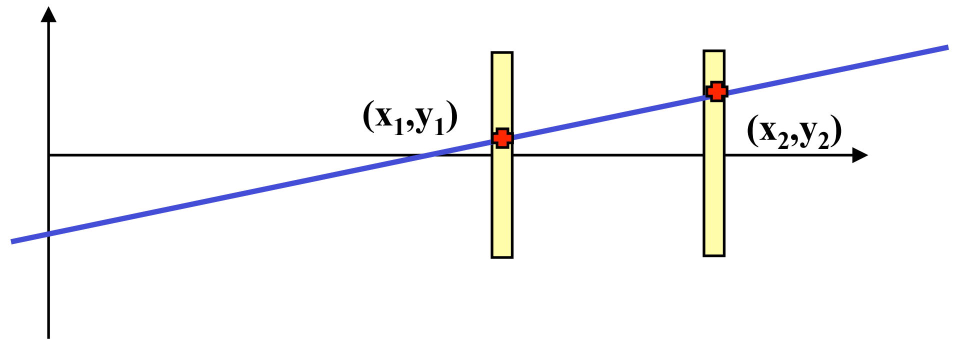
- In proximity of the production vertex, one can ignore the $\kappa x'^2$ term and consider the trajectory a straight line.
- Let's take two detector planes:
  - at positions $\mathbf{x_1}$ and $\mathbf{x_2}$,
  - resolution $\boldsymbol{\sigma_y}$ on the **y**-coordinate measurement.
- The reconstructed trajectory is:

$$y = \frac{y_2 - y_1}{x_2 - x_1} x + \frac{y_1 x_2 - y_2 x_1}{x_2 - x_1}$$

- The uncertainty on the impact parameter is:

$$\sigma_d = \frac{\sqrt{x_2^2 + x_1^2}}{x_2 - x_1} \sigma_y = \sqrt{\frac{n^2 + 1}{(n-1)^2}} \sigma_y$$

- where we introduced the **lever arm**: $\mathbf{n = x_2 / x_1}$



- The **geometrical factor** in front of σ_y is always greater than 1:
  detector resolution must be better than our targeted impact parameter resolution.
- $\mathbf{x_1}$ should be as small as possible. It is usually limited by:
  - beam pipe size
  - radiation damage
  - particle density and background
- $\mathbf{x_2}$ is limited by costs (either financial or operational)

Impact parameter resolution (2)

- Multiple scattering play a key role in the impact parameter resolution.
- Each material layer crossed by the particle before reaching the detector, deflects the particle by a random angle with r.m.s.:

$$\theta_0 = \frac{13.6\ \mathrm{MeV}}{\beta c p} z \sqrt{\frac{l}{X_0}} \left(1 + 0.038 \ln \frac{l}{X_0}\right)$$

where ***l*** is the thickness of the crossed material.

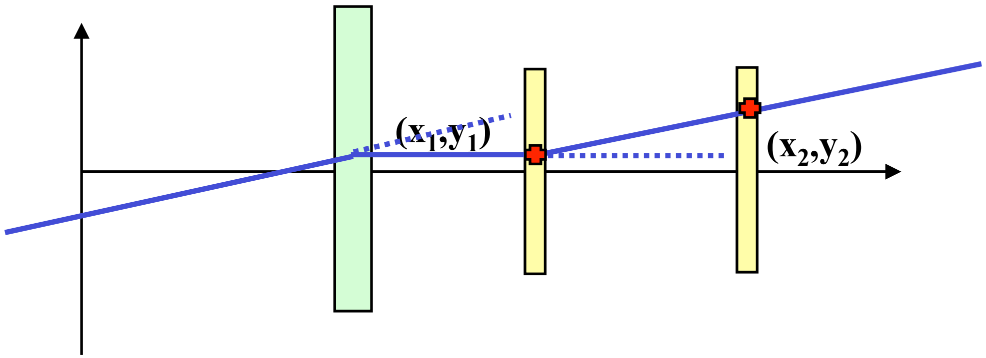
- This deflection translate in an error on the impact parameter of

$$\delta d = R \cdot \theta_0$$

where ***R*** is the distance of the material layer from the interaction point.

- Summing in quadrature all contributions:

$$\sigma_d = \sqrt{\sum_i R_i^2 \theta_{0,i}^2}$$



- The sum is computed over all material layers till the first measured point (included).
- The formula for $\boldsymbol{\theta_0}$ is valid in a plane perpendicular to the trajectory. If the track is tilted by an angle $\boldsymbol{\vartheta}$ with respect to the *x-y* plane, the projected angle is magnified by a factor **1/sin**$\boldsymbol{\vartheta}$.
- Also the crossed thickness ***l*** increases in the same way, providing an additional $\mathbf{1/sin^{1/2}}\boldsymbol{\vartheta}$ factor.

Momentum: *intrinsic* resolution

- Momentum is measured from the bending of the trajectory.
- In collider experiments detectors are put inside the magnetic field.
- Measuring the **sagitta** *s* over a length *L*:

$$s = R\left(1-\cos\frac{\theta}{2}\right) \approx R\frac{\theta^2}{8}$$

$$= \frac{qBL^2}{8p}$$

- numerically:

$$s[\mathrm{m}] = \frac{0.3B[\mathrm{T}]L^2[\mathrm{m}]}{8p[\mathrm{GeV/c}]}$$

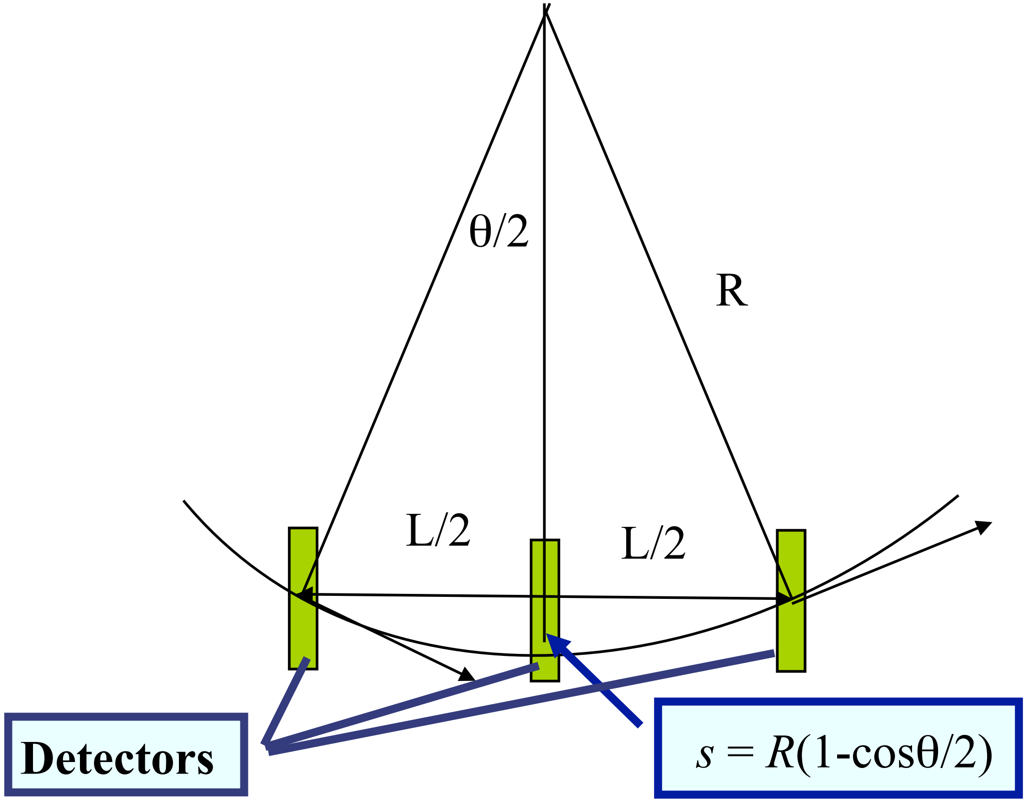
- If measurement by only three detectors:

$$s = y_2 - \frac{1}{2}(y_1 + y_3)$$

$$\sigma_s = \sqrt{3/2}\sigma_y$$

- Momentum resolution is

$$\frac{\sigma_p}{p} = \frac{\sigma_s}{s} = \frac{\sqrt{3/2}\sigma_y 8p}{0.3BL^2}$$



Momentum: multiple scattering

- For multiple scattering deflections in the detector material:

$$\delta y_2 = \frac{L}{2}\delta\theta_1 \quad \delta y_3 = L\delta\theta_1 + \frac{L}{2}\delta\theta_2 \quad \Rightarrow \quad \delta s = \delta y_2 - \frac{1}{2}\delta y_3 = -\frac{L}{2}\delta\theta_2$$

$$\sigma_s = \frac{L}{2}\theta_{\mathrm{ms},2}$$

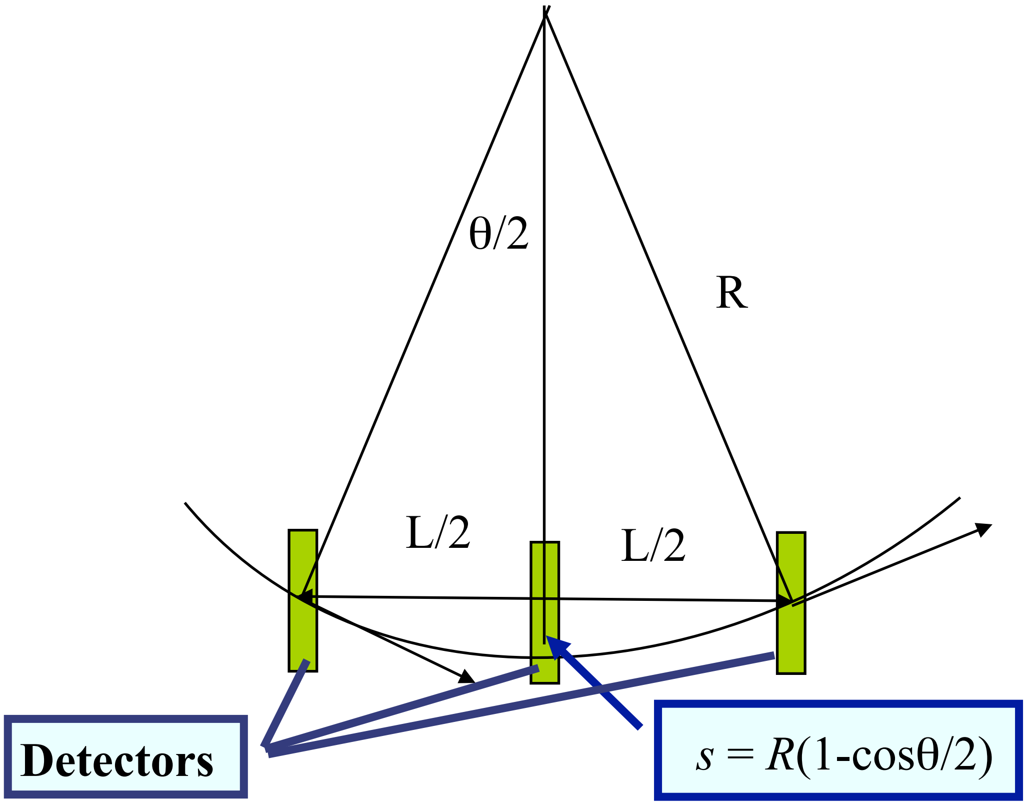
- This is a multiple scatterring contribution to the curvature measurement.
- Adding the two terms, we get:

$$\sigma_s = \sigma_{\mathrm{tracking}} \oplus \frac{\sigma_{\mathrm{MS}}}{p}$$

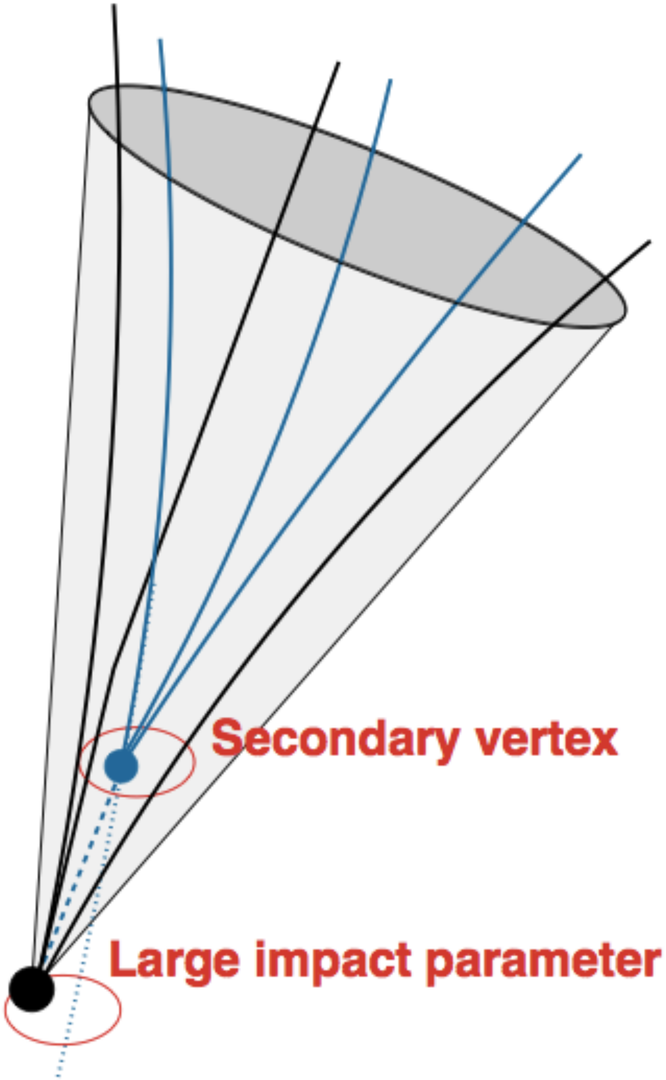
- The relative momentum resolution becomes:

$$\frac{\sigma_p}{p} = \frac{\sigma_s}{s} = \frac{8}{0.3BL^2}\left(p\sigma_{\mathrm{tracking}} \oplus \sigma_{\mathrm{MS}}\right)$$

