Using dispersion relations for hadronic τ-decays: from SM to New Physics

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LANL

Workshop on τ lepton decay

Krakow, September 16, 2013

In collaboration with :

- *V. Bernard (IPN-Orsay), D. R. Boito (TU-Munich),* work in progress
- *M. Antonelli, (INF-LNF Frascati),V. Cirigliano (LANL),*
  A. Lusiani (INF- Scuola Normale Pisa) [ArXiv:1304.8134 [hep-ph]]
- *A. Celis (IFIC-Valencia), V. Cirigliano (LANL),* on ArXiv tomorrow

Outline :

1. Introduction and Motivation
2. Predicting the strange Brs and implications for V_{us}
3. Constraining the LFV couplings of the Higgs with hadronic τ decays
4. Conclusion and Outlook

1. Introduction and Motivation

1.1 Hadronic τ-decays

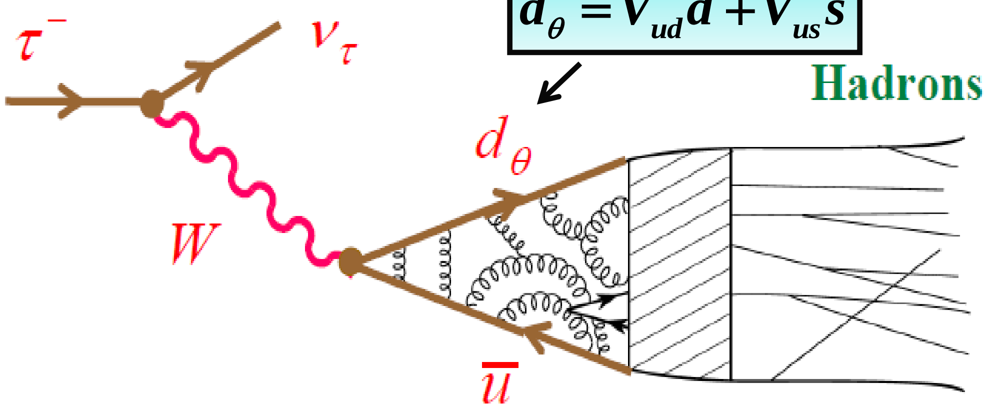
- τ lepton discovered in 1976 by M. Perl et al. (SLAC-LBL group)

PDG'12

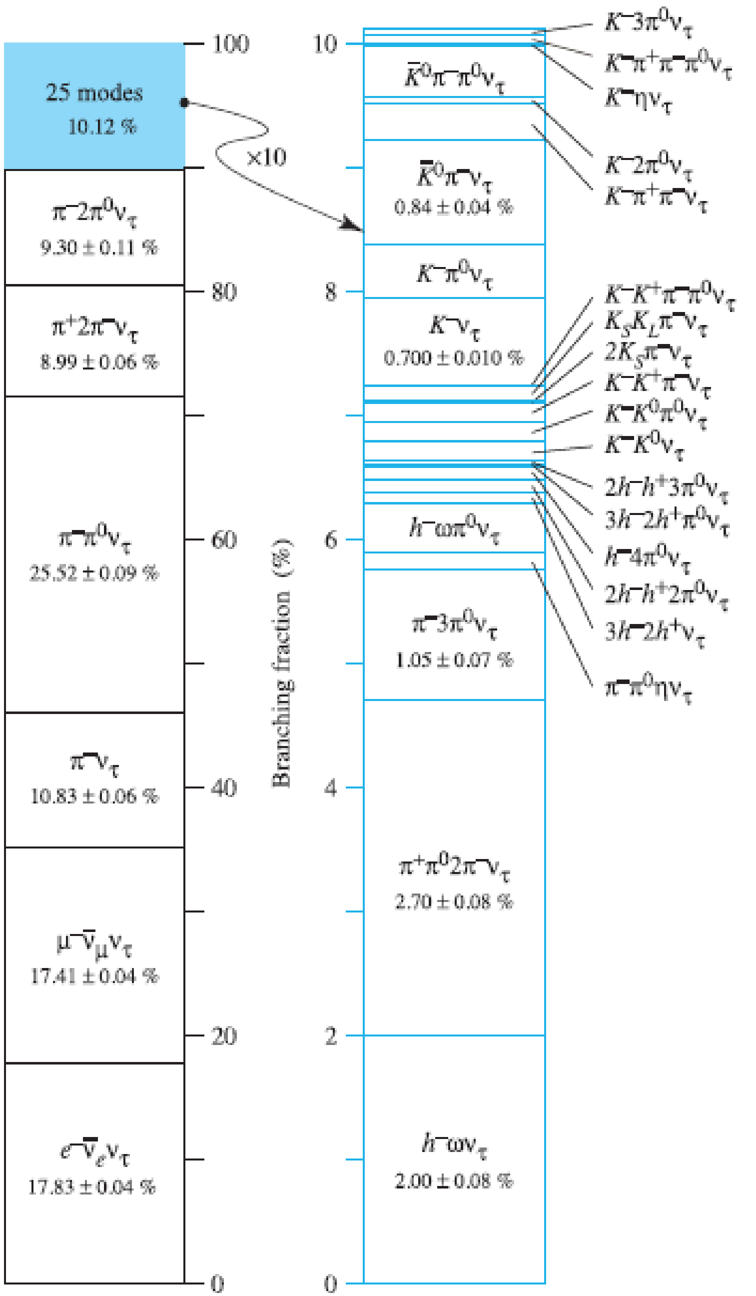
  - Mass : $\boxed{m_\tau = 1.77682(16)\ \mathrm{GeV}}$
  - Lifetime : $\boxed{\tau_\tau = 2.096(10)\cdot 10^{-13}\, s}$

- The only lepton heavy enough to decay into hadrons : lots of semileptonic decays !

$$\boxed{d_\theta = V_{ud}\, d + V_{us}\, s}$$



PDG'12



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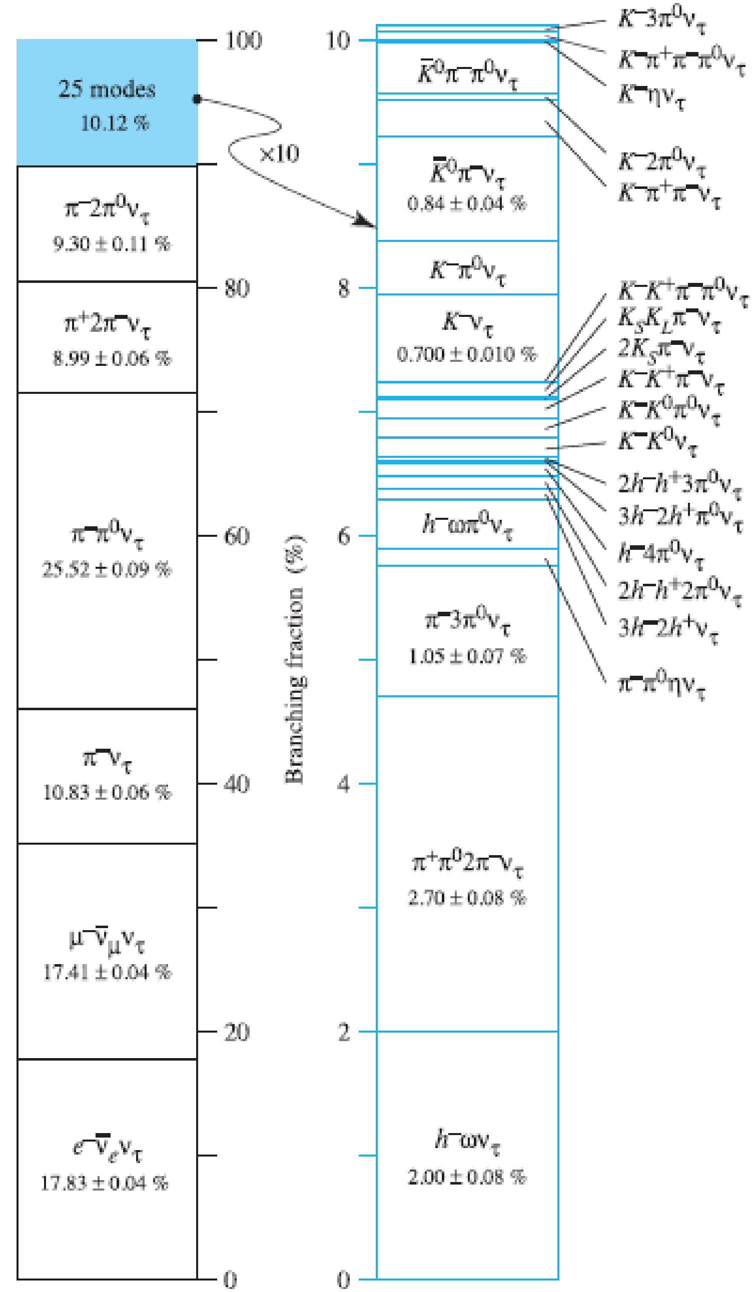
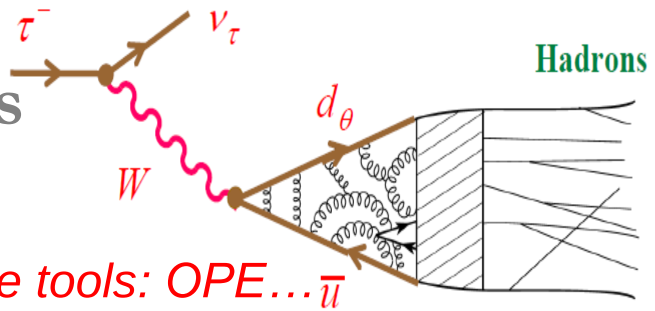
- The only lepton heavy enough to decay into hadrons : lots of semileptonic decays !

