Resonance Chiral Lagrangians and hadronic currents: Theoretical uncertainty

Pablo Roig Garcés (IF-UNAM, México D.F., México)

Collaborators: I. Nugent, T. Przedzinski, O. Shekhovtsova and Z. Was

2nd Workshop tau lepton decays: hadronic currents from Belle BaBar data and LHC signatures
Cracow, 15-22 September 2013

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- Form factors in three meson τ decays

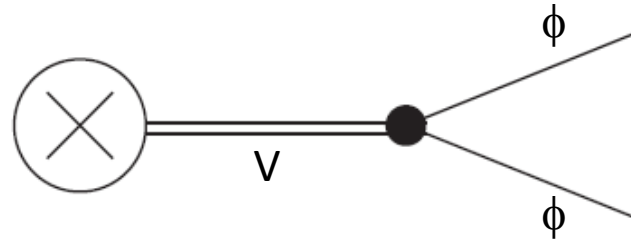- Discussion on the errors
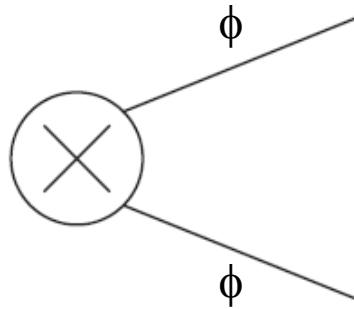
- Ongoing/Future improvements

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Form factors in two meson τ decays
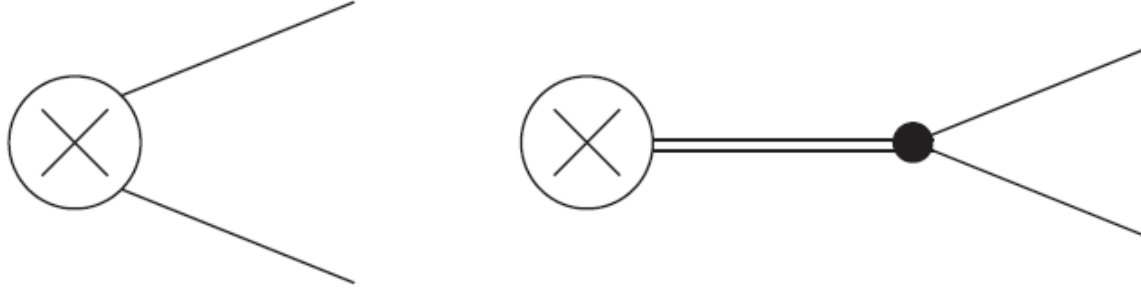
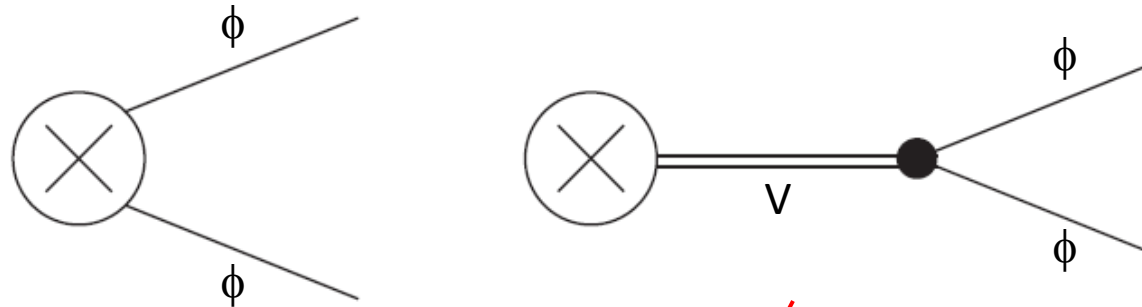# Form factors in two meson τ decays

$\phi = \pi,\ K$

$V = \rho,\ K^*$

Form factors in two meson τ decays
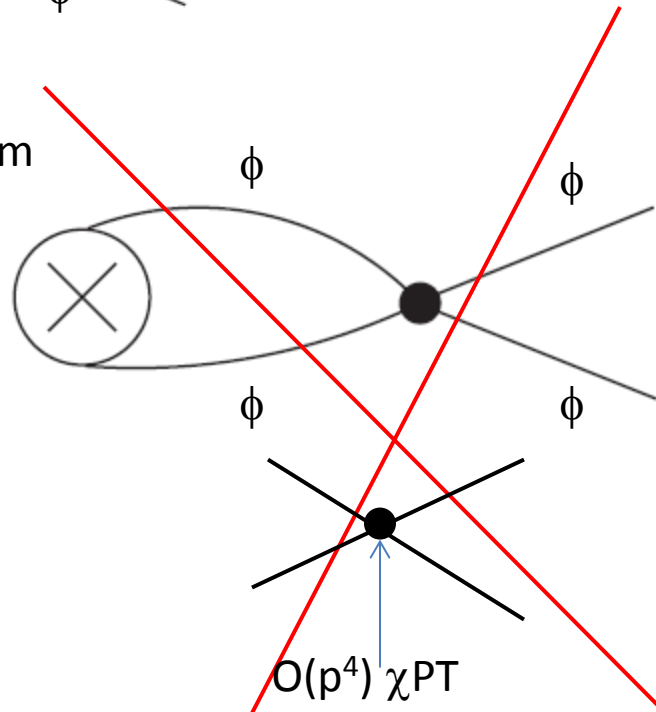
$\phi = \pi, K$

$V = \rho, K^*$

Antisymmetric tensor formalism
for spin-one resonances

Ecker *et al.*
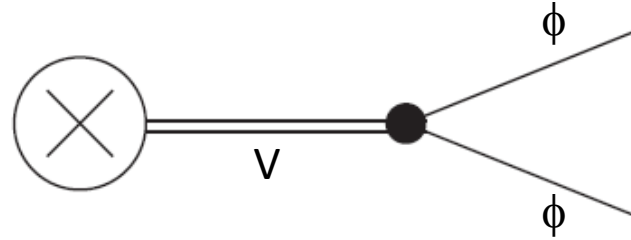
Phys.Lett.B223:425,1989

Nucl.Phys.B321:311,1989

To avoid double counting

$O(p^4)\,\chi PT$

Pablo Roig (UNAM, México D.F.)