  – resolution improves **linearly** with ***B*** and with the **detector point resolution**
  – the improvement is quadratic in ***L***
  – relative momentum resolution:
    - is **constant at low momentum** (MS)
    - **worsens with increasing momentum**



Performance requirements



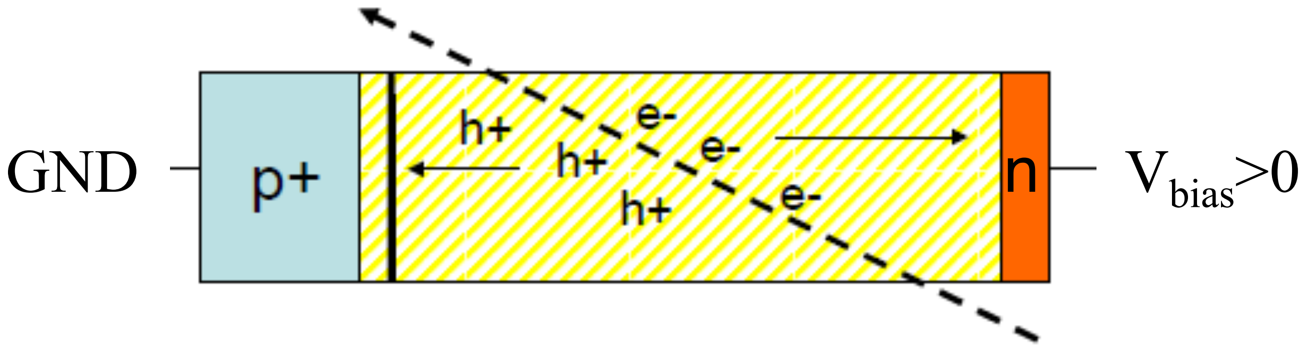
- Very interesting in current experiments are the heavy flavours (*c*, *b*, τ).
  - lifetime of O(10^{-12} s)
  - impact parameters of order of c⟨t⟩~300 μm
  - **need a detector with resolution one order of magnitude better** to detect them with high efficiency and purity.

- In ATLAS (B=2 T, L=1 m) a 200 GeV particle has a sagitta of about 400 μm.
  - to be able to reconstruct accurately new high energy resonances **the sagitta should be reconstructed with few tens of μm precision**.

THE DETECTORS

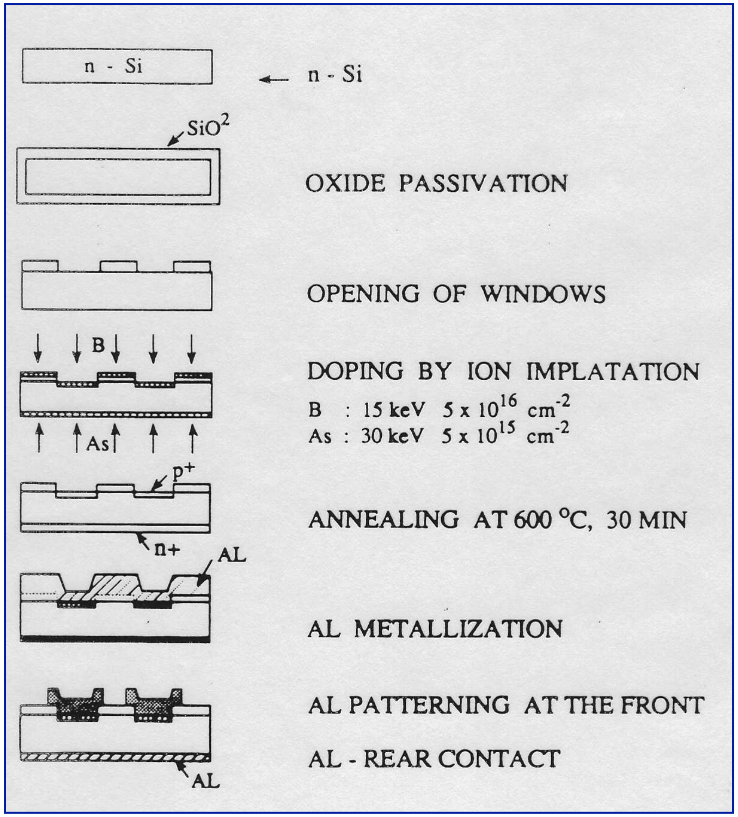
Semiconductor detectors

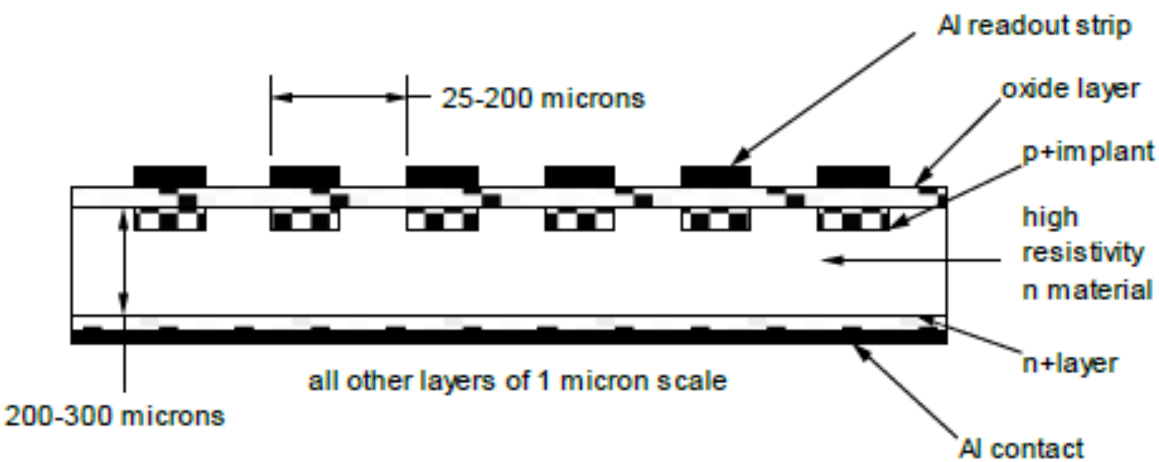
- Semiconductor detectors consists of inversely polarized p-n junctions.
- Depleted region with only static charge density N_D-N_A
  - thickness $W = \sqrt{\mu\rho\varepsilon\left(V_{\text{bias}} + V_{\text{BI}}\right)}$
    μ = carrier mobility 1350 $cm^2V^{-1}s^{-1}$ for e, 450 $cm^2V^{-1}s^{-1}$ for h
    ρ = resisitivity (detector grade Si is 1-10 kΩ/cm)
    ε = dielectric constant, 11.9 ε_0 V_{BI}= *built-in* voltage ~0.5 V
- When a charged particle crosses the detector:
  - collisions excite electrons to the conduction band, creating electron-hole pairs (~3.6 eV/pair, ~80 pairs/μm)
  - the mobile carriers are separated by the junction electric field, generating a current signal of few ns length.



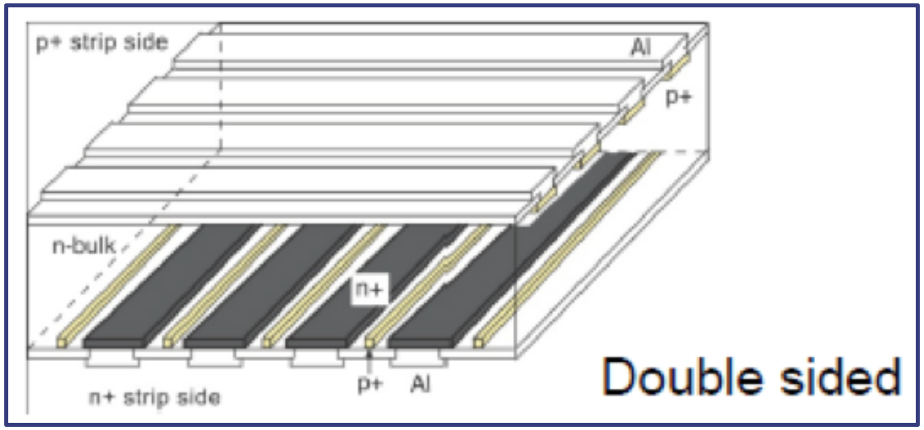
Position sensitive detectors

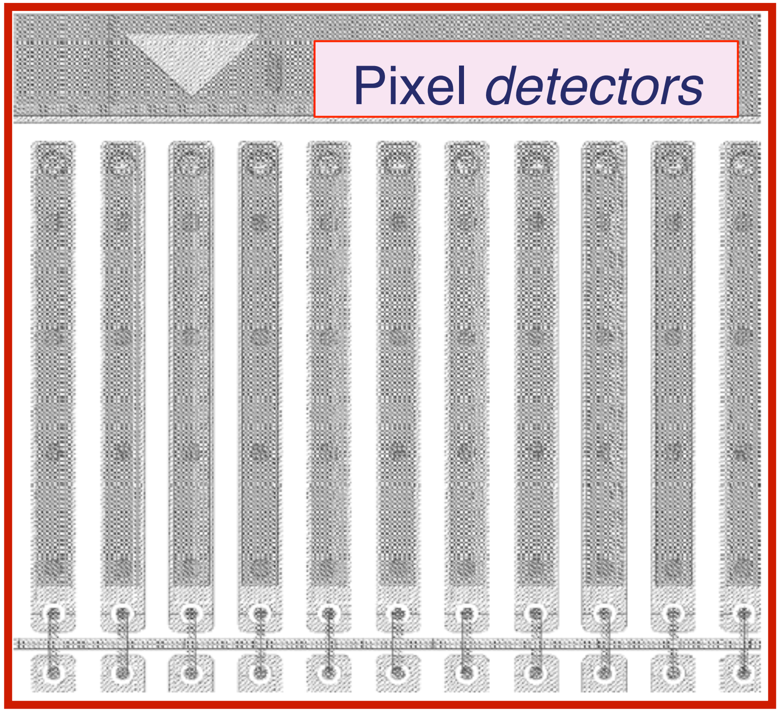
- The first high resolution detectors were silicon microstrip.
- Use of microlithography from semiconductor electronics industry.
- **Fine segmentation** of collecting electrodes: µm level resolution
- Thickness of few hundreds µm: **signal of 10^4 e-h**, detectable with low noise electronics



