Probing τ lepton electromagnetic dipole moments through its leptonic radiative decays

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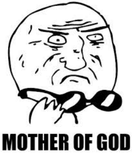
II Workshop Tau Lepton Decays, IFJ PAN, Cracow

hep-ph:1301.5302

work in progress in collaboration with:
S. Eidelman, D. Epifanov, L. Mercolli, C. Ng, M. Passera.

Problem:



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$$a_e = 1\,159\,652\,180.\,73\,(28) \cdot 10^{-12}$$

0.24 parts per billion! Hanneke et al, PRL100 (2008) 120801

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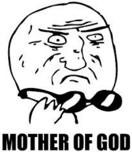

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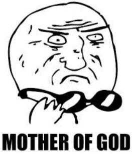

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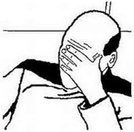


Tau

$$-0.052 \leq a_\tau \leq 0.013$$

Not even a test of LO! $\alpha/(2\pi) \approx 0.00116$ DELPHI - EPJC 35 (2004) 159

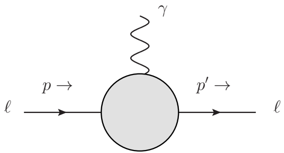
Outline

The SM prediction of tau dipole moments.

Experimental bounds.

Tau Radiative Decays

Interaction two on-shell ℓ with a photon:



$$= \quad \bar{u}(p')\Gamma^{\mu}\left(q^2\right)u(p).$$

Vertex Function:

$$\Gamma^{\mu}\left(q^2\right) = \gamma^{\mu}\ F_1\left(q^2\right)\ + F_A\left(q^2\right)\left(\gamma^{\mu}q^2 - 2m_{\ell}q^{m}u\right)\gamma_5$$
$$+\frac{\sigma^{\mu\nu}}{2m_{\ell}}q_{\nu}\left[\ iF_2\left(q^2\right)\ +\ F_3\left(q^2\right)\ \gamma_5\ \right]$$

- Charge: $F_1\left(q^2 = 0\right) = Q$

Form Factors & Dipole Moments

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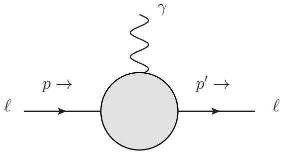

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- $g-2$: $F_2\left(q^2=0\right) = a_\ell$

LO: $a_\ell = \alpha/(2\pi)$ J. Schwinger '48

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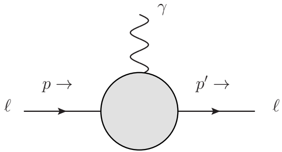

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- $g-2$: $F_2\left(q^2=0\right) = a_\ell$ LO: $a_\ell = \alpha/(2\pi)$ J. Schwinger '48
- EDM: $F_3\left(q^2=0\right) = d_\ell(2m_\ell)/e$

Anomalous Magnetic Moment

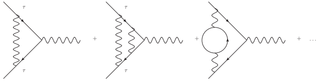
- The magnetic moment for a particle of spin $\vec{S}$

$$\vec{\mu} = g \frac{e}{2m} \vec{S}.$$

- Dirac theory for spin-1/2 fermion predicts

$$(i\partial_\mu - eA_\mu)\gamma^\mu \psi = m\psi \quad \Longrightarrow \quad g = 2 \quad \text{P. Dirac '28}$$

- But g-factor is modified by quantum correction!



$$a = (g-2)/2$$

The SM Prediction for a_τ

The Standard Model prediction of the tau $g-2$ is:

$$
\begin{aligned}
a_\tau^{\mathrm{SM}} = {} & 117\,324\,(2) && \times 10^{-8} && \text{QED} \\
& + 47.4\,(5) && \times 10^{-8} && \text{EW} \\
& + 337.5\,(3.7) && \times 10^{-8} && \text{HLO} \\
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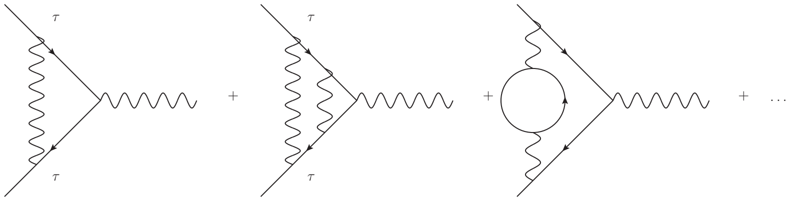