Theoretical uncertainty in RChL hadronic currents

Form factors in two meson τ decays

$\phi = \pi, K$

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ϕ

ϕ

ϕ

ϕ

ϕ

V
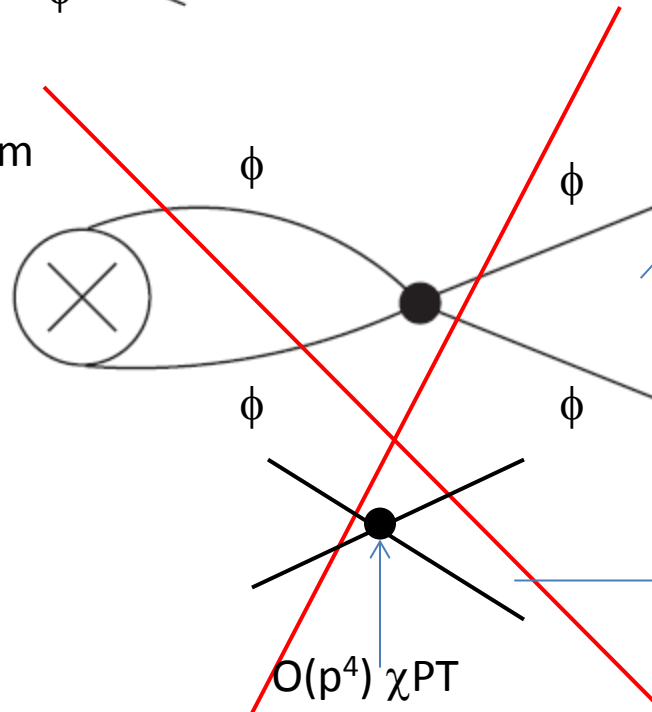
Obtained via the resonances width

Antisymmetric tensor formalism for spin-one resonances

Ecker *et al.*

Phys.Lett.B223:425,1989

Nucl.Phys.B321:311,1989

ϕ

ϕ
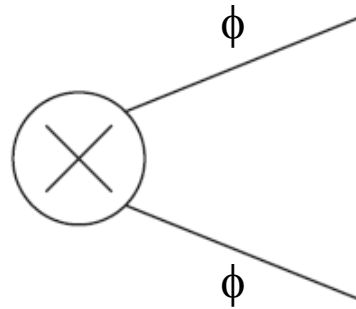
ϕ

ϕ

To avoid double counting

Obtained by integrating resonances out

$O(p^4)\ \chi PT$

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Theoretical uncertainty in RChL hadronic currents

Form factors in two meson τ decays

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Ecker *et al.*

Phys.Lett.B223:425,1989

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$$\mathcal{L}_2[V(1^{--})] = \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f_+^{\mu\nu} \rangle + \frac{iG_V}{2\sqrt{2}} \langle V_{\mu\nu} [u^\mu, u^\nu] \rangle$$

$$u_\mu = i \left\{ u^\dagger (\partial_\mu - ir_\mu) u - u (\partial_\mu - il_\mu) u^\dagger \right\}$$

$$f_\pm^{\mu\nu} = u F_L^{\mu\nu} u^\dagger \pm u^\dagger F_R^{\mu\nu} u$$

$$u(\varphi) = \exp \left\{ i \frac{\Phi}{\sqrt{2}\,F} \right\} \quad \Phi(x) \equiv \frac{1}{\sqrt{2}} \sum_{a=1}^{8} \lambda_a \varphi_a = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix}$$

Form factors in two meson τ decays

$\phi = \pi, K$

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ϕ

ϕ

ϕ

ϕ

V
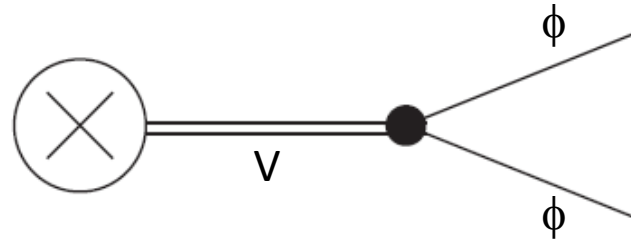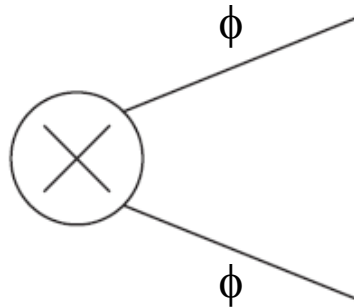
Ecker *et al.*

Phys.Lett.B223:425,1989

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$$F(s)^V = 1 + \frac{F_V G_V}{F^2} \frac{s}{M_V^2 - s}$$

Short-distance constraints

F(s) →0, for s→∞ $\implies$ $F_V G_V / f_\pi^2 = 1$

$$\Downarrow$$

$$F(s)^{VMD} = \frac{M_V^2}{M_V^2 - s}$$

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Theoretical uncertainty in RChL hadronic currents

Form factors in two meson τ decays

ϕ

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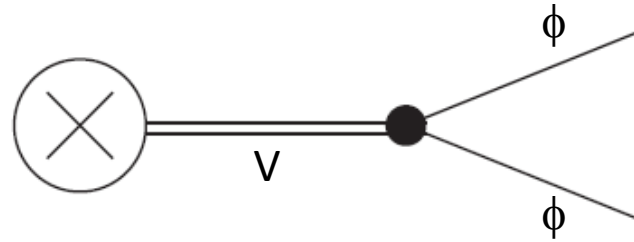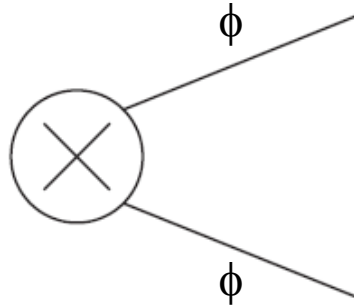
$$F(s)^{VMD} = \frac{M_V^2}{M_V^2 - s}$$

$$\frac{1}{M_V^2 - s} \to \frac{1}{M_V^2 \left[1 + \sum_{P,Q} N_{loop}^{PQ} \frac{s}{96\pi^2 F^2} A_{PQ}(s) \right] - s}$$

$$A(m_P^2/s, m_P^2/\mu^2) = \ln\left(m_P^2/\mu^2\right) + \frac{8m_P^2}{s} - \frac{5}{3} + \sigma_P^3 \ln\left(\frac{\sigma_P + 1}{\sigma_P - 1}\right)$$

$$\sigma_P \equiv \sqrt{1 - 4m_P^2/s}$$

Form factors in two meson τ decays

$\phi = \pi, K$

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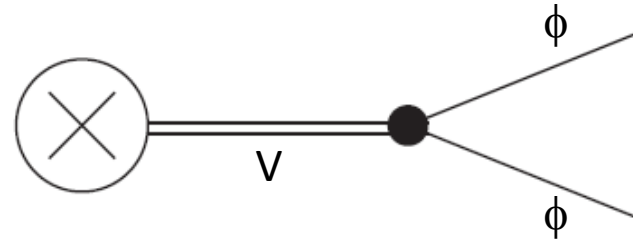
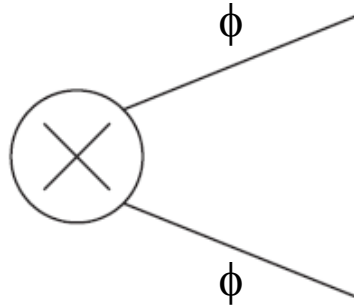$$A(m_P^2/s, m_P^2/\mu^2) = \ln\left(m_P^2/\mu^2\right) + \frac{8 m_P^2}{s} - \frac{5}{3} + \sigma_P^3 \ln\left(\frac{\sigma_P + 1}{\sigma_P - 1}\right)$$

$$\sigma_P \equiv \sqrt{1 - 4 m_P^2/s}$$

Phys.Lett.B412:382-388,1997

Guerrero, Pich

There are several ways of resumming FSI: Omnès exponentiation of Re $A_{PQ}(s)$, dispersion relations, etc…