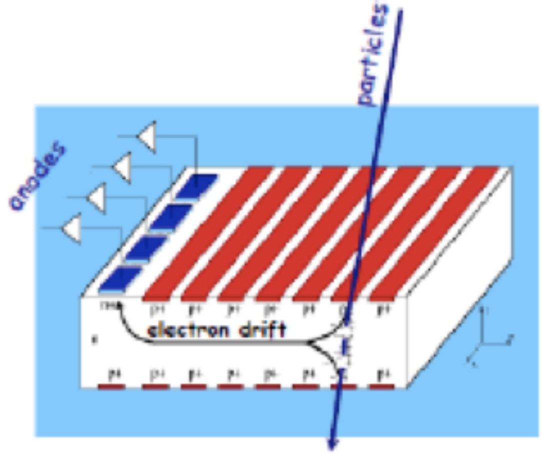


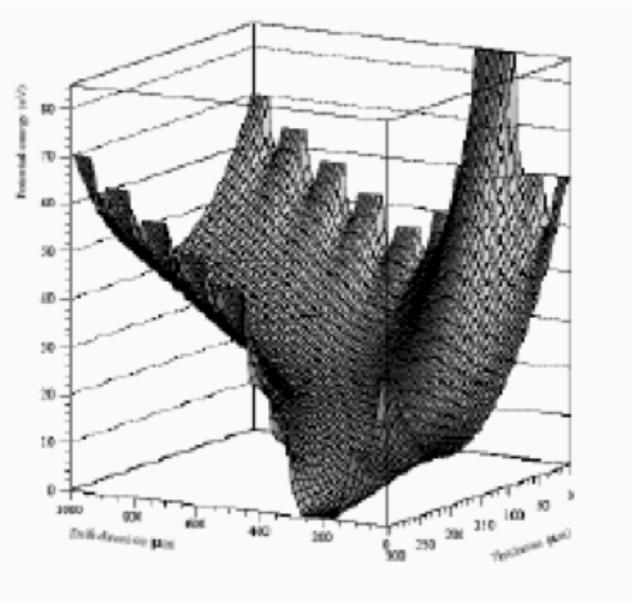
Various types of Si detectors





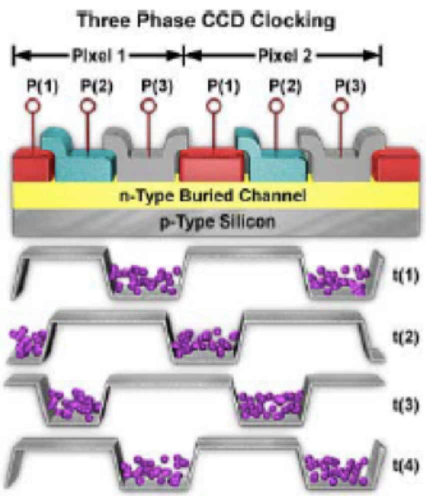
Si drift detectors



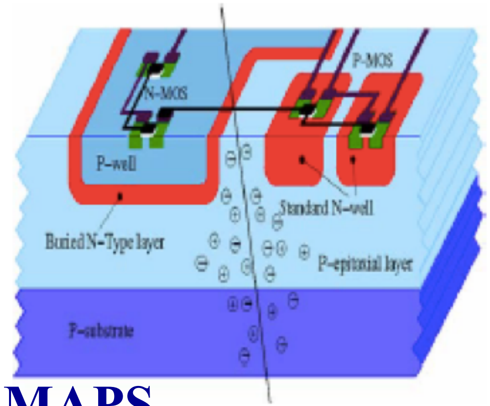


Pixel tecnologies

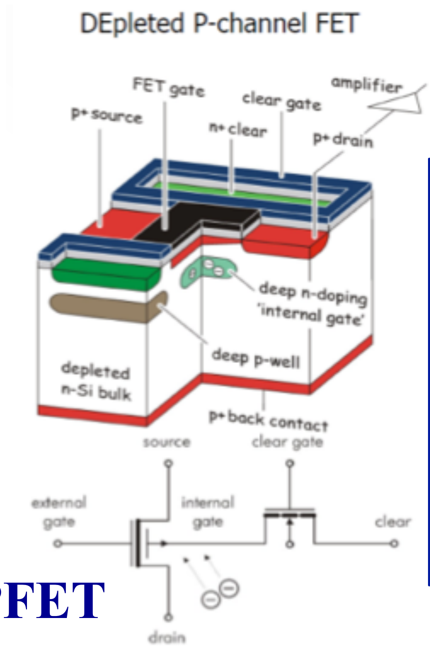
Monolythics



CCD



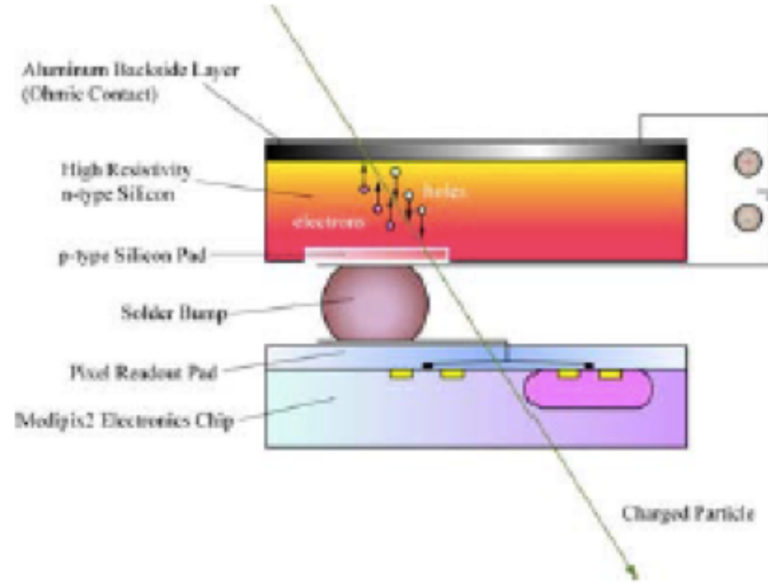
MAPS



DEPFET

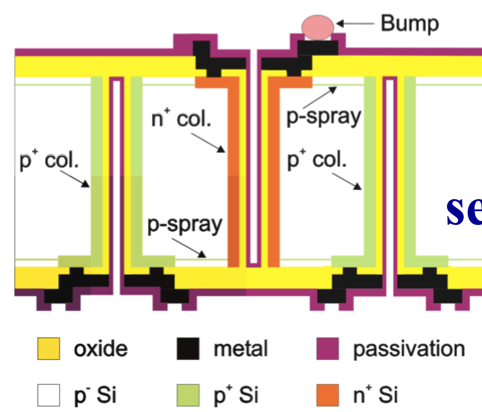
- Integrated readout
- Low pitch
- Low mass
- Relatively slow readout
- Radiation hardness

Hybrids



Bump bonded readout front-end:
- more material
- pitch limited by electronics.

Fast
Rad-hard

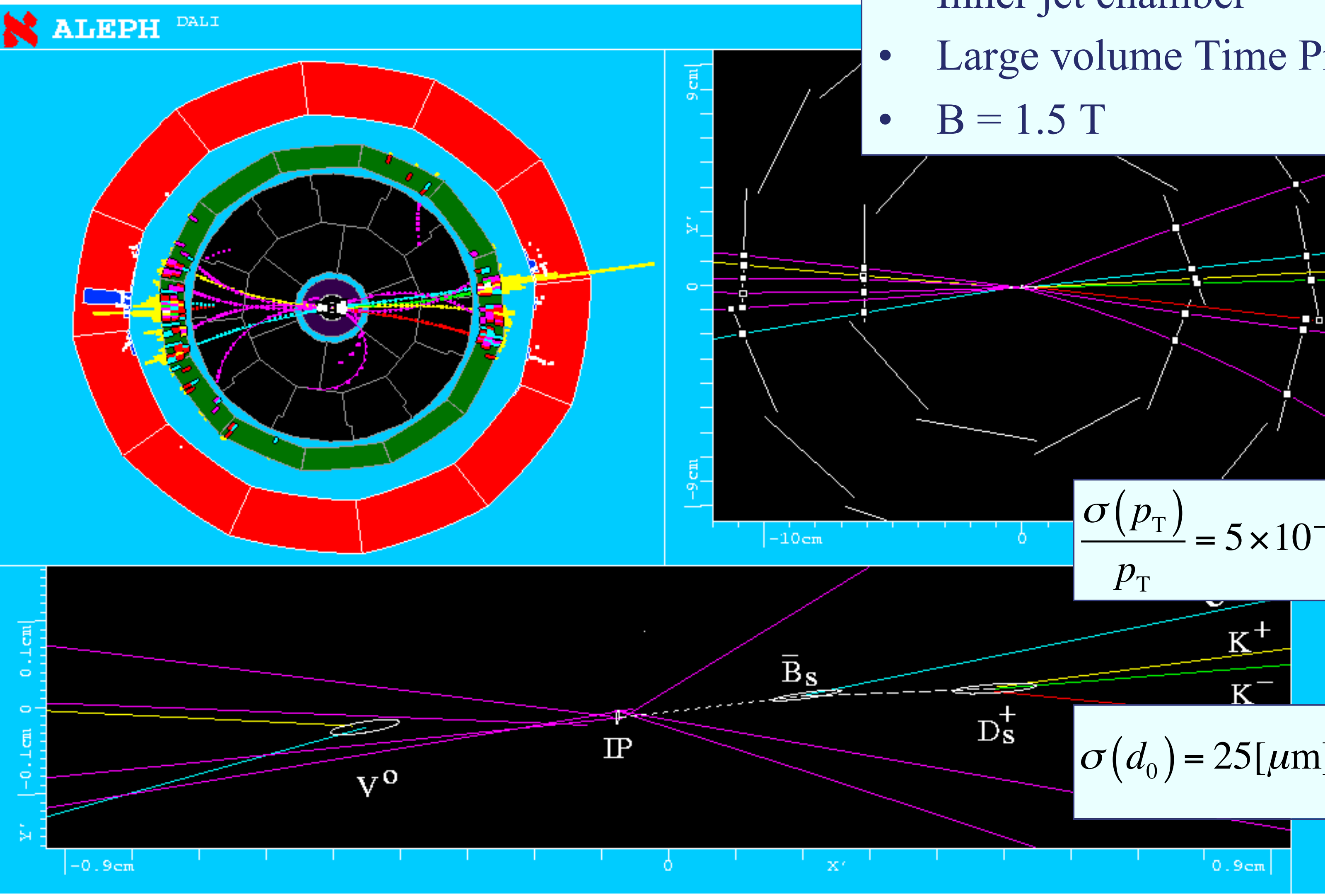


3D sensor

EXAMPLES OF SYSTEMS

ALEPH @ LEP

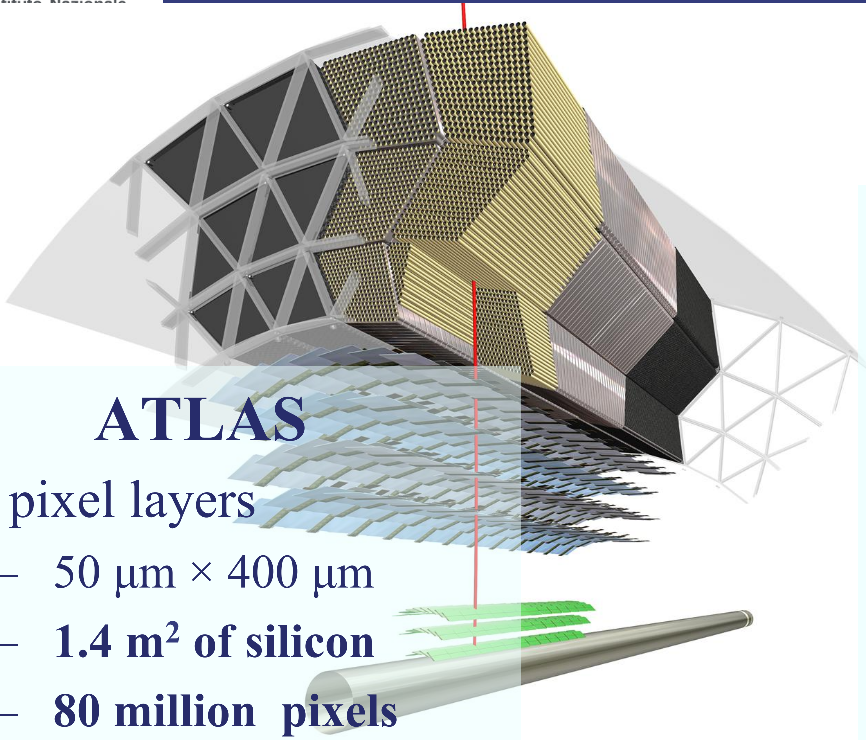
- 2 planes Si microstrips 25 μm pitch
- Inner jet chamber
- Large volume Time Projection Chamber
- B = 1.5 T



$$\frac{\sigma(p_T)}{p_T} = 5\times10^{-3} \oplus 0.6\times10^{-4}[\text{GeV}^{-1}]\, p_T$$

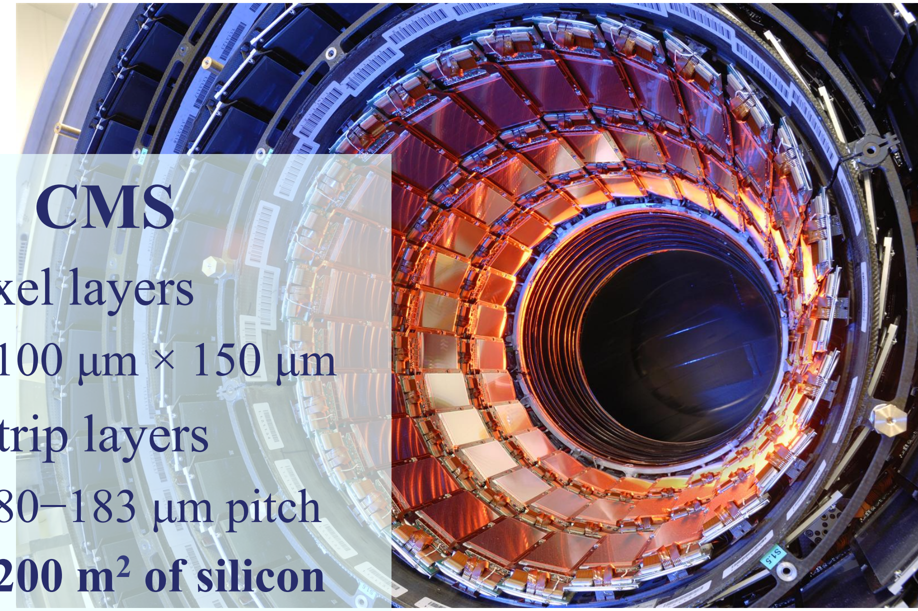
$$\sigma(d_0) = 25[\mu\text{m}] \oplus \frac{95[\mu\text{m GeV}]}{p_T}$$

ATLAS and CMS at LHC



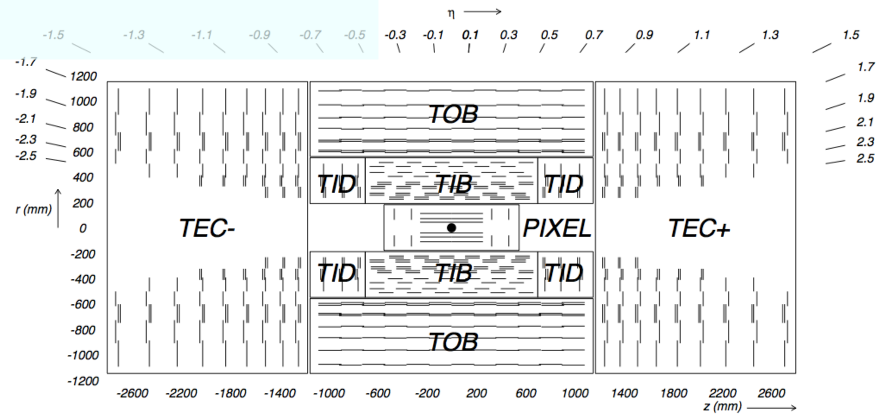
ATLAS

- 3 pixel layers
  - 50 μm × 400 μm
  - **1.4 m^2 of silicon**
  - **80 million pixels**
- 4 strip double-layers
  - 80 μm pitch
  - 400 mrad stereo angle
- Straw tube tracker
  - ~30 points
- B = 2 T



