A *brief* guide to Supersymmetric Models

coepp.org.au

School of Chemistry & Physics
The University of Adelaide
South Australia 5005
Australia

School of Physics
The University of Sydney
New South Wales 2006
Australia

School of Physics
The University of Melbourne
Victoria 3010
Australia

School of Physics
Monash University
Victoria 3800
Australia

Caution

A single lecture is *far* too short to teach Supersymmetry

Last year I gave a brief guide to understanding supersymmetric signals

This year I will focus more on the models, with an overview of the most relevant ones.

Caution

A single lecture is *far* too short to teach Supersymmetry

Last year I gave a brief guide to understanding supersymmetric signals

This year I will focus more on the models, with an overview of the most relevant ones.

In such a short time there will be gaps in the explanations.
Technical details will be suppressed.
So if you don't understand something please don't:

Caution

A single lecture is *far* too short to teach Supersymmetry

Last year I gave a brief guide to understanding supersymmetric signals

This year I will focus more on the models, with an overview of the most relevant ones.

In such a short time there will be gaps in the explanations.
Technical details will be suppressed.
So if you don't understand something please don't:

More extensive lectures can be found at:
http://www.physics.adelaide.edu.au/cssm/seminars/SUSY/

SUperSYmmetry (SUSY)

- A symmetry between fermions and bosons

$Q\,|\text{Boson}\rangle = |\text{Fermion}\rangle$

$Q\,|\text{Fermion}\rangle = |\text{Boson}\rangle$

SUperSYmmetry (SUSY)

- A symmetry between fermions and bosons

Q |Boson⟩ = |Fermion⟩

Q |Fermion⟩ = |Boson⟩

- Extends special relativity, evading Coleman and Mandula "No-Go" theorem.

- The Super Poincare algebra:

P_μ	- translations
$M_{\mu\nu}$	- rotations and boosts
Q_α	- SUSY transformation

$$\begin{aligned}
\left\{Q_\alpha{}^A, \bar{Q}_{\dot{\beta}B}\right\} &= 2\sigma^\mu_{\alpha\dot{\beta}} P_\mu \delta^A{}_B \\
\{Q_\alpha{}^A, Q_\beta{}^B\} &= \left\{\bar{Q}_{\dot{\alpha}A}, \bar{Q}_{\dot{\beta}B}\right\} = 0 \\
[P_\mu, Q_\alpha{}^A] &= [P_\mu, \bar{Q}_{\dot{\alpha}A}] = 0
\end{aligned}$$

SUperSYmmetry (SUSY)

- A symmetry between fermions and bosons

Q |Boson⟩ = |Fermion⟩

Q |Fermion⟩ = |Boson⟩

- Extends special relativity, evading Coleman and Mandula “No-Go” theorem.

- The Super Poincare algebra:

P_μ - translations

$M_{\mu\nu}$ - rotations and boosts

Q_α - SUSY transformation

$$\begin{aligned} \left\{Q_\alpha{}^A, \bar{Q}_{\dot{\beta}B}\right\} &= 2\sigma^\mu_{\alpha\dot{\beta}} P_\mu \delta^A{}_B \\ \left\{Q_\alpha{}^A, Q_\beta{}^B\right\} &= \left\{\bar{Q}_{\dot{\alpha}A}, \bar{Q}_{\dot{\beta}B}\right\} = 0 \\ \left[P_\mu, Q_\alpha{}^A\right] &= \left[P_\mu, \bar{Q}_{\dot{\alpha}A}\right] = 0 \end{aligned}$$

- SUSY = a translation in Superspace.

$$z = (x_\mu, \theta^a, \overline{\theta}_{\dot{a}}).$$

So SUSY is an interesting theoretical idea.

But why would we expect it to be relevant to the LHC?

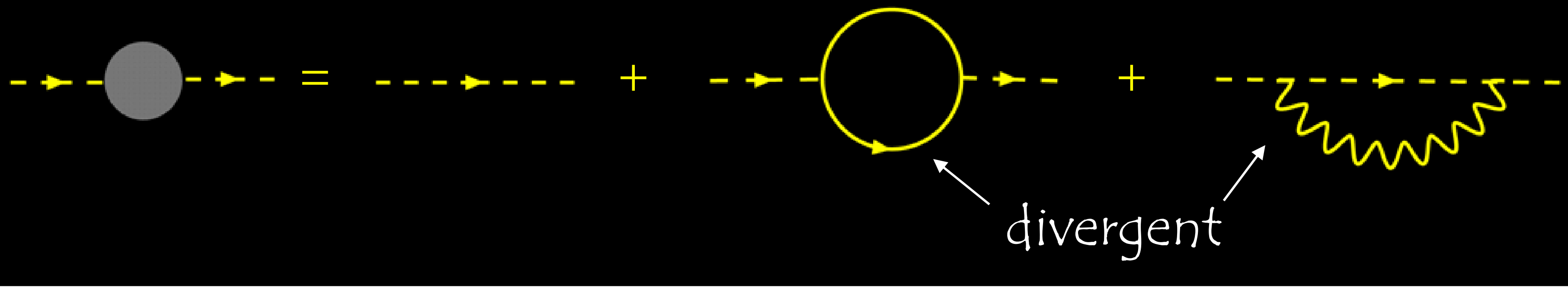
Motivation for SUSY at the LHC

Hierarchy Problem

Hierarchy Problem

- Expect New Physics at Planck Energy (Mass) $(\Lambda \sim 10^{19}\ \text{GeV})$
- Higgs mass sensitive to this scale $(m_h \sim 100\ \text{GeV})$

physical mass = "bare mass" + "loops"



- Cut off integral at Planck Scale (Λ) ← naive approach to renormalisation

$$m_h^2 = m_0^2 - \frac{\lambda_f^2}{8\pi^2}(\Lambda^2 - \int_0^1 dx 2\triangle \ln \frac{\Lambda^2+\triangle}{\triangle})$$

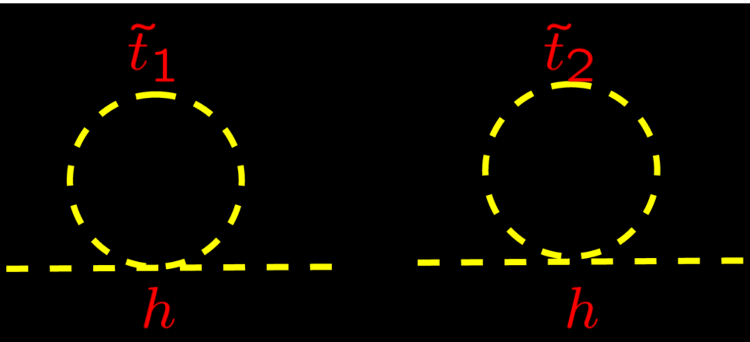
$$m_h^2 = m_0^2 - C\Lambda^2 + \ldots$$

$\Longrightarrow$ Huge Fine tuning!

In Supersymmetry

Bosonic degrees of freedom = Fermionic degrees of freedom.

$\Rightarrow$ Two scalar superpartners for each fermion



$$m_h^2 = m_0^2 - \frac{\lambda_t^2}{8\pi^2}\boxed{(\Lambda^2} - \int_0^1 dx 2\Delta \ln\frac{\Lambda^2+\Delta}{\Delta})$$

$$+\frac{\lambda_{\tilde{t}}}{16\pi^2}\boxed{(2\Lambda^2} - m_{\tilde{t}_1}^2 \ln\frac{\Lambda^2+m_{\tilde{t}_1}^2}{m_{s_1}^2} - m_{\tilde{t}_2}^2 \ln\frac{\Lambda^2+m_{\tilde{t}_2}^2}{m_{\tilde{t}_2}^2})$$

In SUSY $\lambda_{\tilde{t}} = \lambda_t^2$

Quadratic divergences cancelled!

$\Longrightarrow$ No Fine Tuning?

Motivation for SUSY at the LHC

Hierarchy Problem

$m_h^2 = m_0^2 - \frac{\lambda_t^2}{8\pi^2}(\Lambda^2) \quad \Rightarrow$ Huge Fine tuning!

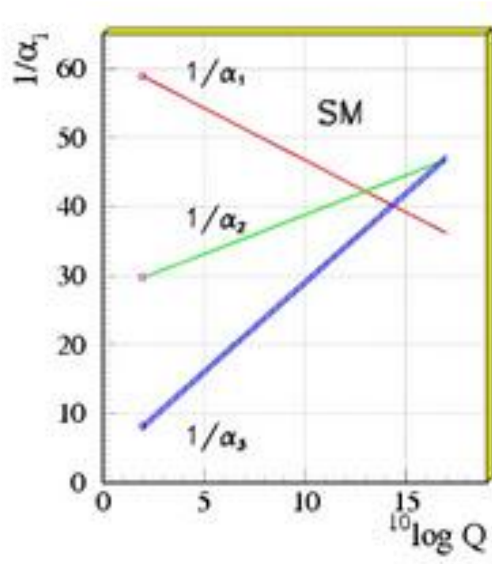
SUSY: $+\frac{\lambda_{\tilde{t}}}{16\pi^2}(2\Lambda^2) \quad \lambda_{\tilde{t}} = \lambda_t^2 \quad \Rightarrow$ Quadratic divergences cancel

Gauge Coupling Unification

GUT matter supermultiplets

$SU(5)$	
$\left(10 \right)_i$	$Q_i,\ u^c_i,\ e^c_i$
$+$	
$\left(5^* \right)_i$	$L_i,\ d^c_i$
$+$	
$\left(1 \right)_i$	N^c_i

GUT matter supermultiplets



$SU(5)$

$\left(10 \right)_i$ $Q_i,\ u_i^c,\ e_i^c$

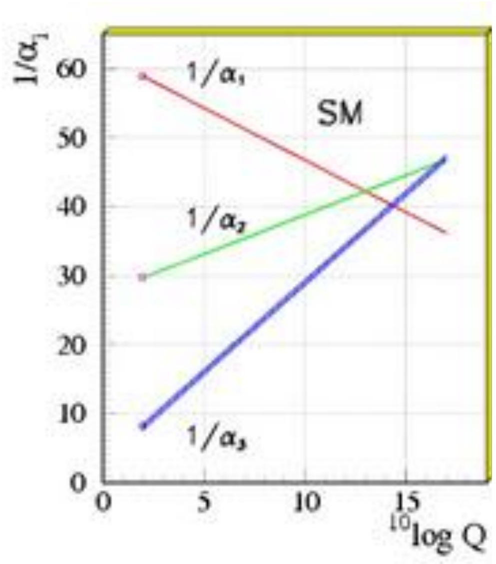
$+$

$\left(5^* \right)_i$ $L_i,\ d_i^c$

$+$

$\left(1 \right)_i$ N_i^c

GUT matter supermultiplets



$SU(5)$

$\left(\ 10\ \right)_i$ $\quad Q_i,\ u^c_i,\ e^c_i$

+

$\left(\ 5^*\ \right)_i$ $\quad L_i,\ d^c_i$

+

$\left(\ 1\ \right)_i$ $\quad N^c_i$

Supermultiplets: each fermion has a scalar superpartner!

squark

quark

$$\hat{Q}_i \left\{ \begin{matrix} \hat{u}_L \\ \hat{d}_L \end{matrix} \right. \left\{ \begin{matrix} \tilde{u}_L & u_L \\ \tilde{d}_L & d_L \end{matrix} \right.$$

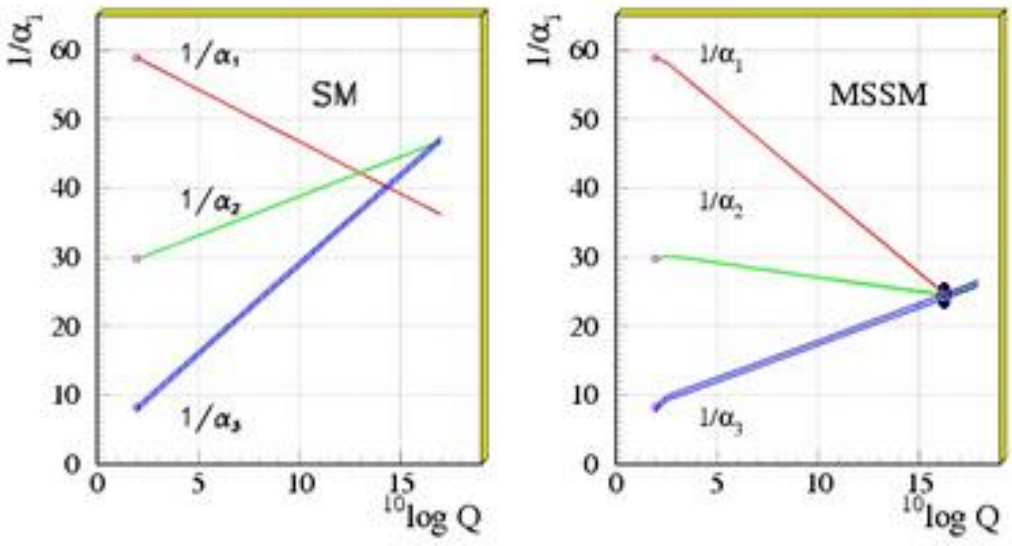
slepton

$$\hat{L}_i \left\{ \begin{matrix} \hat{\nu}_L \\ \hat{l}_L \end{matrix} \right. \left\{ \begin{matrix} \tilde{\nu}_L & \nu_L \\ \tilde{l}_L & l_L \end{matrix} \right.$$

lepton

$$\hat{u^c}_i \in \tilde{u}^*_R\, u^\dagger_R \qquad \hat{d^c}_i \in \tilde{d}^*_R\, d^\dagger_R \qquad \hat{e^c}_i \in \tilde{e}^*_R\, e^\dagger_R$$

Gauge Coupling Unification



SU(N) gauge theory

Gauginos

$$b_N = \frac{11}{3}N - \frac{2}{3}N - \frac{1}{3}n_f - \frac{1}{6}n_s$$

Number of fermions

Number of scalars

U(1) gauge

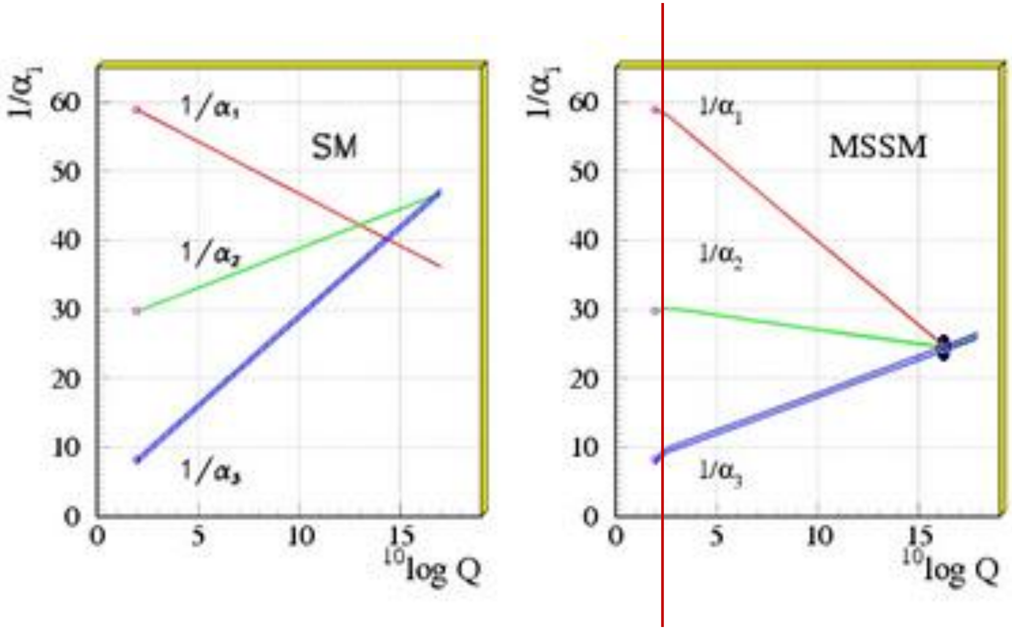
$$b_1 = -\frac{1}{3}\sum_i Y_i^2$$

(matter particles in fundamental representation)

$$\frac{d\alpha_i^{-1}}{d(\log Q)} = \frac{b_i}{2\pi}$$

Gauge Coupling Unification

Running modified
at TeV scale!



SU(N) gauge theory

Gauginos

$$b_N = \frac{11}{3}N - \frac{2}{3}N - \frac{1}{3}n_f - \frac{1}{6}n_s$$

U(1) gauge

$$b_1 = -\frac{1}{3}\sum_i Y_i^2$$

$$\frac{d\alpha_i^{-1}}{d(\log Q)} = \frac{b_i}{2\pi}$$

Number of
fermions

Number of
scalars

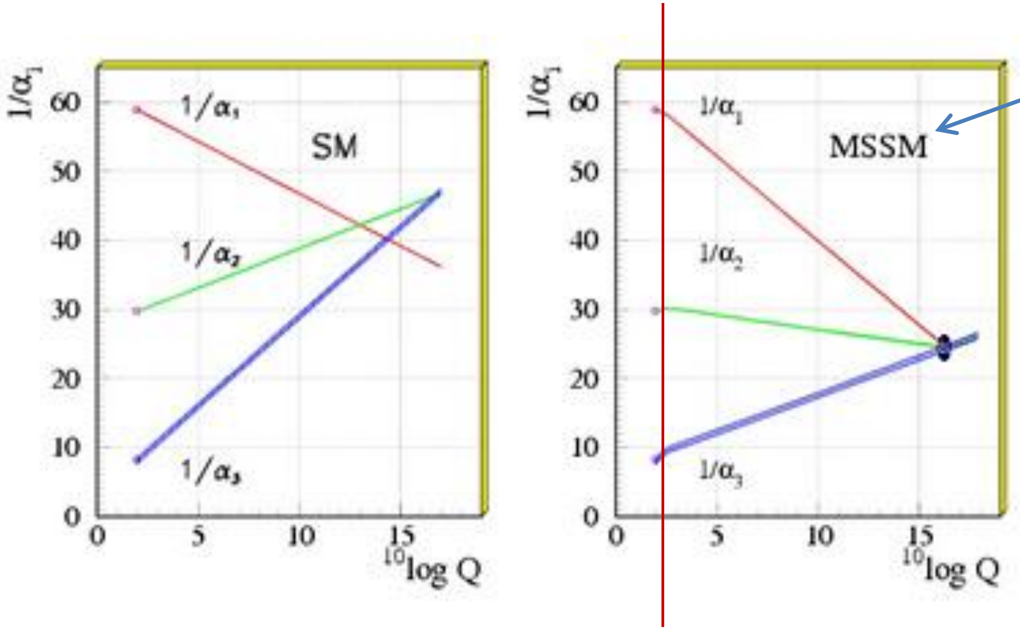
(matter particles in
fundamental
representation)

Gauge Coupling Unification

Running modified at TeV scale!

$\Rightarrow$ TeV scale new physics

Minimal Supersymmetric Standard Model



SU(N) gauge theory

Gauginos

$$b_N = \frac{11}{3}N - \frac{2}{3}N - \frac{1}{3}n_f - \frac{1}{6}n_s$$

U(1) gauge

$$b_1 = -\frac{1}{3}\sum_i Y_i^2$$

$$\frac{d\alpha_i^{-1}}{d(\log Q)} = \frac{b_i}{2\pi}$$

Number of fermions

Number of scalars

(matter particles in fundamental representation)

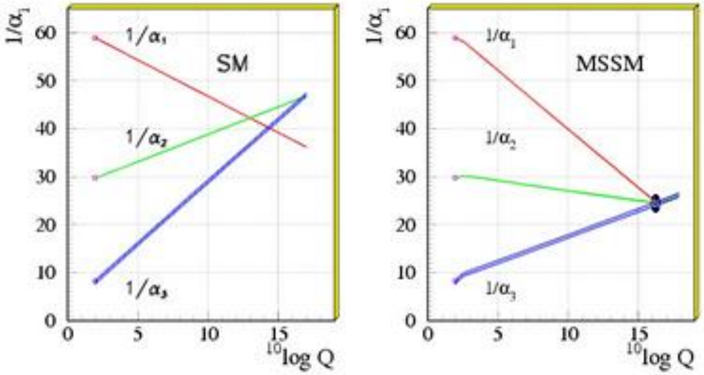
Motivation for SUSY at the LHC

Hierarchy Problem

$$m_h^2 = m_0^2 - \frac{\lambda_t^2}{8\pi^2}(\Lambda^2) \quad \Rightarrow$$ Huge Fine tuning!