  ⇨ Very rich phenomenology

  Test of QCD and EW interactions
- For the tests:
  - Precise measurements needed
  - Hadronic uncertainties under control



1.2 Test of QCD and EW interactions



- Inclusive τ decays : full hadron spectra, *perturbative tools: OPE…*

$\boxed{\tau \rightarrow (\bar{u}d, \bar{u}s)\nu_\tau}$ ⇨ fundamental SM parameters: $\alpha_S(m_\tau)$, $|V_{us}|$, m_s
QCD studies

- Exclusive τ decays : specific hadron spectrum, *non perturbative tools*

$\boxed{\tau \rightarrow (PP, PPP, \ldots)\nu_\tau}$ ⇨ Study of ffs, resonance parameters (M_R, Γ_R)
Hadronization of QCD currents

- τ decays: tool to search for New Physics in inclusive and exclusive decays :

⇨ Unitarity test, CPV, LFV, EDMs, etc.

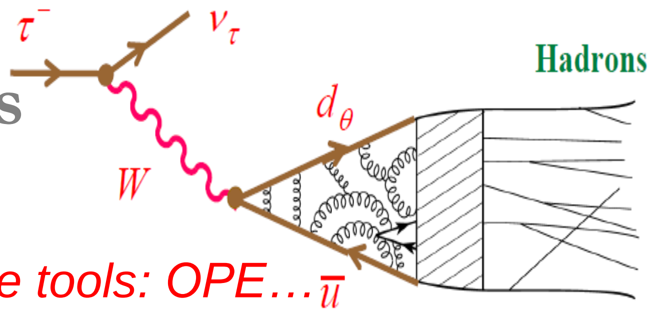
Test of unitarity $\boxed{|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \stackrel{?}{=} 1}$

$0^+ \rightarrow 0^+$ β decays

K_{l3} decays or τ decays

Negligible (B decays)

1.2 Test of QCD and EW interactions



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QCD studies

- Exclusive τ decays : specific hadron spectrum, *non perturbative tools*

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Study of ffs, resonance parameters (M_R, Γ_R)
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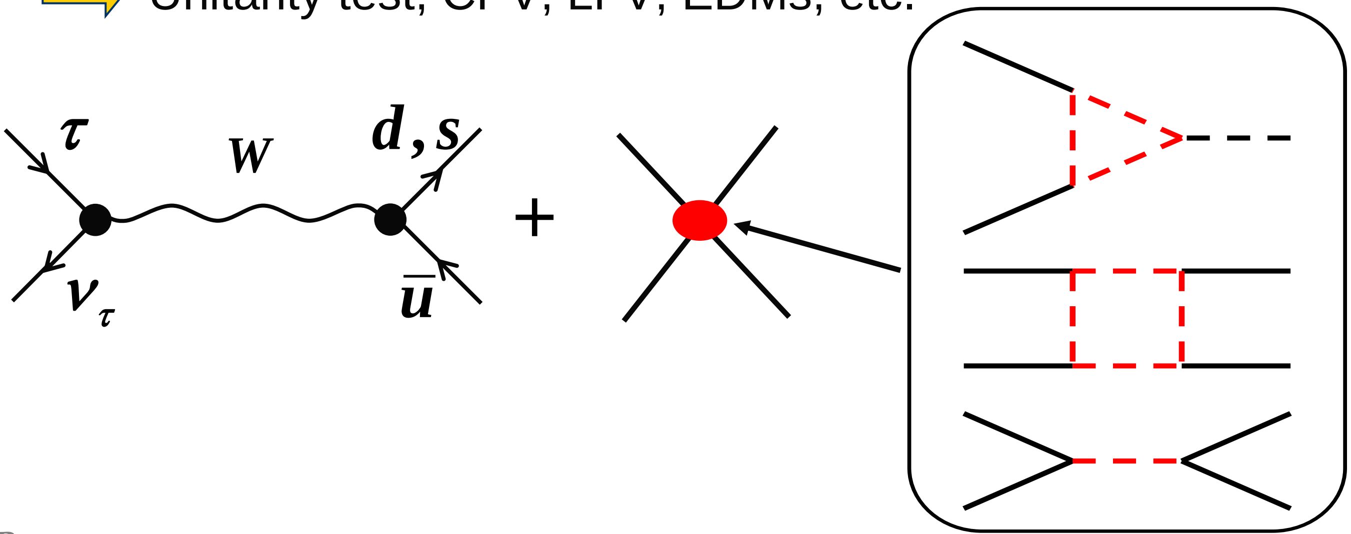


SUSY loops,
Leptoquarks,
Z', Charged Higgs,
Right-Handed
Currents,….

2. Predicting the strange Brs and implication for V_{us}

2.1 Introduction

- $$\boxed{R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e\right)} = R_\tau^{NS} + R_\tau^{S} \approx \left|V_{ud}\right|^2 N_C + \left|V_{us}\right|^2 N_C}$$ naïve QCD prediction

  ⇨ Experimentally $R_\tau = \dfrac{1 - B_e - B_\mu}{B_e} = 3.6291 \pm 0.0086$

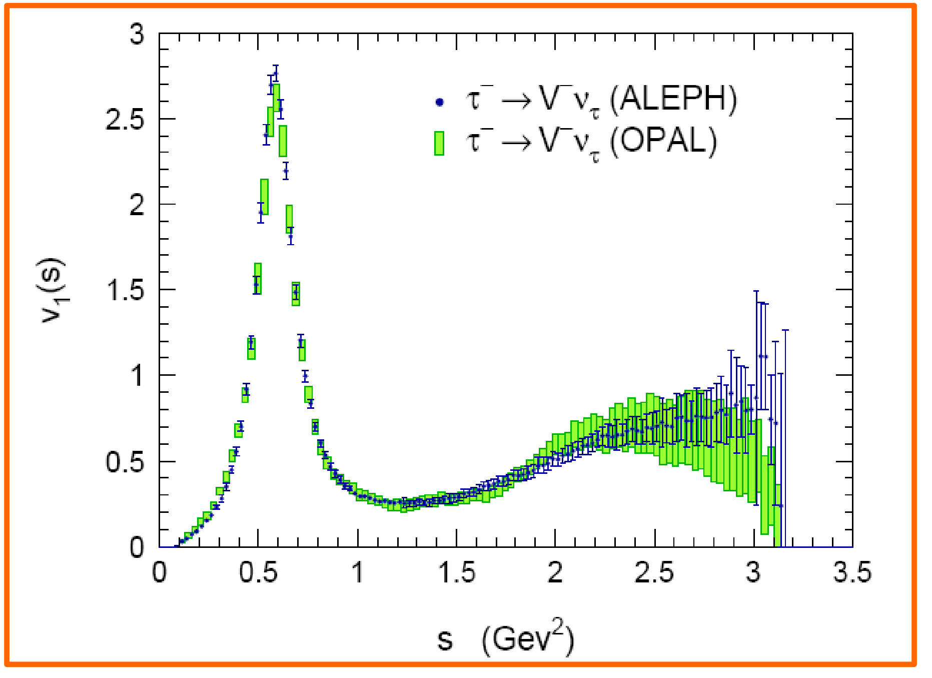
- Difficulty ⇨ QCD corrections : $R_\tau = \left|V_{ud}\right|^2 N_C + \left|V_{us}\right|^2 N_C + O\left(\alpha_S\right)$

- Extraction of the strong coupling constant : $R_\tau^{NS} = f\left(\alpha_S\right)$ ⇨ α_S

  measured calculated

- Determination of V_{us} :

  ⇨ $$\boxed{\left|V_{us}\right|^2 = \frac{R_\tau^S}{\dfrac{R_\tau^{NS}}{\left|V_{ud}\right|^2} - \delta R_{\tau,th}}}$$



2.2 Extraction of V_{us}

- $$\boxed{\delta R_\tau \equiv \frac{R_\tau^{NS}}{|V_{ud}|^2} - \frac{R_\tau^{S}}{|V_{us}|^2}} = N_C S_{EW}\left(\delta_{NP}^{NS} - \delta_{NP}^{S}\right)$$

$SU(3)$ breaking quantity

→ 0 in the $SU(3)$ limit, small, calculable with *OPE*

$$\delta R_\tau = f\left(m_s\right) \Rightarrow \delta R_{\tau,th} = 0.240(32)$$

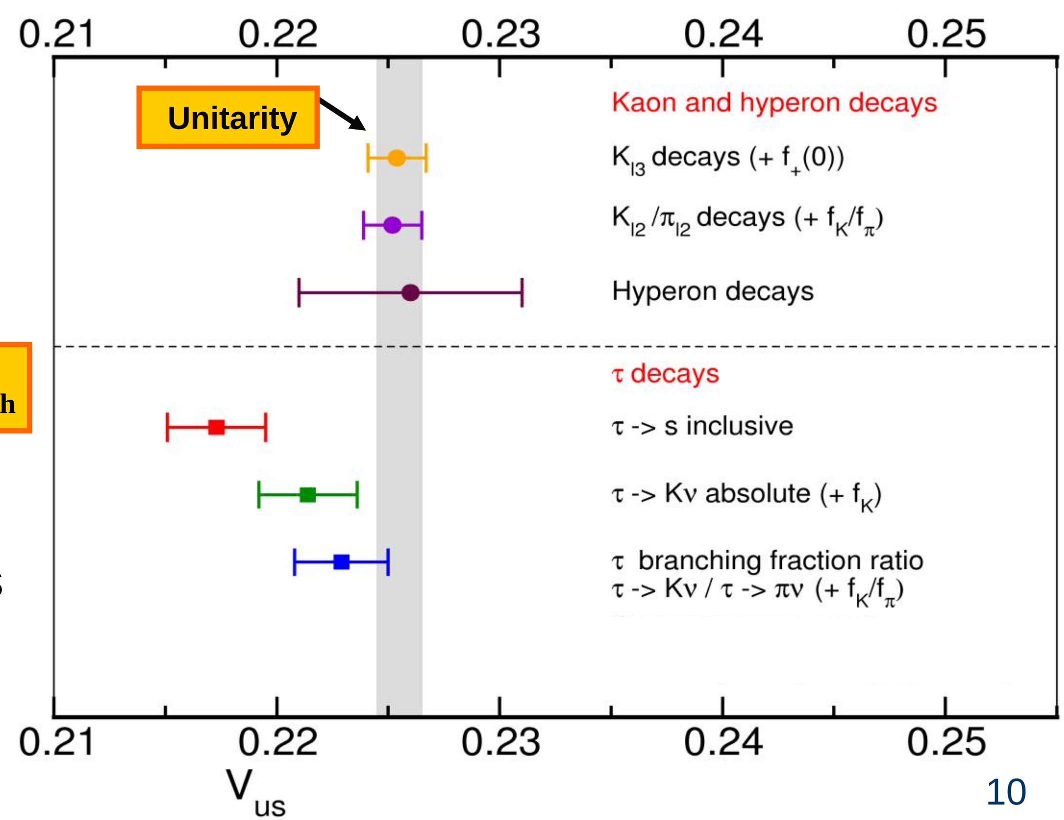
Gamiz, Jamin, Pich, Prades, Schwab'07, Maltman'11