Nuovo Cim.8:316-326,1958

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Form factors in two meson τ decays

$$\langle \pi^-(p_{\pi^-})\pi^0(p_{\pi^0})|\bar{d}\gamma^\mu u|0\rangle = \sqrt{2}(p_{\pi^-} - p_{\pi^0})^\mu F_V^\pi(s)$$

$$F(s)^{\text{VMD}} = \frac{M_\rho^2}{M_\rho^2 - s}$$

Guerrero, Pich '97

$$F(s)_{O(p^4)}^{\text{ChPT}} = 1 + \frac{2L_9^r(\mu)}{f_\pi^2}s - \frac{s}{96\pi^2 f_\pi^2}\left[A(m_\pi^2/s, m_\pi^2/\mu^2) + \frac{1}{2}A(m_K^2/s, m_K^2/\mu^2)\right]$$

$$A(m_P^2/s, m_P^2/\mu^2) = \ln\left(m_P^2/\mu^2\right) + \frac{8m_P^2}{s} - \frac{5}{3} + \sigma_P^3 \ln\left(\frac{\sigma_P+1}{\sigma_P-1}\right) \qquad \sigma_P \equiv \sqrt{1 - 4m_P^2/s}$$

ChPT+VMD

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s} - \frac{s}{96\pi^2 f_\pi^2}\left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}A(m_K^2/s, m_K^2/M_\rho^2)\right]$$

Form factors in two meson τ decays

ChPT+VMD   Guerrero, Pich '97

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Unitarity+Analiticity   Omnés, '58

Form factors in two meson τ decays

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Unitarity+Analiticity Omnés, '58

$O(p^2)$ result for $\delta^1_1(s)$

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s} \exp\left\{ \frac{-s}{96\pi^2 f^2}\left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2} A(m_K^2/s, m_K^2/M_\rho^2) \right] \right\}$$

Form factors in two meson τ decays

ChPT+VMD — Guerrero, Pich '97

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Guerrero, Pich '97

$$\Gamma_\rho(s) = \frac{M_\rho\, s}{96\pi f_\pi^2}\left\{\theta(s - 4m_\pi^2)\,\sigma_\pi^3 + \frac{1}{2}\theta(s - 4m_K^2)\,\sigma_K^3\right\}$$

Gómez-Dumm, Pich, Portolés '00
Phys.Rev.D62:054014,2000

$$= -\frac{M_\rho\, s}{96\pi^2 f_\pi^2}\,\mathrm{Im}\left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}A(m_K^2/s, m_K^2/M_\rho^2)\right]$$

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)}\exp\left\{\frac{-s}{96\pi^2 f_\pi^2}\left[\mathrm{Re}A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}\mathrm{Re}A(m_K^2/s, m_K^2/M_\rho^2)\right]\right\}$$

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Form factors in two meson τ decays

Starting point

Match χPT results to VMD using an Omnés solution for dispersion relation

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp\left\{\frac{-s}{96\pi^2 f_\pi^2}\left[\mathrm{Re}A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}\mathrm{Re}A(m_K^2/s, m_K^2/M_\rho^2)\right]\right\}$$

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Phys.Rev.D57:4136-4141,1998

- χPT up to O(p^4) and leading O(p^6) contributions Guerrero '98
- Right fall-off at high energies

- SU(2)
- Analiticity and unitarity constraints (NNLO)

Phys.Lett.B640:176-181,2006

Idea: Follow the approach of Jamin, Pich, Portolés '06 including excited resonances while retaining (some of) these nice properties

Form factors in two meson τ decays

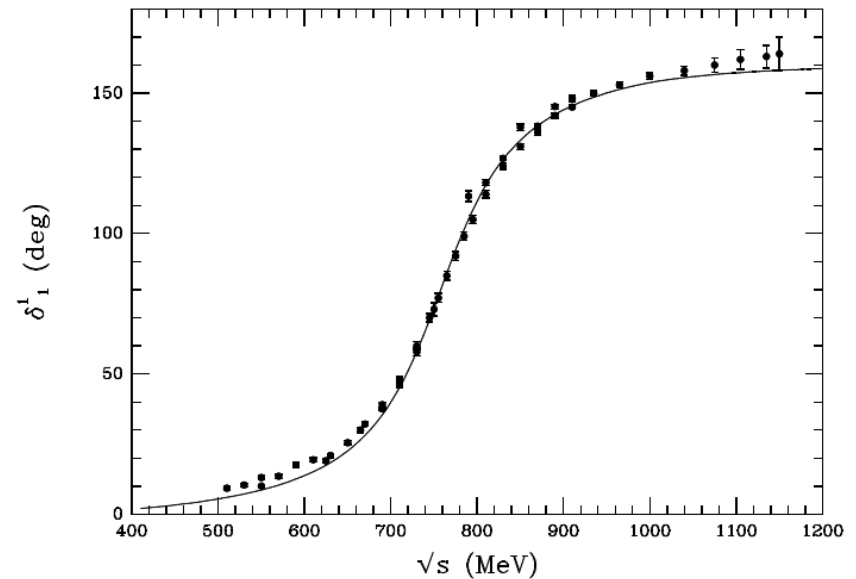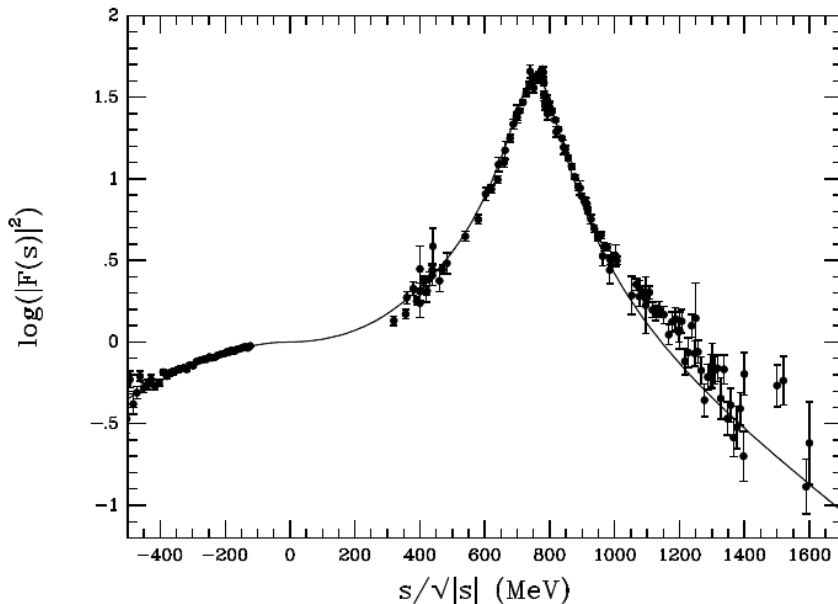Starting point

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Theoretical uncertainty in RChL hadronic currents