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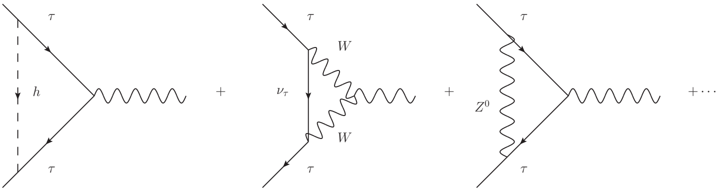

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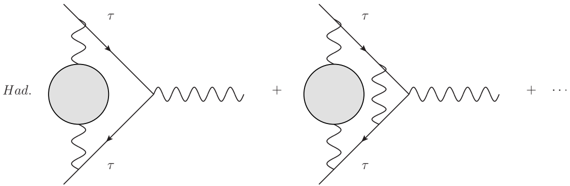

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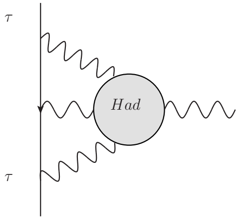

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$$
a_\tau^{\rm SM} = 117\,721\,(5)\ \times 10^{-8}
$$

S. Eidelman, M. Passera, MPLA 22 (2007) 159

Why is a_τ important?

- The tau $g-2$ still to be measured.
- NP associated with a mass scale Λ modify a_ℓ by a contribution

$$a_\ell^{\mathrm{NP}} \approx m_\ell^2/\Lambda^2.$$

- Given $m_\tau^2/m_\mu^2 \sim 283$, a_τ more sensitive than the muon one to EW and NP loop effects.
- If NP effects were of the same order of magnitude as the EW, a_τ would provide a "clean" NP test!

	Electron	Muon	Tau
$a^{\mathrm{EW}}/a^{\mathrm{HAD}}$	1/56	1/45	1/7
$a^{\mathrm{EW}}/\delta a^{\mathrm{HAD}}$	1.6	3	10

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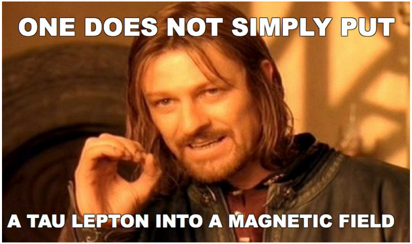
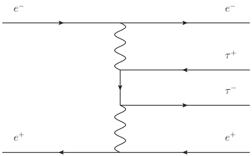
...if only we could measure it!



Experimental Bounds

- Short lifetime $\tau_\tau = 2.9 \cdot 10^{-13}$ s compared to e and μ.
- (Almost?) impossible to measure a_τ via its spin precession.



  - Bounds from $e^+e^- \to e^+e^-\tau^+\tau^-$ total cross-section measurements at LEP 2 (the PDG value):

$$-0.052 \leq a_\tau \leq 0.013$$ DELPHI - EPJC35 (2004) 159

  - Further analysis of LEP1/2 and SLC $e^+e^- \to \tau^+\tau^-$ data, effective Lagrangian approach (95% CL):

$$-0.007 < a_\tau^{\mathrm{NP}} < 0.005$$ González-Sprinberg et al. NPB 582 (2000) 3

The SM Prediction for d_τ

- Electric dipole contribution $F_3\left(q^2\right)$ violates P and T.
- In the SM CKM-phase. EDM at three-loop level.
- SM estimate astonishingly small $|d_\tau^{\rm SM}| \leq 10^{-35} e\cdot \text{cm}$!
- EDM current 95% CL limits from $e^+e^- \to \tau^+\tau^-$:

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$$-2.2 \leq \mathrm{Re}(d_\tau) \leq 4.5\,(10^{-17}\, e \cdot \mathrm{cm})$$
$$-2.5 \leq \mathrm{Im}(d_\tau) \leq 0.8\,(10^{-17}\, e \cdot \mathrm{cm})$$

Belle coll. PLB 551 (2003) 16.

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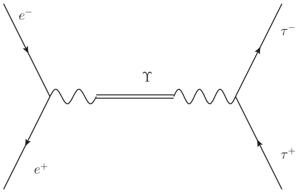
How to improve?

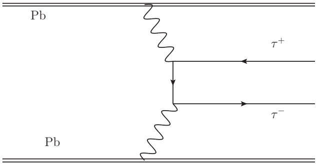
Bernabéu et al. propose the measurement of $F_2(q^2 = M_\Upsilon^2)$ from $e^+e^- \to \tau^+\tau^-$ production at B factories.



- Bounds on $F_2(q^2 = M_\Upsilon^2)$.
- Expected sensitivity for Babar+Belle: $4.6 \cdot 10^{-6}$ with $\mathcal{L} = 2$ ab^{-1}. Bernabéu et al. NPB 790 (2008) 160.