- $$\boxed{|V_{us}|^2 = \frac{R_\tau^S}{\frac{R_\tau^{NS}}{|V_{ud}|^2} - \delta R_{\tau,th}}}$$

→ $$\boxed{|V_{us}| = 0.2173 \pm 0.0020_{\text{exp}} \pm 0.0010_{\text{th}}}$$

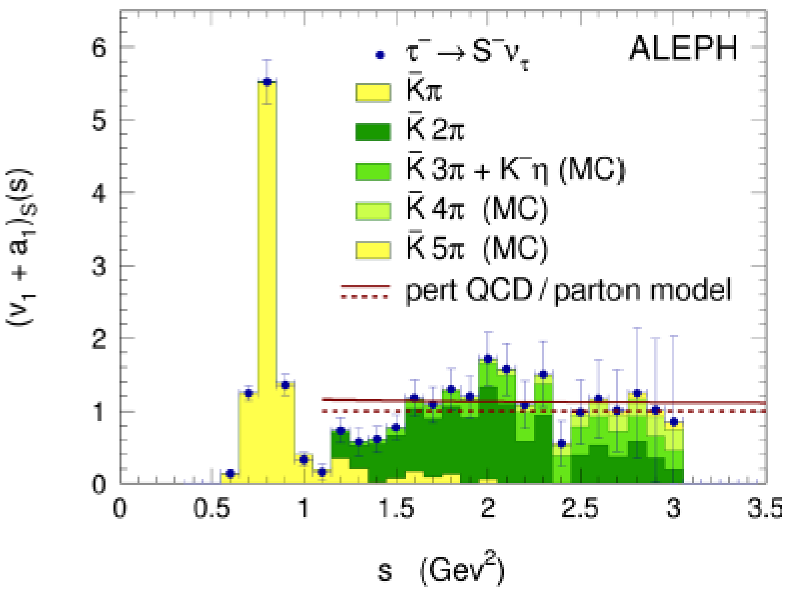
2.6σ away from unitarity!
Dominated by exp. uncertainties contrary to K_{l3}

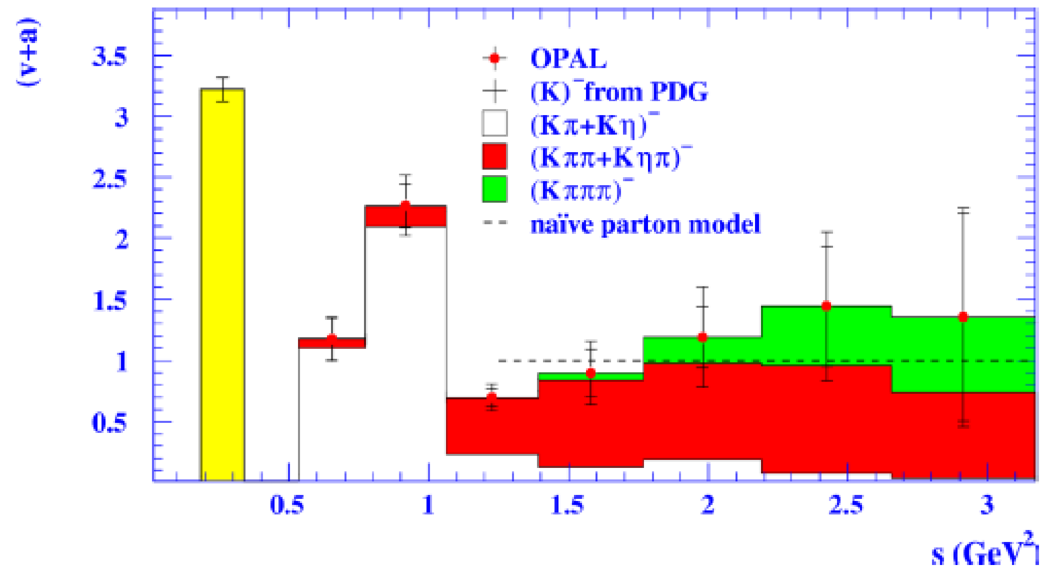
→ Potentially the more precise determination of V_{us}



2.3 τ strange Brs

- Experimental measurements of the strange spectral functions not very precise





New measurements are needed !

- Before B-factories
- With B-factories new measurements :

Smaller $\tau \to$ K branching ratios ⇨ smaller $R_{\tau,S}$ ⇨ smaller $\boxed{V_{us}}$

$$\boxed{R_\tau^S\Big|_{\text{old}} = 0.1686(47)} \Rightarrow \boxed{R_\tau^S\Big|_{\text{new}} = 0.1612(28)}$$

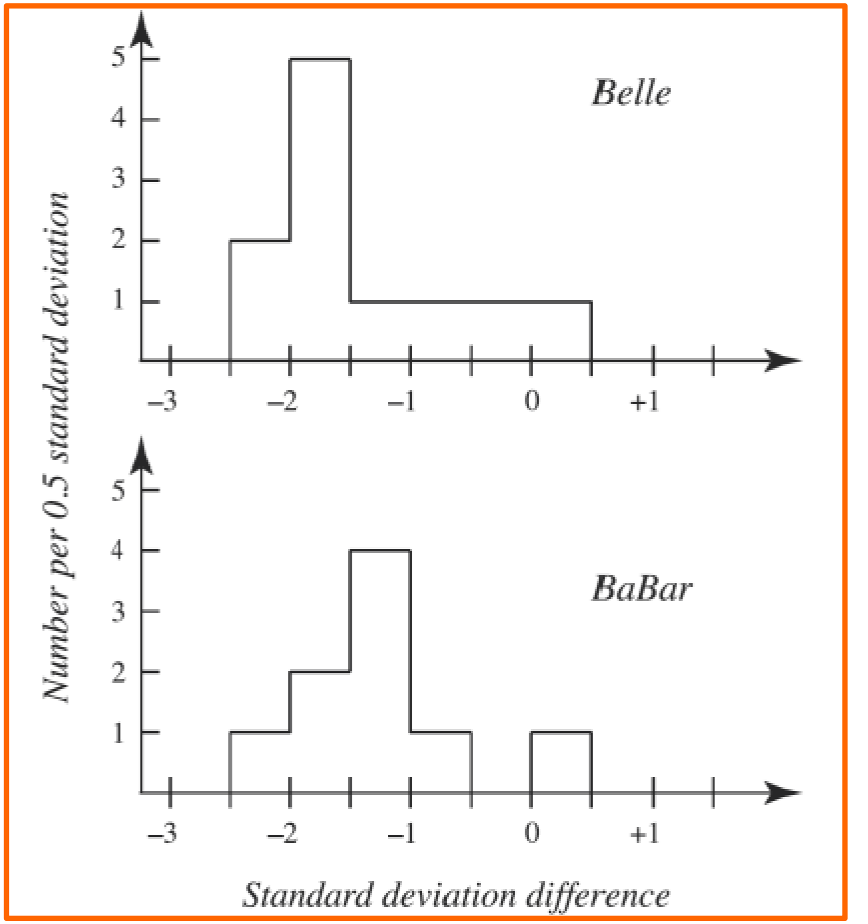
$$\boxed{|V_{us}|_{\text{old}} = 0.2214 \pm 0.0031_{\text{exp}} \pm 0.0010_{\text{th}}} \Rightarrow \boxed{|V_{us}|_{\text{new}} = 0.2173 \pm 0.0020_{\text{exp}} \pm 0.0010_{\text{th}}}$$

2.3 τ strange Brs

- *PDG 2012*: « Eigtheen of the 20 *B*-factory branching fraction measurements are smaller than the non-*B*-factory values. The average normalized difference between the two sets of measurements is -1.30 » (-1.41 for the 11 Belle measurements and -1.24 for the 9 BaBar measurements)

- Measured modes by the 2 B factories:

Mode	BaBar – Belle Normalized Difference (#σ)
$\pi^-\pi^+\pi^-\nu_\tau$ (ex. K^0)	$+1.4$
$K^-\pi^+\pi^-\nu_\tau$ (ex. K^0)	-2.9
$K^-K^+\pi^-\nu_\tau$	-2.9
$K^-K^+K^-\nu_\tau$	-5.4
$\eta\ K^-\nu_\tau$	-1.0
$\phi\ K^-\nu_\tau$	-1.3



2.3 τ strange Brs

- Modes measured in the strange channel for $\tau \to s$:

Branching fraction	HFAG Winter 2012 fit
$\Gamma_{10} = K^-\nu_\tau$	$(0.6955 \pm 0.0096) \cdot 10^{-2}$
$\Gamma_{16} = K^-\pi^0\nu_\tau$	$(0.4322 \pm 0.0149) \cdot 10^{-2}$
$\Gamma_{23} = K^-2\pi^0\nu_\tau$ (ex. K^0)	$(0.0630 \pm 0.0222) \cdot 10^{-2}$
$\Gamma_{28} = K^-3\pi^0\nu_\tau$ (ex. K^0, η)	$(0.0419 \pm 0.0218) \cdot 10^{-2}$
$\Gamma_{35} = \pi^-\overline{K}^0\nu_\tau$	$(0.8206 \pm 0.0182) \cdot 10^{-2}$
$\Gamma_{40} = \pi^-\overline{K}^0\pi^0\nu_\tau$	$(0.3649 \pm 0.0108) \cdot 10^{-2}$
$\Gamma_{44} = \pi^-\overline{K}^0\pi^0\pi^0\nu_\tau$	$(0.0269 \pm 0.0230) \cdot 10^{-2}$
$\Gamma_{53} = \overline{K}^0 h^- h^- h^+\nu_\tau$	$(0.0222 \pm 0.0202) \cdot 10^{-2}$
$\Gamma_{128} = K^-\eta\nu_\tau$	$(0.0153 \pm 0.0008) \cdot 10^{-2}$
$\Gamma_{130} = K^-\pi^0\eta\nu_\tau$	$(0.0048 \pm 0.0012) \cdot 10^{-2}$
$\Gamma_{132} = \pi^-\overline{K}^0\eta\nu_\tau$	$(0.0094 \pm 0.0015) \cdot 10^{-2}$
$\Gamma_{151} = K^-\omega\nu_\tau$	$(0.0410 \pm 0.0092) \cdot 10^{-2}$
$\Gamma_{801} = K^-\phi\nu_\tau(\phi \to KK)$	$(0.0037 \pm 0.0014) \cdot 10^{-2}$
$\Gamma_{802} = K^-\pi^-\pi^+\nu_\tau$ (ex. K^0, ω)	$(0.2923 \pm 0.0068) \cdot 10^{-2}$
$\Gamma_{803} = K^-\pi^-\pi^+\pi^0\nu_\tau$ (ex. K^0, ω, η)	$(0.0411 \pm 0.0143) \cdot 10^{-2}$
$\Gamma_{110} = X_s^-\nu_\tau$	$(2.8746 \pm 0.0498) \cdot 10^{-2}$

HFAG'12

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~70% of the decay modes crossed channels from Kaons!

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~70% of the decay modes crossed channels from Kaons!

Up to ~90% Including the 2π modes

2.4 Prediction of $\tau \to K\nu_\tau$ Br

Antonelli, Cirigliano, Lusiani, E.P. '13

- The Brs of these 3 modes can be predicted using Kaon Brs very precisely measured + form factor information assuming lepton universality

➢ $\tau \to K\nu_\tau$:

$$\boxed{\mathrm{BR}(\tau \to K\nu_\tau) = \frac{m_\tau^3}{2m_K m_\mu^2} \boxed{\frac{S_{\mathrm{EW}}^\tau}{S_{\mathrm{EW}}^K}} \left(\frac{1 - m_K^2/m_\tau^2}{1 - m_\mu^2/m_K^2}\right)^2 \boxed{\frac{\tau_\tau}{\tau_K}} \boxed{\delta_{\mathrm{EM}}^{\tau/K}} \boxed{\mathrm{BR}(K_{\ell 2})}}$$

➢ Inputs needed:
→ Experimental : BR(K_{l2}), lifetimes

→ Theoretical : Short distance EW corrections
Long distance EM corrections

⇨ $\boxed{BR\left(\tau^- \to K^- \nu_\tau\right) = (0.713 \pm 0.003)\%}$

2.5 Prediction of $\tau \to K\pi\nu_\tau$ Br

Antonelli, Cirigliano, Lusiani, E.P. '13

- The Brs of these 3 modes can be predicted using Kaon Brs very precisely measured + form factor information

  - $\tau \to K\pi\nu_\tau$:

$$\boxed{\mathrm{BR}(\tau \to \bar{K}\pi\nu_\tau) = \frac{2m_\tau^5}{m_K^5}\frac{S_{\mathrm{EW}}^\tau}{S_{\mathrm{EW}}^K}\frac{I_K^\tau}{I_K^\ell}\frac{\left(1+\delta_{\mathrm{EM}}^{K\tau}+\tilde{\delta}_{\mathrm{SU(2)}}^{K\pi}\right)^2}{\left(1+\delta_{\mathrm{EM}}^{K\ell}+\delta_{\mathrm{SU(2)}}^{K\pi}\right)^2}\frac{\tau_\tau}{\tau_K}\mathrm{BR}(K \to \pi e\bar{\nu}_e)}$$

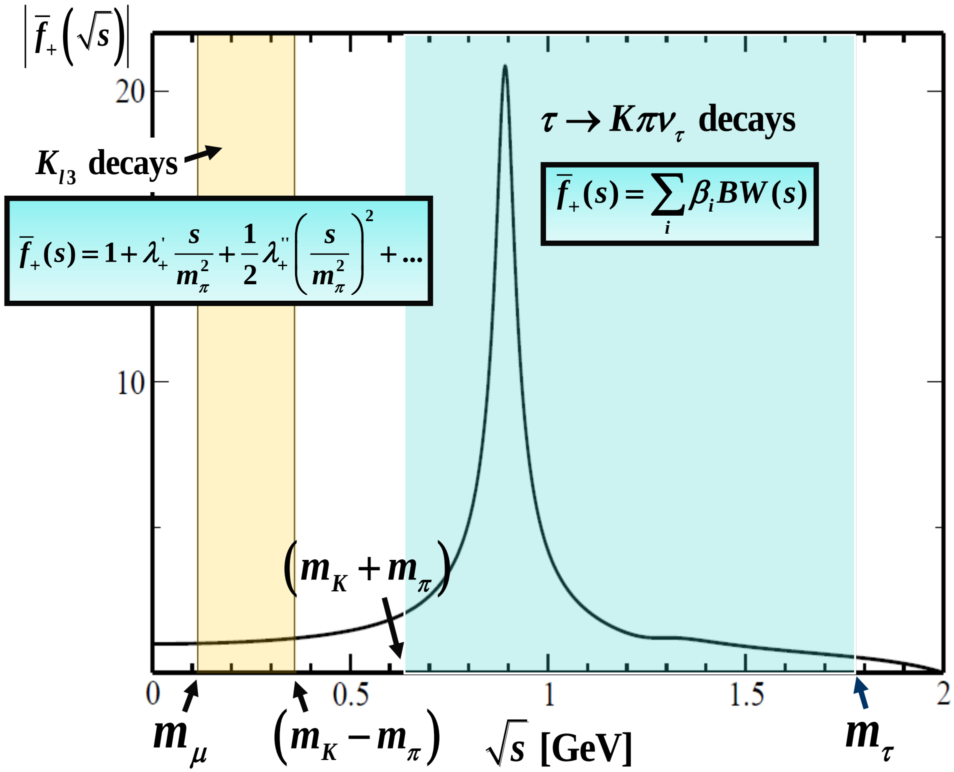
$$\boxed{I_K^{\ell,\tau} = \int ds\, F\left(s, \bar{f}_+(s), \bar{f}_0(s)\right)}$$

  - Inputs needed :
    – The K_{e3} branching ratios, lifetimes
    – Phase space integrals ⇨ use a parametrization for the form factors to determine them from the data
    – The electromagnetic and isospin-breaking corrections

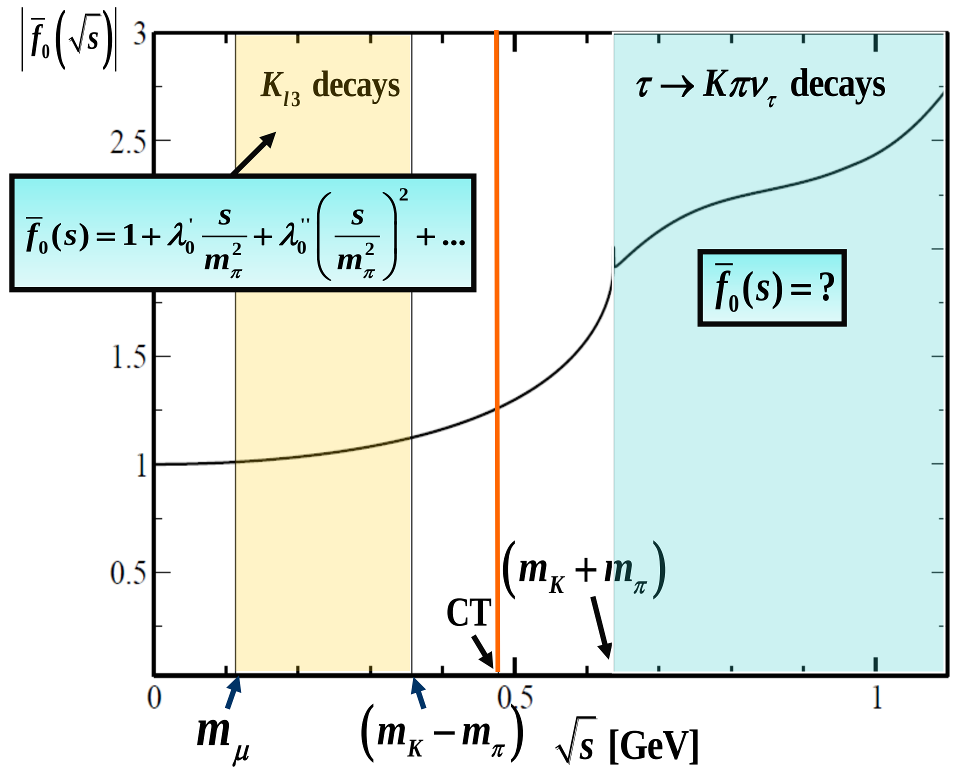
2.5.1 Determination of the Kπ form factors