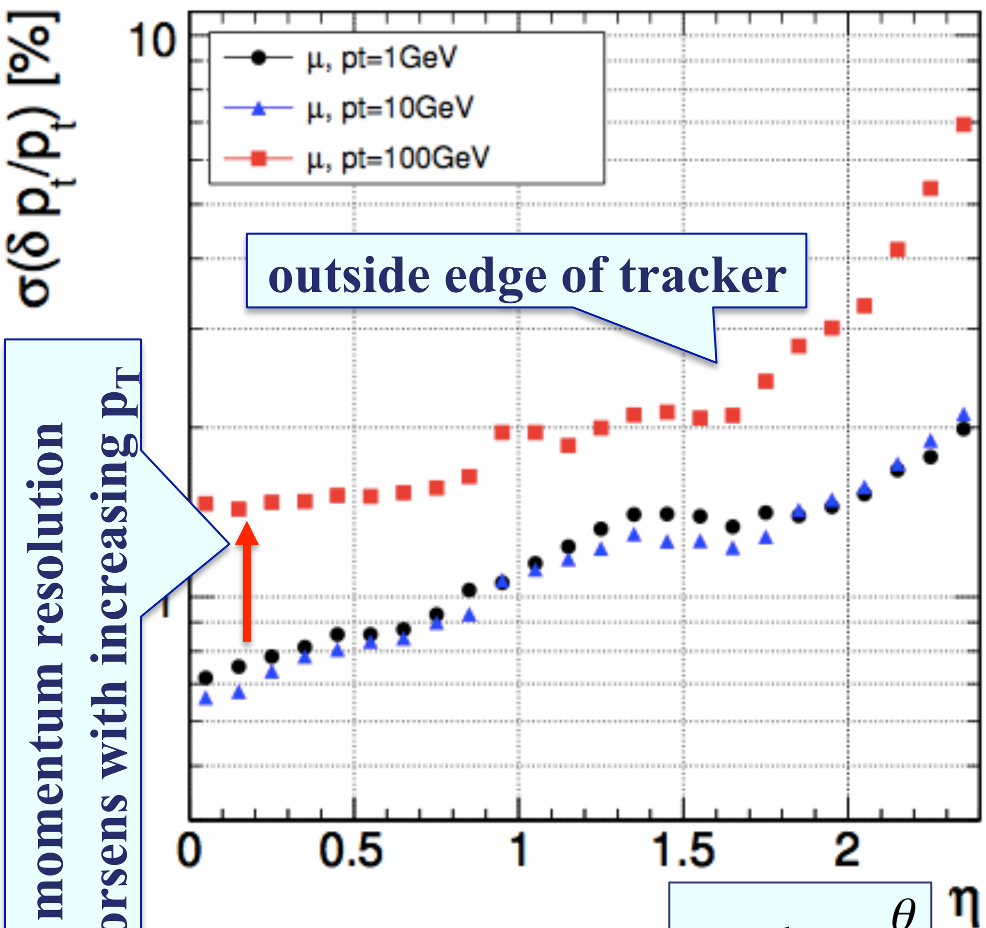
CMS

- 3 pixel layers
  - 100 μm × 150 μm
- 10 strip layers
  - 80−183 μm pitch
  - **200 m^2 of silicon**
  - **>9 million strips**
- B = 4 T



Resolution

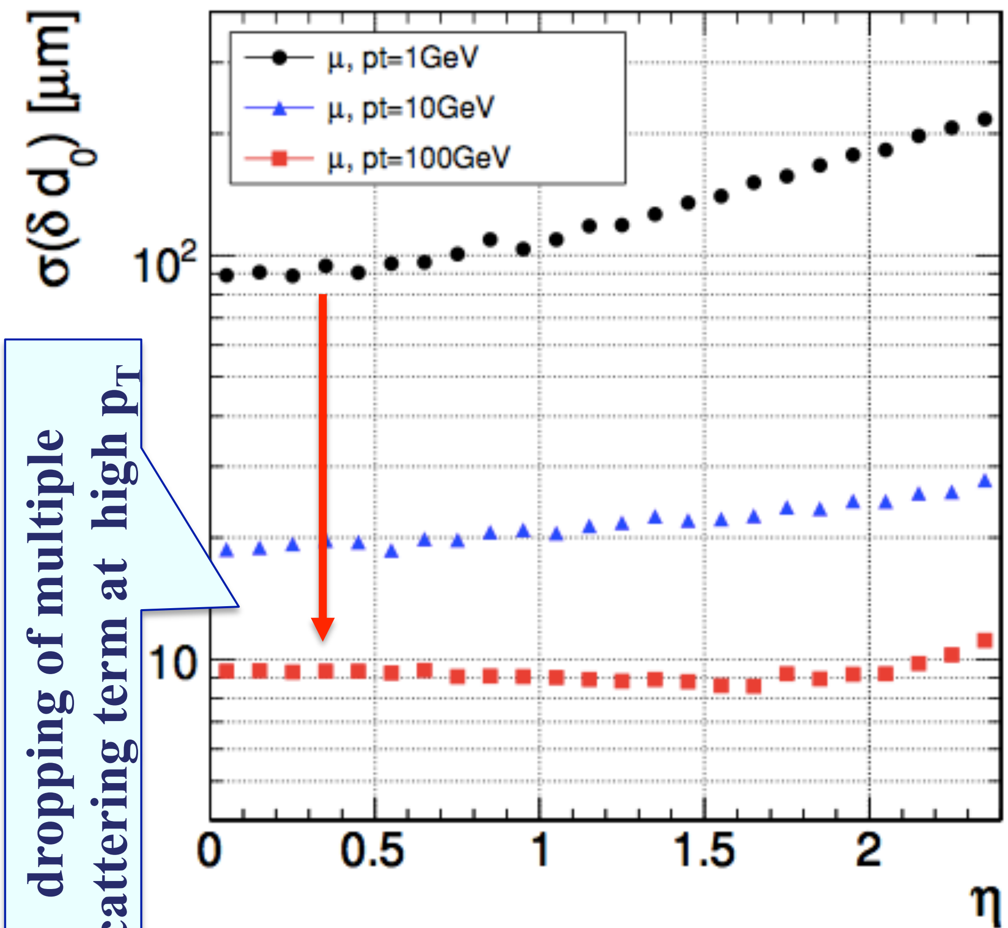
The CMS experiment at the CERN LHC, JIST 3 (2008) S08004



outside edge of tracker

momentum resolution worsens with increasing p_T

$$\eta = -\ln\tan\frac{\theta}{2}$$

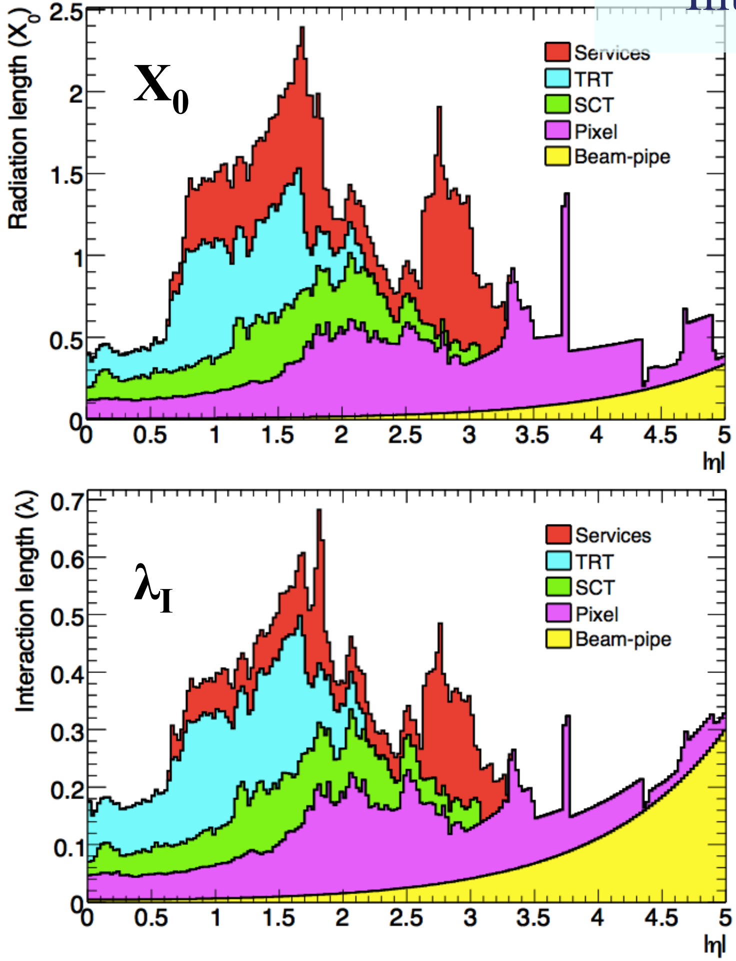


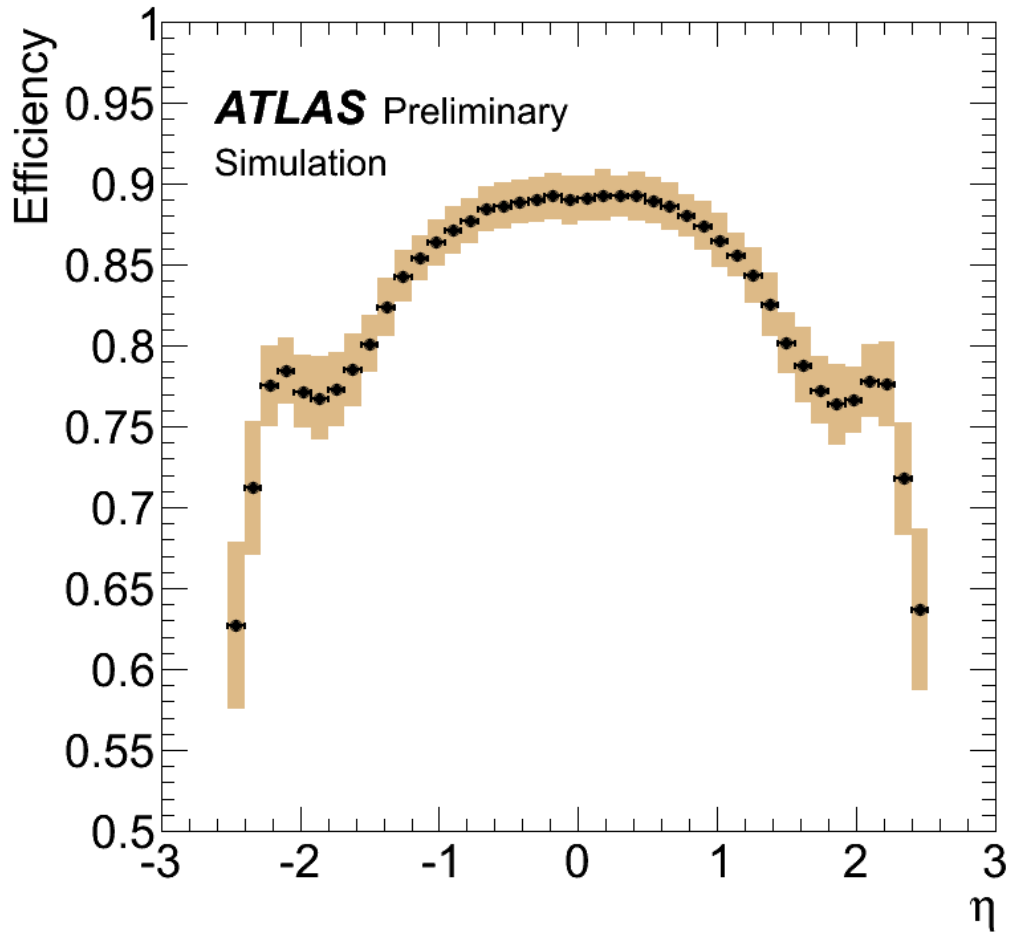
dropping of multiple scattering term at high p_T

Material and efficiency

Material contributes not only to resolution, but also to efficiency:

- Si is almost 100% efficient
- Interactions may deviate the particles, splitting the track.