Possibility to use heavy ion collision at LHC: Pb Pb $\to$ Pb Pb $\tau^+\tau^-$.

- Advantage: Pb Pb $\gamma\gamma \approx$ on-shell.
- Expected sensitivity $\sim 10^{-5}$.
  F. del Aguila et al, PLB 271 (1991) 256



Leptonic Radiative Decays of the Tau

$$\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau \gamma \text{ with } \ell = e, \mu$$

- Suggested by Laursen et al. to search for the a_τ in radiative leptonic τ decays using the phenomenon of radiation zero.
- The tree-level amplitude has $a_\tau = 0$.
- Tree-level amplitude vanishes in the phase space region:

$$\cos(\ell, \gamma) = -1, \quad x = \frac{2E_\ell}{m_\tau} = 1 + \frac{m_\ell^2}{m_\tau^2} \qquad \text{(in the } \tau \text{ r.f).}$$

M. L. Laursen et al. PRD 29 (1984) 2652

Leptonic Radiative Decays of the Tau

$$\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau \gamma \text{ with } \ell = e, \mu$$

Our purpose:

- revisit τ radiative decays,
- extend strategy to d_τ,
- study the dependence of the pol. diff. decay rate on a_τ & d_τ,
- probe a_τ at $\mathcal{O}(10^{-3})$ with minimal assumption,
- provide an alternative determination of a_τ.

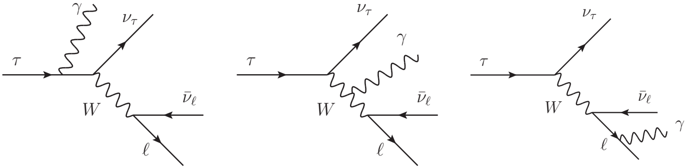
Muon discrepancy δa_μ scaled to a_τ:

- scaled naively $\frac{m_\tau^2}{m_\mu^2}$: $\sim 10^{-6}$.
- there are scenarios in which naive scaling is violated: larger effects $10^{-5} - 10^{-4}$. Giudice, Paradisi, Passera JHEP 1211 (2012) 113

Effective Lagrangian Approach

Effective operators included in the Lagrangian:

$$\mathcal{L}_{\text{eff}} = \left[\; e\frac{\tilde{a}}{4m_\tau}\bar{\tau}\sigma^{\mu\nu}\tau \quad -i\frac{\tilde{d}}{2}\bar{\tau}\sigma^{\mu\nu}\gamma_5\tau \; \right] F_{\mu\nu}$$



$$d\Gamma(\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau \gamma) = d\Gamma_{\text{SM}}$$

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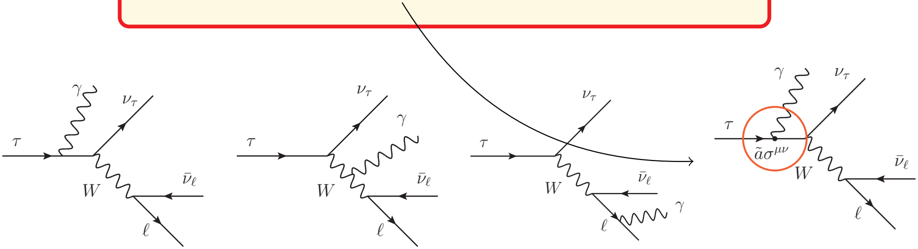

$$d\Gamma(\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau \gamma) = d\Gamma_{\text{SM}} \; + \tilde{a}\, d\Gamma_{\text{a}}$$

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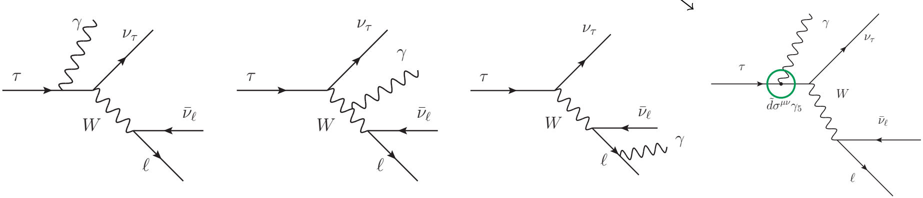


$$d\Gamma(\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau \gamma) = d\Gamma_{\text{SM}} + \tilde{a}\, d\Gamma_{\text{a}} + \tilde{d}\, d\Gamma_{\text{d}}.$$

Terms $\mathcal{O}(\tilde{a}^2)$ & $\mathcal{O}(\tilde{d}^2)$ neglected since $\approx 10^{-6}$.