- Use a *dispersive parametrization* to combine experimental information on K_{l3} $\left(K \to \pi l \nu_l\right)$ and $\tau \to K\pi\nu_\tau$ decays

Vector form factor



Scalar form factor



⇒ Dominance of K*(892) resonance

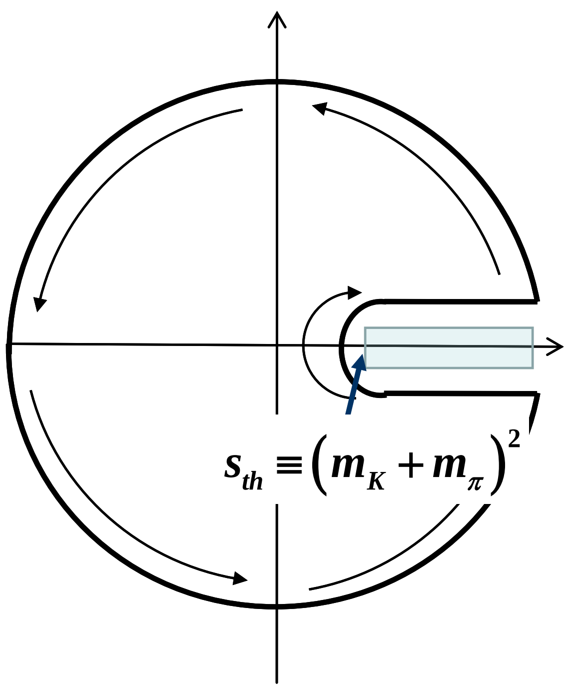
⇒ No obvious dominance of a resonance

Dispersive representation

- Parametrization to analyse both K_{l3} and τ

  ➡ Use dispersion relations

- Omnès representation: ➡ $$\boxed{\bar{f}_{+,0}(s) = \exp\left[\frac{s}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{s'}\frac{\phi_{+,0}(s')}{s'-s-i\varepsilon}\right]}$$



$\phi_{+,0}(s)$: phase of the form factor

- $s < s_{in}$: $\phi_{+,0}(s) = \delta_{K\pi}(s)$

  $K\pi$ scattering phase

- $s \geq s_{in}$: $\phi_{+,0}(s)$ unknown

  ➡ $\phi_{+,0}(s) = \phi_{+,0as}(s) = \pi \pm \pi \quad \left(\bar{f}_{+,0}(s) \to 1/s\right)$

Brodsky & Lepage

- Subtract dispersion relation to weaken the high energy contribution of the phase. Improve the convergence but sum rules to be satisfied!

Determination of the Kπ FFs: Dispersive representation

Bernard, Boito, E.P., in progress

- Dispersion relation with n subtractions in $\bar{s}$:

$$\bar{f}_{+,0}(s) = \exp\left[P_{n-1}(s) + \frac{(s-\bar{s})^n}{\pi} \int_{s_{th}}^{\infty} \frac{ds'}{(s'-\bar{s})^n} \frac{\phi_{+,0}(s')}{s'-s-i\varepsilon} \right]$$

➢ $\bar{f}_0(s)$ ⇨ dispersion relation with 3 subtractions: 2 in s=0 and 1 in s=$\Delta_{K\pi}$

Callan-Treiman

$$\bar{f}_0(s) = \exp\left[\frac{s}{\Delta_{K\pi}} \left(\ln C + (s-\Delta_{K\pi}) \left(\frac{\ln C}{\Delta_{K\pi}} - \frac{\lambda_0'}{m_\pi^2} \right) + \frac{\Delta_{K\pi} s (s-\Delta_{K\pi})}{\pi} \int_{(m_K+m_\pi)^2}^{\infty} \frac{ds'}{s'^2} \frac{\phi_0(s')}{(s'-\Delta_{K\pi})(s'-s-i\varepsilon)} \right) \right]$$

➢ $\bar{f}_+(s)$ ⇨ dispersion relation with 3 subtractions in s=0

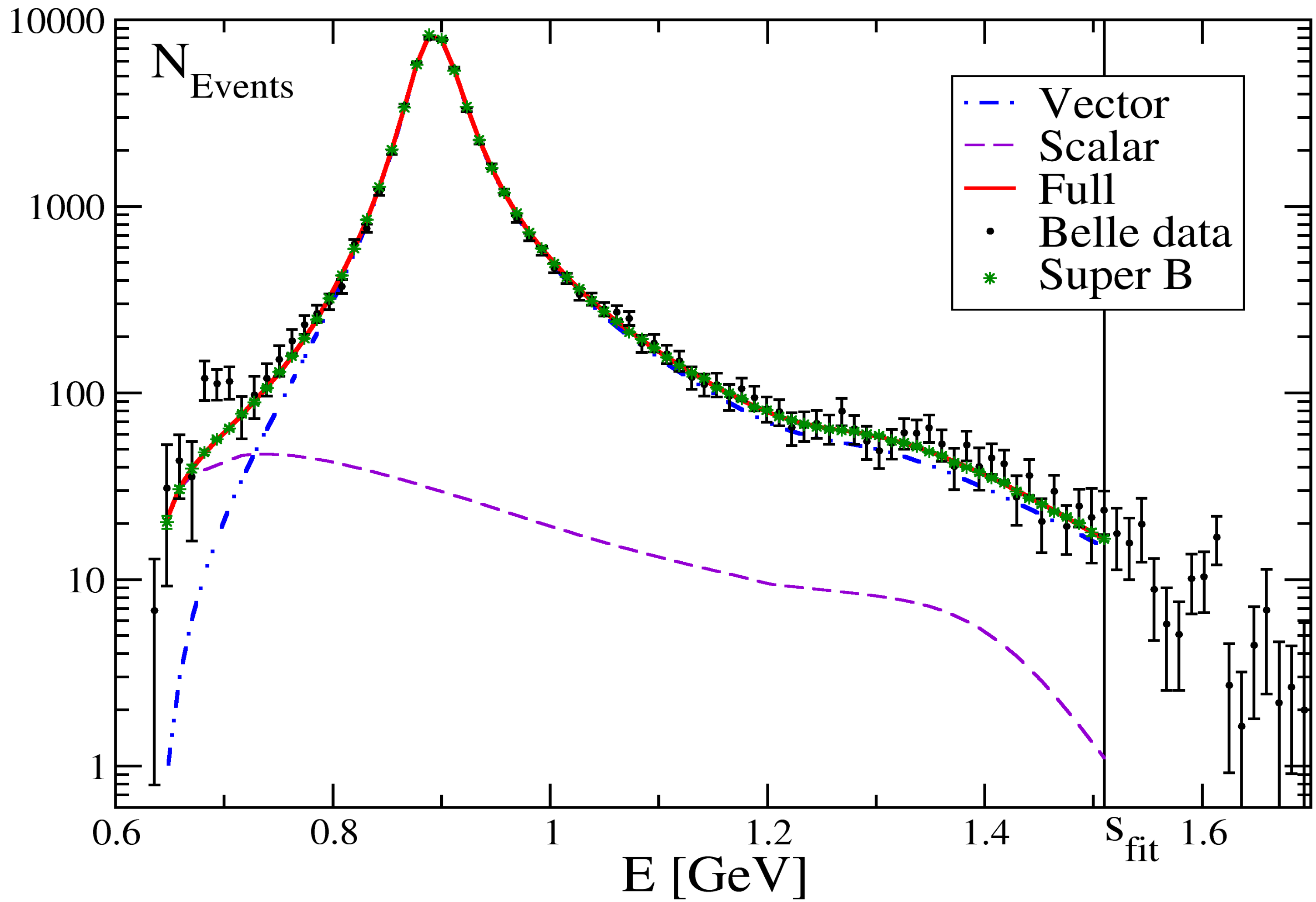
Boito, Escribano, Jamin'09,'10

$$\bar{f}_+(s) = \exp\left[\lambda_+' \frac{s}{m_\pi^2} + \frac{1}{2}\left(\lambda_+'' - \lambda_+'^2\right) \left(\frac{s}{m_\pi^2} \right)^2 + \frac{s^3}{\pi} \int_{(m_K+m_\pi)^2}^{\infty} \frac{ds'}{s'^3} \frac{\phi_+(s')}{(s'-s-i\varepsilon)} \right]$$

Jamin, Pich, Portolés'08

Extracted from a model including
2 resonances K*(892) and K*(1414)

Kπ form factors from $\tau \to K\pi\nu_\tau$ and K_{l3} decays



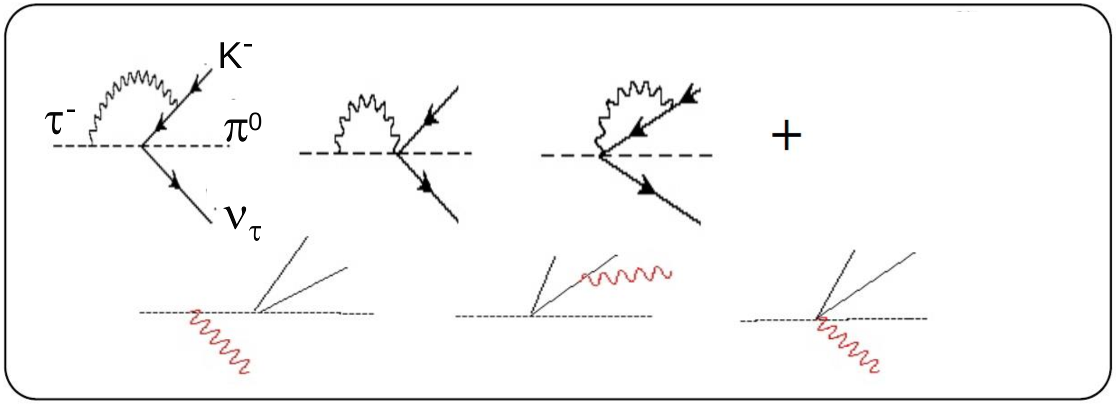
2.5.2 Long-distance electromagnetic corrections

Antonelli, Cirigliano, Lusiani, E.P. '13

- For precise calculations, it is crucial to estimate δ^{τ}_{EM} the long-distance EM corrections to $\tau \to K\pi\nu_\tau$

  Up to now neglected!