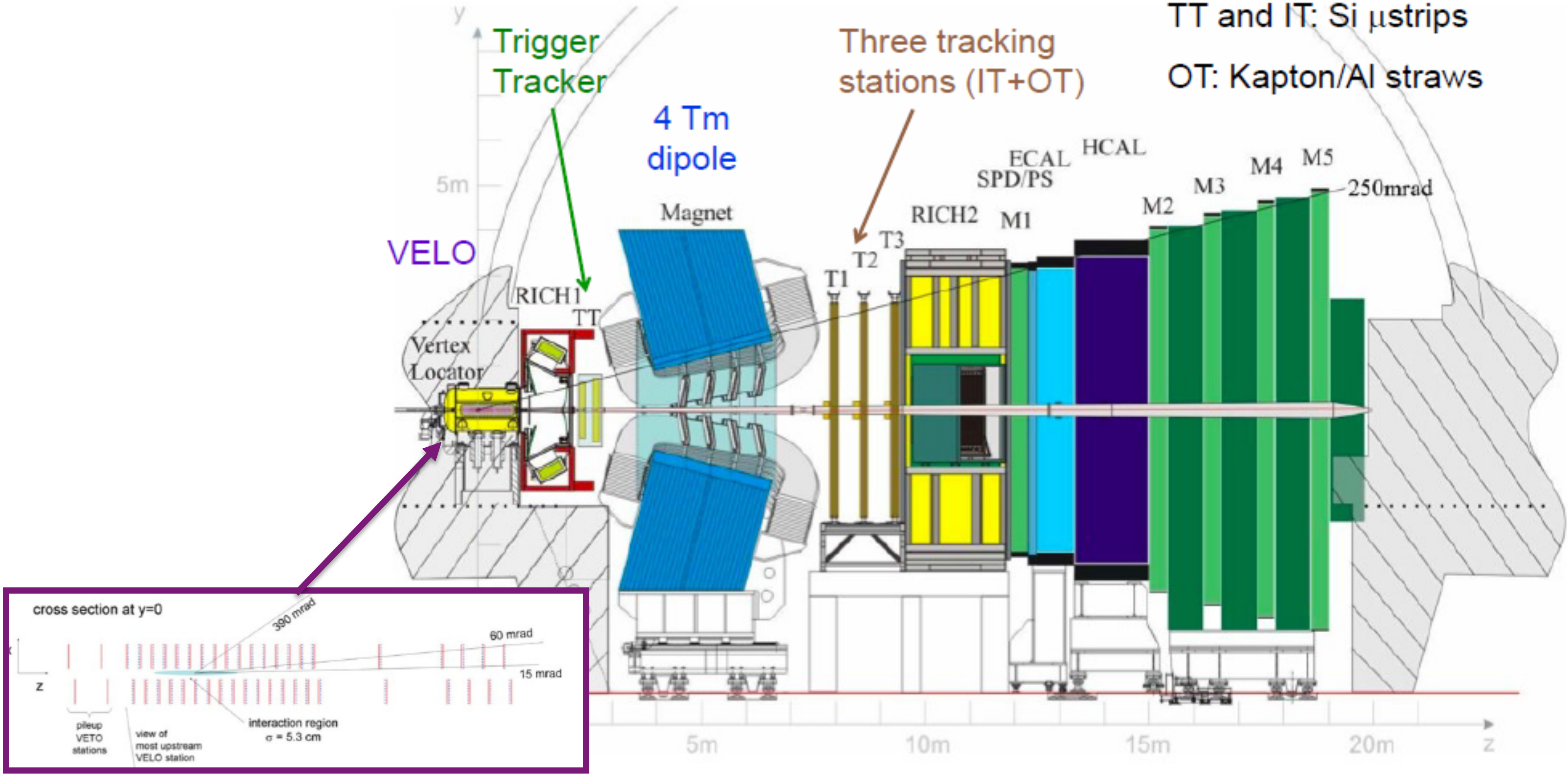
2008 JINST 3 S08005 LHCb Detector

LHCb tracking

VELO: rϕ Si strips

TT and IT: Si μstrips

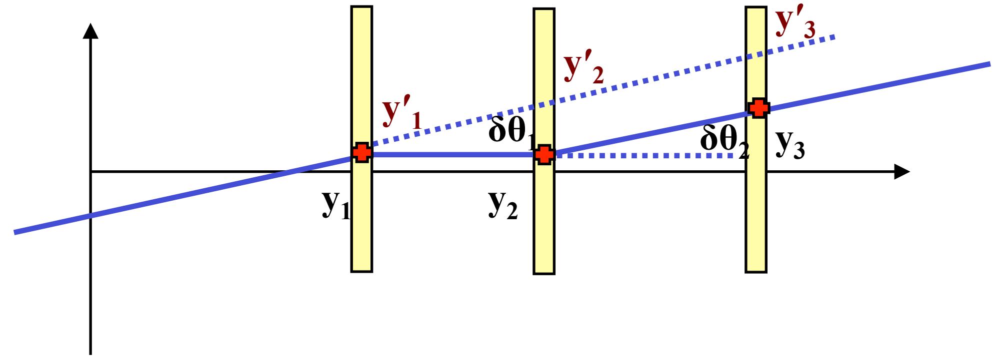
OT: Kapton/Al straws



TRACK FITTING

Track fitting: straight track model

- In our previous examples we used only the minimal number of points.
- Usually more measurements then the minimum:
  - redundancy
  - pattern recognition
  - improved precision



- Simple straight line model: $y = a + bx$ expected crossing points: $y'_i = a + bx_i$
- Best parameters are defined by minimizing the χ^2 of the residuals between the measurements $\boldsymbol{y}_\mathbf{i}$ and the expectations $\boldsymbol{y'}_\mathbf{i}$ from a set of parameters $(\boldsymbol{a},\boldsymbol{b})$.
- If we neglet multiple scattering:

$$\chi^2 = \sum_{i=1,2,3} \frac{(y_i - a - bx_i)^2}{\sigma^2_{y_i}} = (\mathbf{Y} - \mathbf{A}\mathbf{p})^T \mathbf{V}^{-1} (\mathbf{Y} - \mathbf{A}\mathbf{p})$$

inverse of covariance matrix

measurements

model expectation, function of **p**

$$\mathbf{p} = \begin{pmatrix} a \\ b \end{pmatrix} \quad \mathbf{Y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \quad \mathbf{A} = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & x_3 \end{bmatrix} \quad \mathbf{V} = \begin{bmatrix} \sigma^2_{y_1} & 0 & 0 \\ 0 & \sigma^2_{y_2} & 0 \\ 0 & 0 & \sigma^2_{y_3} \end{bmatrix}$$

Track fitting: multiple scattering

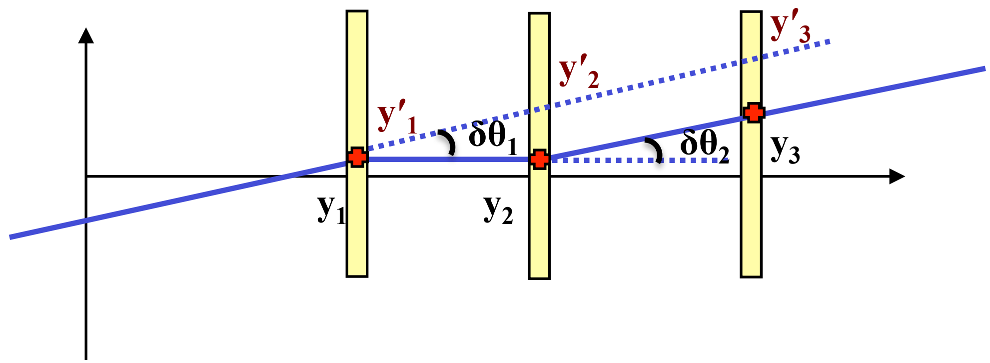
- In reality y_i-y'_i contains contributions from multiple scattering:

$$y_1 = a + bx_1 + \varepsilon_1$$

$$y_2 = a + bx_2 + \varepsilon_2 + (x_2 - x_1)\delta\theta_1$$

$$y_3 = a + bx_3 + \varepsilon_3 + (x_3 - x_1)\delta\theta_1 + (x_3 - x_2)\delta\theta_2$$

ε = measurement error
$\delta\theta$ = m.s. deflection



- The definition of the covariance matrix is: $V_{ij} = \langle (y_i - y'_i)(y_j - y'_j) \rangle$
- Uncertainties are $\langle \varepsilon_i^2 \rangle = \sigma_{y_i}^2, \quad \langle \delta\theta_i^2 \rangle = \theta_{\mathrm{ms},i}^2$
- Error sources are not correlated: $\langle \varepsilon_i \varepsilon_j \rangle = 0, i \neq j; \quad \langle \delta\theta_i \delta\theta_j \rangle = 0, i \neq j; \quad \langle \varepsilon_i \delta\theta_j \rangle = 0$

- Diagonal elements:

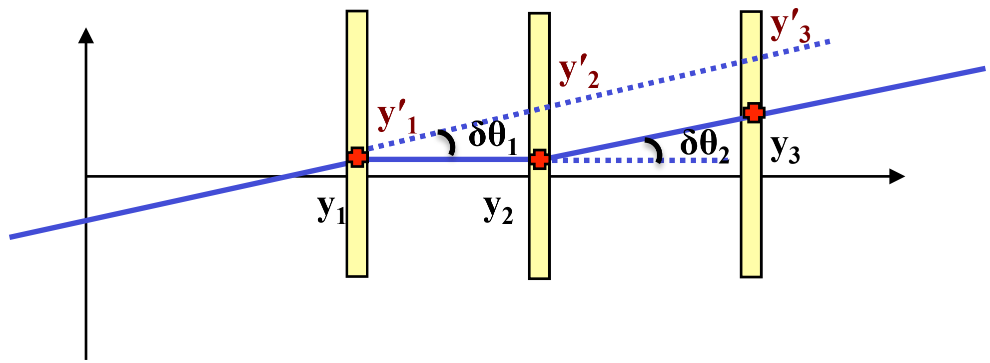
$$V_{11} = \langle \varepsilon_1^2 \rangle = \sigma_{y_1}^2$$

$$V_{22} = \langle (\varepsilon_2 + (x_2 - x_1)\delta\theta_1)^2 \rangle = \langle \varepsilon_2^2 \rangle + 2\langle \varepsilon_2 (x_2 - x_1)\delta\theta_1 \rangle + \langle (x_2 - x_1)^2 \delta\theta_1^2 \rangle = \sigma_{y_2}^2 + (x_2 - x_1)^2 \theta_{\mathrm{ms},1}^2$$

$$V_{33} = \sigma_{y_3}^2 + (x_3 - x_1)^2 \theta_{\mathrm{ms},1}^2 + (x_3 - x_2)^2 \theta_{\mathrm{ms},2}^2$$

Track fitting: multiple scattering

- In reality y_i-y'_i contains contributions from multiple scattering:



ε = measurement error
$\delta\theta$ = m.s. deflection

$$y_1 = a + bx_1 + \varepsilon_1$$

$$y_2 = a + bx_2 + \varepsilon_2 + (x_2 - x_1)\delta\theta_1$$

$$y_3 = a + bx_3 + \varepsilon_3 + (x_3 - x_1)\delta\theta_1 + (x_3 - x_2)\delta\theta_2$$

- The definition of the covariance matrix is: $V_{ij} = \langle (y_i - y'_i)(y_j - y'_j) \rangle$
- Uncertainties are $\langle \varepsilon_i^2 \rangle = \sigma_i^2, \quad \langle \delta\theta_i^2 \rangle = \theta_{\mathrm{ms},i}^2$
- Error sources are not correlated: $\langle \varepsilon_i \varepsilon_j \rangle = 0, i \neq j; \quad \langle \delta\theta_i \delta\theta_j \rangle = 0, i \neq j; \quad \langle \varepsilon_i \delta\theta_j \rangle = 0$

- Non-diagonal elements:

$$V_{12} = V_{13} = 0$$

$$V_{23} = \langle (\varepsilon_2 + (x_2 - x_1)\delta\theta_1)(\varepsilon_3 + (x_3 - x_1)\delta\theta_1 + (x_3 - x_2)\delta\theta_2) \rangle = \langle (x_2 - x_1)(x_3 - x_1)\delta\theta_1^2 \rangle$$

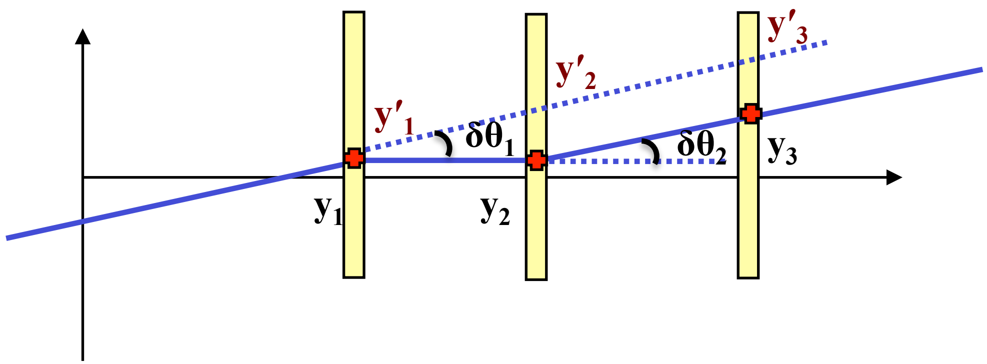
$$= (x_2 - x_1)(x_3 - x_1)\theta_{\mathrm{ms},1}^2$$

Track fitting: multiple scattering

- Finally, the covariance V to be used in the χ^2 minimization is:è

$$\mathbf{V} = \begin{bmatrix} \sigma_{y_1}^2 & 0 & 0 \\ 0 & \sigma_{y_2}^2 & 0 \\ 0 & 0 & \sigma_{y_3}^2 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & (x_2 - x_1)^2 \theta_{\mathrm{ms},1}^2 & (x_3 - x_1)(x_2 - x_1)\theta_{\mathrm{ms},1}^2 \\ 0 & (x_3 - x_1)(x_2 - x_1)\theta_{\mathrm{ms},1}^2 & (x_3 - x_1)^2 \theta_{\mathrm{ms},1}^2 + (x_3 - x_2)^2 \theta_{\mathrm{ms},2}^2 \end{bmatrix}$$

- The second matrix has:
  - **diagonal elements** due to any previous material affecting the trajectory at a given plane.
  - **off-diagonal elements**: present if a previous material layer affect the trajectory in more than one plane.
- In our case:
  - scattering on plane 1
  - affects the position in both plane 2 and plane 3



Global χ^2

- The technique described till now consists in the minimization of a χ^2 involving all measurement points:

$$\chi^2 = (\mathbf{Y} - \mathbf{Ap})^T \mathbf{V}^{-1} (\mathbf{Y} - \mathbf{Ap})$$

  and therefore is indicated as a **global χ^2**:
  - requires the inversion of a NxN covariance matrix (N=number of measurements)
  - has become popular with silicon tracking systems because tracks have few, precise measurements

- Our model assumes the whole track is a straight line:
  - *b* is sort *average* track direction
    - but we are interested in track direction at the production point
  - Multiple scattering is taken into account by giving lower weights to points far away from the interaction region

How can it be improved?