SUSY: $$+\frac{\lambda_{\tilde{t}}}{16\pi^2}(2\Lambda^2) \quad \lambda_{\tilde{t}} = \lambda_t^2 \quad \Rightarrow$$ Quadratic divergences cancel

Gauge Coupling Unification



Dark-Matter / R-parity

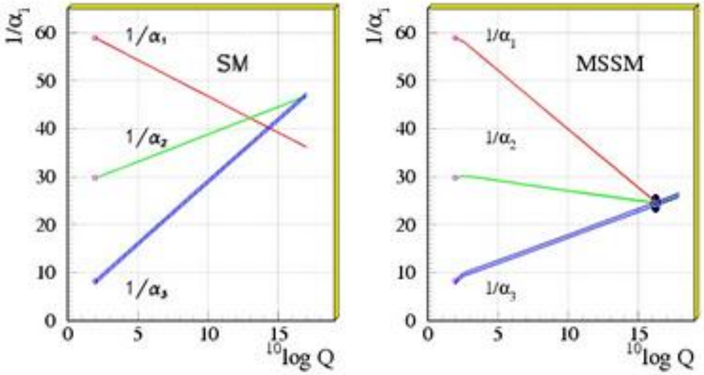
Motivation for SUSY at the LHC

Hierarchy Problem

$m_h^2 = m_0^2 - \frac{\lambda_t^2}{8\pi^2}(\Lambda^2) \quad \Rightarrow$ Huge Fine tuning!

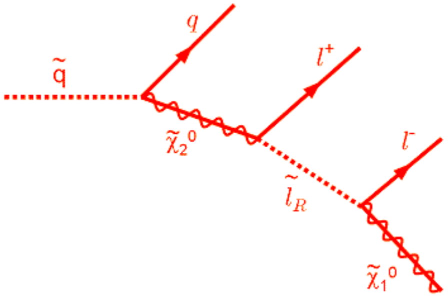
SUSY: $+\frac{\lambda_{\tilde{t}}}{16\pi^2}(2\Lambda^2) \quad \lambda_{\tilde{t}} = \lambda_t^2 \quad \Rightarrow$ Quadratic divergences cancel

Gauge Coupling Unification



Dark-Matter / R-parity

- R-Parity: SM particles even
  SUSY partners odd



Stable LSP
Dark Matter candidate

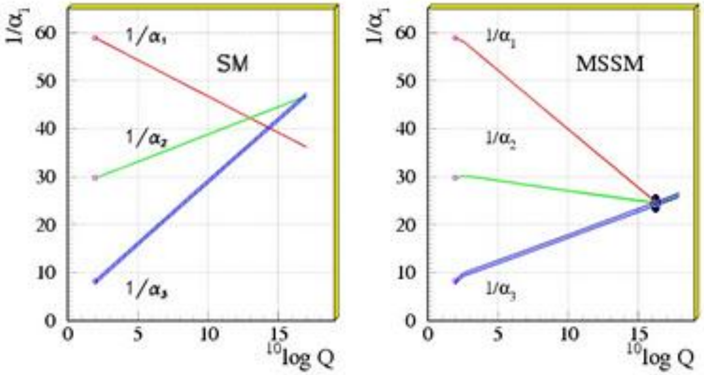
Motivation for SUSY at the LHC

Hierarchy Problem

$$m_h^2 = m_0^2 - \frac{\lambda_t^2}{8\pi^2}(\Lambda^2) \quad \Rightarrow$$ Huge Fine tuning!

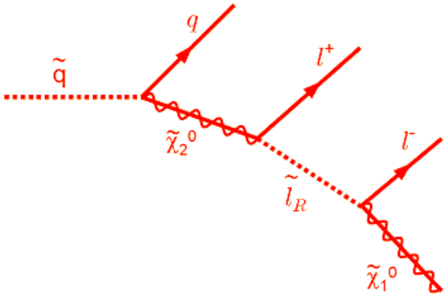
SUSY: $$+\frac{\lambda_{\tilde{t}}}{16\pi^2}(2\Lambda^2) \quad \lambda_{\tilde{t}} = \lambda_t^2 \quad \Rightarrow$$ Quadratic divergences cancel

Gauge Coupling Unification



Dark-Matter / R-parity

- R-Parity: SM particles even
  SUSY partners odd



Stable LSP
Dark Matter candidate

One More Reason

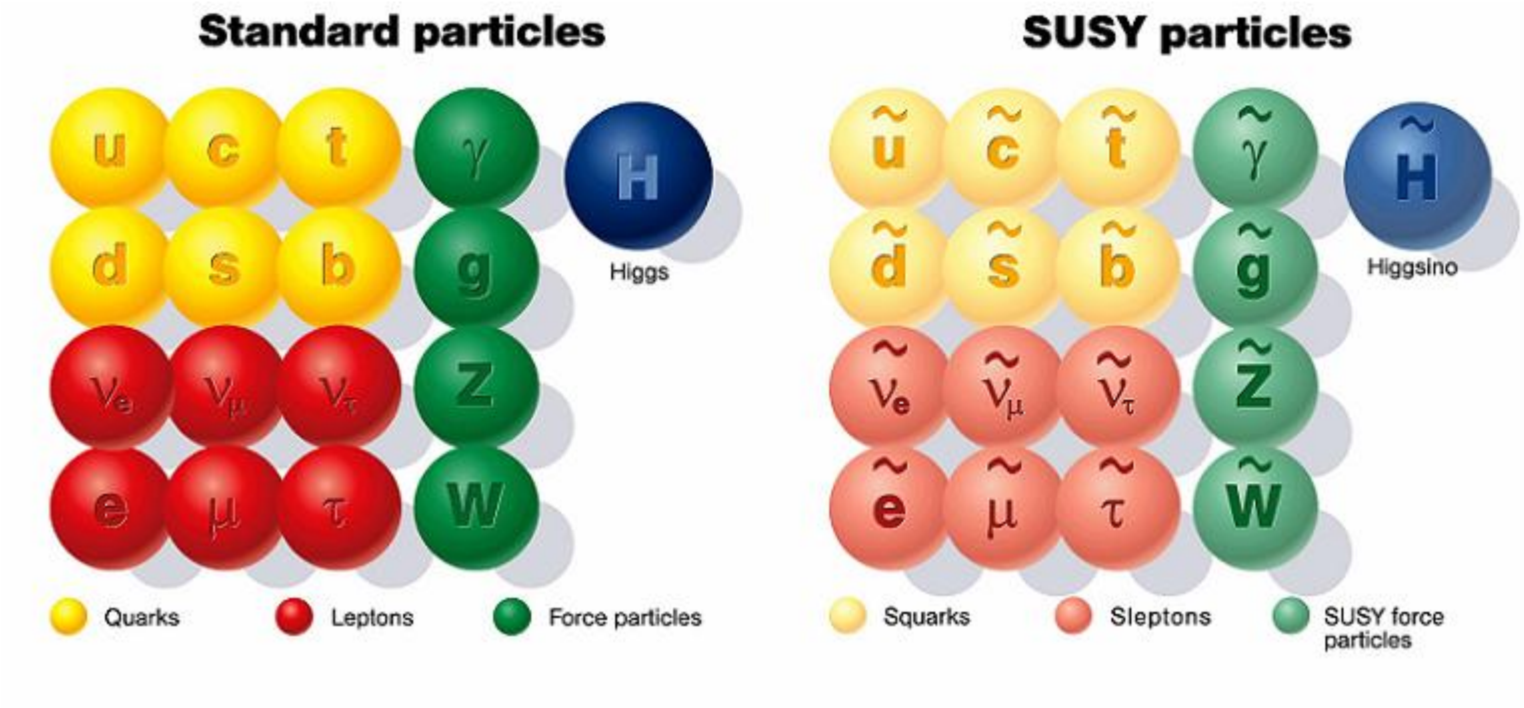
Ideas for new physics for experimentalists/ phenomenologists to work with!

What does a supersymmeric model look like?

Minimal Supersymmetric Standard Model (MSSM)

The MSSM = minimal particle content compatible with known physics, i.e Standard Model particles and properties.

Basic idea: take SM and supersymmetrise:

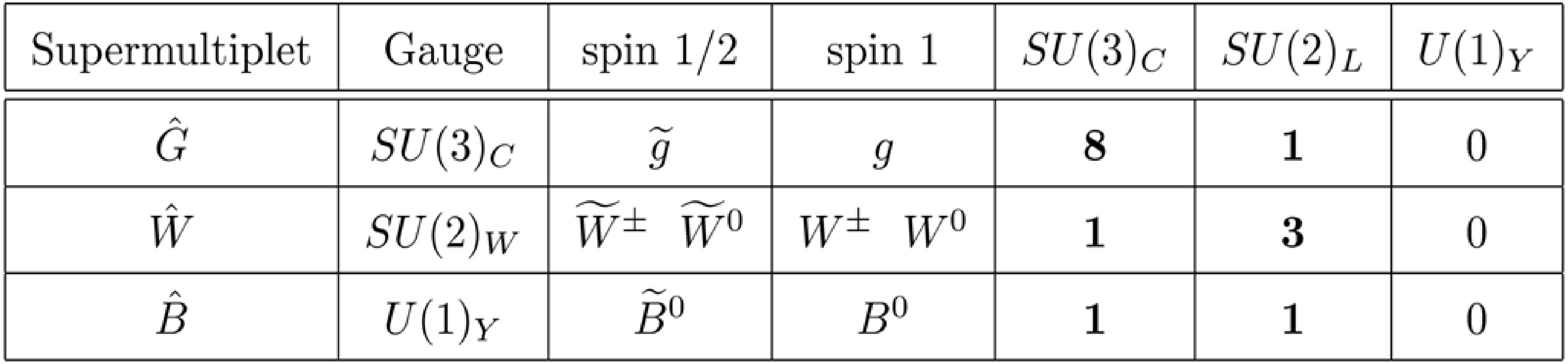


Warning: Image not entirely accurate.

Superfield content of the MSSM

Gauge group is that of SM: $G_{SM} \equiv \overset{\text{Strong}}{\overset{\downarrow}{SU(3)}} \times \overset{\text{Weak}}{\overset{\downarrow}{SU(2)}} \times \overset{\text{hypercharge}}{\overset{\downarrow}{U(1)_Y}}$

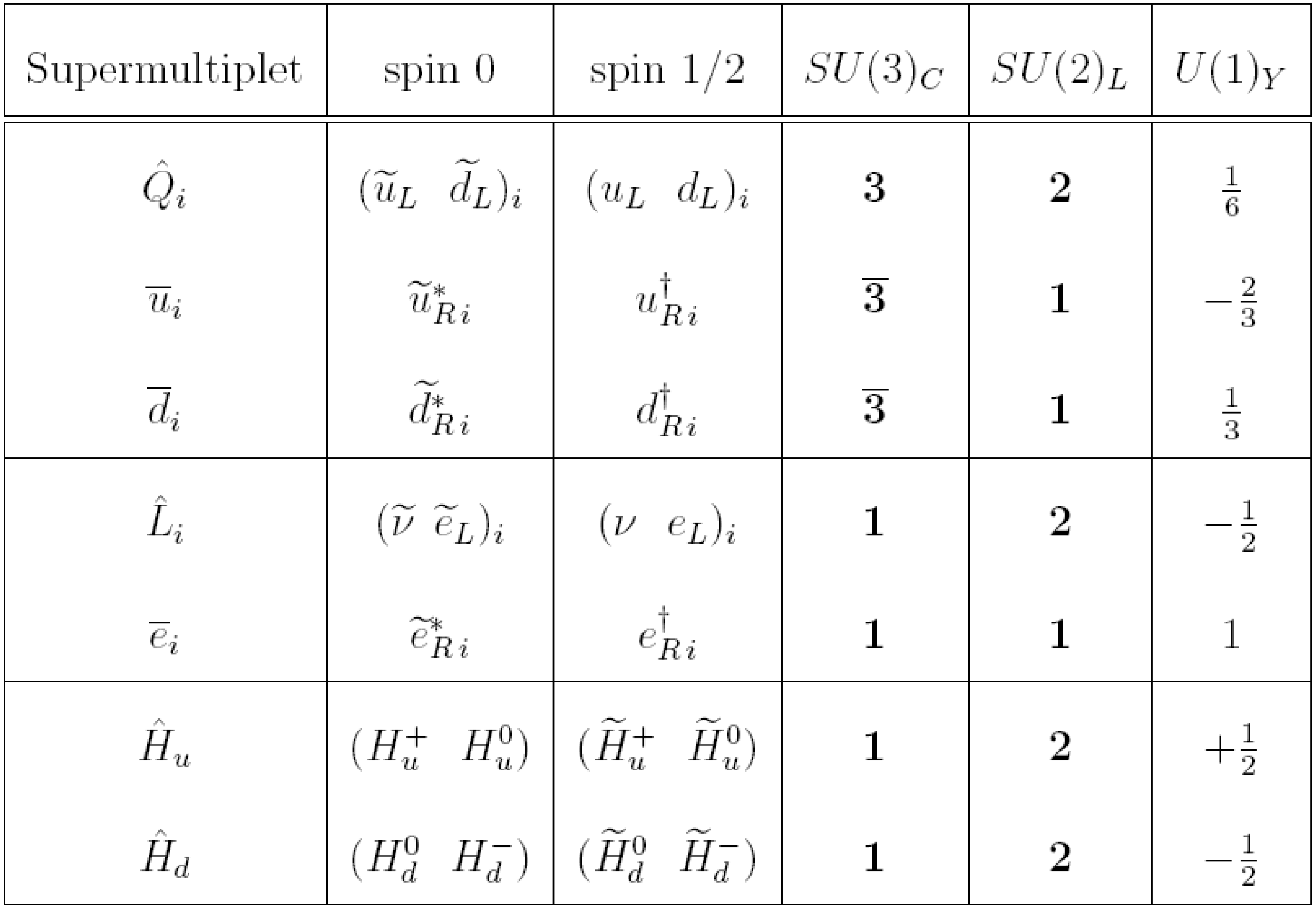
Vector superfields of the MSSM



Supermultiplet	Gauge	spin 1/2	spin 1	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$\hat{G}$	$SU(3)_C$	$\widetilde{g}$	g	$\mathbf{8}$	$\mathbf{1}$	0
$\hat{W}$	$SU(2)_W$	$\widetilde{W}^\pm \quad \widetilde{W}^0$	$W^\pm \quad W^0$	$\mathbf{1}$	$\mathbf{3}$	0
$\hat{B}$	$U(1)_Y$	$\widetilde{B}^0$	B^0	$\mathbf{1}$	$\mathbf{1}$	0

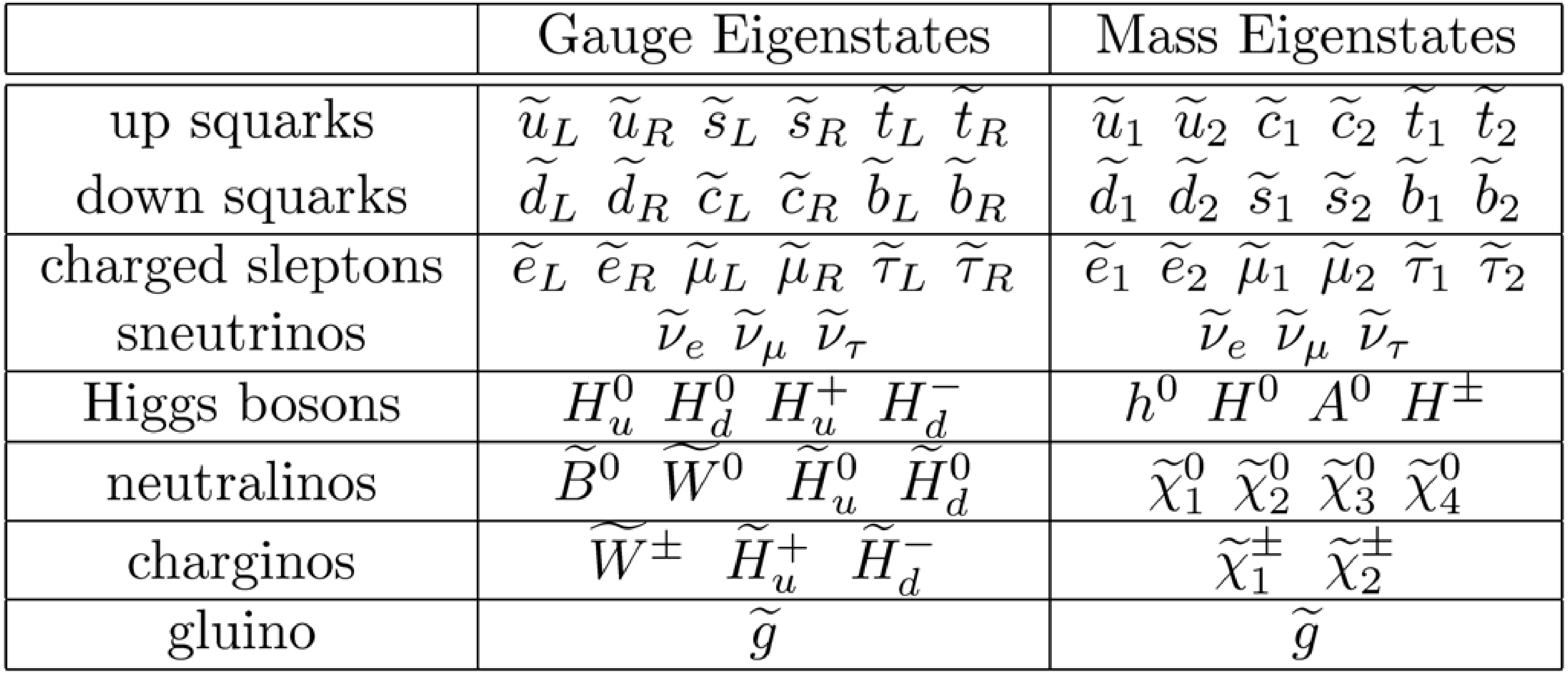
Gauge supermultiplets of the MSSM, and gauge group representations.