- Adapt the calculations of $\tau^- \to \pi^- \pi^0 \nu_\tau$ *Cirigliano, Ecker, Neufeld'02*



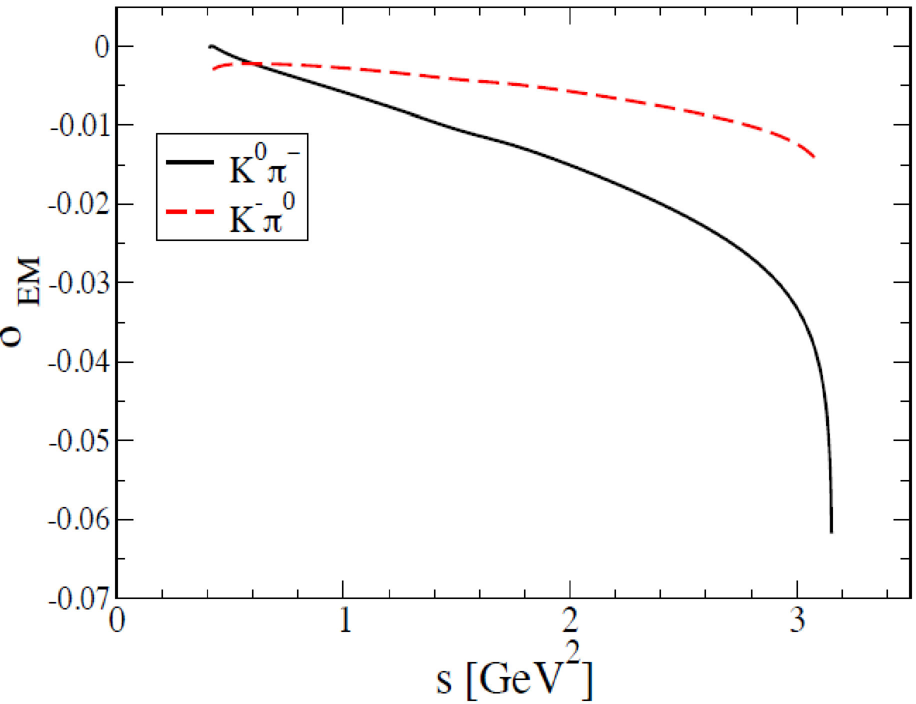
→ ChPT to O(p^2e^2)

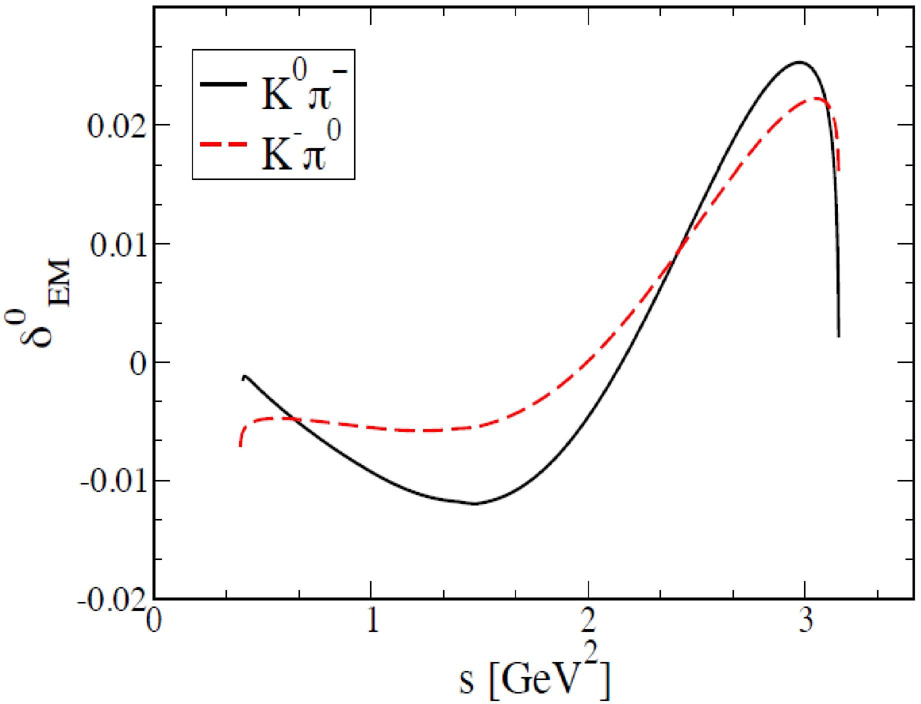
→ Counter-terms neglected

2.5.2 Long-distance electromagnetic corrections

Antonelli, Cirigliano, Lusiani, E.P. '13

- Form factors corrections:





$$\delta_{\mathrm{EM}}^{\bar{K}^0\tau} = (-0.15 \pm 0.2)\,\%$$

$$\delta_{\mathrm{EM}}^{K^-\tau} = (-0.2 \pm 0.2)\,\%$$

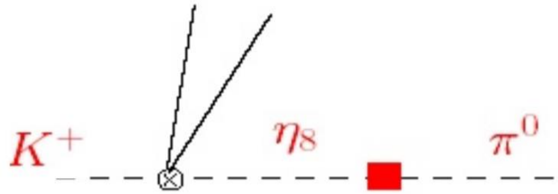
to be compared

Mode	$\delta_{\mathrm{EM}}^{K\ell}$ (%)
K^0_{e3}	0.495 ± 0.110
$K^{\pm}_{e3}$	0.050 ± 0.125
$K^0_{\mu 3}$	0.700 ± 0.110
$K^{\pm}_{\mu 3}$	0.008 ± 0.125

2.5.3 Isospin breaking corrections

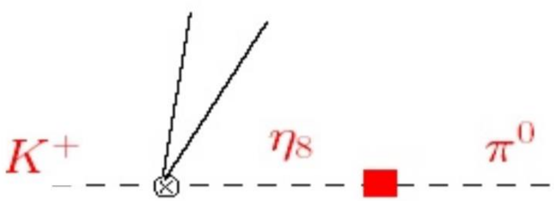
$$\delta^{K\pi}_{\mathrm{SU(2)}} = \frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} - 1$$

Antonelli, Cirigliano, Lusiani, E.P. '13



+ IB in one loop graphs + CT

approximated by



+ IB in the K*- to Kπ coupling

$$\Rightarrow \quad \frac{f_+^{K^-\pi^0}(s)}{f_+^{K^0\pi^-}(s)} = \left(1+\sqrt{3}\varepsilon\right)\left(1+\tilde{g}\,\frac{m_K^2}{(4\pi F_\pi)^2}\,\frac{s}{m_{K^*}^2}\,\varepsilon\right)$$

with $\varepsilon = \frac{\sqrt{3}}{4}\frac{m_d - m_u}{m_s - \hat{m}}$

$\tilde{g} \in [-2, 2] \Rightarrow \tilde{\delta}^{K\pi}_{\mathrm{SU(2)}} = \pm 0.5\%$

ε from *FLAG* $\Rightarrow \tilde{\delta}^{K\pi}_{\mathrm{SU(2)}} = \left(2.9 \pm 0.4_{\mathrm{mixing}} \pm 0.5\right)\%$