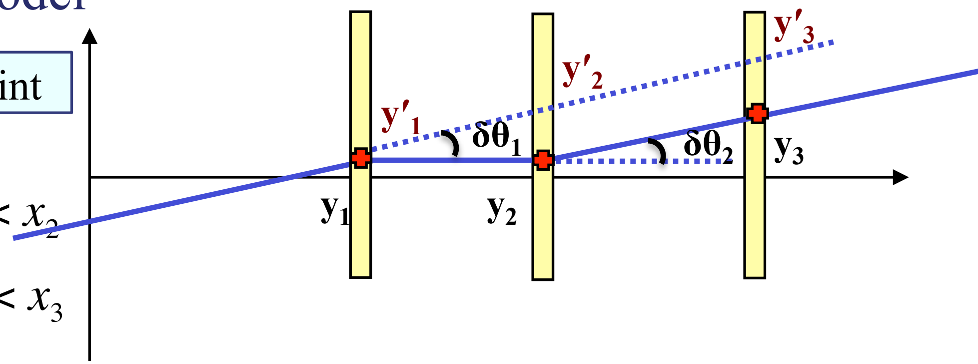
Global χ^2

- Insert scattering angles as part of the track model

$$y(x)=\begin{cases} a+bx & x<x_1 \\ a+bx+\delta\theta_1(x-x_1) & x_1<x<x_2 \\ a+bx+\delta\theta_1(x-x_1)+\delta\theta_2(x-x_2) & x_2<x<x_3 \end{cases}$$

b is track direction at interaction point

track direction changes along *x*



  - Additional parameters, with expectation value 0 and r.m.s. θ_{ms}
- The same $\chi^2=(\mathbf{Y}-\mathbf{Ap})^T\mathbf{V}^{-1}(\mathbf{Y}-\mathbf{Ap})$ holds, but with the modified matrices:

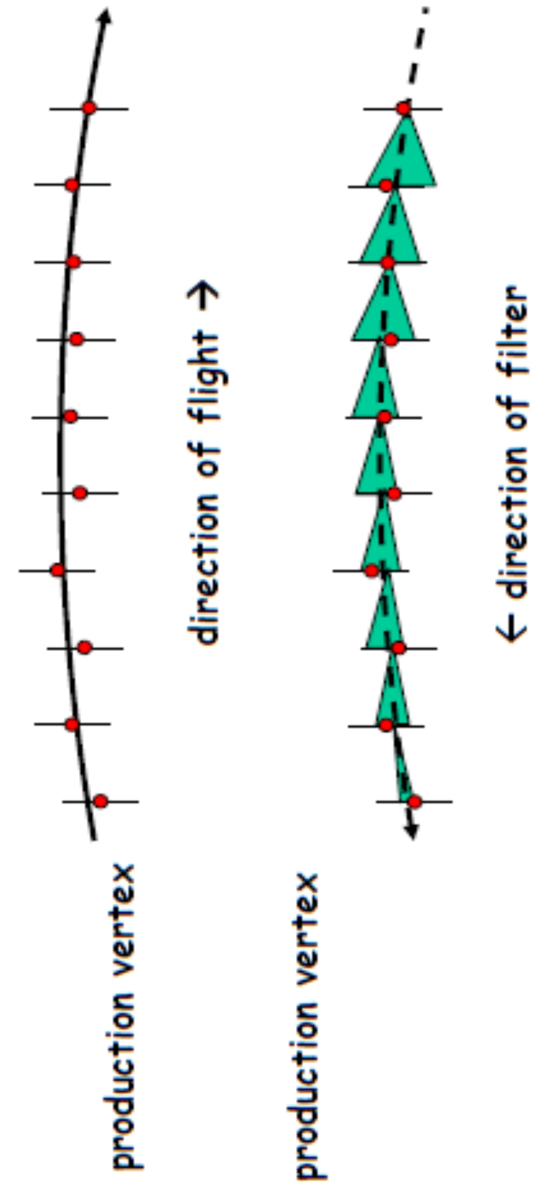
$$\mathbf{p}=\begin{pmatrix} a \\ b \\ \delta\theta_1 \\ \delta\theta_2 \end{pmatrix} \quad \mathbf{Y}=\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ 0 \\ 0 \end{pmatrix} \quad \mathbf{A}=\begin{bmatrix} 1 & x_1 & 0 & 0 \\ 1 & x_2 & x_2-x_1 & 0 \\ 1 & x_3 & x_3-x_1 & x_3-x_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{V}=\begin{bmatrix} \sigma_{y_1}^2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{y_2}^2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{y_3}^2 & 0 & 0 \\ 0 & 0 & 0 & \theta_{\mathrm{ms},1}^2 & 0 \\ 0 & 0 & 0 & 0 & \theta_{\mathrm{ms},2}^2 \end{bmatrix}$$

  - The number of degrees of freedom does not change
  - Estimate multiple scattering instead of putting it into the weights

Kalman filter

- Step-by-step updating procedure:
  - use initial estimation of track parameters
  - extrapolate to next measured point
  - compare extrapolation with measurement
  - derive updated track parameters
- Continue adding all points one at the time.

- For each point invert a matrix of size equal to the track parameters
  - computation time is Nd^3 instead of N^3
- Comparison allows for rejection of outliers
  - can also be used during pattern recognition



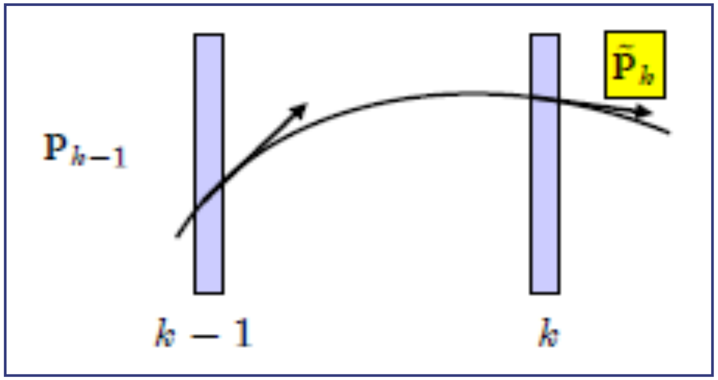
Kalman filter

- Only providing basic idea of Kalman filtering
  - one iteration of the fit, from detector plane *k*-1 to *k*
  - see bibliography for more details
- At plane *k*-1 we have an estimation of the track parameters $\mathbf{p}_{k\text{-}1}$, with their covariance matrix $\mathbf{C}_{k\text{-}1}$.
- Extrapolate to plane *k*:

$$\tilde{\mathbf{p}}_k = \mathbf{f}\left(\mathbf{p}_{k-1}\right) \qquad \mathbf{F} = \frac{\partial \mathbf{f}}{\partial \mathbf{p}}\left(\mathbf{p}_{k-1}\right)$$

$$\tilde{\mathbf{C}}_k = \mathbf{F}\mathbf{C}_k\mathbf{F}^{\mathrm{T}} + \mathbf{M}_{ms}$$

- $\mathbf{M}_{ms}$ includes the multiple scattering uncertainty in the extrapolation.
- On surface *k* we have some measurements $\mathbf{m}_k$ with covariance $\mathbf{V}_k$.



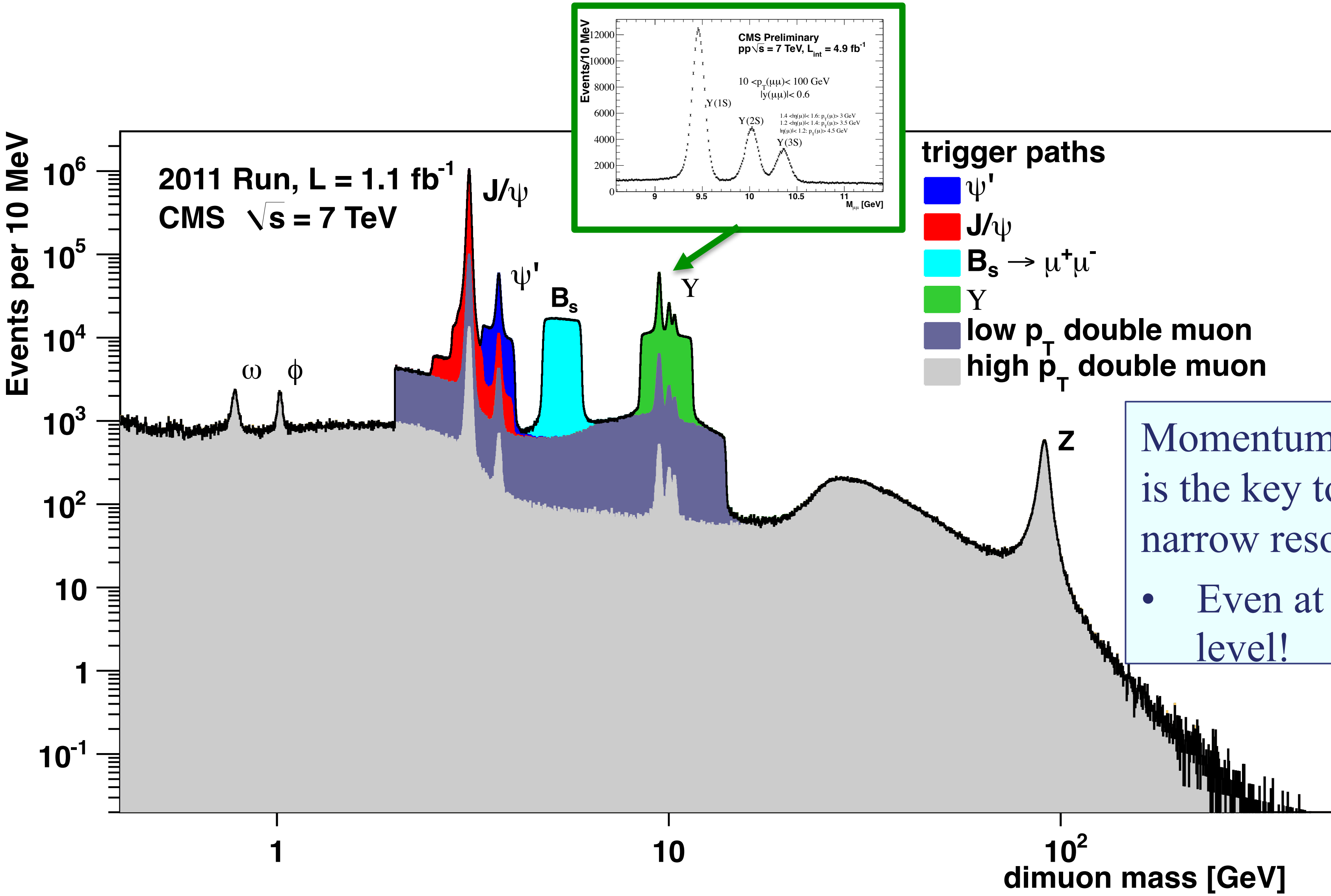
- The updated parameters $\mathbf{p}_k$ are obtained my minimizing a χ^2 including:
  - comparison of $\mathbf{m}_k$ with expectations $\mathbf{y}_k(\mathbf{p}_k)$ from the track model
  - the extrapolated parameters

$$\chi^2 = \left(\mathbf{m}_k - \mathbf{y}_k(\mathbf{p}_k)\right)^{\mathrm{T}} \mathbf{V}_k^{-1}\left(\mathbf{m}_k - \mathbf{y}_k(\mathbf{p}_k)\right) + \left(\tilde{\mathbf{p}}_k - \mathbf{p}_k\right)^{\mathrm{T}} \tilde{\mathbf{C}}_k^{-1}\left(\tilde{\mathbf{p}}_k - \mathbf{p}_k\right)$$

- Try to develop the concrete expressions for a linear track fit:
  - solutions in the back-up slides