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Our formula

$$
\begin{aligned}
F_V^-(s) &= \frac{M_\rho^2 + s(\gamma e^{i\phi_1} + \delta e^{i\phi_2})}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)}\exp\left[\frac{-s}{96\pi^2 F_\pi^2}\left(\Re A_\pi(s) + \Re A_K(s)/2\right)\right] \\
&\quad - \frac{\gamma s e^{i\phi_1}}{M_{\rho'}^2 - s - iM_{\rho'}\Gamma_{\rho'}(s)}\exp\left[\frac{-s\Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3 \sigma_\pi^3(M_{\rho'}^2)}\Re A_\pi(s)\right] \\
&\quad - \frac{\delta s e^{i\phi_2}}{M_{\rho''}^2 - s - iM_{\rho''}\Gamma_{\rho''}(s)}\exp\left[\frac{-s\Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3 \sigma_\pi^3(M_{\rho''}^2)}\Re A_\pi(s)\right].
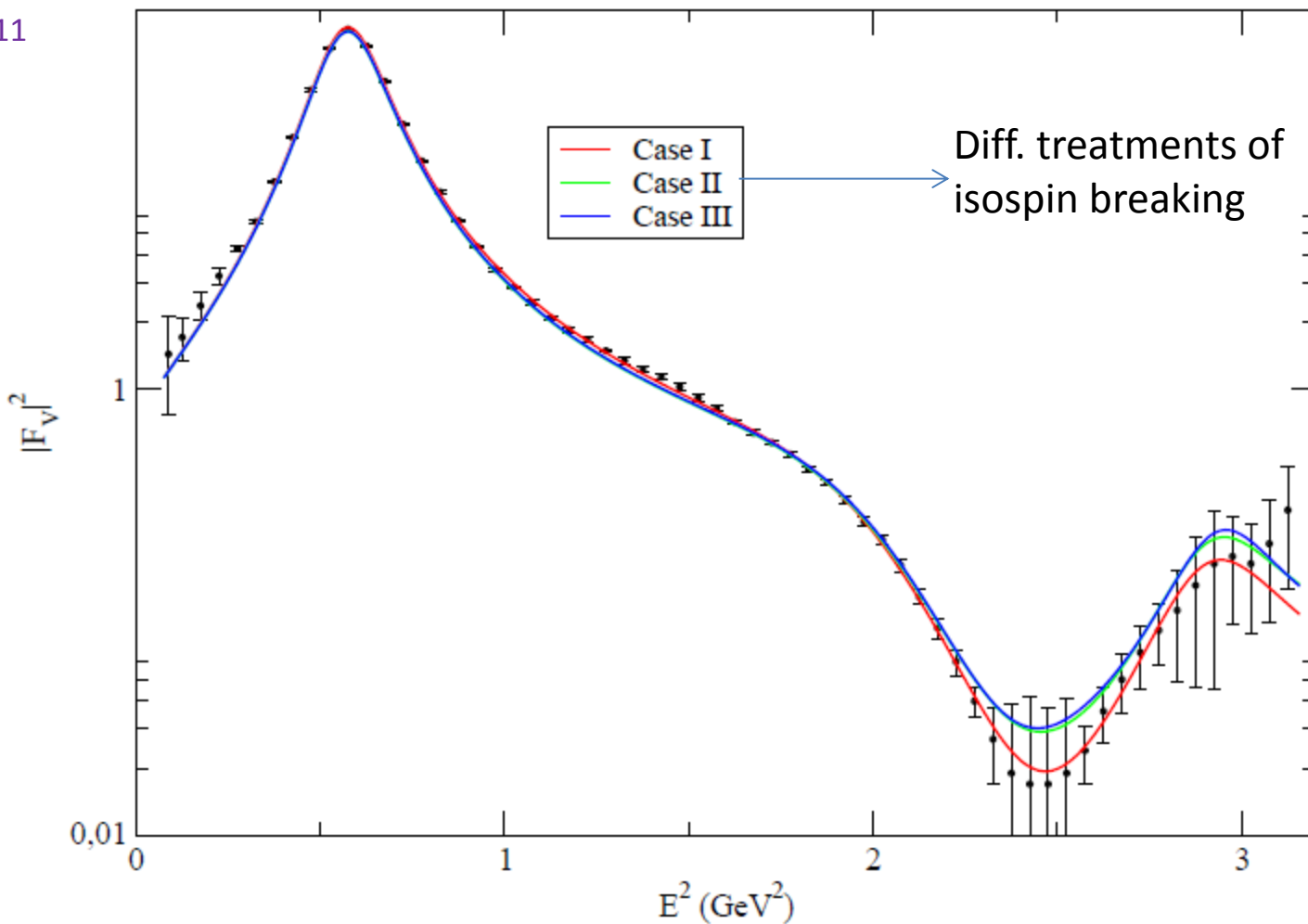\end{aligned}
$$

Roig '11

e-Print: **arXiv:1112.0962**

- χPT up to $O(p^4)$ and leading $O(p^6)$ contributions Guerrero '98
- Right fall-off at high energies
- SU(2)

- Analiticity and unitarity constraints (NNLO)
- (Phenomenological) contribution of ρ' + ρ''

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Form factors in two meson τ decays

Starting point

Guerrero, Pich '97

Match χPT results to VMD using an Omnés solution for dispersion relation

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp\left\{\frac{-s}{96\pi^2 f_\pi^2}\left[\mathrm{Re}A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}\mathrm{Re}A(m_K^2/s, m_K^2/M_\rho^2)\right]\right\}$$

Our formula

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F_V^-(s) &= \frac{M_\rho^2 + s(\gamma e^{i\phi_1} + \delta e^{i\phi_2})}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)}\exp\left[\frac{-s}{96\pi^2 F_\pi^2}(\Re eA_\pi(s) + \Re eA_K(s)/2)\right] \\
&- \frac{\gamma s e^{i\phi_1}}{M_{\rho'}^2 - s - iM_{\rho'}\Gamma_{\rho'}(s)}\exp\left[\frac{-s\Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3 \sigma_\pi^3(M_{\rho'}^2)}\Re eA_\pi(s)\right] \\
&- \frac{\delta s e^{i\phi_2}}{M_{\rho''}^2 - s - iM_{\rho''}\Gamma_{\rho''}(s)}\exp\left[\frac{-s\Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3 \sigma_\pi^3(M_{\rho''}^2)}\Re eA_\pi(s)\right].
\end{aligned}$$

Roig '11

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This is what is included in TAUOLA right now

Form factors in two meson τ decays

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Diff. treatments of isospin breaking

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Form factors in two meson τ decays

On the inclusion of excited resonances

Our formula $\boxed{F_V^-(s)}$
$$
\begin{aligned}
F_V^-(s) &= \frac{M_\rho^2 + s(\gamma e^{i\phi_1} + \delta e^{i\phi_2})}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp\left[\frac{-s}{96\pi^2 F_\pi^2}\left(\Re eA_\pi(s) + \Re eA_K(s)/2\right)\right] \\
&\quad - \frac{\gamma s e^{i\phi_1}}{M_{\rho'}^2 - s - iM_{\rho'}\Gamma_{\rho'}(s)} \exp\left[\frac{-s\Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3 \sigma_\pi^3(M_{\rho'}^2)}\Re eA_\pi(s)\right] \\
&\quad - \frac{\delta s e^{i\phi_2}}{M_{\rho''}^2 - s - iM_{\rho''}\Gamma_{\rho''}(s)} \exp\left[\frac{-s\Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3 \sigma_\pi^3(M_{\rho''}^2)}\Re eA_\pi(s)\right].
\end{aligned}
$$