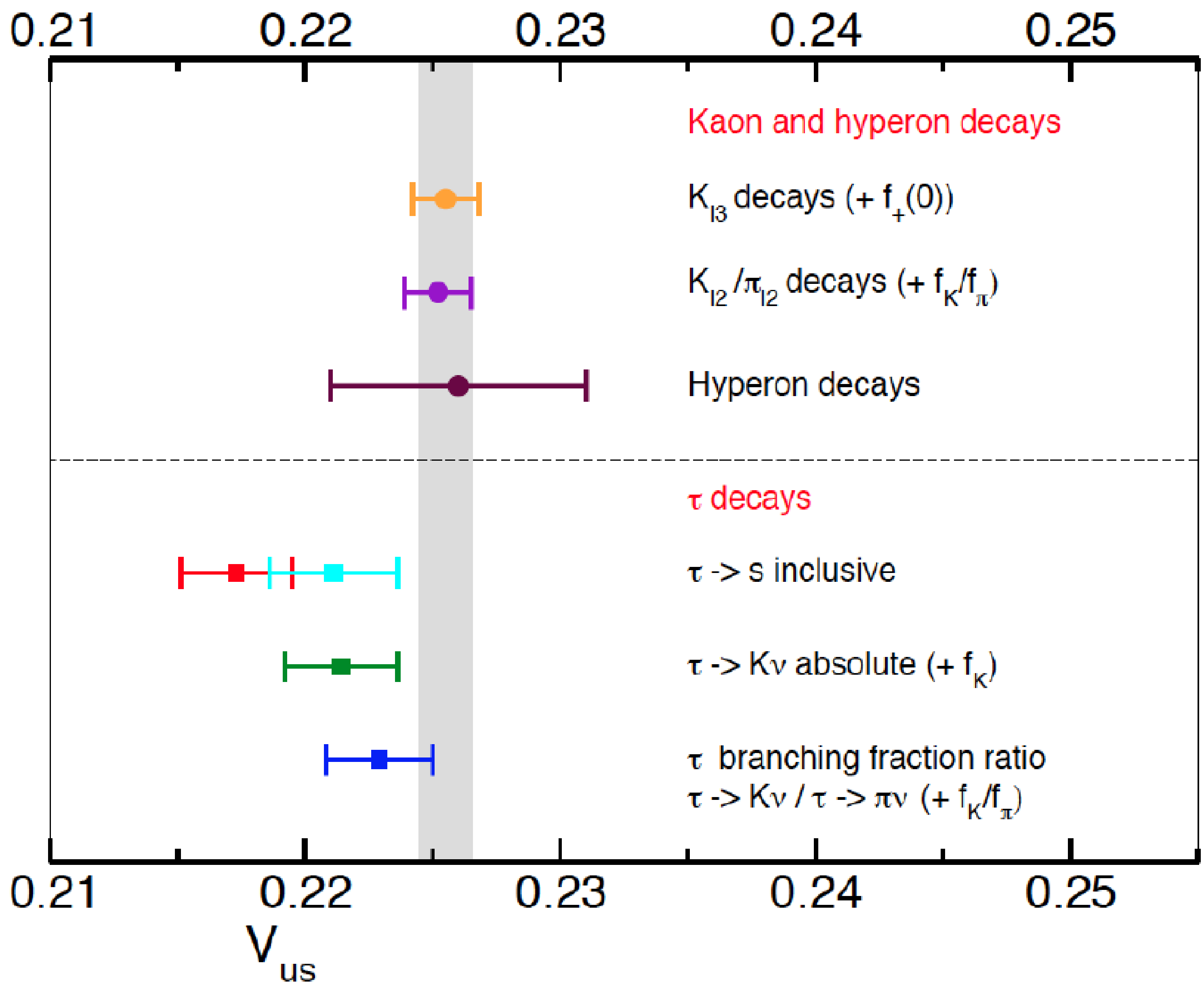
2.6 Prediction of τ strange Brs and V_{us}

Antonelli, Cirigliano, Lusiani, E.P. '13

Mode	BR	% err	BR(K_{e3})	τ_K	τ_τ	I_K^τ / I_K^e	Δ_{EM}	$\Delta_{\mathrm{SU(2)}}$
$\tau^- \to \bar{K}^0 \pi^- \nu_\tau$	0.8569 ± 0.0293	3.42	0.22	0.41	0.35	3.34	0.46	0
$\tau^- \to K^- \pi^0 \nu_\tau$	0.4709 ± 0.0178	3.79	0.06	0.12	0.34	3.60	0.47	1.00

Branching fraction	HFAG Winter 2012 fit	Prediction
$\Gamma_{10} = K^- \nu_\tau$	$(0.6955 \pm 0.0096) \cdot 10^{-2}$	$(0.713 \pm 0.003) \cdot 10^{-2}$
$\Gamma_{16} = K^- \pi^0 \nu_\tau$	$(0.4322 \pm 0.0149) \cdot 10^{-2}$	$(0.4709 \pm 0.0178) \cdot 10^{-2}$
$\Gamma_{35} = \pi^- \bar{K}^0 \nu_\tau$	$(0.8206 \pm 0.0182) \cdot 10^{-2}$	$(0.8569 \pm 0.0293) \cdot 10^{-2}$
$\Gamma_{110} = X_s^- \nu_\tau$	$(2.8746 \pm 0.0498) \cdot 10^{-2}$	$(2.9714 \pm 0.0561) \cdot 10^{-2}$

$$\boxed{|V_{us}| = 0.2173 \pm 0.0022} \Rightarrow \boxed{|V_{us}| = 0.2211 \pm 0.0025}$$



0.21
0.22
0.23
0.24
0.25
Kaon and hyperon decays
K_{l3} decays (+ $f_+(0)$)
K_{l2}/π_{l2} decays (+ f_K/f_π)
Hyperon decays
τ decays
τ -> s inclusive
τ -> Kν absolute (+ f_K)
τ branching fraction ratio
τ -> Kν / τ -> πν (+ f_K/f_π)
V_{us}

3. Constraining the LFV couplings of the Higgs with hadronic τ decays

Celis, Cirigliano, E.P.'13

3.1 Introduction

- Discovery of a 125 GeV scalar particle :
  Standard Higgs? ⇒ Need to study its properties

- Consider the possibility of non-standard LFV couplings of the Higgs
  ⇒ arise in several models

Goudelis, Lebedev,Park'11
Davidson, Grenier'10

- Conveniently parametrized by effective interaction

$$\boxed{\mathcal{L}_Y = -m_i \bar{f}_L^i f_R^i - Y_{ij} \left(\bar{f}_L^i f_R^i \right) \varphi + h.c. + \ldots}$$

Harnick, Koop, Zupan'12
Blankenburg, Ellis, Isidori'12
McKeen, Pospelov, Ritz'12
Arhrib, Cheng, Kong'12

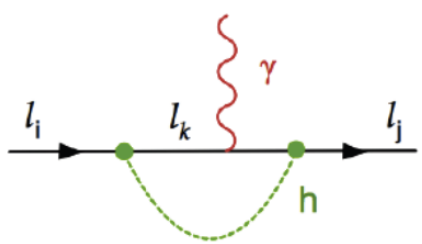
- In the SM : $Y_{ij}^{h_{SM}} = \frac{m_i}{v} \delta_{ij}$

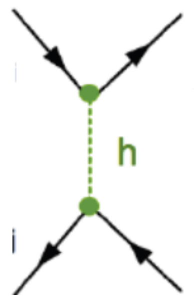
- In full generality parametrization of the Yukawas
  - $Y_{ii}^{\varphi} = y_i^{\varphi} \frac{m_i}{v}$
  - Assumption: CP conservation ⇒ Y_{ij}^{φ} → *real* for $\boxed{\varphi \equiv h}$ CP-even Higgs; → *Imaginary* for $\boxed{\varphi \equiv A}$ CP-odd Higgs

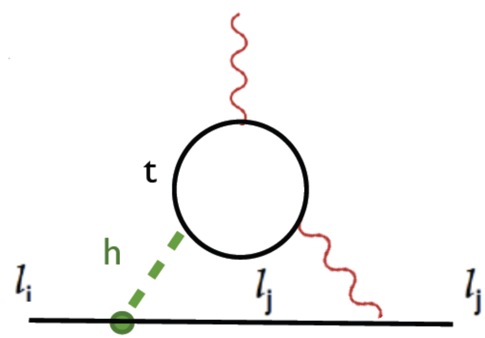
3.1.1 Introduction

- $\mathcal{L}_Y \xrightarrow{\text{EFT}}$ $\boxed{\mathcal{L}_{d=6} = -\frac{\lambda_\ell^{'ij}}{\Lambda^2}\left(\bar{f}_L^{\,i} f_R^{\,j}\right) H \left(H^\dagger H\right) + h.c. + \ldots}$

- $\mathcal{L}_Y$ mediates LFV Higgs and generates at low energy
  - 4 fermion operators
  - Dipole (loops)







3.1.2 Constraints on LFV Higgs couplings

Harnick, Koop, Zupan'12

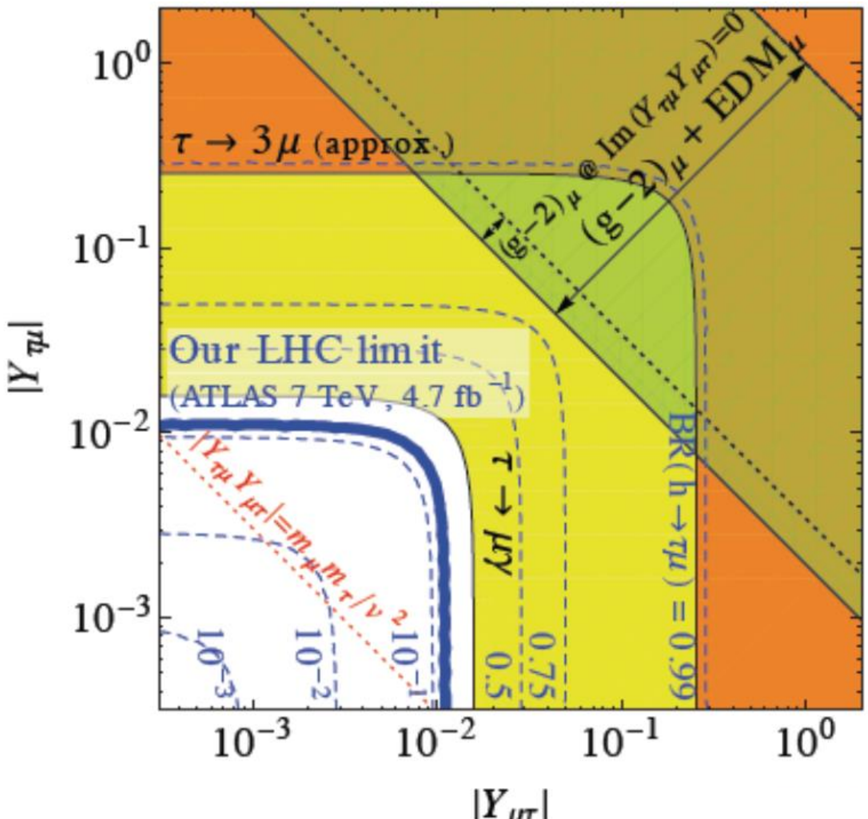
- Results :