MSSM Chiral Superfield Content



Supermultiplet	spin 0	spin 1/2	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$\hat{Q}_i$	$(\widetilde{u}_L \;\; \widetilde{d}_L)_i$	$(u_L \;\; d_L)_i$	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{6}$
$\overline{u}_i$	$\widetilde{u}^*_{R\,i}$	$u^\dagger_{R\,i}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-\frac{2}{3}$
$\overline{d}_i$	$\widetilde{d}^*_{R\,i}$	$d^\dagger_{R\,i}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{3}$
$\hat{L}_i$	$(\widetilde{\nu} \;\; \widetilde{e}_L)_i$	$(\nu \;\; e_L)_i$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$
$\overline{e}_i$	$\widetilde{e}^*_{R\,i}$	$e^\dagger_{R\,i}$	$\mathbf{1}$	$\mathbf{1}$	1
$\hat{H}_u$	$(H_u^+ \;\; H_u^0)$	$(\widetilde{H}_u^+ \;\; \widetilde{H}_u^0)$	$\mathbf{1}$	$\mathbf{2}$	$+\frac{1}{2}$
$\hat{H}_d$	$(H_d^0 \;\; H_d^-)$	$(\widetilde{H}_d^0 \;\; \widetilde{H}_d^-)$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$

Mass eigenstates of the MSSM (and SUSY jargon)



	Gauge Eigenstates	Mass Eigenstates
up squarks	$\widetilde{u}_L\ \widetilde{u}_R\ \widetilde{s}_L\ \widetilde{s}_R\ \widetilde{t}_L\ \widetilde{t}_R$	$\widetilde{u}_1\ \widetilde{u}_2\ \widetilde{c}_1\ \widetilde{c}_2\ \widetilde{t}_1\ \widetilde{t}_2$
down squarks	$\widetilde{d}_L\ \widetilde{d}_R\ \widetilde{c}_L\ \widetilde{c}_R\ \widetilde{b}_L\ \widetilde{b}_R$	$\widetilde{d}_1\ \widetilde{d}_2\ \widetilde{s}_1\ \widetilde{s}_2\ \widetilde{b}_1\ \widetilde{b}_2$
charged sleptons	$\widetilde{e}_L\ \widetilde{e}_R\ \widetilde{\mu}_L\ \widetilde{\mu}_R\ \widetilde{\tau}_L\ \widetilde{\tau}_R$	$\widetilde{e}_1\ \widetilde{e}_2\ \widetilde{\mu}_1\ \widetilde{\mu}_2\ \widetilde{\tau}_1\ \widetilde{\tau}_2$
sneutrinos	$\widetilde{\nu}_e\ \widetilde{\nu}_\mu\ \widetilde{\nu}_\tau$	$\widetilde{\nu}_e\ \widetilde{\nu}_\mu\ \widetilde{\nu}_\tau$
Higgs bosons	$H_u^0\ H_d^0\ H_u^+\ H_d^-$	$h^0\ H^0\ A^0\ H^\pm$
neutralinos	$\widetilde{B}^0\ \ \widetilde{W}^0\ \ \widetilde{H}_u^0\ \ \widetilde{H}_d^0$	$\widetilde{\chi}_1^0\ \widetilde{\chi}_2^0\ \widetilde{\chi}_3^0\ \widetilde{\chi}_4^0$
charginos	$\widetilde{W}^\pm\ \ \widetilde{H}_u^+\ \ \widetilde{H}_d^-$	$\widetilde{\chi}_1^\pm\ \ \widetilde{\chi}_2^\pm$
gluino	$\widetilde{g}$	$\widetilde{g}$

SUSY partners of SM particles are “sparticles”.

Scalar partners of SM fermions are “sfermions” → “squarks” and “sleptons”.

Wino, Bino, Higgsinos → Neutralinos

(charged) Wino, Higgsinos → Charginos

How do we understand the phenomenology of a SUSY model?

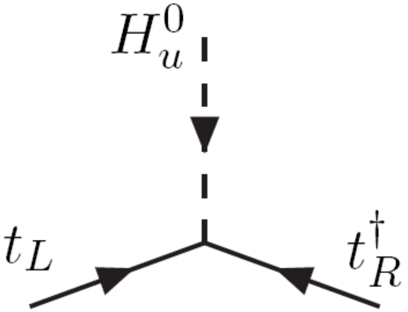
The physics of a SUSY model is given by the

Superpotential
+
Gauge structure
+
Soft breaking

MSSM Superpotential

$$\boxed{\mathcal{W}_{MSSM} = \epsilon_{\alpha\beta}(y_{\mathrm{u}}^{ij}\hat{H}_{\mathrm{u}}^{\alpha}\hat{\bar{\mathrm{u}}}_i\hat{Q}_j^{\beta} - y_{\mathrm{d}}^{ij}\hat{H}_{\mathrm{d}}^{\alpha}\hat{\bar{\mathrm{d}}}_i\hat{Q}_j^{\beta} - y_{\mathrm{e}}^{ij}\hat{H}_{\mathrm{d}}^{\alpha}\hat{\bar{\mathrm{e}}}_i\hat{L}_j^{\beta} + \mu\hat{H}_{\mathrm{u}}^{\alpha}\hat{H}_{\mathrm{d}}^{\beta})}$$

$$L_{\mathcal{W}} = \sum_i = -\left|\frac{\partial\mathcal{W}(\phi)}{\partial\phi_i}\right|^2 - \sum_{i,j}\psi_i\psi_j\frac{\partial^2\mathcal{W}(\phi)}{\partial\phi_i\partial\phi_j} + h.c.$$

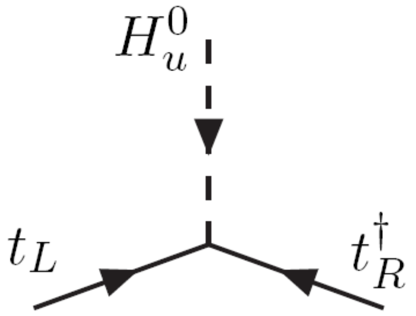


SM-like Yukawa coupling H-f-f

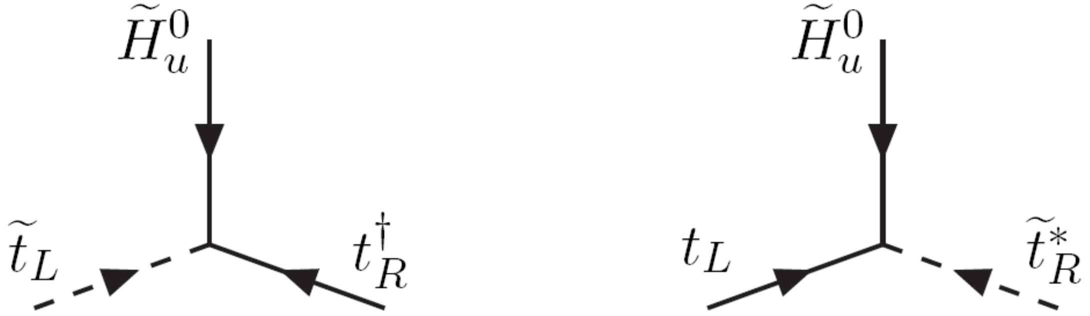
MSSM Superpotential

$$\boxed{\mathcal{W}_{MSSM} = \epsilon_{\alpha\beta}(y_{\mathrm{u}}^{ij}\hat{H}_{\mathrm{u}}^{\alpha}\hat{\bar{\mathrm{u}}}_i\hat{Q}_j^{\beta} - y_{\mathrm{d}}^{ij}\hat{H}_{\mathrm{d}}^{\alpha}\hat{\bar{\mathrm{d}}}_i\hat{Q}_j^{\beta} - y_{\mathrm{e}}^{ij}\hat{H}_{\mathrm{d}}^{\alpha}\hat{\bar{\mathrm{e}}}_i\hat{L}_j^{\beta} + \mu\hat{H}_{\mathrm{u}}^{\alpha}\hat{H}_{\mathrm{d}}^{\beta})}$$

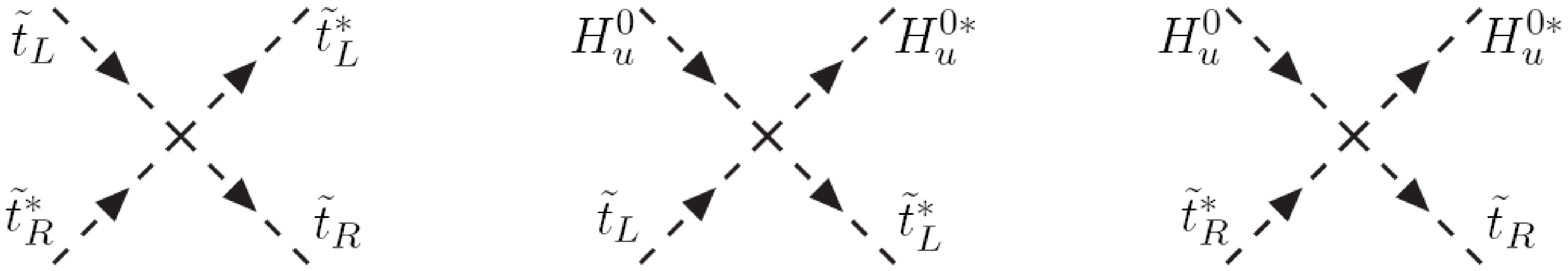
$$L_{\mathcal{W}} = \sum_i = -\left|\frac{\partial\mathcal{W}(\phi)}{\partial\phi_i}\right|^2 - \sum_{i,j}\psi_i\psi_j\frac{\partial^2\mathcal{W}(\phi)}{\partial\phi_i\partial\phi_j} + h.c.$$



SM-like Yukawa coupling H-f-f



Higgs-squark-quark couplings with same Yukawa coupling!

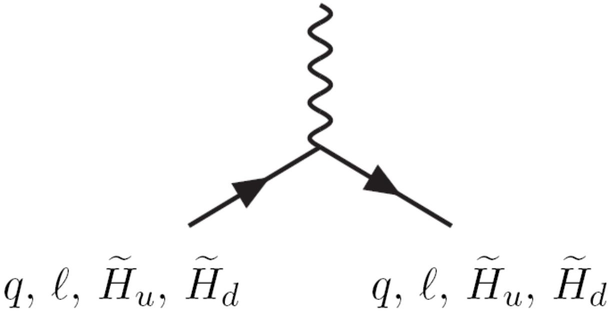


Quartic scalar couplings from same Yukawa coupling

SUSY Gauge interactions

For gauge invariant Lagrangian : in usual kinetic terms, giving:

derivatives ⟶ covariant derivatives $\sum |D_\mu \phi_i|^2 + i\overline{\psi}_i \not{D} \psi_i$

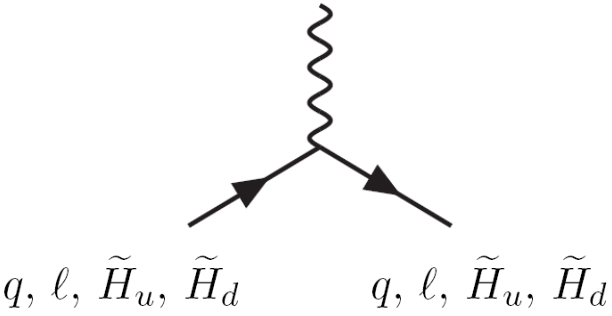


gauge-fermion-fermion vertex

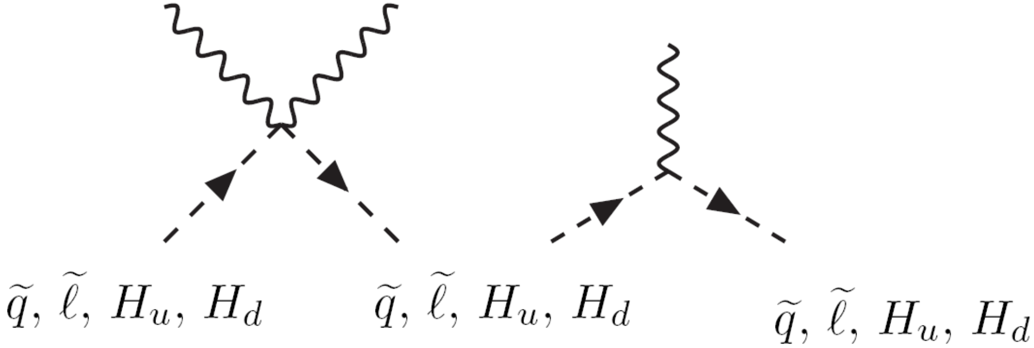
SUSY Gauge interactions

For gauge invariant Lagrangian : in usual kinetic terms, giving:

derivatives → covariant derivatives $\sum_i |D_\mu\phi_i|^2 + i\overline{\psi}_i \not{D}\psi_i$

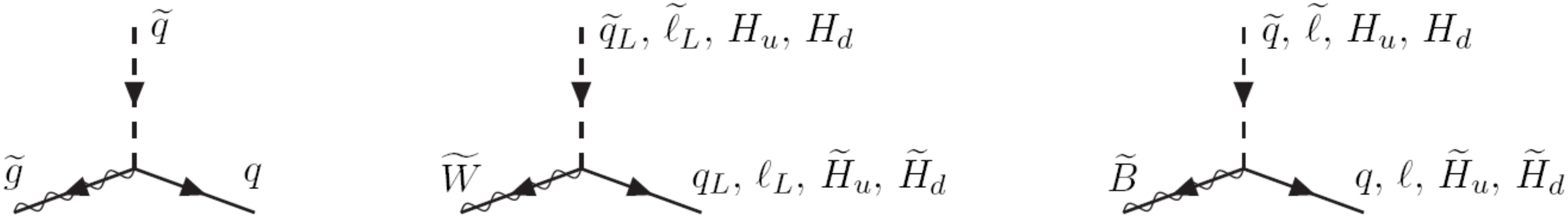


gauge-fermion-fermion vertex



Scalar-gauge vertices

Add Supersymmetric terms to keep SUSY invariant $ig\sqrt{2}(\phi^*\lambda\psi - \overline{\lambda\psi}\phi)$



Gaugino interactions from Kahler potential

All arise from something called the "Kahler Potential" $K = \sum_i \Phi_i^\dagger e^{2gV}\Phi_i$

Soft SUSY breaking

- Soft = doesn’t break dimensionless coupling relations

  ⇒ Maintain solutions to Hierarchy problem + gauge coupling unification

Dimension 3 or less operators only

Soft SUSY breaking

- Soft = doesn’t break dimensionless coupling relations

  ⇒ Maintain solutions to Hierarchy problem + gauge coupling unification

Dimension 3 or less operators only

$$
\begin{aligned}
-\mathcal{L}^{MSSM}_{soft} &= \frac{1}{2}\left[M_3\lambda_{\tilde{g}}\lambda_{\tilde{g}} + M_2\widetilde{W}^a\widetilde{W}^a + M_1\widetilde{B}\widetilde{B} + \text{h.c.}\right] \\
&+ \epsilon_{\alpha\beta}[B\mu H_d^\alpha H_u^\beta - a_{u_{ij}} H_u^\alpha\, \widetilde{\overline{u}}_i\, \widetilde{Q}_j^\beta + a_{d_{ij}} H_d^\alpha\, \widetilde{\overline{d}}_i\, \widetilde{Q}_j^\beta + a_{e_{ij}} H_d^\alpha\, \widetilde{\overline{e}}_i\, \widetilde{L}_j^\beta + \text{h.c.}] \\
&+ m^2_{H_d}|H_d|^2 + m^2_{H_u}|H_u|^2 + \widetilde{Q}_i^\alpha m^2_{Q_{ij}} \widetilde{Q}_j^{\alpha *} \\
&+ \widetilde{L}_i^\alpha m^2_{L_{ij}} \widetilde{L}_j^{\alpha *} + \widetilde{\overline{u}}^*_{Ri} m^2_{u_{ij}} \widetilde{\overline{u}}_j + \widetilde{\overline{d}}^*_i m^2_{d_{ij}} \widetilde{\overline{d}}_j + \widetilde{\overline{e}}^*_i m^2_{e_{ij}} \widetilde{\overline{e}}_j .
\end{aligned}
$$

Soft SUSY breaking

- Soft = doesn't break dimensionless coupling relations

  $\Rightarrow$ Maintain solutions to Hierarchy problem + gauge coupling unification

Dimension 3 or less operators only

$$
\begin{aligned}
-\mathcal{L}^{MSSM}_{soft} \;=\;& \frac{1}{2}\left[M_3\lambda_{\tilde{g}}\lambda_{\tilde{g}} + M_2\widetilde{W}^a\widetilde{W}^a + M_1\widetilde{B}\widetilde{B} + \text{h.c.}\right] \\
+\;& \epsilon_{\alpha\beta}[B\mu H_d^\alpha H_u^\beta - a_{u_{ij}} H_u^\alpha \,\widetilde{\bar{u}}_i\, \widetilde{Q}_j^\beta + a_{d_{ij}} H_d^\alpha \,\widetilde{\bar{d}}_i\, \widetilde{Q}_j^\beta + a_{e_{ij}} H_d^\alpha \,\widetilde{\bar{e}}_i\, \widetilde{L}_j^\beta + \text{h.c.}] \\
+\;& m^2_{H_d}|H_d|^2 + m^2_{H_u}|H_u|^2 + \widetilde{Q}_i^\alpha m^2_{Q_{ij}} \widetilde{Q}_j^{\alpha *} \\
+\;& \widetilde{L}_i^\alpha m^2_{L_{ij}} \widetilde{L}_j^{\alpha *} + \widetilde{\bar{u}}^*_{Ri} m^2_{u_{ij}} \widetilde{\bar{u}}_j + \widetilde{\bar{d}}^*_i m^2_{d_{ij}} \widetilde{\bar{d}}_j + \widetilde{\bar{e}}^*_i m^2_{e_{ij}} \widetilde{\bar{e}}_j .
\end{aligned}
$$

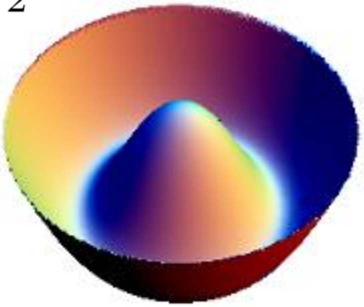
→ These soft breaking masses describe all possible ways supersymmetry could be broken softly without any assumptions about the breaking mechanism

→ Different breaking mechanisms can constrain the phenomenology further

What about the Higgs boson
and electroweak symmetry breaking?

Electroweak Symmetry Breaking (EWSB)

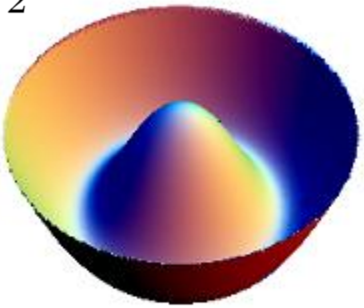
Recall in the SM the Higgs potential is: $V(\phi) = \mu^2\phi^\dagger\phi + |\lambda|\left(\phi^\dagger\phi\right)^2$



$$\frac{\partial V}{\partial \phi} \quad = \quad 0 \Rightarrow \mu^2 = -2|\lambda|\phi^\dagger\phi \qquad \Rightarrow \phi^\dagger\phi = -\frac{\mu^2}{2|\lambda|} \equiv v^2$$

Electroweak Symmetry Breaking (EWSB)

Recall in the SM the Higgs potential is: $V(\phi) = \mu^2 \phi^\dagger \phi + |\lambda| \left(\phi^\dagger \phi\right)^2$



$$\frac{\partial V}{\partial \phi} \quad = \quad 0 \Rightarrow \mu^2 = -2|\lambda|\phi^\dagger\phi \qquad \Rightarrow \phi^\dagger \phi = -\frac{\mu^2}{2|\lambda|} \equiv v^2$$

→ Vacuum Expectation Value (VEV)

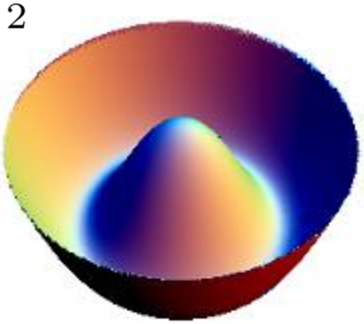
ϕ is an $SU(2)$ doublet with two complex components $\phi = \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix}$

Underlying SU(2) invariance $\Rightarrow$ the direction of the VEV in SU(2) space is arbitrary.