Roig '11

$$\gamma \equiv -F_V'G_V'/F^2 \qquad \delta \equiv -F_V''G_V''/F^2 \qquad F_V G_V + F_V'G_V' + F_V''G_V'' + ... = F^2$$

$$\mathcal{L}_2[V(1^{--})] = \frac{\boxed{F_V}}{2\sqrt{2}}\langle V_{\mu\nu}f_+^{\mu\nu}\rangle + \frac{i\boxed{G_V}}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle$$

Form factors in two meson τ decays

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Our formula $\boxed{F_V^-(s)}$ =

Roig '11

$$F_V^-(s) = \frac{M_\rho^2 + s(\gamma e^{i\phi_1} + \delta e^{i\phi_2})}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)}\exp\left[\frac{-s}{96\pi^2 F_\pi^2}(\Re eA_\pi(s) + \Re eA_K(s)/2)\right]$$

$$-\frac{\gamma s e^{i\phi_1}}{M_{\rho'}^2 - s - iM_{\rho'}\Gamma_{\rho'}(s)}\exp\left[\frac{-s\Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3\sigma_\pi^3(M_{\rho'}^2)}\Re eA_\pi(s)\right]$$

$$-\frac{\delta s e^{i\phi_2}}{M_{\rho''}^2 - s - iM_{\rho''}\Gamma_{\rho''}(s)}\exp\left[\frac{-s\Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3\sigma_\pi^3(M_{\rho''}^2)}\Re eA_\pi(s)\right].$$

$$\gamma \equiv -F_V'G_V'/F^2 \qquad \delta \equiv -F_V''G_V''/F^2 \qquad F_VG_V + F_V'G_V' + F_V''G_V'' + ... = F^2$$

$$\mathcal{L}_2[V(1^{--})] = \frac{\boxed{F_V}}{2\sqrt{2}}\langle V_{\mu\nu}f_+^{\mu\nu}\rangle + \frac{i\boxed{G_V}}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle$$

$\longrightarrow$ Easy to implement for two meson modes. For three meson modes a number of new couplings (involving new operator structures) appear. At what stage shall we include them?

Form factors in two meson τ decays

Kη (Escribano et. al. '13)

• The procedure for other two meson decay channels is analogous (Although SFF relevant in some decay channels).

$$F_{PQ}^V(s) = F^{VMD}(s) \exp\left[\sum_{P,Q} N_{loop}^{PQ} \frac{-s}{96\pi^2 F^2} Re A_{PQ}(s)\right]$$

$$F_+^{K\pi}(s) = \left[\frac{M_{K_*}^2 + \gamma s}{M_{K_*}^2 - s - iM_{K_*}\Gamma_{K_*}(s)} - \frac{\gamma s}{M_{K_{*\prime}}^2 - s - iM_{K_{*\prime}}\Gamma_{K_{*\prime}}(s)}\right] e^{\frac{3}{2}Re\left[\tilde{H}_{K\pi}(s) + \tilde{H}_{K\eta}(s)\right]}$$

Jamin, Pich, Portolés '06

$$\Gamma_{K_*}(q^2) = \frac{M_{K_*} q^2}{128\pi F^2}\left[\lambda^{3/2}\left(1, \frac{m_K^2}{q^2}, \frac{m_\pi^2}{q^2}\right)\theta(q^2 - thr_{K\pi}) + \lambda^{3/2}\left(1, \frac{m_K^2}{q^2}, \frac{m_\eta^2}{q^2}\right)\theta(q^2 - thr_{K\eta})\right]$$

$$\langle\pi^-(p)|\bar{s}\gamma^\mu u|K^0(k)\rangle = \left[(k+p)^\mu - \frac{m_K^2 - m_\pi^2}{q^2}(k-p)^\mu\right]F_+(q^2) + \frac{m_K^2 - m_\pi^2}{q^2}(k-p)^\mu F_0(q^2)$$

Jamin, Oller, Pich '01, '06

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Form factors in two meson τ decays

• The procedure for other two meson decay channels is analogous (Although SFF relevant in some decay channels).

• An alternative procedure for resummation is described at 'Ongoing/Future improvements'

$$F(s)^{VMD} = \frac{M_V^2}{M_V^2 - s - i M_V \Gamma_V(s)}$$

$$F_{PQ}^V(s) = F^{VMD}(s) \exp\left[\sum_{P,Q} N_{loop}^{PQ} \frac{-s}{96\pi^2 F^2} Re A_{PQ}(s)\right]$$

$$A(m_P^2/s, m_P^2/\mu^2) = \ln\left(m_P^2/\mu^2\right) + \frac{8m_P^2}{s} - \frac{5}{3} + \sigma_P^3 \ln\left(\frac{\sigma_P + 1}{\sigma_P - 1}\right) \qquad \sigma_P \equiv \sqrt{1 - 4m_P^2/s}$$

High-E constraints in two meson τ decays

- The **RχT** form factors must satisfy the **QCD high-energy** behaviour in this limit.

- To be satisfied at **LO in 1/N$_C$** (∞ # of states). However, phenomenology suggests that the effect of higher orders and excitations on these constraints is pretty small:

Guerrero-Pich '97

$\pi\pi$
$$\sum_i F_V^i G_V^i = F_\pi^2 \xrightarrow[N_C \to \infty]{} F_V G_V = F_\pi^2 \longrightarrow L_9 = \frac{F_V G_V}{2M_\rho^2} = \frac{F_\pi^2}{2M_\rho^2} = 7.2 \cdot 10^{-3}$$
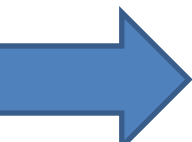
Jamin et. al. '06, '08, '10

$K\pi$
$K\eta$
$$\gamma = -\frac{F_V' G_V'}{F^2} = \frac{F_V G_V}{F^2} - 1 \xrightarrow[N_C \to \infty]{} \gamma = 0 \quad \& \quad \gamma = -0.05 \pm 0.02$$

Escribano et. al. '13

(See talk by Sergi González-Solís)

Error of neglecting heavier resonances and higher orders contribution $\leq$ 5% in two meson modes (assumably higher in many meson modes because richer dynamics)
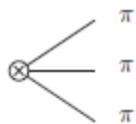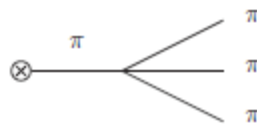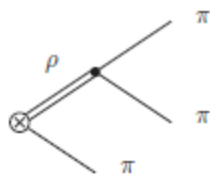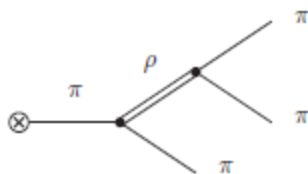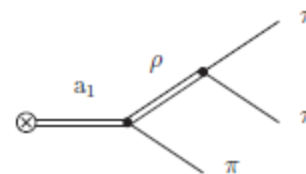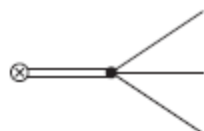
Pablo Roig (UNAM, México D.F.)   Theoretical uncertainty in RChL hadronic currents

