Fits to $\tau \to K\pi\nu_\tau$ decays

Exclusive decay spectra

Viable information can be obtained from the decay spectra for exclusive τ-decay channels.

A first step in this direction is a reliable description of the $\tau \to \nu_\tau K\pi$ decay spectrum:

(MJ, Pich, Portolés 2006/08)
(Boito, Escribano, MJ 2008)
(Passemar et al. 2006-11)

$$\frac{d\Gamma_{K\pi}}{d\sqrt{s}} = \frac{G_F^2 |V_{us}|^2 M_\tau^3}{32\pi^3 s} \left(1 - \frac{s}{M_\tau^2}\right)^2 \times$$

$$\left[\left(1 + 2\,\frac{s}{M_\tau^2}\right) q_{K\pi}^3 \, |F_+^{K\pi}(s)|^2 + \frac{3\Delta_{K\pi}^2}{4s}\, q_{K\pi} |F_0^{K\pi}(s)|^2\right].$$

To this end the $K\pi$ vector and scalar form factors $F_+^{K\pi}(s)$ and $F_0^{K\pi}(s)$ are required as an input.

$K\pi$ form factors

A description of the $K\pi$ vector form factor can be obtained within chiral perturbation theory with resonances (RχPT):

$$F_+^{K\pi}(s) = \frac{m_{K^*}^2}{m_{K^*}^2 - s - \kappa\,\mathrm{Re}\widetilde{H}_{K\pi}(s) - i m_{K^*}\gamma_{K^*}(s)}\,.$$

The parameters of this model, namely m_{K^*} and γ_{K^*}, can be fitted from experimental data for p-wave $K\pi$ scattering, or from the τ data.

The physical parameters M_{K^*} and Γ_{K^*} can be inferred from the pole of $F_+^{K\pi}(s)$ in the complex s-plane.

Also a second resonance contribution can easily be included.

In the elastic, single-channel case a subtracted dispersive representation is available which is related to the Omnès solution:

$$\frac{F_+^{K\pi}(s)}{F_+^{K\pi}(0)} = \exp\left(\frac{\alpha_1 s}{M_\pi^2} + \frac{\alpha_2 s^2}{2M_\pi^4} + \frac{s^3}{\pi}\int\limits_{s_{K\pi}}^{s_{\rm cut}} \frac{\delta_{K\pi}(s')}{(s')^3(s'-s-i0)}\,ds'\right)$$

where $\delta_{K\pi}(s)$ is the elastic P-wave $K\pi$ phase shift.

The two subtraction constants α_1 and α_2 are related to slope and curvature of the vector form factor:

$$\lambda_+^{'} = \alpha_1 \,, \qquad \lambda_+^{''} = \alpha_2 + \alpha_1^2 \,.$$

$K\pi$ form factors

The scalar form factor $F_0^{K\pi}(s)$ can be obtained from a dispersion relation analysis of S-wave $K\pi$ scattering data.

(MJ, Oller, Pich 2000/02)

The scalar form factors are defined by:

$$i\,\langle\Omega|\,\partial^{\mu}(\bar{s}\gamma_{\mu}u)\,|\Gamma\rangle = \frac{\Delta_{K\pi}}{\sqrt{2}}\,C_{\Gamma}\,F_{\Gamma}(s)$$

where the C_{Γ} are Clebsch-Gordan coefficients and

$$\Delta_{K\pi} = M_K^2 - M_\pi^2\,.$$

As yet, the form factors $F_i(s)$ have not been measured directly. Thus an indirect determination is required.

From unitarity we have the following relation:

$$\mathrm{Im} F_k(s) = \sum_i \sigma_i(s)\, F_i(s)\, t_0^{ik}(s)^*$$

with $t_0^{ik}(s)$: S-wave $I=1/2$ scattering amplitudes.

The $F_i(s)$ also satisfy dispersion relations. In the 2-channel case with $F_1 \equiv F_{K\pi}$ and $F_3 \equiv F_{K\eta'}$:

$$F_1(s) = \frac{1}{\pi}\int_{s_1}^{\infty} \frac{\sigma_1 F_1\, t_0^{11}(s')^*}{(s'-s-i0)}\, ds' + \frac{1}{\pi}\int_{s_3}^{\infty} \frac{\sigma_3 F_3\, t_0^{13}(s')^*}{(s'-s-i0)}\, ds'$$

$$F_3(s) = \frac{1}{\pi}\int_{s_1}^{\infty} \frac{\sigma_1 F_1\, t_0^{13}(s')^*}{(s'-s-i0)}\, ds' + \frac{1}{\pi}\int_{s_3}^{\infty} \frac{\sigma_3 F_3\, t_0^{33}(s')^*}{(s'-s-i0)}\, ds'$$

Numerical analysis

Several ingredients are required for solving the set of coupled integral equations:

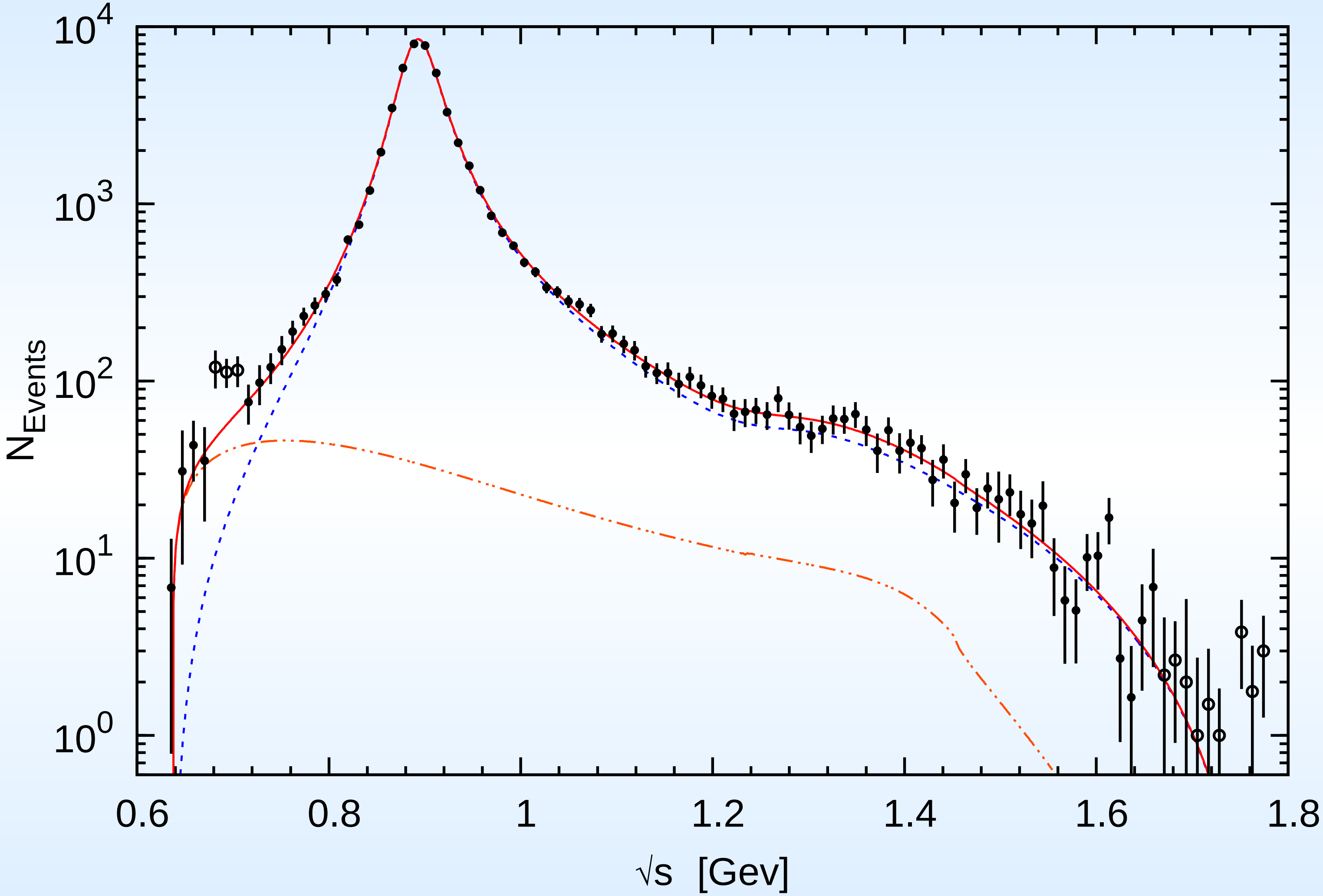
- An input for the scattering amplitudes is obtained by fitting an Ansatz from resonance ChPT to experimental data for S-wave $K\pi$ scattering. (M.J., Oller, Pich 2000/02)

- Two integration constants are also required. These can be chosen to be: $F_{K\pi}(0) = 0.972 \pm 0.012$ and

$$\frac{F_{K\pi}(\Delta_{K\pi})}{F_{K\pi}(0)} = \frac{F_K}{F_\pi F_{K\pi}(0)} + \frac{\Delta_{\rm CT}}{F_{K\pi}(0)} = 1.2346(53)$$

- The last relation follows from the ratio of leptonic K and π decays, as well as $|V_{us}|F_{K\pi}(0)$ from K_{l3} decays.

Fit to BELLE spectrum



$M_{K^*} = 892.0 \pm 0.5$ MeV, $\Gamma_{K^*} = 46.5 \pm 1.1$ MeV

As a prediction of the model, we obtain the slope and the curvature of the vector form factor $F_+^{K\pi}(s)$:

$$\lambda'_+ = (25.49 \pm 0.31) \cdot 10^{-3}\,, \qquad \lambda''_+ = (12.22 \pm 0.14) \cdot 10^{-4}\,.$$

The can be compared to earlier experimental determinations:

Collaboration	$\lambda'_+\,[10^{-3}]$	$\lambda''_+\,[10^{-3}]$
ISTRA 04	24.9 ± 1.6	0.84 ± 0.41
KTEV 04	20.64 ± 1.75	3.20 ± 0.69
NA48 04	28.0 ± 2.4	0.2 ± 0.5
KLOE 06	25.5 ± 1.8	1.4 ± 0.8

Conclusions

- ☞ Employing a dispersive representation of the $K\pi$ form factors, a satisfactory description of the $\tau \to \nu_\tau K\pi$ decay spectrum can be obtained.
- ☞ While a coupled-channel analysis is available for the scalar $K\pi$ form factors, our model for the vector form factors is purely elastic.
- ☞ To my mind, fits to experimental data should be done in a two-way approach: on the one hand, experimentalists can try to fit theoretical models provided by theorists.
- ☞ On the other hand, it would be very useful if unfolded distributions with correlations would be made available.

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Thank You for Your attention !