Any choice breaks $SU(2) \times U(1)_Y$ in the vacuum, choosing $\langle \phi \rangle_0 = \begin{pmatrix} 0 \\ v \end{pmatrix}$

Electroweak Symmetry Breaking (EWSB)

Recall in the SM the Higgs potential is: $V(\phi) = \mu^2\phi^\dagger\phi + |\lambda|\left(\phi^\dagger\phi\right)^2$



$$\frac{\partial V}{\partial \phi} = 0 \Rightarrow \mu^2 = -2|\lambda|\phi^\dagger\phi \quad \Rightarrow \phi^\dagger\phi = -\tfrac{\mu^2}{2|\lambda|} \equiv v^2$$

→ Vacuum Expectation Value (VEV)

ϕ is an $SU(2)$ doublet with two complex components $\phi = \left(\begin{array}{c} \phi^1 \\ \phi^2 \end{array}\right)$

Underlying SU(2) invariance $\Rightarrow$ the direction of the VEV in SU(2) space is arbitrary.

Any choice breaks $SU(2) \times U(1)_Y$ in the vacuum, choosing $\langle\phi\rangle_0 = \left(\begin{array}{c} 0 \\ v \end{array}\right)$

→ All $SU(2) \times U(1)_Y$ generators broken: $\sigma^i\langle\phi\rangle_0 \neq 0, \qquad Y\langle\phi\rangle_0 \neq 0.$

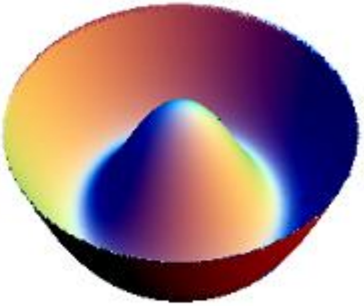
But for this choice $Q\langle\phi\rangle_0 = \frac{1}{2}(\sigma^3 + Y)\langle\phi\rangle_0 = 0.$

→ $\phi = \left(\begin{array}{c} \phi^1 \\ \phi^2 \end{array}\right)$ can be written as $\phi = \left(\begin{array}{c} \phi^+ \\ \phi^0 \end{array}\right)$

Showing the components' charge under unbroken generator Q

EWSB

Recall in the SM the Higgs potential is: $V(\phi) = \mu^2\phi^\dagger\phi + |\lambda|\left(\phi^\dagger\phi\right)^2$



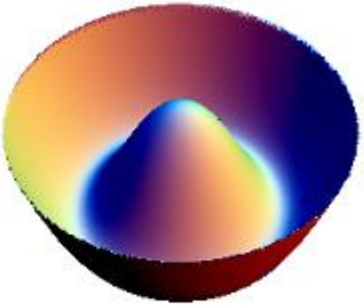
$$\frac{\partial V}{\partial \phi} = 0 \Rightarrow \mu^2 = -2|\lambda|\phi^\dagger\phi \quad \Rightarrow \phi^\dagger\phi = -\frac{\mu^2}{2|\lambda|} \equiv v^2$$

The MSSM Higgs Potential

$$\boxed{\begin{aligned} V_{\mathrm{H}} = {} & (m_{H_d}^2 + |\mu|^2)(|H_d^0|^2 + |H_d^-|^2) + (m_{H_u}^2 + |\mu|^2)(|H_u^+|^2 + |H_u^0|^2) \\ & + B\mu(H_u^+H_d^- - H_u^0H_d^0 + \mathrm{h.c.}) + \frac{1}{8}(g^2 + g'^2)\left(|H_d^0|^2 + |H_d^-|^2 - |H_u^+|^2 - |H_u^0|^2\right)^2 \\ & + \frac{1}{2}g^2(H_u^{+*}H_d^0 + H_u^{0*}H_d^-)(H_u^+H_d^{0*} + H_u^0H_d^{-*}) \end{aligned}}$$

EWSB

Recall in the SM the Higgs potential is: $V(\phi) = \mu^2 \phi^\dagger \phi + |\lambda| \left(\phi^\dagger \phi\right)^2$



$$\frac{\partial V}{\partial \phi} = 0 \Rightarrow \mu^2 = -2|\lambda| \phi^\dagger \phi \quad \Rightarrow \phi^\dagger \phi = -\frac{\mu^2}{2|\lambda|} \equiv v^2$$

The MSSM Higgs Potential

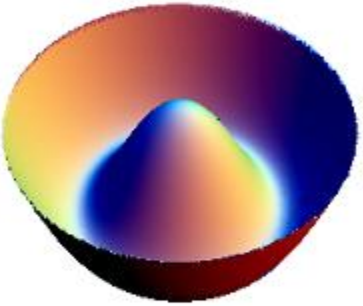
$$\begin{aligned} V_{\mathrm{H}} \quad = \quad & (m_{H_d}^2 + |\mu|^2)(|H_d^0|^2 + |H_d^-|^2) + (m_{H_u}^2 + |\mu|^2)(|H_u^+|^2 + |H_u^0|^2) \\ & + B\mu(H_u^+ H_d^- - H_u^0 H_d^0 + \mathrm{h.c.}) + \frac{1}{8}(g^2 + g'^2)\left(|H_d^0|^2 + |H_d^-|^2 - |H_u^+|^2 - |H_u^0|^2\right)^2 \\ & + \frac{1}{2} g^2 (H_u^{+\,*} H_d^0 + H_u^{0\,*} H_d^-)(H_u^+ H_d^{0\,*} + H_u^0 H_d^{-\,*}) \end{aligned}$$

As in the SM, underlying $SU(2)_W$ invariance means we can choose one component of one doublet to have no VEV:

Choose: $\langle H_u^+ \rangle_0 = 0$

EWSB

Recall in the SM the Higgs potential is: $V(\phi) = \mu^2\phi^\dagger\phi + |\lambda|\left(\phi^\dagger\phi\right)^2$



$$\frac{\partial V}{\partial \phi} = 0 \Rightarrow \mu^2 = -2|\lambda|\phi^\dagger\phi \quad \Rightarrow \phi^\dagger\phi = -\frac{\mu^2}{2|\lambda|} \equiv v^2$$

The MSSM Higgs Potential

$$\begin{aligned} V_{\rm H} \;=\; & (m^2_{H_d} + |\mu|^2)(|H_d^0|^2 + |H_d^-|^2) + (m^2_{H_u} + |\mu|^2)(|H_u^+|^2 + |H_u^0|^2) \\ & + B\mu(H_u^+ H_d^- - H_u^0 H_d^0 + \text{h.c.}) + \frac{1}{8}(g^2 + g'^2)\left(|H_d^0|^2 + |H_d^-|^2 - |H_u^+|^2 - |H_u^0|^2\right)^2 \\ & + \frac{1}{2}g^2(H_u^{+\,*} H_d^0 + H_u^{0\,*} H_d^-)(H_u^+ H_d^{0\,*} + H_u^0 H_d^{-\,*}) \end{aligned}$$

As in the SM, underlying $SU(2)_W$ invariance means we can choose one component of one doublet to have no VEV:

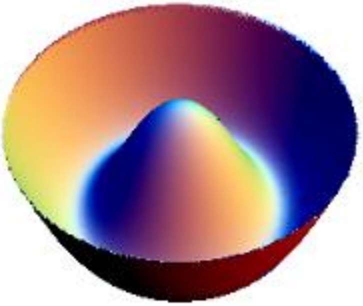
Choose: $\langle H_u^+ \rangle_0 = 0$

$$\Rightarrow \boxed{B\mu + \tfrac{1}{2}g^2 H_u^{0\,*} H_d^{0\,*} = 0} \text{ OR } H_d^- = 0 \qquad \Rightarrow \langle H_d^- \rangle_0 = 0$$

bad for stable EWSB

EWSB

Recall in the SM the Higgs potential is: $V(\phi) = \mu^2 \phi^\dagger \phi + |\lambda| \left(\phi^\dagger \phi\right)^2$



$$\frac{\partial V}{\partial \phi} = 0 \Rightarrow \mu^2 = -2|\lambda|\phi^\dagger \phi \quad \Rightarrow \phi^\dagger \phi = -\frac{\mu^2}{2|\lambda|} \equiv v^2$$

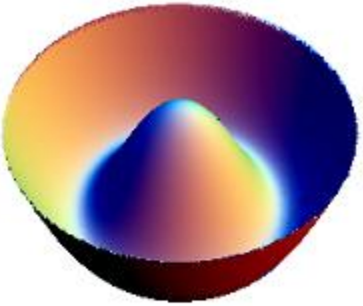
The MSSM Higgs Potential

$$\boxed{\begin{aligned} V_{\rm H} = \; & \overbrace{(m_{H_d}^2 + |\mu|^2)}^{m_1^2} |H_d^0|^2 + \overbrace{(m_{H_u}^2 + |\mu|^2)}^{m_2^2} |H_u^0|^2 - B\mu(H_u^0 H_d^0 + \text{h.c.}) \\ & + \frac{1}{8}(g^2 + g'^2)\left(|H_d^0|^2 - |H_u^0|^2\right)^2 \end{aligned}}$$

EWSB

Recall in the SM the Higgs potential is: $V(\phi) = \mu^2 \phi^\dagger \phi + |\lambda| \left(\phi^\dagger \phi\right)^2$

$$\frac{\partial V}{\partial \phi} = 0 \Rightarrow \mu^2 = -2|\lambda|\phi^\dagger\phi \qquad \Rightarrow \phi^\dagger\phi = -\frac{\mu^2}{2|\lambda|} \equiv v^2$$



The MSSM Higgs Potential

$$\boxed{\begin{aligned} V_{\mathrm{H}} = & \overbrace{(m_{H_d}^2 + |\mu|^2)}^{m_1^2}|H_d^0|^2 + \overbrace{(m_{H_u}^2 + |\mu|^2)}^{m_2^2}|H_u^0|^2 - B\mu(H_u^0 H_d^0 + \mathrm{h.c.}) \\ & + \frac{1}{8}(g^2 + g'^2)\left(|H_d^0|^2 - |H_u^0|^2\right)^2 \end{aligned}}$$

For successful EWSB:

$$\begin{aligned} (m_{H_d}^2 + m_{H_u}^2 + 2|\mu|^2) & \geq 2B\mu \\ (m_{H_d}^2 + |\mu|^2)(m_{H_u}^2 + |\mu|^2) & \leq (B\mu)^2 \end{aligned}$$

EWSB conditions

$$\begin{aligned} V_{\mathrm{H}} &= \overbrace{(m^2_{H_d} + |\mu|^2)}^{m_1^2}|H^0_d|^2 + \overbrace{(m^2_{H_u} + |\mu|^2)}^{m_2^2}|H^0_u|^2 - B\mu(H^0_u H^0_d + \mathrm{h.c.}) \\ &\quad + \frac{1}{8}(g^2 + g'^2)\left(|H^0_d|^2 - |H^0_u|^2\right)^2 \end{aligned}$$

For successful EWSB:

$$\begin{aligned} (m^2_{H_d} + m^2_{H_u} + 2|\mu|^2) &\geq 2B\mu \\ (m^2_{H_d} + |\mu|^2)(m^2_{H_u} + |\mu|^2) &\leq (B\mu)^2 \end{aligned}$$

$$\begin{aligned} m_1^2 v_d &= B\mu v_u + \frac{1}{8}(g^2 + g'^2)(v_u^2 - v_d^2)v_d \\ m_2^2 v_u &= B\mu v_d - \frac{1}{8}(g^2 + g'^2)(v_u^2 - v_d^2)v_u \end{aligned}$$

$$\begin{aligned} \langle H_u \rangle &= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_u \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v\sin\beta \end{pmatrix} \\ \langle H_d \rangle &= \frac{1}{\sqrt{2}}\begin{pmatrix} v_d \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} v\cos\beta \\ 0 \end{pmatrix} \end{aligned}$$

With: $\tan\beta = \frac{v_u}{v_d}$

EWSB conditions

$$\begin{aligned} V_{\mathrm{H}} &= \overbrace{(m_{H_d}^2 + |\mu|^2)}^{m_1^2}|H_d^0|^2 + \overbrace{(m_{H_u}^2 + |\mu|^2)}^{m_2^2}|H_u^0|^2 - B\mu(H_u^0 H_d^0 + \mathrm{h.c.}) \\ &\quad + \frac{1}{8}(g^2 + g'^2)\left(|H_d^0|^2 - |H_u^0|^2\right)^2 \end{aligned}$$

For successful EWSB:

$$\begin{aligned} (m_{H_d}^2 + m_{H_u}^2 + 2|\mu|^2) &\geq 2B\mu \\ (m_{H_d}^2 + |\mu|^2)(m_{H_u}^2 + |\mu|^2) &\leq (B\mu)^2 \end{aligned}$$

$$\begin{aligned} m_1^2 v_d &= B\mu v_u + \frac{1}{8}(g^2 + g'^2)(v_u^2 - v_d^2)v_d \\ m_2^2 v_u &= B\mu v_d - \frac{1}{8}(g^2 + g'^2)(v_u^2 - v_d^2)v_u \end{aligned}$$

$$\begin{aligned} \langle H_u \rangle &= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_u \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v\sin\beta \end{pmatrix} \\ \langle H_d \rangle &= \frac{1}{\sqrt{2}}\begin{pmatrix} v_d \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} v\cos\beta \\ 0 \end{pmatrix} \end{aligned}$$

With: $\tan\beta = \frac{v_u}{v_d}$ $\quad M_Z^2 = \frac{g^2+g'^2}{4}(v_u^2 + v_d^2) = \frac{g^2+g'^2}{4}(v^2)$

$$\begin{aligned} \sin(2\beta) &= \frac{2B\mu}{m_{H_u}^2 + m_{H_u}^2 + 2|\mu^2|} \\ M_Z^2 &= \frac{|m_{H_d}^2 - m_{H_u}^2|}{\sqrt{1 - \sin^2(2\beta)}} - m_{H_u}^2 - m_{H_d}^2 - 2|\mu|^2 \end{aligned}$$

MSSM Higgs Sector

Two complex
Higgs doublets

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}, \; H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix} \qquad \Rightarrow$$

8 degrees of freedom

MSSM Higgs Sector

Two complex Higgs doublets $H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}, \; H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$ $\Rightarrow$ 8 degrees of freedom

Gauge bosons Z^0, W^+, W^- Swallow 3 degrees of freedom

→ 5 physical Higgs bosons $h, \quad H, \quad A^0 \quad H^\pm$

MSSM Higgs Sector

Two complex
Higgs doublets $H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}, \; H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$ $\Rightarrow$ 8 degrees of freedom

Gauge bosons Z^0, W^+, W^- Swallow 3 degrees of freedom

→ 5 physical Higgs bosons $h, \quad H, \quad A^0 \quad H^\pm$

Light Higgs Upper bound: $m_h \leq M_Z |\cos 2\beta|$

Consequence of quartic coupling fixed in terms of gauge couplings
(compare with free λ parameter in SM)

MSSM Higgs Sector

Two complex Higgs doublets $H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}, \; H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$ $\Rightarrow$ 8 degrees of freedom

Gauge bosons Z^0, W^+, W^- Swallow 3 degrees of freedom

→ 5 physical Higgs bosons $h, \quad H, \quad A^0 \quad H^\pm$

Light Higgs Upper bound: $m_h \leq M_Z |\cos 2\beta|$

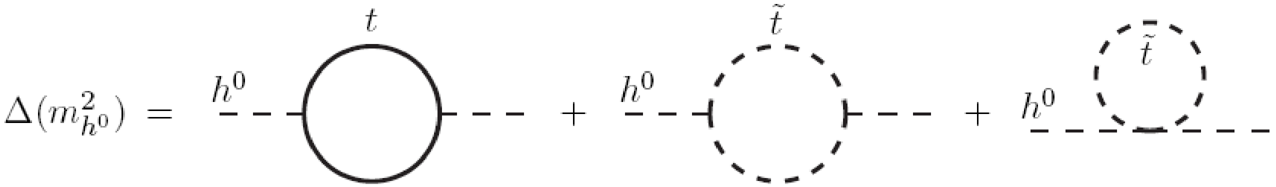
Consequence of quartic coupling fixed in terms of gauge couplings

(compare with free λ parameter in SM)

→ Radiative corrections significantly raise this

Including radiative corrections $m_{h^0} \lesssim 135 \text{ GeV}$

→ $$\Delta(m_{h^0}^2) = \frac{3}{4\pi^2} \cos^2\alpha \; y_t^2 m_t^2 \ln\left(m_{\tilde t_1} m_{\tilde t_2} / m_t^2\right) .$$



Constranied MSSM (cMSSM)

From SUperGRAvity (SUGRA)

Take minimal set of couplings:

Universal soft scalar mass: $m_0^2 = \kappa \frac{|\langle F \rangle|^2}{M_{Pl}^2}$

Universal soft gaugino mass: $M_{1/2} = f \frac{\langle F \rangle}{m_{Pl}}$

Universal soft trilinear mass: $A = \frac{\alpha \langle F \rangle}{M_{Pl}}$

Universal soft bilinear mass: $B = \frac{\beta \langle F \rangle}{M_{Pl}}$

Gauge coupling Unification Scale

$$\underbrace{g_i(M_X) = g_0 \quad y_i(M_X) = y_i}_{\text{Fixed}}$$

$$m_i^2(M_X) = m_0 \quad M_i(M_X) = M_{1/2} \quad A_i(M_X) = A_0$$

$$\underbrace{\mu(M_X) = \mu \quad B\mu(M_X) = B\mu}$$

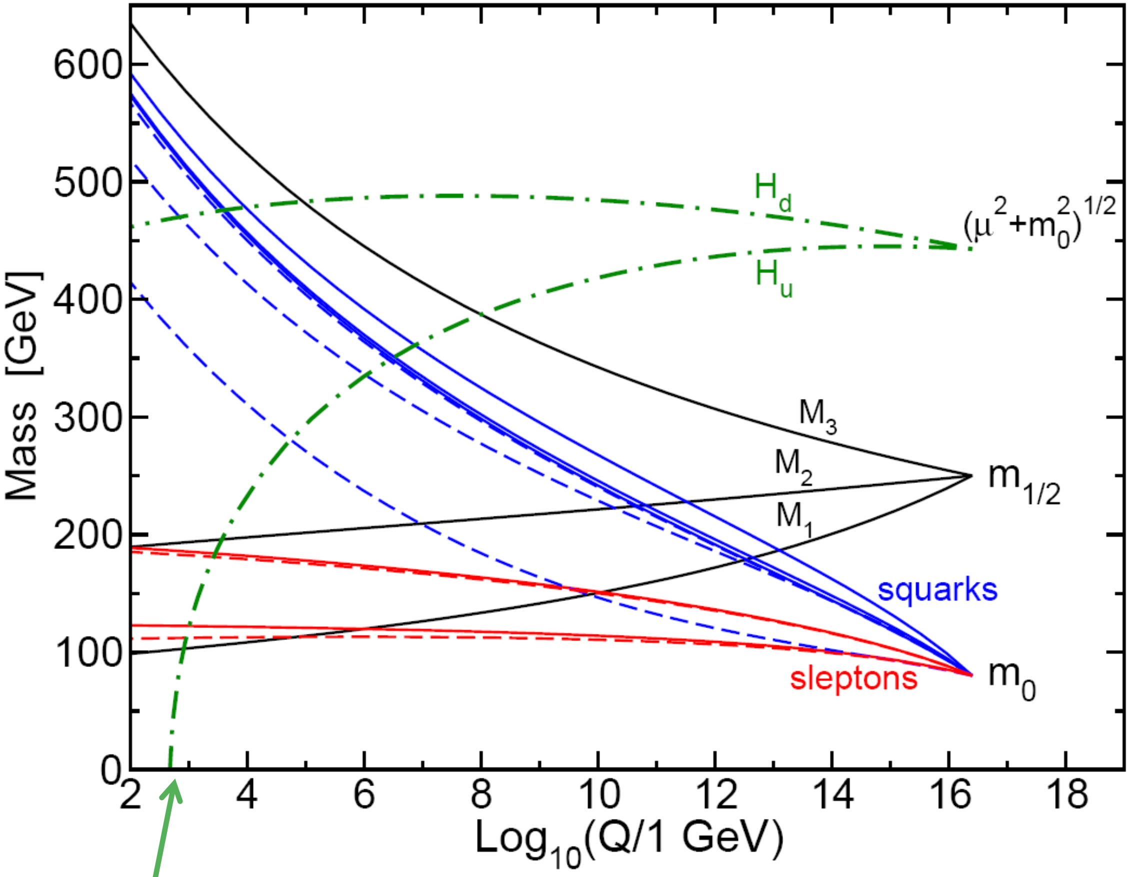
Traded for M_Z and tan β
(via EWSB conditions)

$$\tan\beta = \text{ratio of vevs} = \frac{v_u}{v_d} \qquad \langle H_u \rangle = v_u \quad \langle H_d \rangle = v_d$$

Free parameters: $\{m_0, M_{1/2}, A, \tan\beta, \text{sgn}(\mu)\}$

Renormalisation Group flow

Renormalisation group equations (RGEs)
connect soft masses (at M_X)
to the EW scale.