Form factors in three meson τ decays

$$\tau \rightarrow \pi\pi\pi\nu_\tau$$

$$\tau \rightarrow KK\pi\nu_\tau$$

Pablo Roig (UNAM, México D.F.)   Theoretical uncertainty in RChL hadronic currents

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Only one independent (relevant) form factor

Only axial-vector current

Three independent (relevant) FFs
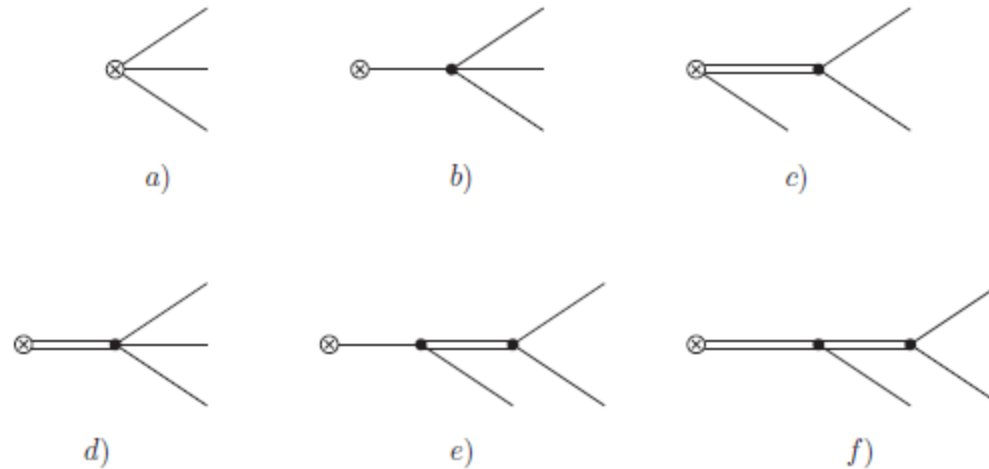
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Both axial-vector and vector current

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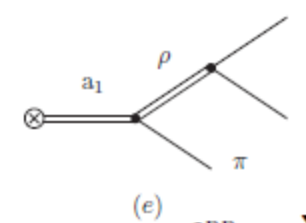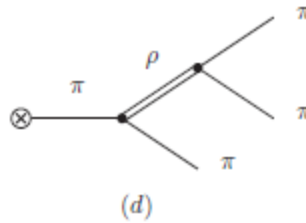Only one independent (relevant) form factor

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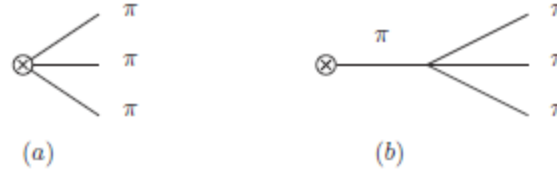
Phys.Lett.B685:158-164,2010
Gómez-Dumm, Roig, Pich, Portolés '09

$$\mathcal{L}_4^{\mathrm{V}} = \sum_{i=1}^{5} \frac{g_i}{M_V} \mathcal{O}_{\mathrm{VPPP}}^i + \sum_{i=1}^{7} \frac{c_i}{M_V} \mathcal{O}_{\mathrm{VJP}}^i$$

$$\mathcal{L}_2^{\mathrm{RR}} = \sum_{i=1}^{5} \lambda_i \mathcal{O}_{\mathrm{VAP}}^i + \sum_{i=1}^{4} d_i \mathcal{O}_{\mathrm{VVP}}^i$$
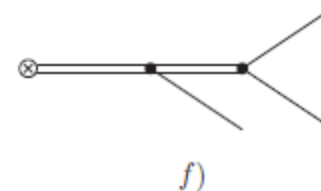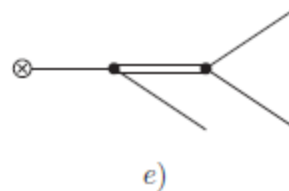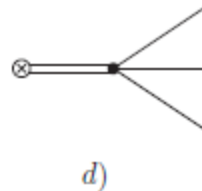
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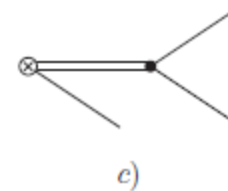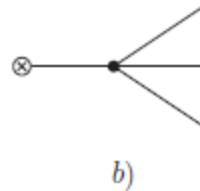
Gómez-Dumm, Roig, Pich, Portolés '09
Phys.Rev.D81:034031,2010

Pablo Roig (UNAM, México D.F.)

Theoretical uncertainty in RChL hadronic currents

Form factors in three meson τ decays

$$F_{\pm i} = \pm (F_i^\chi + F_i^{\rm R} + F_i^{\rm RR}) \,, \qquad i = 1, 2 \qquad\qquad F_2(Q^2, s, t) = F_1(Q^2, t, s)$$

$$F_1^\chi(Q^2, s, t) = -\frac{2\sqrt{2}}{3F}$$

$$F_1^{\rm R}(Q^2, s, t) = \frac{\sqrt{2}\, F_V\, G_V}{3\, F^3} \left[\frac{3\, s}{s - M_V^2} - \left(\frac{2G_V}{F_V} - 1 \right) \left(\frac{2Q^2 - 2s - u}{s - M_V^2} + \frac{u - s}{t - M_V^2} \right) \right]$$

$$F_1^{\rm RR}(Q^2, s, t) = \frac{4\, F_A\, G_V}{3\, F^3} \frac{Q^2}{Q^2 - M_A^2} \left[- (\lambda' + \lambda'') \frac{3\, s}{s - M_V^2} + H(Q^2, s) \frac{2Q^2 + s - u}{s - M_V^2} + H(Q^2, t) \frac{u - s}{t - M_V^2} \right]$$

$$H(Q^2, x) = - \lambda_0 \frac{m_\pi^2}{Q^2} + \lambda' \frac{x}{Q^2} + \lambda''$$

Relations from short-distance **QCD**:

$$\begin{cases} F_V\, G_V = F^2 \\ F_V^2 - F_A^2 = F^2 \\ F_V^2\, M_V^2 = F_A^2\, M_A^2 \end{cases}$$

$$\lambda' = \frac{F^2}{2\sqrt{2}\, F_A\, G_V} = \frac{M_A}{2\sqrt{2}\, M_V}\,,$$

$$\lambda'' = \frac{2\, G_V - F_V}{2\sqrt{2}\, F_A} = \frac{M_A^2 - 2M_V^2}{2\sqrt{2}\, M_V\, M_A}$$

$$4\, \lambda_0 = \lambda' + \lambda'' = \frac{M_A^2 - M_V^2}{\sqrt{2}\, M_V\, M_A}\,,$$