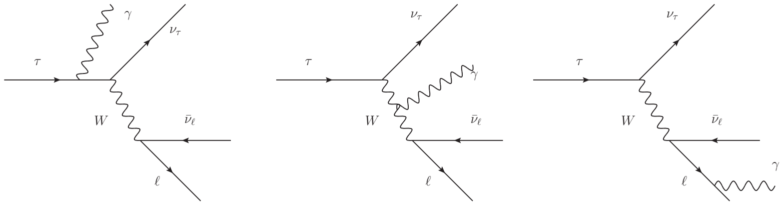
QED NLO Corrections to Tau Radiative Decay

- Since we want to probe a_τ and d_τ at $\mathcal{O}(10^{-3})$, terms of the same order cannot be neglected in the decay rate.
- Leading contribution from W-boson propagator, $(m_\tau/M_W)^2 \approx 5 \cdot 10^{-4}$.
- $d\Gamma$ at order α^2, i.e. we must include the QED NLO corrections.

QED NLO Corrections to Tau Radiative Decay

We computed the polarized differential decay rate up to order α^2, including the effective dipole moments couplings.

$$d\Gamma = d\Gamma_0 + \frac{m_\tau^2}{m_W^2} d\Gamma_W + \tilde{a}\, d\Gamma_{\mathrm{a}} + \tilde{d}\, d\Gamma_{\mathrm{d}} + \frac{\alpha}{\pi} d\Gamma_{\mathrm{NLO}}$$



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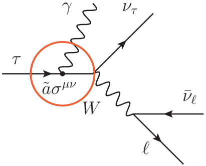

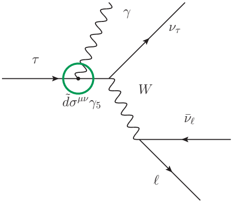

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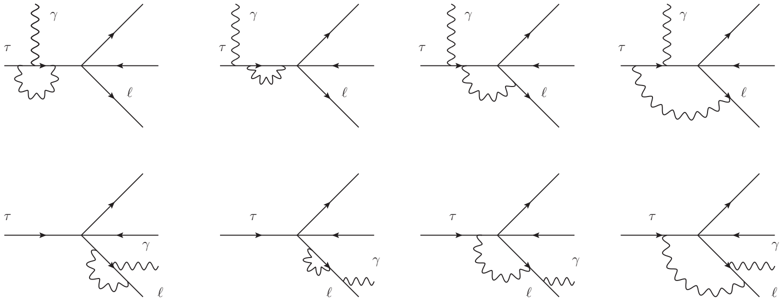

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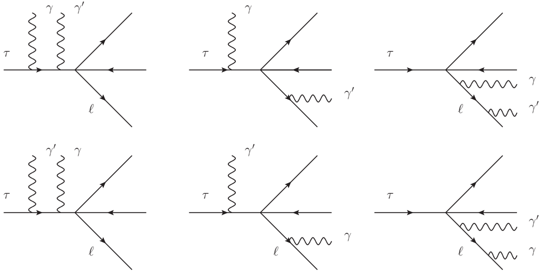

The $\tilde{a}$ and $\tilde{d}$ Parameters

- QED NLO corrections are included $d\Gamma$,
- The effective Lagrangian $\mathcal{L}_{\text{eff}}$ predicts:

$$a_\tau = F_2^{(\text{QED})}(0) + \tilde{a} = \frac{\alpha}{2\pi} + \tilde{a} + \mathcal{O}\left(\alpha^2\right)$$

$$d_\tau = \frac{e}{2m_\tau} F_3^{(\text{QED})}(0) + \tilde{d} = \tilde{d} + \mathcal{O}\left(\alpha^2\right) \frac{e}{2m_\tau}$$

- $\tilde{a}$ parametrizes all non-QED contributions to the $g-2$ of $\mathcal{O}(\alpha)$.
- $\tilde{d}$ non-QED contribution to the EDM of $\mathcal{O}\left(\alpha\right) \cdot \frac{e}{2m_\tau}$.

Opportunities and Challenges

- Phase space integration kills $d\Gamma_a$ & $d\Gamma_a$ because of IR singularities associated to $d\Gamma_0$:

CLEO coll. PRL84(2000)830	our results
$\mathrm{Br}(\tau^- \to \mu^- \overline{\nu}_\mu \nu_\tau \gamma)\Big\vert_{E\gamma > 10\mathrm{MeV}} = 0.361\,(38)\%$	$\left[0.367 + (8.6 \cdot 10^{-4})\,\tilde{a}\right]\%$
$\mathrm{Br}(\tau^- \to e^- \overline{\nu}_e \nu_\tau \gamma)\Big\vert_{E\gamma > 10\mathrm{MeV}} = 1.75\,(18)\%$	$\left[1.837 + (8.2 \cdot 10^{-4})\,\tilde{a}\right]\%$

- A full phase space analysis is needed.
- Precise measurement of differential rate. High statistics required.
- Feasibility for the search of the tau dipole moments Belle and future Belle-II is currently under study.
- What can LHC tell us about tau dipole moments?