$m_2^2 < 0 \Rightarrow$ EWSB conditions satisfied!

Finding the CMSSM mass Spectrum

GUT scale boundary conditions

$$\begin{aligned} g_i(M_X) &= g_0 \\ m_i^2(M_X) &= m_0 \quad M_i(M_X) = M_{1/2} \quad A_i(M_X) = A_0 \end{aligned}$$

Solve simultaniously via iteration
(use two loop RGES)

Low energy boundary conditions

SM inputs, e.g. $m_t, m_b, m_\tau, m_Z, m_W, \alpha_i$

+

EWSB Constraints $(\mu, B\mu) \to (\tan\beta, m_Z)$ + 1 loop tadpoles

Add one loop self eneregies to predict pole masses.
Dominant two loop corrections for lightest Higgs mass

Exploring the MSSM mass Spectrum

Public codes:

Softsusy (Ben Allanach): http://softsusy.hepforge.org/

Spheno (Werner Porod): https://spheno.hepforge.org/

SUSPECT (Djouadi, Kneur, Moultaka):
http://www.lpta.univmontp2.fr/users/kneur/Suspect/

ISASUSY (Baer, Paige, Tata, Protopescu)
Part of ISAJET: http://www.nhn.ou.edu/~isajet/

Beyond the MSSM

Non-minimal Supersymmetry

The fundamental motivations for Supersymmetry are:

- The hierarchy problem (fine tuning)
- Gauge Coupling Unification
- Dark matter

None of these require Supersymmetry to be realised in a minimal form.

MSSM is not the only model we can consider.

The μ problem and singlet extensions

- The MSSM superpotential contains a mass scale, μ

$$W_{MSSM} = Y_u \hat{\bar{Q}}_L \hat{H}_u \hat{u}_R - Y_d \hat{\bar{Q}}_L \cdot \hat{H}_d \hat{d}_R - Y_e \hat{\bar{E}} \cdot \hat{H}_d \hat{d}_R - \mu \hat{H}_u \hat{H}_d$$

apriori this is not connected to SUSY breaking or EWSB

The μ problem and singlet extensions

- The MSSM superpotential contains a mass scale, μ

$$W_{MSSM} = Y_u \hat{\bar{Q}}_L \hat{H}_u \hat{u}_R - Y_d \hat{\bar{Q}}_L \cdot \hat{H}_d \hat{d}_R - Y_e \hat{\bar{E}} \cdot \hat{H}_d \hat{d}_R - \mu \hat{H}_u \hat{H}_d$$

apriori this is not connected to SUSY breaking or EWSB

- TeV scale soft SUSY breaking masses are generated
$\Rightarrow$ EWSB naturally driven by radiative corrections.

Requires

$$\mu \approx 0.1 - 1 \text{ TeV}$$

Why should μ be related to this scale?

The μ problem and singlet extensions

- The MSSM superpotential contains a mass scale, μ

$$W_{MSSM} = Y_u \hat{\bar{Q}}_L \hat{H}_u \hat{u}_R - Y_d \hat{\bar{Q}}_L \cdot \hat{H}_d \hat{d}_R - Y_e \hat{\bar{E}} \cdot \hat{H}_d \hat{d}_R - \mu \hat{H}_u \hat{H}_d$$

apriori this is not connected to SUSY breaking or EWSB

- TeV scale soft SUSY breaking masses are generated

$\Rightarrow$ EWSB naturally driven by radiative corrections.

Requires

$$\mu \approx 0.1 - 1 \text{ TeV}$$

Why should μ be related to this scale?

Add $\hat{S}$ - SM-gauge singlet , $\Rightarrow \lambda S H_u H_d$ is allowed

$$\langle H_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_d \\ 0 \end{pmatrix}, \qquad \langle H_u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_u \end{pmatrix}, \qquad \langle S \rangle = \frac{s}{\sqrt{2}}$$

⇒ $\mu_{\text{eff}} H_u H_d$ with $\mu_{\text{eff}} = \lambda \langle S \rangle$

The μ problem and singlet extensions

- The MSSM superpotential contains a mass scale, μ

$$W_{MSSM} = Y_u \widehat{\bar{Q}}_L \widehat{H}_u \widehat{u}_R - Y_d \widehat{\bar{Q}}_L \cdot \widehat{H}_d \widehat{d}_R - Y_e \widehat{\bar{E}} \cdot \widehat{H}_d \widehat{d}_R - \mu \widehat{H}_u \widehat{H}_d$$

apriori this is not connected to SUSY breaking or EWSB

- TeV scale soft SUSY breaking masses are generated

$\Rightarrow$ EWSB naturally driven by radiative correc tions.

Requires

$$\mu \approx 0.1 - 1 \text{ TeV}$$

Why should μ be related to this scale?

Add $\widehat{S}$ - SM-gauge singlet , $\Rightarrow \lambda S H_u H_d$ is allowed

$$\langle H_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_d \\ 0 \end{pmatrix}, \qquad \langle H_u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_u \end{pmatrix}, \qquad \langle S \rangle = \frac{s}{\sqrt{2}}$$

➡ $\mu_{\text{eff}} H_u H_d$ with $\mu_{\text{eff}} = \lambda \langle S \rangle$

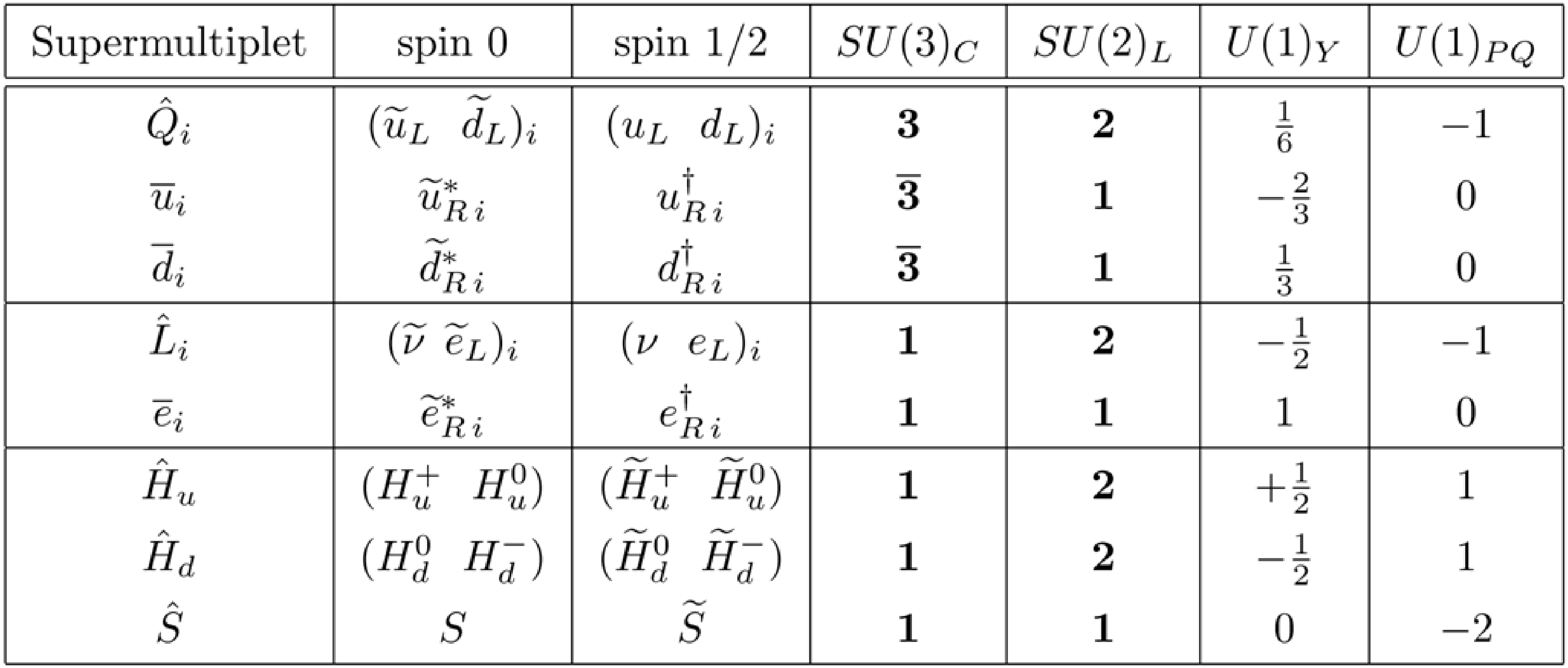
If μ not generated or forbidden, and radiative corrections $\Rightarrow s = \mathcal{O}(TeV)$

Problem solved!

So our superpotential so far is $$\mathcal{W} = \underbrace{Y_u \bar{Q}_L H_u u_R - Y_d \bar{Q}_L \cdot H_d d_R - Y_e \bar{E} \cdot H_d d_R}_{\text{Yukawa terms}} - \underset{\substack{\uparrow \\ \text{effective } \mu\text{–term}}}{\lambda S H_u H_d}$$

So our superpotential so far is $\mathcal{W} = \underbrace{Y_u \bar{Q}_L H_u u_R - Y_d \bar{Q}_L \cdot H_d d_R - Y_e \bar{E} \cdot H_d d_R}_{\text{Yukawa terms}} - \underset{\substack{\uparrow \\ \text{effective } \mu\text{–term}}}{\lambda S H_u H_d}$

But this has a (global) Peccei-Quinn symmetry $\hat{\Psi}_i \rightarrow e^{iQ_i^{PQ}\theta}\hat{\Psi}_i$



Supermultiplet	spin 0	spin 1/2	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_{PQ}$
$\hat{Q}_i$	$(\widetilde{u}_L \ \ \widetilde{d}_L)_i$	$(u_L \ \ d_L)_i$	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{6}$	-1
$\overline{u}_i$	$\widetilde{u}^*_{R\,i}$	$u^\dagger_{R\,i}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-\frac{2}{3}$	0
$\overline{d}_i$	$\widetilde{d}^*_{R\,i}$	$d^\dagger_{R\,i}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{3}$	0
$\hat{L}_i$	$(\widetilde{\nu} \ \widetilde{e}_L)_i$	$(\nu \ \ e_L)_i$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	-1
$\overline{e}_i$	$\widetilde{e}^*_{R\,i}$	$e^\dagger_{R\,i}$	$\mathbf{1}$	$\mathbf{1}$	1	0
$\hat{H}_u$	$(H_u^+ \ \ H_u^0)$	$(\widetilde{H}_u^+ \ \ \widetilde{H}_u^0)$	$\mathbf{1}$	$\mathbf{2}$	$+\frac{1}{2}$	1
$\hat{H}_d$	$(H_d^0 \ \ H_d^-)$	$(\widetilde{H}_d^0 \ \ \widetilde{H}_d^-)$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	1
$\hat{S}$	S	$\widetilde{S}$	$\mathbf{1}$	$\mathbf{1}$	0	-2

So our superpotential so far is $\mathcal{W} = \underbrace{Y_u \bar{Q}_L H_u u_R - Y_d \bar{Q}_L \cdot H_d d_R - Y_e \bar{E} \cdot H_d d_R}_{\text{Yukawa terms}} - \overset{}{\lambda S H_u H_d}$

$\uparrow$ effective μ–term

But this has a (global) Peccei-Quinn symmetry $\hat{\Psi}_i \rightarrow e^{iQ_i^{PQ}\theta}\hat{\Psi}_i$

Supermultiplet	spin 0	spin 1/2	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_{PQ}$
$\hat{Q}_i$	$(\widetilde{u}_L \;\; \widetilde{d}_L)_i$	$(u_L \;\; d_L)_i$	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{6}$	-1
$\overline{u}_i$	$\widetilde{u}^*_{R\,i}$	$u^\dagger_{R\,i}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-\frac{2}{3}$	0
$\overline{d}_i$	$\widetilde{d}^*_{R\,i}$	$d^\dagger_{R\,i}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{3}$	0
$\hat{L}_i$	$(\widetilde{\nu} \;\; \widetilde{e}_L)_i$	$(\nu \;\; e_L)_i$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	-1
$\overline{e}_i$	$\widetilde{e}^*_{R\,i}$	$e^\dagger_{R\,i}$	$\mathbf{1}$	$\mathbf{1}$	1	0
$\hat{H}_u$	$(H_u^+ \;\; H_u^0)$	$(\widetilde{H}_u^+ \;\; \widetilde{H}_u^0)$	$\mathbf{1}$	$\mathbf{2}$	$+\frac{1}{2}$	1
$\hat{H}_d$	$(H_d^0 \;\; H_d^-)$	$(\widetilde{H}_d^0 \;\; \widetilde{H}_d^-)$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	1
$\hat{S}$	S	$\widetilde{S}$	$\mathbf{1}$	$\mathbf{1}$	0	-2

$S \rightarrow \langle S \rangle \Rightarrow \mu_{eff} H_u H_d$ and breaks $U(1)_{PQ}$.

$\longrightarrow$ **massless axion!**

$$\mathcal{W} = Y_u \bar{Q}_L H_u u_R - Y_d \bar{Q}_L \cdot H_d d_R - Y_e \bar{E} \cdot H_d d_R - \lambda S H_u H_d \longrightarrow$$

massless axion!

NMSSM Chiral Superfield Content

[Dine, Fischler and Srednicki] [Ellis, Gunion, Haber, Roszkowski, Zwirner]

Supermultiplet	spin 0	spin 1/2	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_{PQ}$
$\hat{Q}_i$	$(\widetilde{u}_L \ \ \widetilde{d}_L)_i$	$(u_L \ \ d_L)_i$	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{6}$	-1
$\overline{u}_i$	$\widetilde{u}^*_{R\,i}$	$u^\dagger_{R\,i}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-\frac{2}{3}$	0
$\overline{d}_i$	$\widetilde{d}^*_{R\,i}$	$d^\dagger_{R\,i}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{3}$	0
$\hat{L}_i$	$(\widetilde{\nu} \ \widetilde{e}_L)_i$	$(\nu \ \ e_L)_i$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	-1
$\overline{e}_i$	$\widetilde{e}^*_{R\,i}$	$e^\dagger_{R\,i}$	$\mathbf{1}$	$\mathbf{1}$	1	0
$\hat{H}_u$	$(H_u^+ \ \ H_u^0)$	$(\widetilde{H}_u^+ \ \ \widetilde{H}_u^0)$	$\mathbf{1}$	$\mathbf{2}$	$+\frac{1}{2}$	1
$\hat{H}_d$	$(H_d^0 \ \ H_d^-)$	$(\widetilde{H}_d^0 \ \ \widetilde{H}_d^-)$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	1
$\hat{S}$	S	$\widetilde{S}$	$\mathbf{1}$	$\mathbf{1}$	0	-2

$$\mathcal{W}_{NMSSM} = Y_u \bar{Q}_L H_u u_R - Y_d \bar{Q}_L \cdot H_d d_R - Y_e \bar{E} \cdot H_d d_R - \underset{\uparrow}{\lambda} S H_u H_d + \frac{1}{3} \underset{\nwarrow}{\kappa} S^3$$

μ_{eff}

PQ breaking

The superpotential of the Next-to-Minimal Supersymmetric Standard Model (NMSSM) is

[Dine, Fischler and Srednicki]
[Ellis, Gunion, Haber, Roszkowski, Zwirner]

$$\mathcal{W}_{NMSSM} = \overbrace{Y_u \hat{\bar{Q}}_L \hat{H}_u \hat{u}_R - Y_d \hat{\bar{Q}}_L \cdot \hat{H}_d \hat{d}_R - Y_e \hat{\bar{E}} \cdot \hat{H}_d \hat{d}_R}^{\text{Yukawa terms as in MSSM}} + \overset{\substack{\text{effective } \mu\text{−term} \\ \downarrow}}{\lambda \hat{S} \hat{H}_u \hat{H}_d} + \overset{\substack{\text{PQ breaking term} \\ \swarrow}}{\frac{1}{3}\kappa \hat{S}^3}$$

$$+\mu \hat{H}_u \hat{H}_d + \xi_F \hat{S} + \mu' \hat{S}^2 \quad (Z_3 \text{ violating})$$

The superpotential of the Next-to-Minimal Supersymmetric Standard Model (NMSSM) is

[Dine, Fischler and Srednicki]
[Ellis, Gunion, Haber, Roszkowski, Zwirner]

$$\mathcal{W}_{NMSSM} = \overbrace{Y_u \hat{\bar{Q}}_L \hat{H}_u \hat{u}_R - Y_d \hat{\bar{Q}}_L \cdot \hat{H}_d \hat{d}_R - Y_e \hat{\bar{E}} \cdot \hat{H}_d \hat{d}_R}^{\substack{\text{Yukawa terms} \\ \text{as in MSSM}}} + \overset{\substack{\text{effective } \mu\text{–term} \\ \downarrow}}{\lambda \hat{S} \hat{H}_u \hat{H}_d} + \overset{\substack{\text{PQ breaking term} \\ \swarrow}}{\frac{1}{3} \kappa \hat{S}^3}$$

$$+\mu \hat{H}_u \hat{H}_d + \xi_F \hat{S} + \mu' \hat{S}^2 \quad (Z_3 \text{ violating})$$

We also need new soft supersymmetry breaking terms in the Lagrangian:

$$-\mathcal{L}_{\text{soft}} \supset m_S^2 |S|^2 + [\lambda A_\lambda S H_u H_d + \frac{1}{3} \kappa A_\kappa S^3 + \text{h.c.}]$$

$$[+m_3^2 H_u H_d + \frac{1}{2} m_S'^2 S^2 + \xi_S S + \text{h.c.}] \ (Z_3 \text{ violating})$$

The superpotential of the Next-to-Minimal Supersymmetric Standard Model (NMSSM) is

[Dine, Fischler and Srednicki]
[Ellis, Gunion, Haber, Roszkowski, Zwirner]

$$\mathcal{W}_{NMSSM} = \overbrace{Y_u \hat{\bar{Q}}_L \hat{H}_u \hat{u}_R - Y_d \hat{\bar{Q}}_L \cdot \hat{H}_d \hat{d}_R - Y_e \hat{\bar{E}} \cdot \hat{H}_d \hat{d}_R}^{\text{Yukawa terms as in MSSM}} + \overset{\substack{\text{effective } \mu\text{–term} \\ \downarrow}}{\lambda \hat{S} \hat{H}_u \hat{H}_d} + \overset{\substack{\text{PQ breaking term} \\ \swarrow}}{\frac{1}{3} \kappa \hat{S}^3}$$
$$+\mu \hat{H}_u \hat{H}_d + \xi_F \hat{S} + \mu' \hat{S}^2 \quad (Z_3 \text{ violating})$$

We also need new soft supersymmetry breaking terms in the Lagrangian:

$$-\mathcal{L}_{\text{soft}} \supset m_S^2 |S|^2 + [\lambda A_\lambda S H_u H_d + \frac{1}{3} \kappa A_\kappa S^3 + \text{h.c.}]$$
$$[+m_3^2 H_u H_d + \frac{1}{2} m_S'^2 S^2 + \xi_S S + \text{h.c.}] \; (Z_3 \text{ violating})$$

Extended Higgs sector: 3 CP-even Higgs, 2 CP-odd Higgs (new real and imagnary scalar S)

Extended Neutralino sector: 5 neutralinos (singlino, the new fermion component of S)

Exploring the NMSSM spectrum

Public codes:

NMSPEC (Ellwanger, Hugonie)
http://www.th.u-psud.fr/NMHDECAY/nmssmtools.html (part of nmssmtools)

Spheno (Werner Porod): https://spheno.hepforge.org/

NEW

Softsusy (Ben Allanach, PA, Lewis Tunstall, Alexander Voigt, A.W. Williams):
(hopefully) to be released this month.