Pablo Roig (UNAM, México D.F.) Theoretical uncertainty in RChL hadronic currents

Form factors in three meson τ decays

Gómez-Dumm, Roig, Pich, Portolés '09

$\Gamma a_1(Q^2)$:

$$\Gamma_{a_1}(Q^2) = \Gamma_{a_1}^\pi(Q^2)\,\theta(Q^2 - 9m_\pi^2) + \Gamma_{a_1}^K(Q^2)\,\theta(Q^2 - (2m_K + m_\pi)^2) ,$$

$$\Gamma_{a_1}^{\pi,K}(Q^2) = \frac{-S}{192(2\pi)^3 F_A^2 M_{a_1}} \left(\frac{M_{a_1}^2}{Q^2} - 1\right)^2 \int ds\,dt\, T_{1+}^{\pi,K\mu}\, T_{1+\,\mu}^{\pi,K*} .$$

Pablo Roig (UNAM, México D.F.)   Theoretical uncertainty in RChL hadronic currents

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$$\frac{1}{M_\rho^2 - q^2 - iM_\rho\Gamma_\rho(q^2)} \longrightarrow \frac{1}{1+\beta_{\rho'}}\left[\frac{1}{M_\rho^2 - q^2 - iM_\rho\Gamma_\rho(q^2)} + \frac{\beta_{\rho'}}{M_{\rho'}^2 - q^2 - iM_{\rho'}\Gamma_{\rho'}(q^2)}\right]$$

Form factors in three meson τ decays

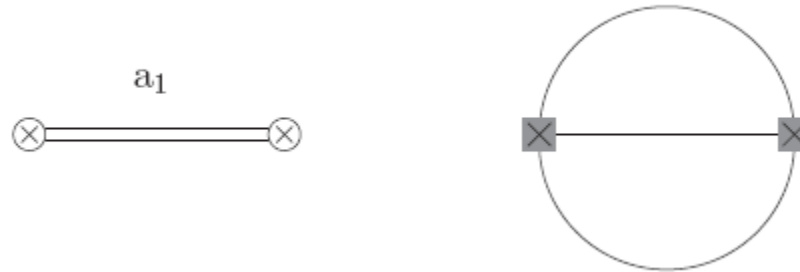Gómez-Dumm, Roig, Pich, Portolés '09

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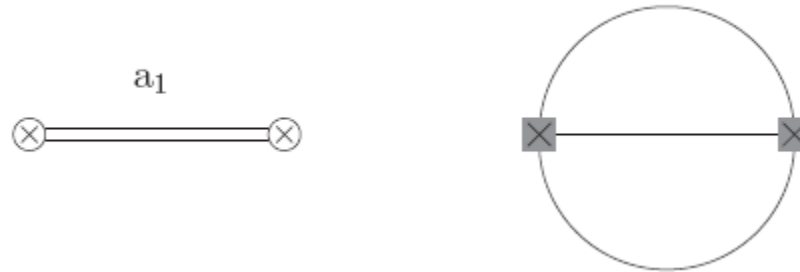
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Inclusion from a Lagrangian would imply 3 coups. instead of $\beta_{\rho'}$

F_V', G_V', F_A'

Discussion on the errors

- $\varepsilon(1/N_c) \sim 1/3$?

't Hooft '74, Witten '79

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LO in $1/N_c$ **QCD**

Pablo Roig (UNAM, México D.F.)   Theoretical uncertainty in RChL hadronic currents

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JHEP 0408:042,2004   JHEP 0701:039,2007   JHEP 0807:014,2008   JHEP 1102:109,2011   Phys.Rev.D75:114011,2007

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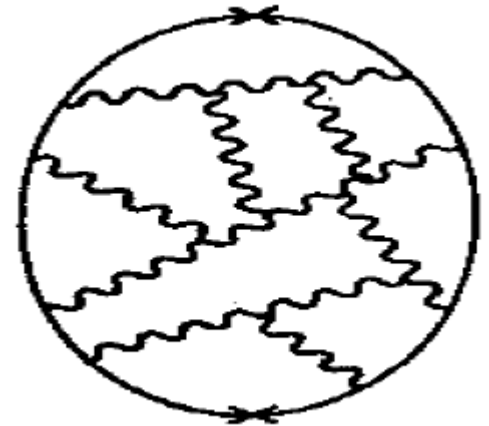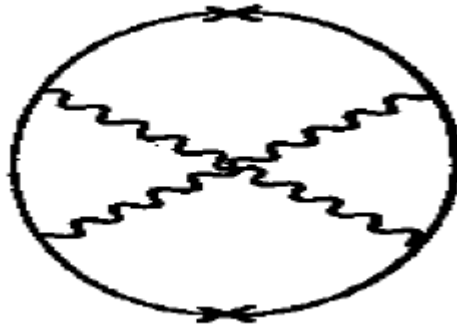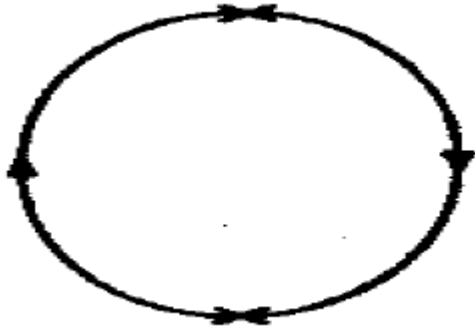→ We cannot specify the expansion parameter ($\sim 1/N_c$)

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Gómez-Dumm & Roig '12, '13   Escribano, González-Solís, Roig '13

Phys.Rev. D86 (2012) 076009   Eur. Phys. J. C (2013) 73: 2528

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Alternatively:

Exact Unitarity

$$F_V(s) = \frac{M_V^2}{M_V^2\left[1 + \sum_{P,Q} N_{loop}^{PQ} \frac{s}{96\pi^2 F^2} A_{PQ}(s)\right] - s}$$

$$\delta^{PQ}(s) = Im\left[F_V^{PQ}(s)\right] / Re\left[F_V^{PQ}(s)\right]$$

$$F_V^{PQ}(s) = \exp\left\{\alpha_1 s + \alpha_2 s^2 + \frac{s^3}{\pi}\int_{s_{thr}}^{s_{cut}} ds' \frac{\delta^{PQ}(s')}{s'^3(s'-s-i\epsilon)}\right\}$$

Tiny differences in observables between both approaches

Jamin, Pich, Portolés '06, '08 Boito, Escribano, Jamin '07; Dumm, Roig '13; Escribano, González-Solís, Roig '13

Pablo Roig (UNAM, México D.F.)   Theoretical uncertainty in RChL hadronic currents

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$$F_I^{\text{scal}}(x) = F_I^{\text{resonant}}(x) + R_I^{\text{scal}}(x)$$

$$
\begin{aligned}
F_+ &= F_+^\chi + F_+^R + F_+^{RR} + \sqrt{2}\left[R_0^{\text{scal}}(s) + R_0^{\text{scal}}(t)\right] + R_2^{\text{scal}}(s) + R_2^{\text{scal}}(t), \\
F_- &= -(F_+^\chi + F_+^R + F_+^{RR}) - \left[R_0^{\text{scal}}(s) + R_0^{\text{scal}}(t)\right] + \sqrt{2}\left[R_2^{\text{scal}}(s) + R_2^{\text{scal}}(t)\right]
\end{aligned}
$$