Summary and Outlook

- It is desirable to get a more precise measurement of the τ dipole moments.
- We propose to improve the PDG values through the study of leptonic radiative decays.
- The possibility to measure a_τ in radiative decays at Belle is under investigation.
- A feasibility study for the search of a_τ & d_τ at LHC is needed.

Thanks!

QED NLO Corrections: Backup

We computed the polarized decay rate up to order α^2, including the effective coupling of $\tilde{a}$.

$$d\Gamma = d\Gamma_0 + \frac{m_\tau^2}{m_W^2} d\Gamma_W + \tilde{a}\, d\Gamma_{\mathrm{a}} + \tilde{d}\, d\Gamma_{\mathrm{d}} + \frac{\alpha}{\pi} d\Gamma_{\mathrm{NLO}}$$

- Neutrinos momenta integrated over the phase space.
- W-boson propagator effects can be neglected in $d\Gamma_{\mathrm{NLO}}$ since $\mathcal{O}\left(\alpha^2 \frac{m_\tau^2}{m_W^2}\right)$.
- Fermi four-fermion interaction used in the computation of $d\Gamma_1$.
- Agree with previous results:
  T. Kinoshita, A. Sirlin PRL2(1959)177;
  Y. Kuno, Y. Okada, RMP73(2001)151;
  A. Fischer et al. PRD49(1994)3426;
  A. B. Arbuzov PLB597(2004)285.

Decay Rate Formula

The total differential decay for a polarized τ lepton in the tau r.f. is

$$\frac{d^3\Gamma}{dx\, dy\, d\cos\theta} = \frac{\alpha\, G_F^2 m_\tau^5}{2(4\pi)^6} y\sqrt{x^2-4r^2}\,\Big\{G(x,y,\theta_{\ell\gamma}) + y\cos\theta_\gamma K(x,y,\theta_{\ell\gamma})$$

$$+\cos\theta_\ell\sqrt{x^2-4r^2}\,J(x,y,\theta_{\ell\gamma}) + \tilde{d}_\tau m_\tau y\sqrt{x^2-4r^2}\,\hat{p}_l\cdot(\hat{p}_\gamma\times\vec{n})\,G_{\rm d}(x,y,z)\Big\}$$

where

$$x=\frac{2E_\ell}{m_\tau}, \quad y=\frac{2E_\gamma}{m_\tau}, \quad r=\frac{m_\ell}{m_\tau},$$

θ_ℓ (θ_γ) is the angle between the lepton (photon) momentum and the polarization vector $\vec{n}$.

$$G = G_0 + \frac{m_\tau^2}{m_W^2}G_W + \tilde{a}\,G_{\rm a} + \frac{\alpha}{\pi}G_{\rm NLO}$$

The functions J and K have the same structure.

$$G_0 = -\frac{64\pi^2}{3y^2z^2}\Big[r^4\left(6xy^2+6y^3-6y^2z-8y^2\right)+r^2\big(-4x^2y^2-6x^2yz$$
$$-8xy^3+2xy^2z+6xy^2+6xyz^2+8xyz+6xz^2-4y^4+5y^3z+6y^3$$
$$-2y^2z^2-6y^2z-3yz^3+6yz^2-6z^3-8z^2\big)+4x^3yz+8x^2y^2z$$
$$-8x^2yz^2-6x^2yz-4x^2z^2+6xy^3z-8xy^2z^2-6xy^2z+6xyz^3$$
$$-2xyz^2+8xz^3+6xz^2+2y^4z-2y^3z^2-3y^3z+2y^2z^3-2y^2z^2$$
$$-2yz^4+5yz^3+6yz^2-6z^3\Big]$$

Kinoshita & Sirlin ('59), Kuno & Okada ('01)

$$G_a = -\frac{64\pi^2}{3yz}\Big[xyz+2xz^2+y^2z+yz^2-yz-2z^3-2z^2+r^2\left(-y^2+yz-3z^2\right)\Big]$$

$$G_{aa} = -\frac{16\pi^2}{3}\left[2x^2+2xy-2xz-3x-yz+2z+r^2\left(-3x-2y+4\right)\right]$$