Supersymmetric Models

- Minimal Supersymmetric Standard Model (MSSM)

$$\mathcal{W}_{MSSM} = Y_u\bar{Q}_L H_u u_R - Y_d\,\bar{Q}_L\cdot H_d\,d_R - Y_e\,\bar{E}\cdot H_d\,d_R - \mu H_u H_d$$

- Next to Minimal Supersymmetric Standard Model (NMSSM)

[Dine, Fischler and Srednicki] [Ellis, Gunion, Haber, Roszkowski, Zwirner]

$$\mathcal{W}_{NMSSM} = Y_u\bar{Q}_L H_u u_R - Y_d\,\bar{Q}_L\cdot H_d\,d_R - Y_e\,\bar{E}\cdot H_d\,d_R - \lambda S H_u H_d + \frac{1}{3}\kappa S^3$$

$$\hookrightarrow \mu_{eff} = \lambda\langle S\rangle$$

NMSSM is minimal matter extension.

We can also extend the gauge group of the SM.

Supersymmetric Models

- Minimal Supersymmetric Standard Model (MSSM)

$$\mathcal{W}_{MSSM} = Y_u \bar{Q}_L H_u u_R - Y_d \bar{Q}_L \cdot H_d d_R - Y_e \bar{E} \cdot H_d d_R - \mu H_u H_d$$

- Next to Minimal Supersymmetric Standard Model (NMSSM)

[Dine, Fischler and Srednicki] [Ellis, Gunion, Haber, Roszkowski, Zwirner]

$$\mathcal{W}_{NMSSM} = Y_u \bar{Q}_L H_u u_R - Y_d \bar{Q}_L \cdot H_d d_R - Y_e \bar{E} \cdot H_d d_R - \lambda S H_u H_d + \frac{1}{3} \kappa S^3$$

$$\hookrightarrow \mu_{eff} = \lambda \langle S \rangle$$

NMSSM is minimal matter extension.

We can also extend the gauge group of the SM.

What if the $U(1)_{PQ}$ was a local rather global symmetry?

Supersymmetric Models

- Minimal Supersymmetric Standard Model (MSSM)

$$\mathcal{W}_{MSSM} = Y_u \bar{Q}_L H_u u_R - Y_d \bar{Q}_L \cdot H_d d_R - Y_e \bar{E} \cdot H_d d_R - \mu H_u H_d$$

- Next to Minimal Supersymmetric Standard Model (NMSSM)

[Dine, Fischler and Srednicki] [Ellis, Gunion, Haber, Roszkowski, Zwirner]

$$\mathcal{W}_{NMSSM} = Y_u \bar{Q}_L H_u u_R - Y_d \bar{Q}_L \cdot H_d d_R - Y_e \bar{E} \cdot H_d d_R - \lambda S H_u H_d + \frac{1}{3} \kappa S^3$$

$$\hookrightarrow \mu_{eff} = \lambda \langle S \rangle$$

NMSSM is minimal matter extension.

We can also extend the gauge group of the SM.

What if the $U(1)_{PQ}$ was a local rather global symmetry?

When a local $U(1)'$ is broken
Z′ eats the massless axion to become massive vector boson!

USSM Chiral Superfield Content

Supermultiplet	spin 0	spin 1/2	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_{PQ}$
$\hat{Q}_i$	$(\widetilde{u}_L \ \ \widetilde{d}_L)_i$	$(u_L \ \ d_L)_i$	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{6}$	-1
$\overline{u}_i$	$\widetilde{u}^*_{R\,i}$	$u^\dagger_{R\,i}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-\frac{2}{3}$	0
$\overline{d}_i$	$\widetilde{d}^*_{R\,i}$	$d^\dagger_{R\,i}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{3}$	0
$\hat{L}_i$	$(\widetilde{\nu} \ \widetilde{e}_L)_i$	$(\nu \ \ e_L)_i$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	-1
$\overline{e}_i$	$\widetilde{e}^*_{R\,i}$	$e^\dagger_{R\,i}$	$\mathbf{1}$	$\mathbf{1}$	1	0
$\hat{H}_u$	$(H_u^+ \ \ H_u^0)$	$(\widetilde{H}_u^+ \ \ \widetilde{H}_u^0)$	$\mathbf{1}$	$\mathbf{2}$	$+\frac{1}{2}$	1
$\hat{H}_d$	$(H_d^0 \ \ H_d^-)$	$(\widetilde{H}_d^0 \ \ \widetilde{H}_d^-)$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	1
$\hat{S}$	S	$\widetilde{S}$	$\mathbf{1}$	$\mathbf{1}$	0	-2

$$\mathcal{W}_{USSM} = \underbrace{Y_u \bar{Q}_L H_u u_R - Y_d \bar{Q}_L \cdot H_d d_R - Y_e \bar{E} \cdot H_d d_R}_{\text{Yukawa terms}} - \lambda S \underset{\underset{\text{effective } \mu\text{–term}}{\uparrow}}{H_u} H_d$$

Problem: to avoid gauge anomalies $\sum_i Q_i^{U(1)} = 0$

USSM Chiral Superfield Content

Supermultiplet	spin 0	spin 1/2	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)'$
$\hat{Q}_i$	$(\widetilde{u}_L \ \ \widetilde{d}_L)_i$	$(u_L \ \ d_L)_i$	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{6}$	Q'_Q
$\overline{u}_i$	$\widetilde{u}^*_{R\,i}$	$u^\dagger_{R\,i}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-\frac{2}{3}$	Q'_u
$\overline{d}_i$	$\widetilde{d}^*_{R\,i}$	$d^\dagger_{R\,i}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{3}$	Q'_d
$\hat{L}_i$	$(\widetilde{\nu} \ \widetilde{e}_L)_i$	$(\nu \ \ e_L)_i$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	Q'_L
$\overline{e}_i$	$\widetilde{e}^*_{R\,i}$	$e^\dagger_{R\,i}$	$\mathbf{1}$	$\mathbf{1}$	1	Q'_e
$\hat{H}_u$	$(H_u^+ \ \ H_u^0)$	$(\widetilde{H}_u^+ \ \ \widetilde{H}_u^0)$	$\mathbf{1}$	$\mathbf{2}$	$+\frac{1}{2}$	Q'_{H_u}
$\hat{H}_d$	$(H_d^0 \ \ H_d^-)$	$(\widetilde{H}_d^0 \ \ \widetilde{H}_d^-)$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	Q'_{H_d}
$\hat{S}$	S	$\widetilde{S}$	$\mathbf{1}$	$\mathbf{1}$	0	Q'_S

$$\mathcal{W}_{USSM} = \underbrace{Y_u \bar{Q}_L H_u u_R - Y_d \bar{Q}_L \cdot H_d \, d_R - Y_e \bar{E} \cdot H_d \, d_R}_{\text{Yukawa terms}} - \lambda S \underset{\substack{\uparrow \\ \text{effective } \mu\text{–term}}}{H_u H_d}$$

Problem: to avoid gauge anomalies $\sum_i Q_i^{U(1)} = 0$

Charges not specified in the definition of the USSM

U(1) extended Supersymmetric Standard Model (USSM)

$$\mathcal{W}_{USSM} = \underbrace{Y_u \bar{Q}_L H_u u_R - Y_d \bar{Q}_L \cdot H_d \, d_R - Y_e \bar{E} \cdot H_d \, d_R}_{\text{Yukawa terms}} - \underset{\substack{\uparrow \\ \text{effective } \mu\text{–term}}}{\lambda S H_u H_d}$$

Modified Gauge sector, new Z'

Modified Higgs sector: 3 CP-even Higgs,
2 CP-odd Higgs (new real and imaginary scalar S)

Modified Neutralino sector: 6 neutralinos:
(new singlino + Zprimino)

Supersymmetric Models

- Minimal Supersymmetric Standard Model (MSSM)

$$\mathcal{W}_{MSSM} = Y_u \bar{Q}_L H_u u_R - Y_d \bar{Q}_L \cdot H_d d_R - Y_e \bar{E} \cdot H_d d_R - \mu H_u H_d$$

[Dine, Fischler and Srednicki]
[Ellis, Gunion, Haber, Roszkowski, Zwirner]

- Next to Minimal Supersymmetric Standard Model (NMSSM)

$$\mathcal{W}_{NMSSM} = Y_u \bar{Q}_L H_u u_R - Y_d \bar{Q}_L \cdot H_d d_R - Y_e \bar{E} \cdot H_d d_R - \lambda S H_u H_d + \frac{1}{3} \kappa S^3$$

$$\hookrightarrow \mu_{eff} = \lambda \langle S \rangle$$

- U(1) extended Supersymmetric Standard Model (USSM)

E_6 inspired models

For anomaly cancelation, one can use complete E_6 matter multiplets

New $U(1)'$ from E_6

E_6 inspired models

For anomaly cancelation, one can use complete E_6 matter multiplets

New $U(1)'$ from E_6

$$\begin{tikzcd}
E_6 \arrow[r] & SO(10)\times U(1)_\psi \arrow[dr] & & \\
& & SU(5)\times U(1)_\chi \arrow[dr] & \\
& & & SU(3)_C\times SU(2)_W\times U(1)_Y
\end{tikzcd}$$

$$SU(3)\times SU(2)\times U(1)_Y\times U(1)'$$

$$U(1)' = \cos\theta\, U(1)_\chi + \sin\theta\, U(1)_\psi$$

E_6 inspired models

For anomaly cancelation, one can use complete E_6 matter multiplets

New $U(1)'$ from E_6

$$E_6 \to SO(10) \times U(1)_\psi$$

$$\hookrightarrow SU(5) \times U(1)_\chi$$

$$\hookrightarrow SU(3)_C \times SU(2)_W \times U(1)_Y$$

$$SU(3) \times SU(2) \times U(1)_Y \times U(1)'$$

$$U(1)' = \cos\theta\, U(1)_\chi + \sin\theta\, U(1)_\psi$$

- Matter from 3 complete generations of E_6

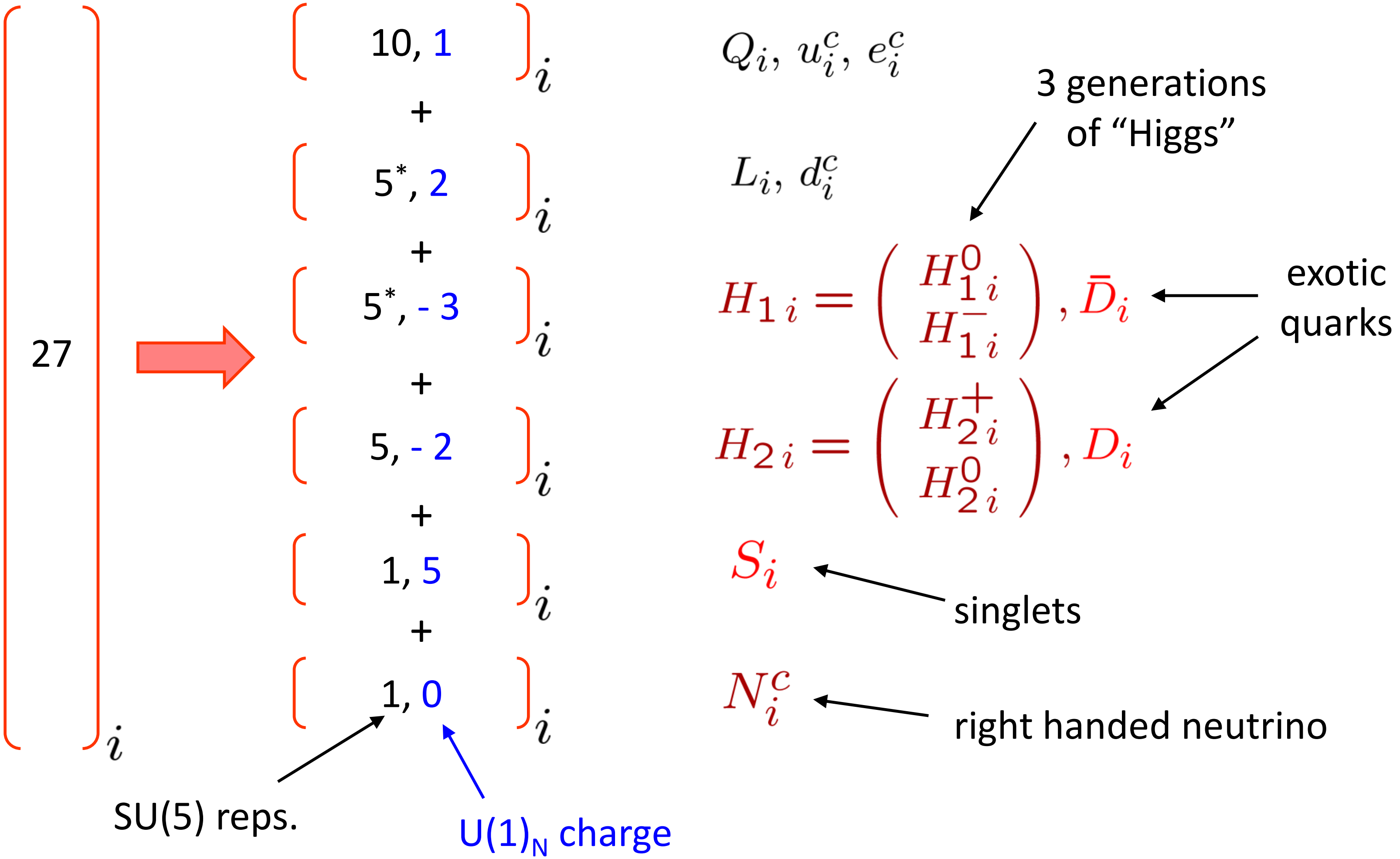
  $\Rightarrow$ automatic cancellation of gauge anomalies!

- In the E_6SSM $\tan\theta = \sqrt{15}$ $\Rightarrow$ right-handed neutrino is a gauge singlet

Exceptional Supersymmetric Standard Model (E$_6$SSM)

[Phys.Rev. D73 (2006) 035009 , Phys.Lett. B634 (2006) 278-284 S.F.King, S.Moretti & R. Nevzorov]

All the SM matter fields are contained in one 27-plet of E$_6$ per generation.



$$\left[\,27\,\right]_i \Rightarrow \left[\,10,\ 1\,\right]_i + \left[\,5^*,\ 2\,\right]_i + \left[\,5^*,\ -3\,\right]_i + \left[\,5,\ -2\,\right]_i + \left[\,1,\ 5\,\right]_i + \left[\,1,\ 0\,\right]_i$$

$Q_i,\ u^c_i,\ e^c_i$

$L_i,\ d^c_i$

$H_{1\,i} = \begin{pmatrix} H^0_{1\,i} \\ H^-_{1\,i} \end{pmatrix},\ \bar{D}_i$

$H_{2\,i} = \begin{pmatrix} H^+_{2\,i} \\ H^0_{2\,i} \end{pmatrix},\ D_i$

S_i

N^c_i

3 generations of “Higgs”

exotic quarks

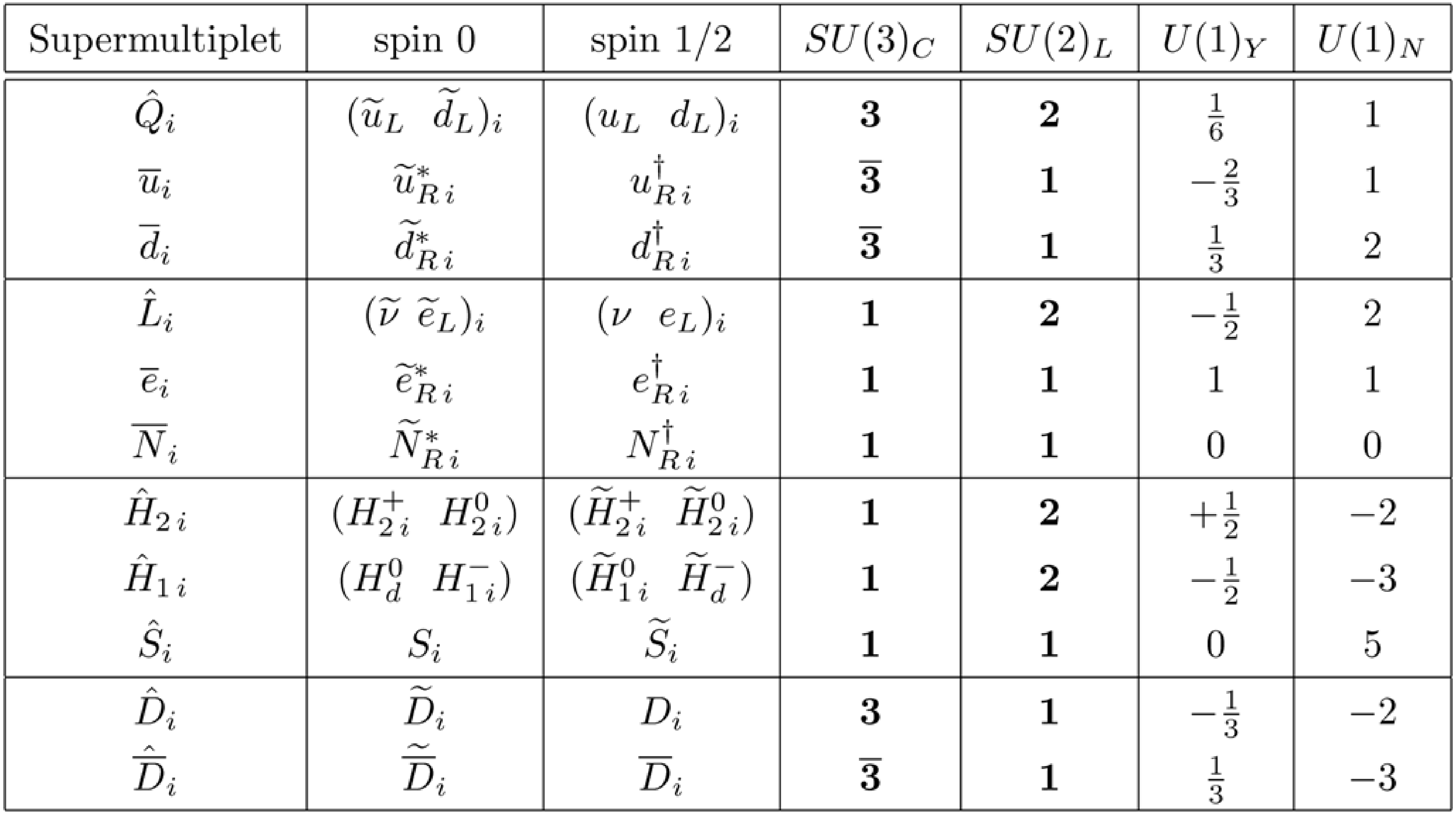
singlets

right handed neutrino

SU(5) reps.