Isidori, Maiani, Nicolacci, Pacetti

JHEP 0605 (2006) 049

$$R_0(x) = \left\{\frac{\alpha_0}{Q^2} + \frac{\alpha_1}{Q^4}(x - M_{f_0}^2) + \mathcal{O}\left[(x - M_{f_0}^2)^2\right]\right\} e^{i\delta_0(x)}$$

Schenk **Nucl.Phys. B363 (1991) 97-116**
Colangelo, Gasser, Leutywler
Nucl.Phys. B603 (2001) 125-179

$$\tan\delta_I(x) = \sigma_\pi(x)\left(A_0^I + B_0^I q^2 + C_0^I q^4 + D_0^I q^6\right)\frac{4m_\pi^2 - x_0^I}{x - x_0^I}$$

As temporary solution we have used a phenomenological parametrization (see Olga's talk)

Pablo Roig (UNAM, México D.F.)   Theoretical uncertainty in RChL hadronic currents

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  López Castro et. al. **Phys.Rev.D74:071301,2006**
  Cirigliano, Ecker, Neufeld
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  Phys.Lett.B513:361-370,2001
  JHEP 0208:002,2002

- Some important remaining modes: $\pi\pi\pi\pi$, $K\pi\pi$, SFF in $K\pi$ ...
  Jamin, Oller, Pich '01,'06

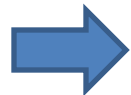 **Nucl.Phys.B622:279-308,2002**   **Phys.Rev.D74:074009,2006**

Pablo Roig (UNAM, México D.F.)   Theoretical uncertainty in RChL hadronic currents

Ongoing/Future improvements
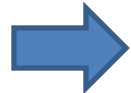**(through common work with experimentalists: D. Epifanov, I. Nugent, …)**

Ongoing/Future improvements

(through common work with experimentalists: D. Epifanov, I. Nugent, …)

- $\varepsilon(1/N_c) \sim 1/3$

• Resummation in the two meson modes. ε below 3%.
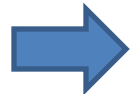
•FSI in three meson modes. Relevant in $d\Gamma/ds$ (~ 10%).

•SU(3) breaking terms in the Lagrangian. (KKπ, ~ 30%)

Moussallam
Eur.Phys.J.C53:401-412,2008

•Spin zero resonance contributions. ($\pi\pi\pi$)

•Excited resonance contribution in KKπ channels.

Bijnens, Colangelo, Talavera
JHEP 9805:014,1998

• Complete O(p^6) χPT, i.e. NNLO, in $\pi\pi$ channels.

López Castro et. al. **Phys.Rev.D74:071301,2006**

Cirigliano, Ecker, Neufeld

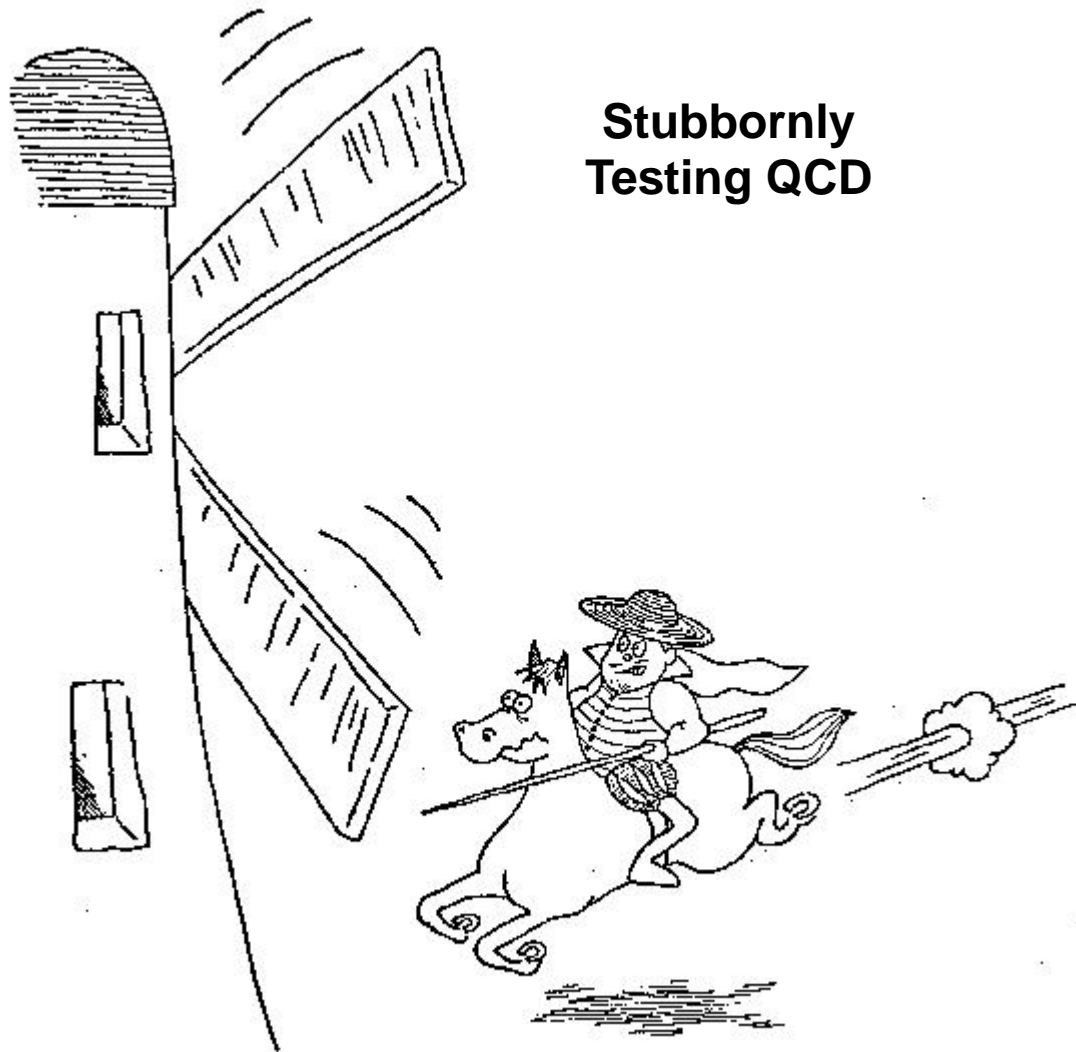•SU(2) breaking in $\pi\pi$ channels.
Phys.Lett.B513:361-370,2001
JHEP 0208:002,2002

•Some important remaining modes: $\pi\pi\pi\pi$, K$\pi\pi$, SFF in Kπ … Jamin, Oller, Pich '01,'06

•Remaining two meson modes: $\pi\eta^{(')},K\eta^{(')}$ Escribano, González-Solís, Roig '13

Pablo Roig (UNAM, México D.F.)   Theoretical uncertainty in RChL hadronic currents

Stubbornly Testing QCD

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