APPLICATIONS

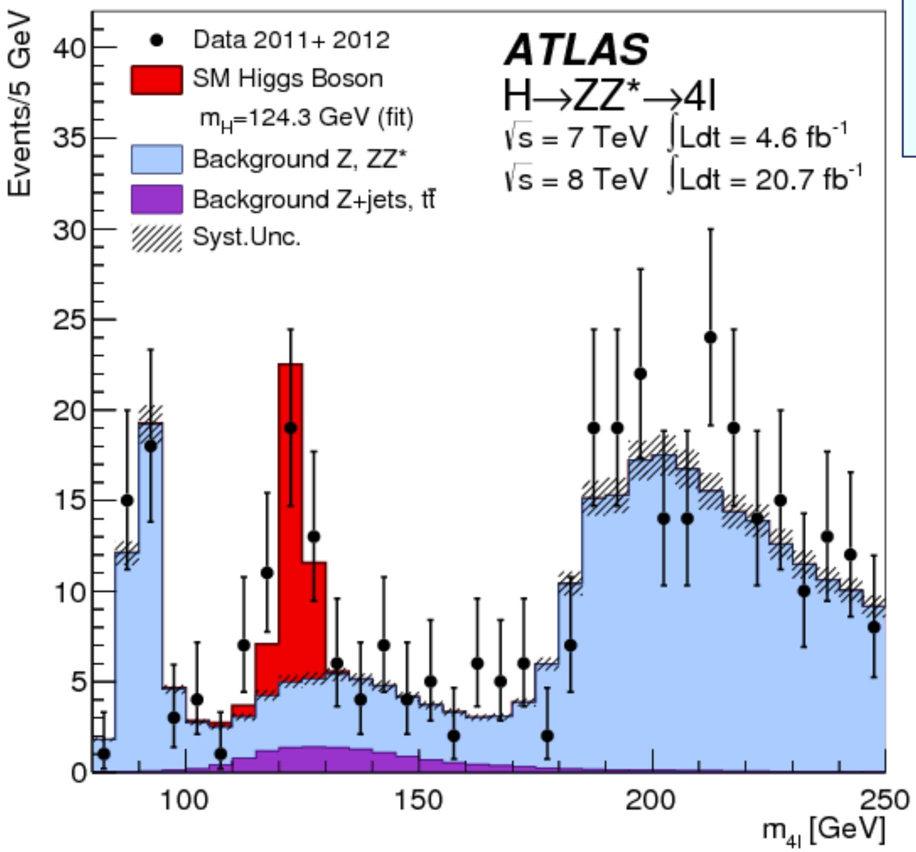
Invariant mass reconstruction



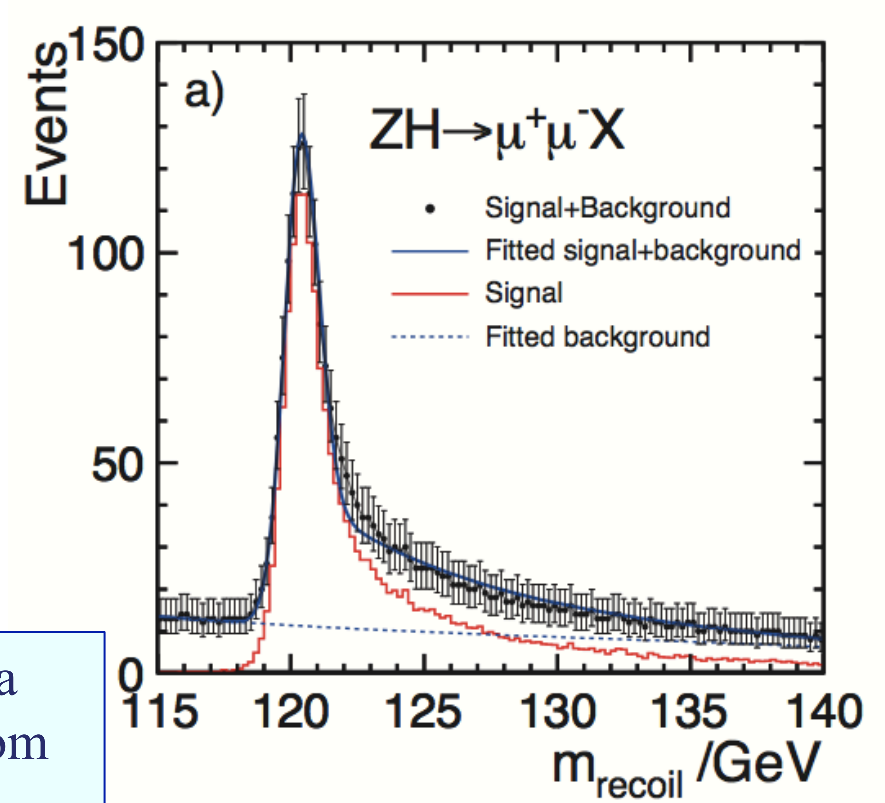
Momentum resolution is the key to assess narrow resonances.

- Even at the trigger level!

Higgs and momentum resolution

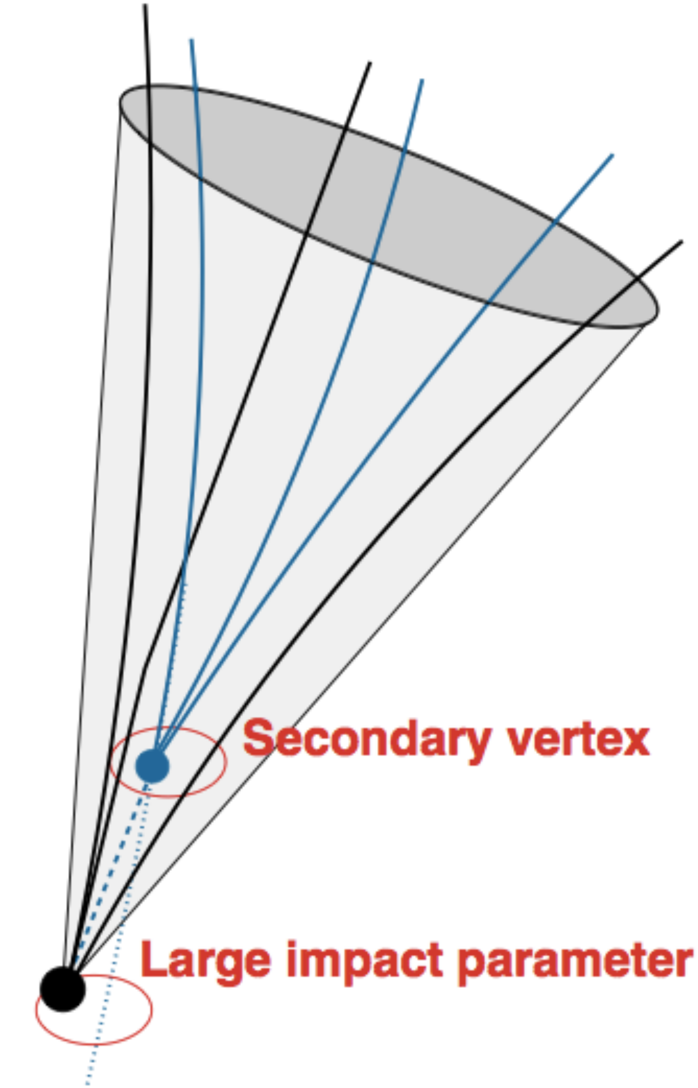


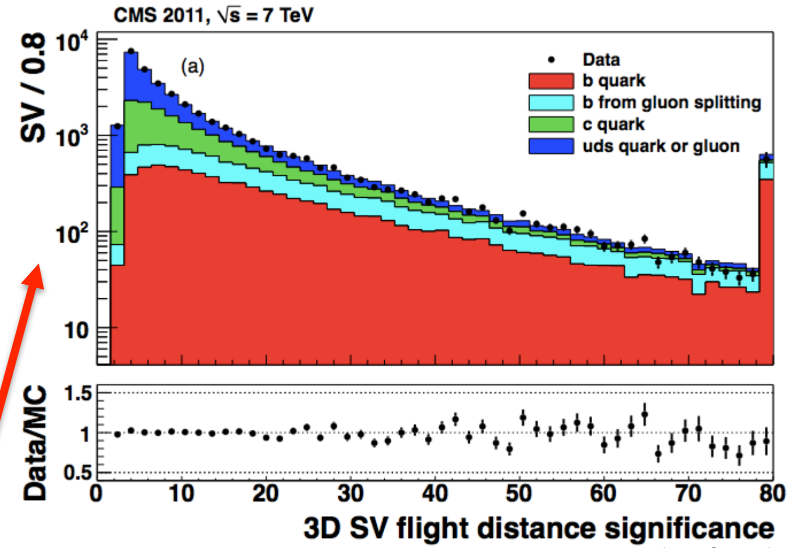
Separation of the Higgs signal in the H→ZZ decay at the LHC rely on accurate curvature determination.



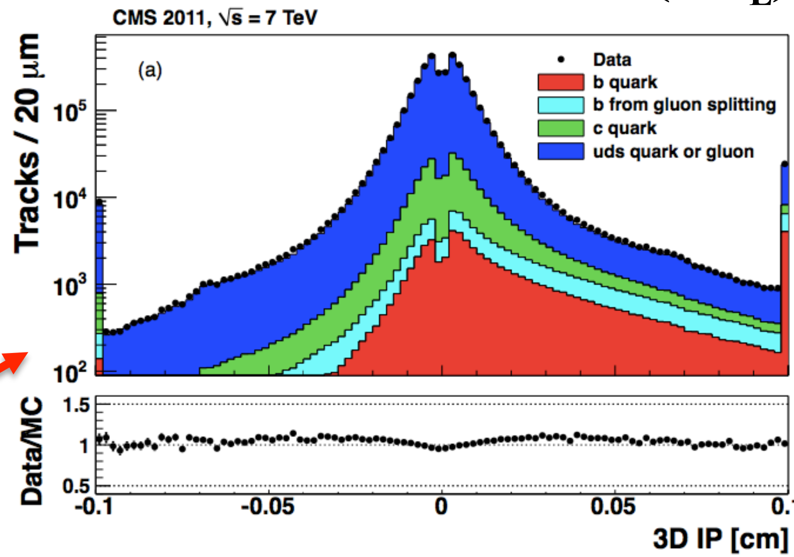
Future experiments at the ILC can perform a measurement of the total production rate from mass of the system recoiling agaist the Z

Identification of short lived particles





(L/σ_L)



quark b ~ 1.5 ps

quark c ~ 0.5 ps

τ lepton 0.29 ps

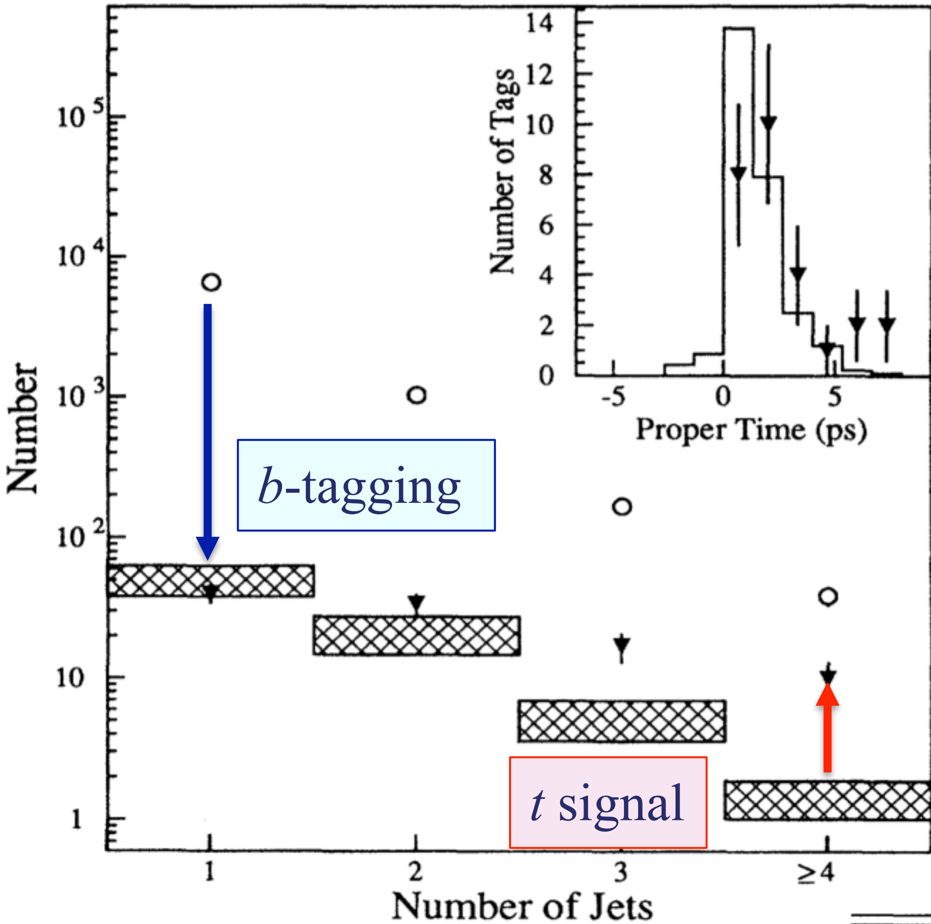
Decay length:

~ γβct

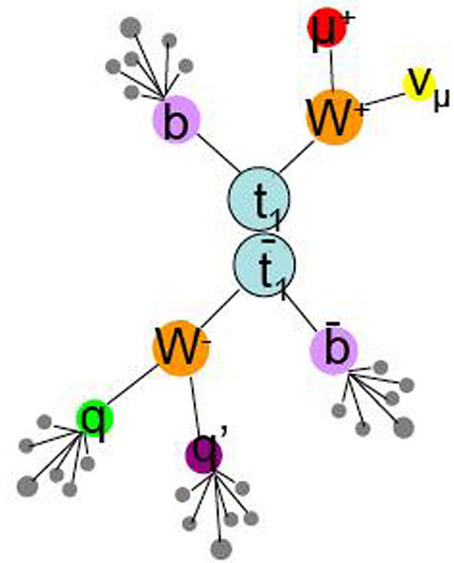
Impact parameter:

~ cτ

Top quark discovery



CDF Collaboration, Phys. Rev. Lett. 74 (1995) 2626

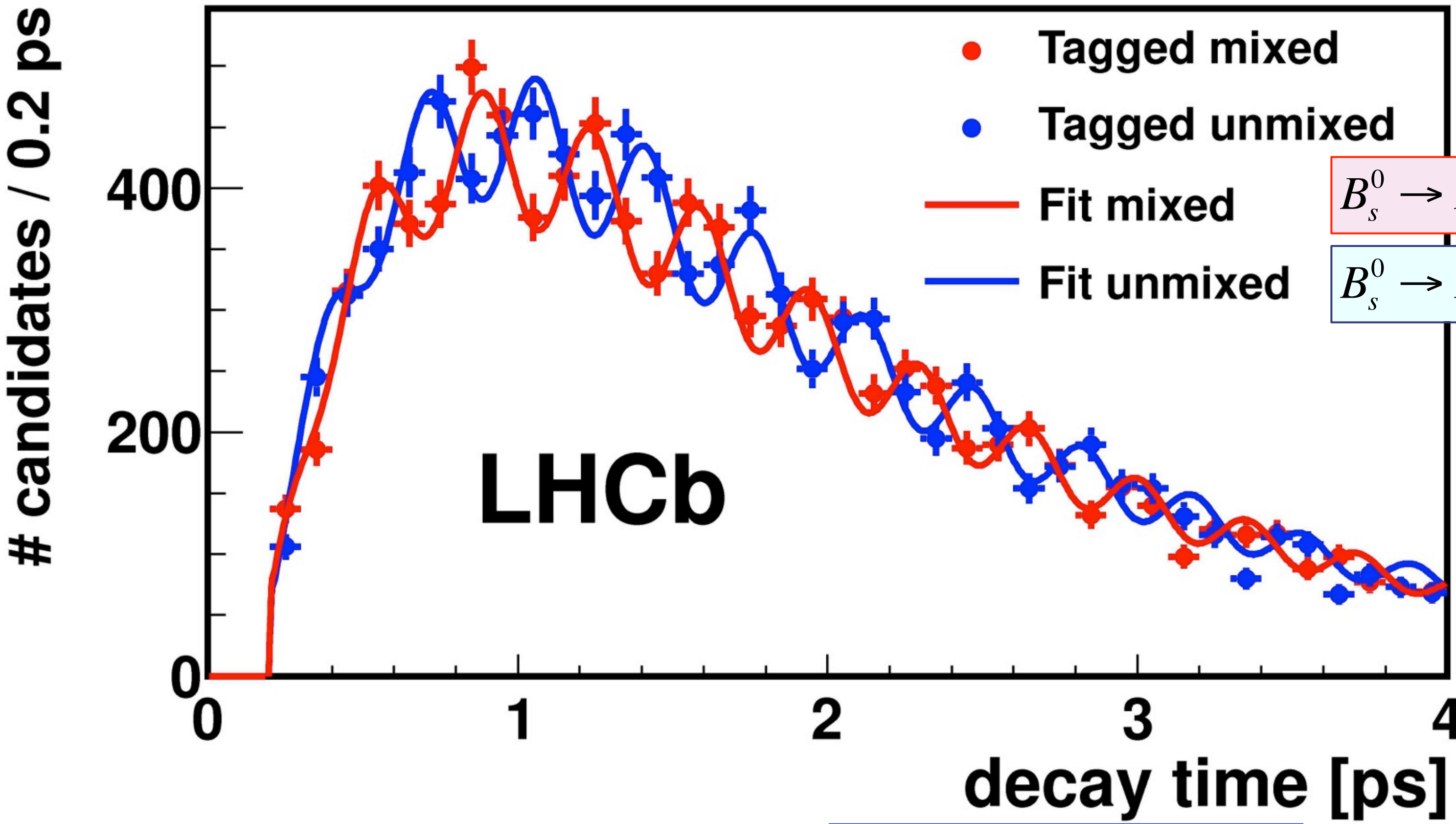


Topology:

- 2 *b*-jets
- 2 jets from *W* decay
- a leptonic *W* decay

Channel	SVX	SLT	Dilepton
Observed	27 tags	23 tags	6 events
Expected background	6.7 ± 2.1	15.4 ± 2.0	1.3 ± 0.3
Background probability	2×10^{-5}	6×10^{-2}	3×10^{-3}

B^0_s oscillations



$B^0_s \to \bar{B}^0_s \to D^+_s \pi^-$

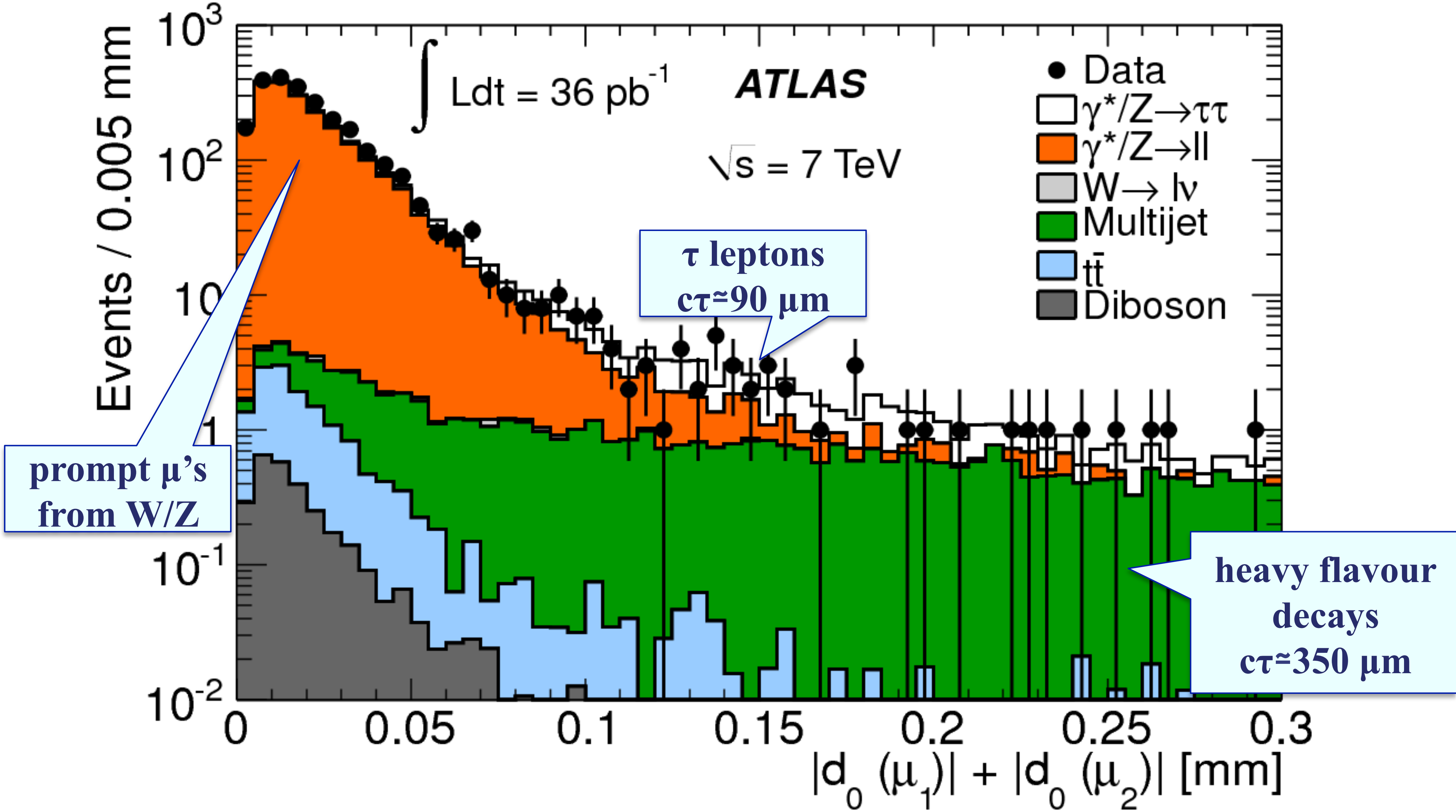
$B^0_s \to D^-_s \pi^+$

$$t = \frac{L}{\gamma\beta c} = \frac{L}{p/m}$$

impact parameter resolution

momentum resolution

Z→ττ→μμ+4ν



Events / 0.005 mm
$\int Ldt = 36\ pb^{-1}$
ATLAS
$\sqrt{s}$ = 7 TeV
Data
γ*/Z→ττ
γ*/Z→ll
W→ lν
Multijet
$t\bar{t}$
Diboson
τ leptons
cτ≈90 μm
prompt μ's
from W/Z
heavy flavour
decays
cτ≈350 μm
|d0 (μ1)| + |d0 (μ2)| [mm]

Conclusions

- I hope to have provided you with a quick overview of the very basics of charged particle tracking:
  - how it works
  - why it is useful
  - ...and why Si detectors are great at that!
- Many topics not addressed here:
  - detector technologies just shortly listed
  - front-end electronics and position reconstruction (beyond just electrode segmentation)
  - no mention of radiation damage
  - pattern recognition and vertex reconstruction
  - future intelligent trigger systems

All of these are very active and challenging research areas

Example

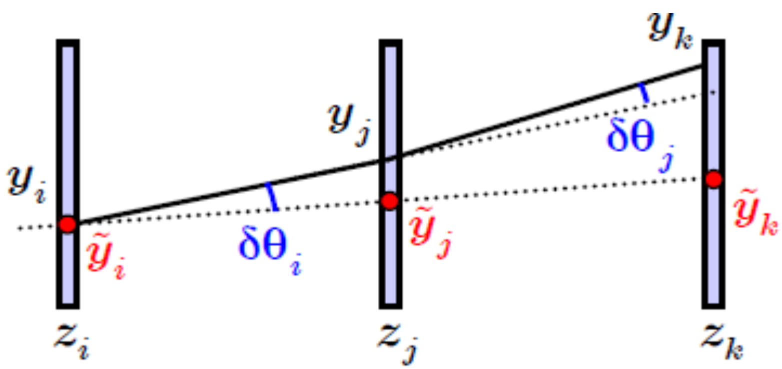
KALMAN FILTER FOR "STRAIGHT" TRACKS

Kalman filter: example

- Initial estimate of track parameters using *j, k*

$$y = a_j + b_j(z - z_j)$$

$$\mathbf{p}_j = \begin{pmatrix} a_j \\ b_j \end{pmatrix} = \begin{pmatrix} y_j \\ \dfrac{y_k - y_j}{z_k - z_j} \end{pmatrix} \qquad \mathbf{C}_j = \begin{pmatrix} \sigma_{y_j}^2 & -\dfrac{\sigma_{y_j}^2}{z_k - z_j} \\ -\dfrac{\sigma_{y_j}^2}{z_k - z_j} & \dfrac{\sigma_{y_j}^2 + \sigma_{y_k}^2}{(z_k - z_j)^2} \end{pmatrix}$$



- Extrapolate to point *i*: $\quad y = a_j + b_j(z - z_j) \Rightarrow y = a_i + b_i(z - z_i)$

$$\tilde{\mathbf{p}}_i = \begin{pmatrix} \tilde{a}_i \\ \tilde{b}_i \end{pmatrix} = \begin{pmatrix} a_j - b_j(z_j - z_i) \\ b_j \end{pmatrix}$$

$$\tilde{\mathbf{C}}_j = \frac{1}{(z_k - z_j)^2} \begin{pmatrix} (z_k - z_i)^2 \sigma_{y_j}^2 + (z_j - z_i)^2 \sigma_{y_k}^2 & -(z_k - z_i)\sigma_{y_j}^2 - (z_j - z_i)\sigma_{y_k}^2 \\ -(z_k - z_i)\sigma_{y_j}^2 - (z_j - z_i)\sigma_{y_k}^2 & \sigma_{y_j}^2 + \sigma_{y_k}^2 \end{pmatrix} + \theta_{p,j}^2 \begin{pmatrix} (z_j - z_i)^2 & z_j - z_i \\ z_j - z_i & 1 \end{pmatrix}$$

which gives contribution to the χ^2 for the parameters at *i*: $\qquad \mathbf{p}_i = \begin{pmatrix} a_i \\ b_i \end{pmatrix}$

$$\chi^2 = (\tilde{\mathbf{p}}_i - \mathbf{p}_i)^T \tilde{\mathbf{C}}^{-1} (\tilde{\mathbf{p}}_i - \mathbf{p}_i)$$

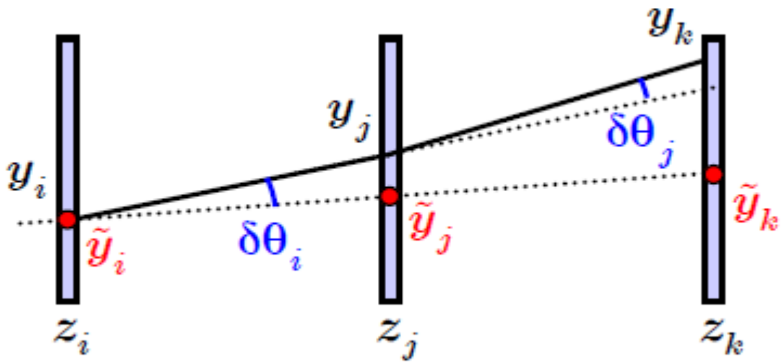
Kalman filter: example

- The measurement at *i* gives the term:

$$y = a_i + b_i(z - z_i)$$

$$\mathbf{H}_i = (1 \quad 0) \qquad \mathbf{H}_i \mathbf{p}_j - y_i = a_i - y_i$$

$$\chi^2 = \frac{(y_i - a_i)^2}{\sigma_{y_i}^2}$$



- And the new parameters are obtained by the minimization of:

$$\chi^2 = (\tilde{\mathbf{p}}_i - \mathbf{p}_i)^T \tilde{\mathbf{C}}^{-1} (\tilde{\mathbf{p}}_i - \mathbf{p}_i) + \frac{(y_i - a_i)^2}{\sigma_{y_i}^2}$$

- Which can be put in the general χ^2 form: $\chi^2 = (\mathbf{Y} - \mathbf{A}\mathbf{p})^T \mathbf{V}^{-1} (\mathbf{Y} - \mathbf{A}\mathbf{p})$

$$\mathbf{p} = \begin{pmatrix} a_i \\ b_i \end{pmatrix} \quad \mathbf{Y} = \begin{pmatrix} \tilde{a}_i \\ \tilde{b}_i \\ y_i \end{pmatrix} \quad \mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \mathbf{V} = \begin{bmatrix} \tilde{\mathbf{C}}_i & \mathbf{0} \\ \mathbf{0} & \sigma_{y_i}^2 \end{bmatrix}$$

whose solution is:

$$\mathbf{p} = (\mathbf{A}^T \mathbf{W} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{W} \mathbf{Y} \quad \mathbf{C}_i = (\mathbf{A}^T \mathbf{W} \mathbf{A})^{-1} \quad \mathbf{W} = \mathbf{V}^{-1} = \begin{bmatrix} \tilde{\mathbf{C}}_i^{-1} & \mathbf{0} \\ \mathbf{0} & 1/\sigma_{y_i}^2 \end{bmatrix}$$

Kalman filter: example

- And finally, going to the interaction point:

$$y = a_0 + b_0 z$$

$$\mathbf{p}_0 = \begin{pmatrix} a_0 \\ b_0 \end{pmatrix} = \begin{pmatrix} a_i - b_i z_i \\ b_i \end{pmatrix}$$

$$\mathbf{C}_0 = \begin{pmatrix} 1 & -z_i \\ 0 & 1 \end{pmatrix} \mathbf{C}_i \begin{pmatrix} 1 & 0 \\ -z_i & 1 \end{pmatrix} + \theta_{p,i}^2 \begin{pmatrix} z_i^2 & -z_i \\ -z_i & 1 \end{pmatrix}$$