U(1)$_N$ charge

E_6SSM Chiral Superfield Content



Supermultiplet	spin 0	spin 1/2	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_N$
$\hat{Q}_i$	$(\tilde{u}_L \ \ \tilde{d}_L)_i$	$(u_L \ \ d_L)_i$	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{6}$	1
$\overline{u}_i$	$\tilde{u}^*_{R\,i}$	$u^\dagger_{R\,i}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-\frac{2}{3}$	1
$\overline{d}_i$	$\tilde{d}^*_{R\,i}$	$d^\dagger_{R\,i}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{3}$	2
$\hat{L}_i$	$(\tilde{\nu} \ \tilde{e}_L)_i$	$(\nu \ \ e_L)_i$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	2
$\overline{e}_i$	$\tilde{e}^*_{R\,i}$	$e^\dagger_{R\,i}$	$\mathbf{1}$	$\mathbf{1}$	1	1
$\overline{N}_i$	$\tilde{N}^*_{R\,i}$	$N^\dagger_{R\,i}$	$\mathbf{1}$	$\mathbf{1}$	0	0
$\hat{H}_{2\,i}$	$(H^+_{2\,i} \ \ H^0_{2\,i})$	$(\tilde{H}^+_{2\,i} \ \ \tilde{H}^0_{2\,i})$	$\mathbf{1}$	$\mathbf{2}$	$+\frac{1}{2}$	-2
$\hat{H}_{1\,i}$	$(H^0_d \ \ H^-_{1\,i})$	$(\tilde{H}^0_{1\,i} \ \ \tilde{H}^-_d)$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	-3
$\hat{S}_i$	S_i	$\tilde{S}_i$	$\mathbf{1}$	$\mathbf{1}$	0	5
$\hat{D}_i$	$\tilde{D}_i$	D_i	$\mathbf{3}$	$\mathbf{1}$	$-\frac{1}{3}$	-2
$\hat{\overline{D}}_i$	$\tilde{\overline{D}}_i$	$\overline{D}_i$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{3}$	-3

Note: In it's usual form there are also two extra SU(2) doublets included for single step gauge coupling unification, but these are negleected here for simplicity.

Supersymmetric Models

- Minimal Supersymmetric Standard Model (MSSM)

$$\mathcal{W}_{MSSM} = Y_u \bar{Q}_L H_u u_R - Y_d \bar{Q}_L \cdot H_d d_R - Y_e \bar{E} \cdot H_d d_R - \mu H_u H_d$$

- Next to Minimal Supersymmetric Standard Model (NMSSM)

[Dine, Fischler and Srednicki]
[Ellis, Gunion, Haber, Roszkowski, Zwirner]

$$\mathcal{W}_{NMSSM} = Y_u \bar{Q}_L H_u u_R - Y_d \bar{Q}_L \cdot H_d d_R - Y_e \bar{E} \cdot H_d d_R - \lambda S H_u H_d + \frac{1}{3} \kappa S^3$$

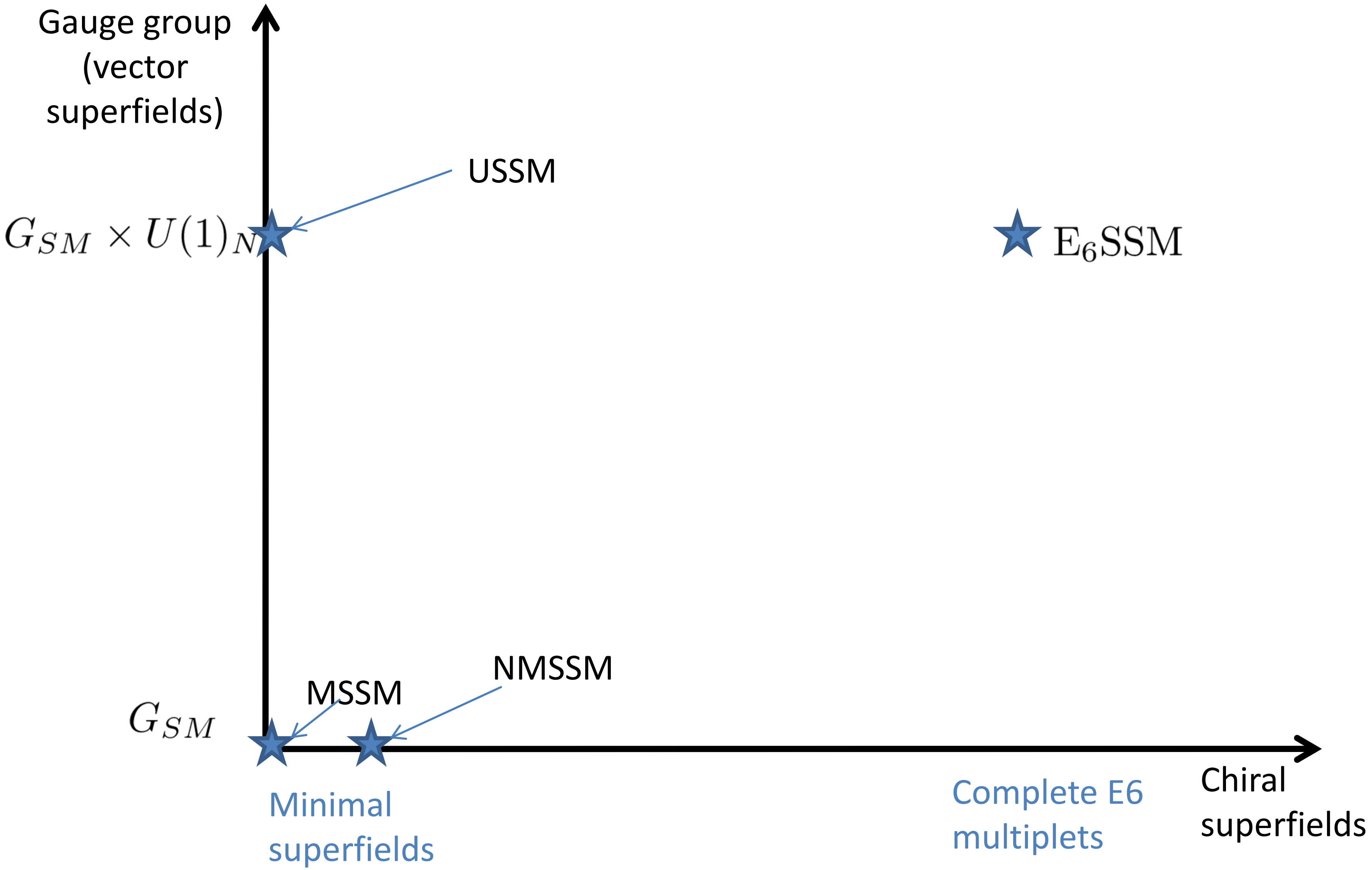
$$\to \mu_{eff} = \lambda \langle S \rangle$$

- U(1) extended Supersymmetric Standard Model (USSM)

- Exceptional Supersymmetric Standard Model (E_6SSM)

[S.F. King, S. Moretti, R. Nevzrov, Phys.Rev. D73 (2006) 035009]

SUSY Model space



Exploring exotic SUSY models

Private codes:

Most models have no public spectrum generators

In many cases there are private spectrum generators, e.g. cE6SSM generator (PA)
If a model interests you, can contact the authors and ask if there is a code.

Such codes normally come with health warnings and no documentation

Public general model spectrum generators :

SARAH (Florian Staub): http://sarah.hepforge.org/
Mathematica package for obtaining feynman rules, self energies, RGEs,
Latest version outputs a SPHENO like fortran spectrum generator.

NEW

Flexible Supersymmetry (PA, Jae-hyon Park, Alexander Voigt, Dominik Stockinger):
Mathematica metacode using SARAH package
Create modern, fast and flexible spectrum generator in C++ (somewhat softsusy like)
(hopefully) to be released this ~~summer~~ (southern hemisphere) winter.

Thank you for listening

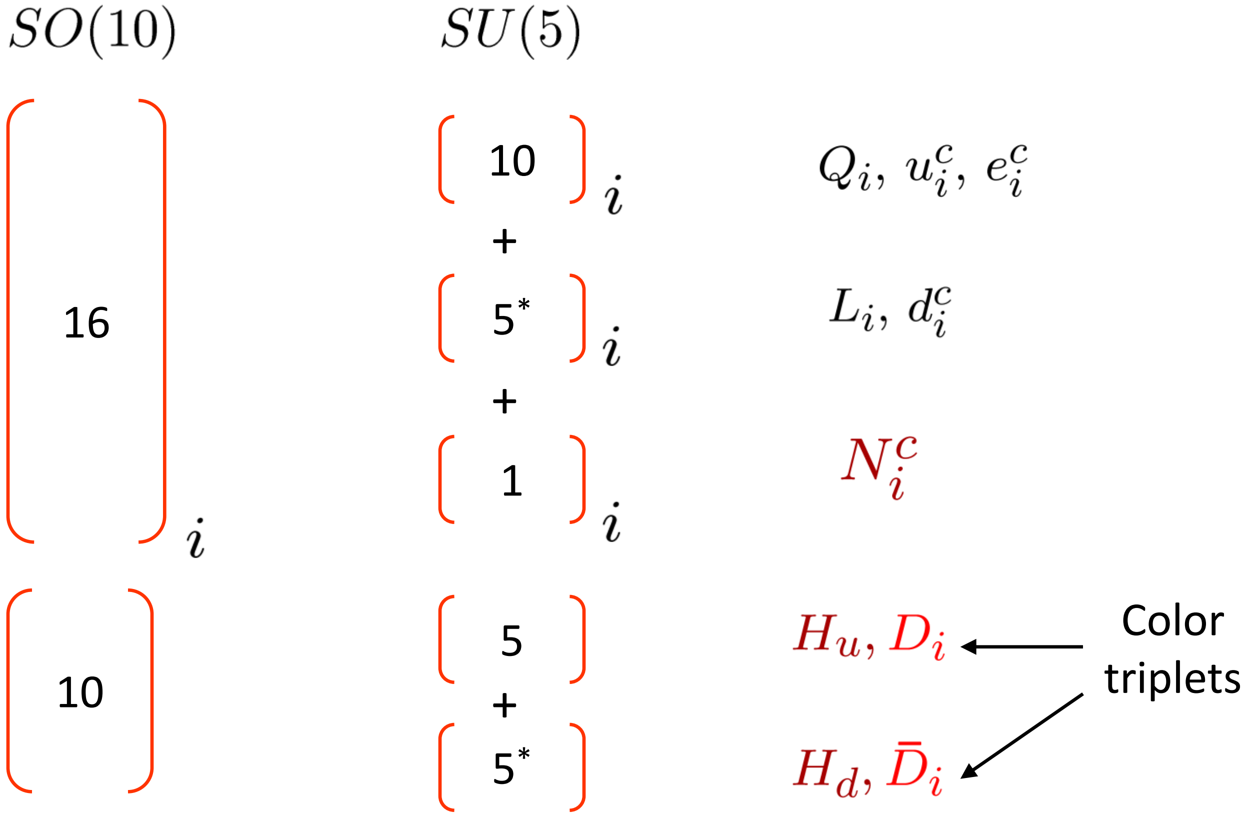
Back up slides

Beyond the CMSSM
(Relaxing high scale constraints)

Non-universal Higgs MSSM (NUHM)

$$\boxed{\begin{aligned} g_i(M_X) &= g_0 \;\; m_i^2(M_X) = m_0 \;\; m_{H_u}^2(M_X) = m_{H_d}(M_X) \neq m_0 \\ M_i(M_X) &= M_{1/2} \;\; A_i(M_X) = A_0 \;\; B\mu(M_X) = B\mu \end{aligned}}$$

Motivated since Higgs bosons do not fit into the same SU(5) or SO(10) GUT multiplets:



$SO(10)$	$SU(5)$		
$(16)_i$	$(10)_i$	$Q_i,\, u_i^c,\, e_i^c$	
	+ $(5^*)_i$	$L_i,\, d_i^c$	
	+ $(1)_i$	N_i^c	
(10)	(5)	$H_u,\, D_i$	Color triplets
	+ (5^*)	$H_d,\, \bar{D}_i$	

Beyond the CMSSM

(Relaxing high scale constraints)

Non-universal Higgs MSSM (NUHM)

$$\begin{aligned} g_i(M_X) &= g_0 \;\; m_i^2(M_X) = m_0 \;\; m_{H_u}^2(M_X) = m_{H_d}(M_X) \neq m_0 \\ M_i(M_X) &= M_{1/2} \;\; A_i(M_X) = A_0 \;\; B\mu(M_X) = B\mu \end{aligned}$$

Motivated since Higgs bosons do not fit into the same SU(5) or SO(10) GUT multiplets:

Very mild modification to the CMSSM

Impact: Higgs masses not linked to other scalar masses so strongly

⟶ easier to fit EWSB constraints and other observables

Beyond the CMSSM

(Relaxing high scale constraints)

For universal gauginos we have a (one loop) relation:

$$\frac{d}{dt}g_i = \frac{b_i}{(4\pi)^2}g_i^3 \qquad \frac{d}{dt}M_i = 2\frac{b_i}{(4\pi)^2}g_i^2 M_i$$

$$\Rightarrow \frac{d}{dt}g_i^{-2} = -2\frac{b_i}{(4\pi)^2} \qquad \frac{d}{dt}\left(\frac{M_i}{g_i^2}\right) = 0 \qquad \Rightarrow M_i(t) = \frac{g_i^2}{g_0^2}M_{1/2}$$

$$\Rightarrow \text{the ratio } M_1 : M_2 : M_3 \text{ is fixed to } 0.15 : 0.25 : 0.7$$

Testable predictions for gaugino universality!

Non-universal Gaugino masses

$$\boxed{\begin{aligned} g_i(M_X) &= g_0 \;\; m_i^2(M_X) = m_0 \;\; M_1(M_X) \neq M_2(M_X) \neq M_3(M_X) \\ A_i(M_X) &= A_0 \;\; B\mu(M_X) = B\mu \end{aligned}}$$

Breaks ratio ⟶ get different gaugino mass patterns: $M_i(t) = \frac{g_i^2}{g_0^2}M_i(M_X)$

One can also ignore the universality more parameters to consider the model with less prejudice, e.g. pMSSM

$$\boxed{\begin{aligned} &m_{Q_3}, m_{Q_1}, m_{L_3}, m_{L_1}, m_{u_3}, m_{u_1}, m_{d_3}, m_{d_1}, m_{e_3}, m_{e_1} \\ &\qquad M_1, M_2, M_3, A_t, A_b, A_\tau, \mu, M_A, \tan\beta \end{aligned}}$$

Gauge Mediation

Two things ignored in the discussion of the cMSSM!

1) Flavour diagonal assumption for soft masses, not well motivated

In general supergravity mediation
⇒ large flavour mixing masses
⇒ leading to flavour changing processes

cMSSM just assumes flavour diagonal without motivation

2) The breakdown of supersymmetry leads to a massless goldstino.

(extension of goldstone's theorem)

In local supersymmetry (i.e. Supergravity) gravitino 'eats' goldstino

Super-Higgs mechanism

⟶ Massive gravitino

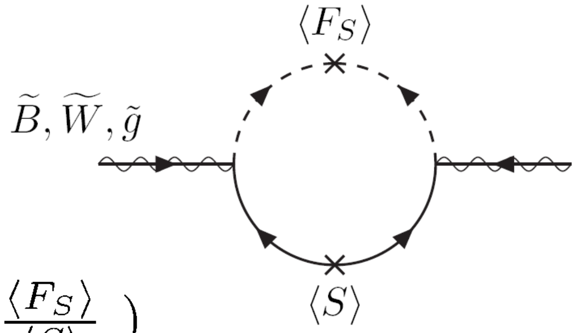
Gauge Mediation

In gauge mediated symmetry breaking the SUSY breaking is transmitted from the hidden sector via SM gauge interactions of heavy messenger fields.

Chiral Messenger fields couple to Hidden sector

$$\mathcal{W}_{mess} = \sum_i \lambda_i S \Phi_i \overline{\Phi}_i$$

→ SUSY breaking in messenger spectrum



SM Gauge interactions couple them to visible sector

Loops from gauge interactions with virtual messengers

→ flavour diagonal soft masses.

Loop diagram: →
$$M_a = \frac{\alpha_a}{4\Pi} \Lambda \quad (\Lambda = \frac{\langle F_S \rangle}{\langle S \rangle} \;)$$

Two loop diagrams:
$$m^2_{\phi_i}(M_m) = 2\Lambda^2 N_5 \sum_{a=1}^{3} C_a(i) \left(\frac{g_a^2}{16\pi^2} \right)^2$$

$$A_i \approx 0$$

Light gravitino
$$m_{3/2} = \frac{1}{\sqrt{3} M_{Pl}} \left(\sum_i |\langle F_i \rangle|^2 \right)^{\frac{1}{2}}$$

→ Soft mass relations imposed at messenger scale

→ Non-universal soft gaugino masses since they depend on gauge interactions!

Minimal Gauge Mediated SUSY Breaking (mGMSB)

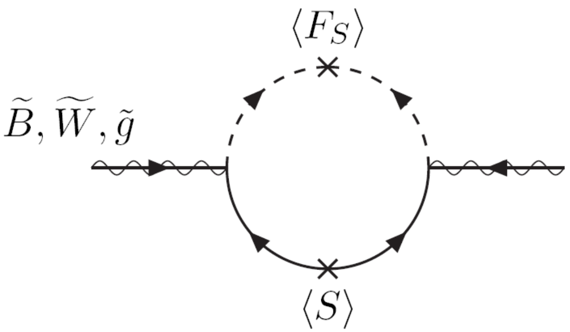
- Messenger fields form Complete SU(5) representations
- Assumptions ($\frac{\langle F_S \rangle}{\lambda_i \langle S \rangle^2}$ small) $\Rightarrow$ messenger couplings λ_i don't affect spectrum.

$$\Lambda = \frac{\langle F_S \rangle}{\langle S \rangle}$$

Number of SU(5) multiplets

$$\mathcal{W}_{mess} = \sum_i \lambda_i S \Phi_i \overline{\Phi}_i$$

$$\Lambda, \quad M_m, \quad N_5, \quad \underbrace{\tan\beta, \quad \mathrm{sgn}(\mu)}$$



Messenger scale

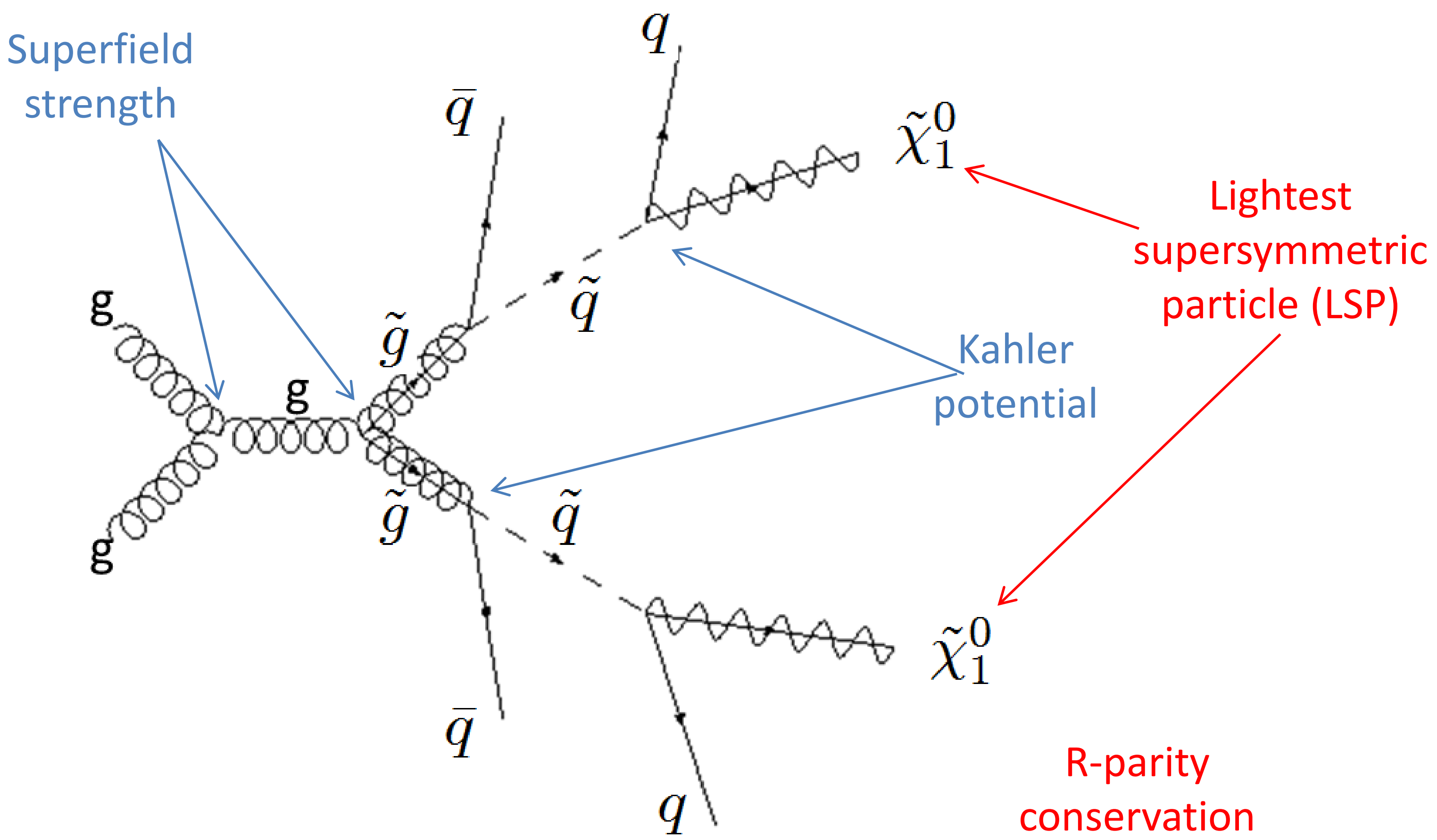
From EWSB as in CMSSM

$$M_a(M_m) = \frac{g_a^2}{(4\pi)^2} \Lambda N_5$$

$$m_{\phi_i}^2(M_m) = 2\Lambda^2 N_5 \sum_{a=1}^{3} C_a(i) \left(\frac{g_a^2}{16\pi^2} \right)^2 \qquad A_i \approx 0$$

A SUSY signature at the LHC

Gluino pair production → Cascade Decay



Superfield strength

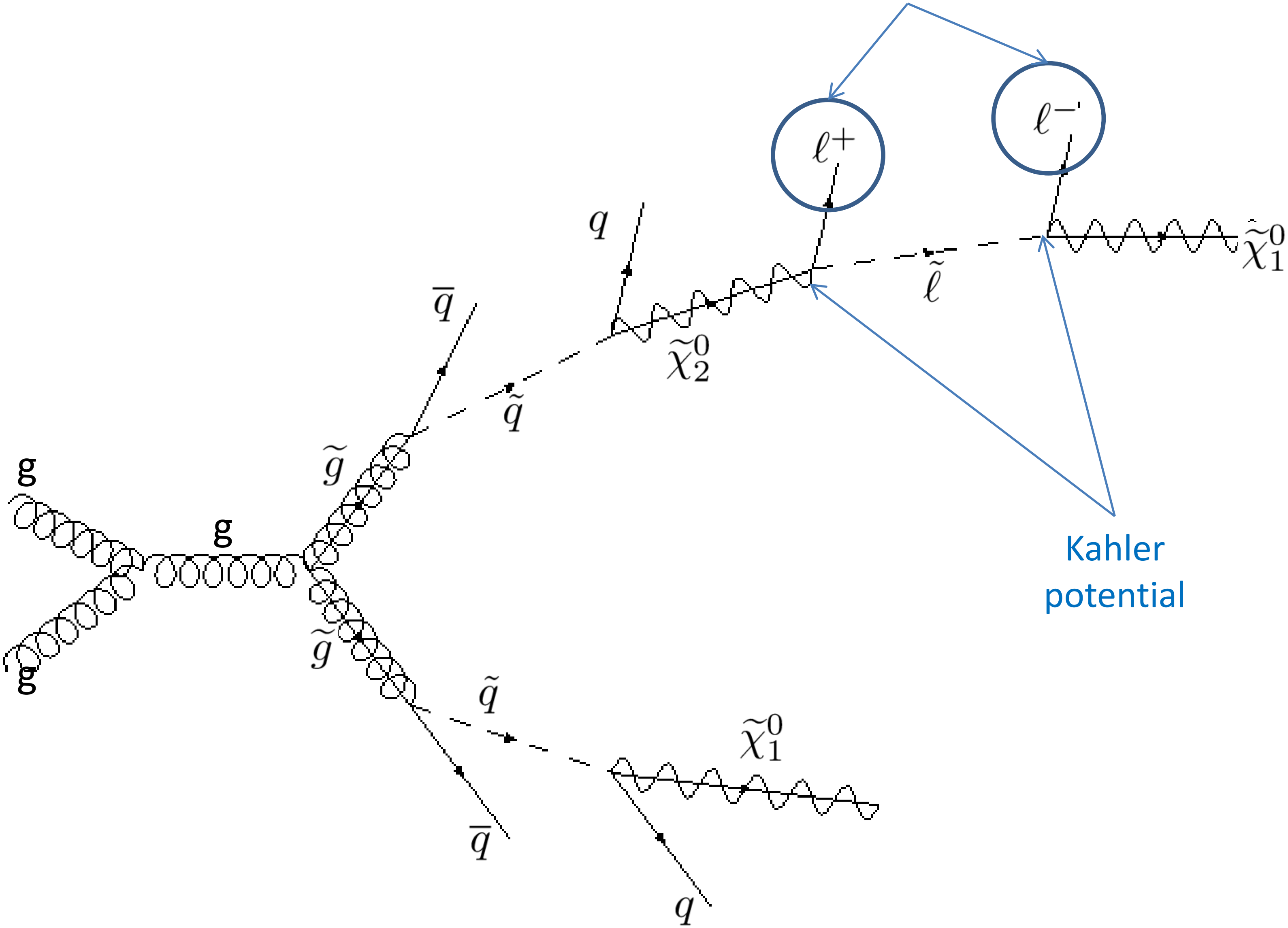
Kahler potential

Lightest supersymmetric particle (LSP)

R-parity conservation signal

Contributes to: $pp \to q\bar{q}q\bar{q} + E_T^{Miss} + X$

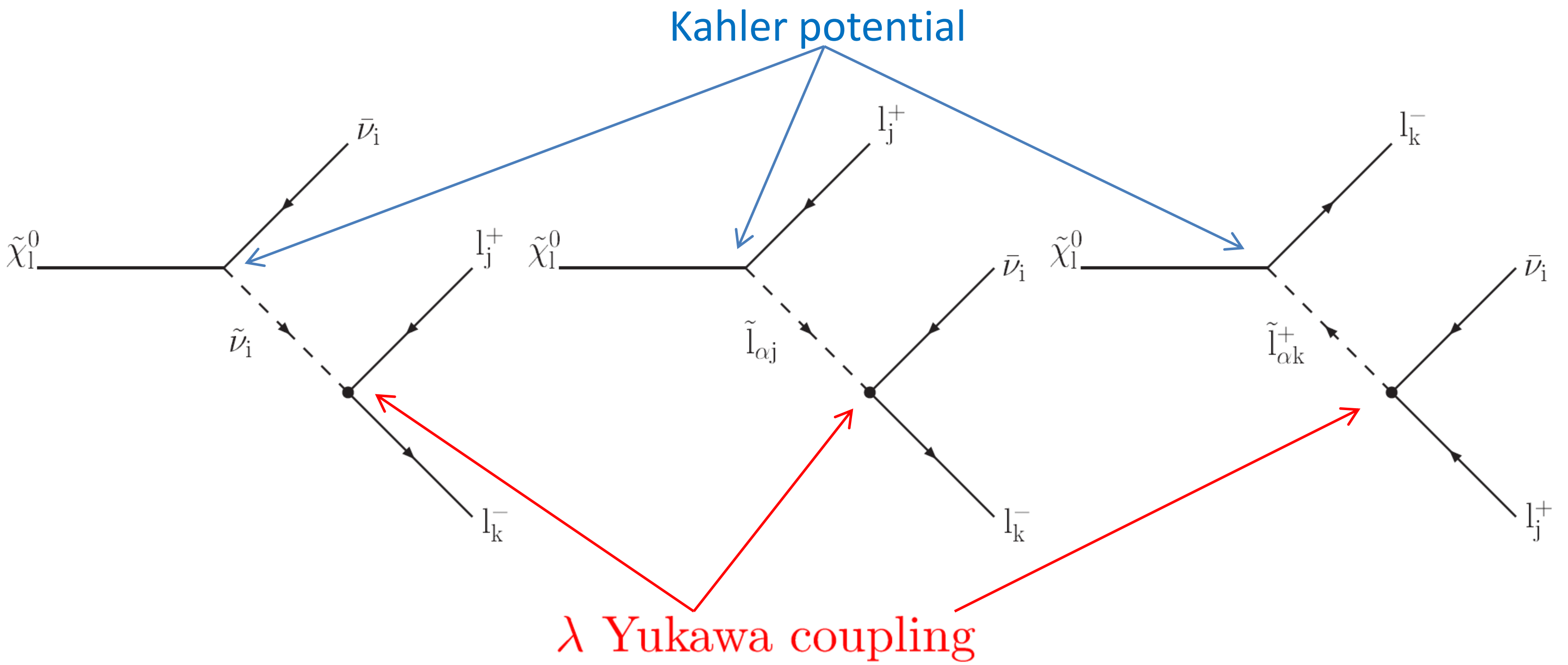
Opposite sign leptons.



Kahler potential

R-parity violating MSSM

$$\begin{aligned}\mathcal{W}^{RPV}_{MSSM} &= \epsilon_{\alpha\beta}(y_{\mathrm{u}}^{ij}\hat{H}_{\mathrm{u}}^{\alpha}\overline{u}_i\hat{Q}_j^{\beta} - y_{\mathrm{d}}^{ij}\hat{H}_{\mathrm{d}}^{\alpha}\overline{d}_i\hat{Q}_j^{\beta} - y_{\mathrm{e}}^{ij}\hat{H}_{\mathrm{d}}^{\alpha}\overline{e}_i\hat{L}_j^{\beta} + \mu\hat{H}_{\mathrm{u}}^{\alpha}\hat{H}_{\mathrm{d}}^{\beta}) \\ &\quad + \frac{1}{2}\lambda^{ijk}L_iL_j\overline{e}_k + \lambda'^{ijk}L_iQ_j\overline{d}_k + \mu'^{i}L_iH_u + \frac{1}{2}\lambda''^{ijk}\overline{u}_i\overline{d}_j\overline{d}_k\end{aligned}$$



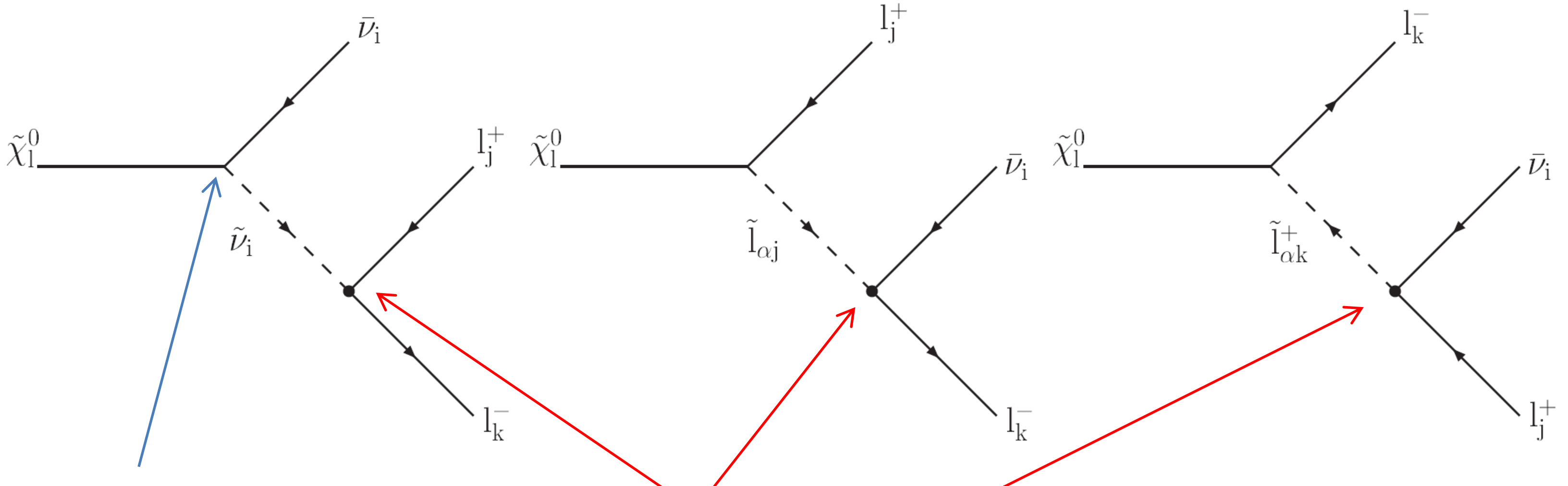
Kahler potential

λ Yukawa coupling

→ Multi-leptons + jets + MET

R-parity violating MSSM

$$\begin{aligned}\mathcal{W}^{RPV}_{MSSM} &= \epsilon_{\alpha\beta}(y^{ij}_{\mathrm{u}}\hat{H}^{\alpha}_{\mathrm{u}}\overline{u}_i\hat{Q}^{\beta}_j - y^{ij}_{\mathrm{d}}\hat{H}^{\alpha}_{\mathrm{d}}\overline{d}_i\hat{Q}^{\beta}_j - y^{ij}_{\mathrm{e}}\hat{H}^{\alpha}_{\mathrm{d}}\overline{e}_i\hat{L}^{\beta}_j + \mu\hat{H}^{\alpha}_{\mathrm{u}}\hat{H}^{\beta}_{\mathrm{d}}) \\ &\quad + \frac{1}{2}\lambda^{ijk}L_iL_j\overline{e}_k + \lambda'^{ijk}L_iQ_j\overline{d}_k + \mu'^{i}L_iH_u + \frac{1}{2}\lambda''^{ijk}\overline{u}_i\overline{d}_j\overline{d}_k\end{aligned}$$

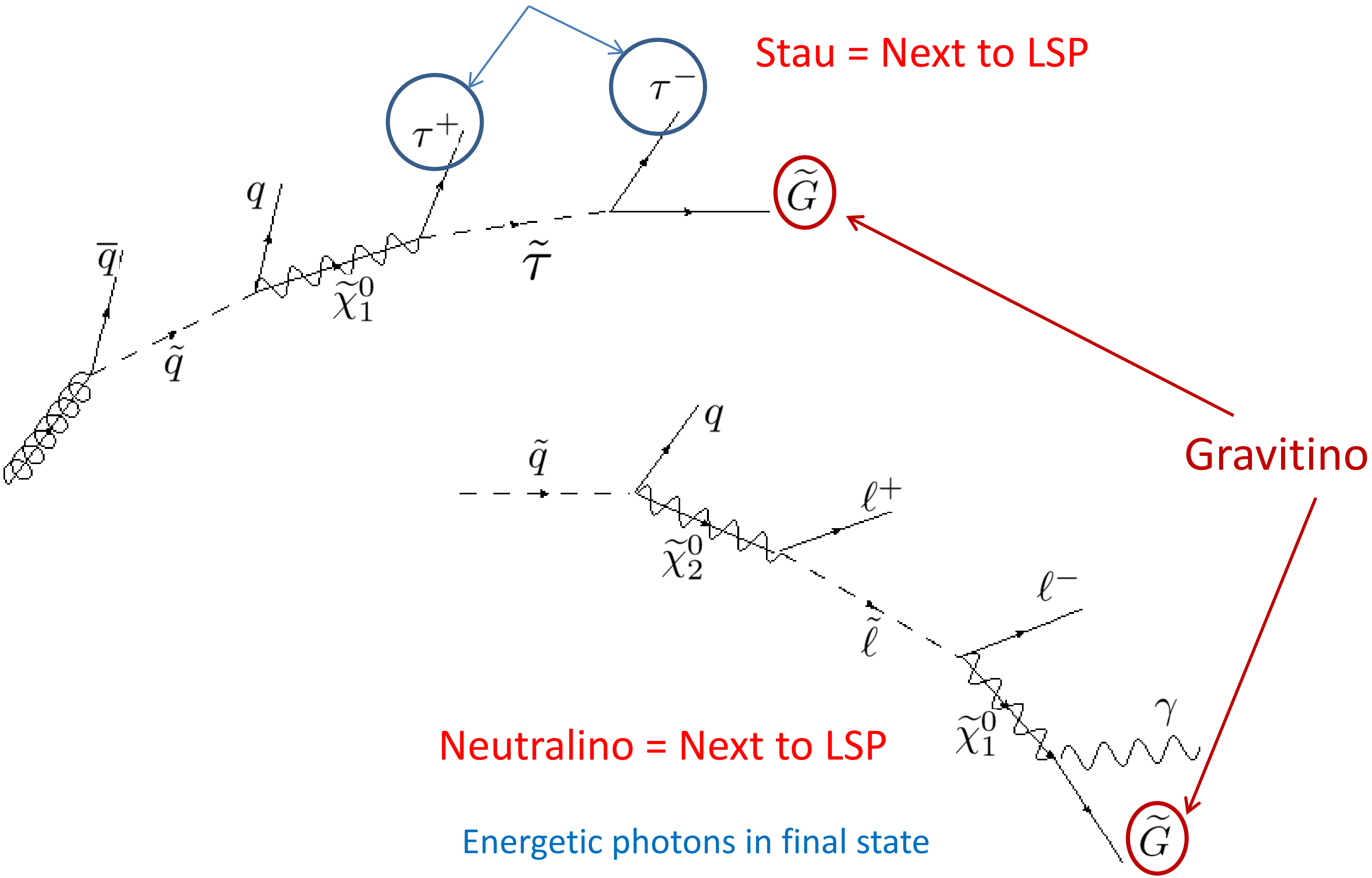


Kahler potential

λ' Yukawa coupling

Supersymmetric particles	Couplings		
	λ_{ijk}	λ'_{ijk}	λ''_{ijk}
$\widetilde{\chi}^0$	$\ell_i^+\bar{\nu}_j\ell_k^-$, $\ell_i^-\nu_j\ell_k^+$, $\bar{\nu}_i\ell_j^+\ell_k^-$, $\nu_i\ell_j^-\ell_k^+$	$\ell_i^+\bar{u}_jd_k$, $\ell_i^-u_j\bar{d}_k$, $\bar{\nu}_i\bar{d}_jd_k$, $\nu_id_j\bar{d}_k$	$\bar{u}_i\bar{d}_j\bar{d}_k$, $u_id_jd_k$
$\widetilde{\chi}^+$	$\ell_i^+\ell_j^+\ell_k^-$, $\ell_i^+\bar{\nu}_j\nu_k$ $\bar{\nu}_i\ell_j^+\nu_k$, $\nu_i\nu_j\ell_k^+$	$\ell_i^+\bar{d}_jd_k$, $\ell_i^+\bar{u}_ju_k$ $\bar{\nu}_i\bar{d}_ju_k$, $\nu_iu_j\bar{d}_k$	$u_id_ju_k$, $u_iu_jd_k$ $\bar{d}_i\bar{d}_j\bar{d}_k$

Gravitino LSP / Gauge Mediated SUSY Breaking



Taus : gives dilepton final state

Stau = Next to LSP

Gravitino

Neutralino = Next to LSP

Energetic photons in final state