Channel	BR 90% CL	$\sqrt{\left\|Y^h_{ij}\right\|^2 + \left\|Y^h_{ji}\right\|^2}$
$\mu \to e\gamma$	$< 2.4 \times 10^{-12}$	$< 3.6 \times 10^{-6}$
$\mu \to 3e$	$< 1 \times 10^{-12}$	$\lesssim 3.1 \times 10^{-5}$
$\tau \to e\gamma$	$< 3.3 \times 10^{-8}$	< 0.014
$\tau \to 3e$	$< 2.7 \times 10^{-8}$	$\lesssim 0.12$
$\tau \to \mu\gamma$	$< 4.4 \times 10^{-8}$	< 0.016
$\tau \to 3\mu$	$< 2.1 \times 10^{-8}$	$\lesssim 0.25$

- Bounds from flavour factories : *MEG*, *Belle*, *Babar* and *LHCb* for $\tau \to 3\mu$

- Strong constraint from $\tau \to \mu\gamma$ loop induced process, very sensitive to UV completion ⇨ Model dependent

- From LHC : best constraints on $h \to \tau\mu$, $h \to \tau e$

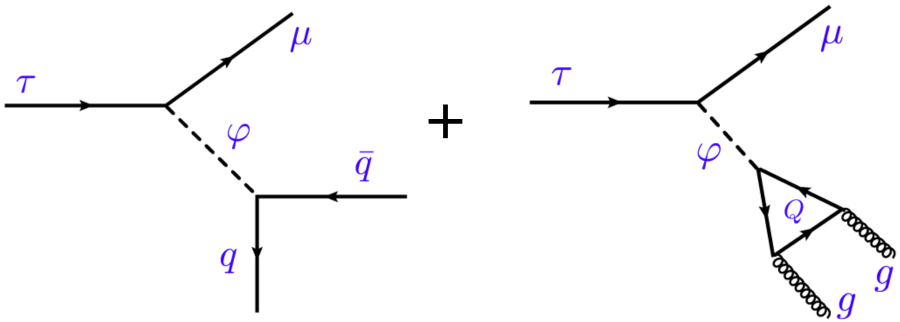


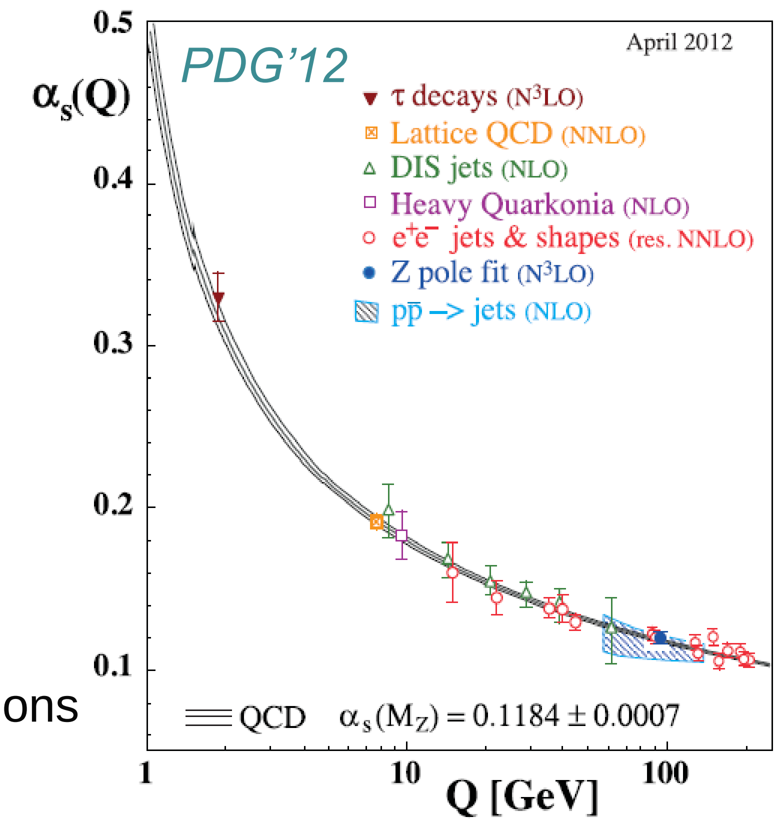
N.B.: Diagonal couplings set to the SM values

3.1.3 Constraints from hadronic τ decays ($\tau \to \mu\pi\pi$)

- Most of the time not taken into account but important because tree level Higgs exchange $\Rightarrow$ less sensitive to UV completion

- Contribution from tree level Higgs exchange





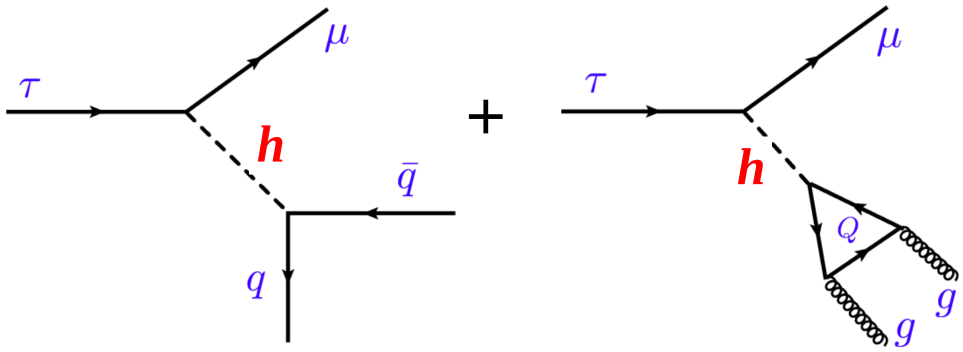
- Complementary analysis between *LHC* and *flavour physics* : *crossed channel*!

  $\Rightarrow$ two different energy scales :
  - LHC: perturbative QCD
  - Flavour factories: intermediate energy, use of ChPT + dispersion relations

$\Rightarrow$ Very interesting processes to look at!

3.2 Constraints from τ → μππ

- Tree level Higgs exchange



- $\mathcal{L}_Y$ ⇨ $$\boxed{\mathcal{L}^h_{eff} \simeq -\frac{h}{v}\left(\sum_{q=u,d,s} y^h_q\, m_q\, \bar{q}\, q - \sum_{q=c,b,t} \frac{\alpha_s}{12\pi}\, y^h_q\, G^a_{\mu\nu} G^{\mu\nu}_a\right)}$$

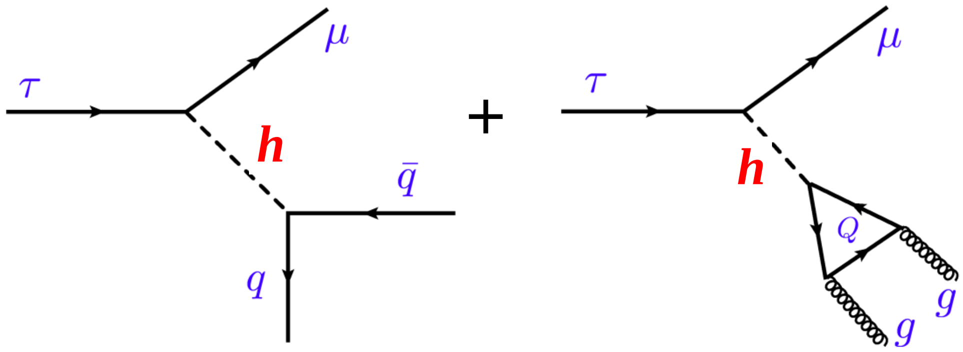
- Problem : Have the hadronic part under control, ChPT not valid at these energies!

⇨ Use *form factors* determined with *dispersion relations* matched at low energy to *CHPT*

Dreiner, Hanart, Kubis, Meissner'13

3.2.1 Constraints from $\tau \to \mu\pi\pi$

- Tree level Higgs exchange



- $\mathcal{L}_Y$ ⇨ $$\boxed{\mathcal{L}^h_{eff} \simeq -\frac{h}{v}\left(\sum_{q=u,d,s} y^h_q\, m_q\, \bar{q}\, q - \sum_{q=c,b,t} \frac{\alpha_s}{12\pi}\, y^h_q\, G^a_{\mu\nu} G^{\mu\nu}_a\right)}$$

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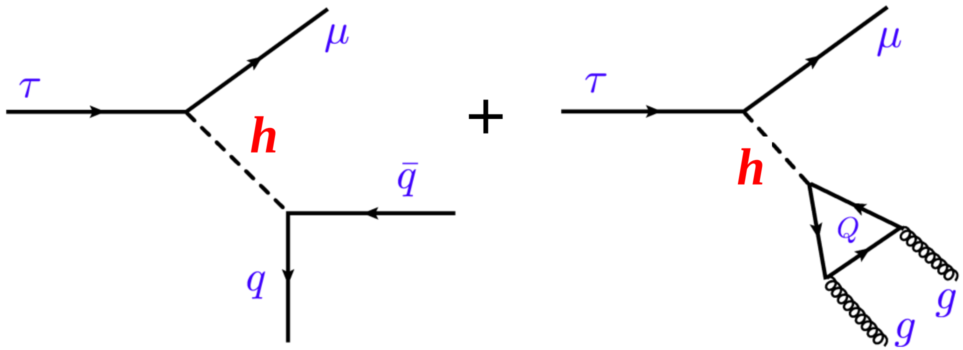
3.2.1 Constraints from $\tau \to \mu\pi\pi$

- Tree level Higgs exchange



$$\frac{d\Gamma(\tau \to \mu\pi^+\pi^-)}{d\sqrt{s}} = \frac{(m_\tau^2 - s)^2\sqrt{s-4m_\pi^2}}{256\pi^3 m_\tau^3}\frac{\left(|Y_{\tau\mu}^h|^2 + |Y_{\mu\tau}^h|^2\right)}{M_h^4 v^2}\left|\mathcal{K}_\Delta \Delta_\pi(s) + \mathcal{K}_\Gamma \Gamma_\pi(s) + \mathcal{K}_\theta \theta_\pi(s)\right|^2$$

$$f\left(y_q^h\right)$$

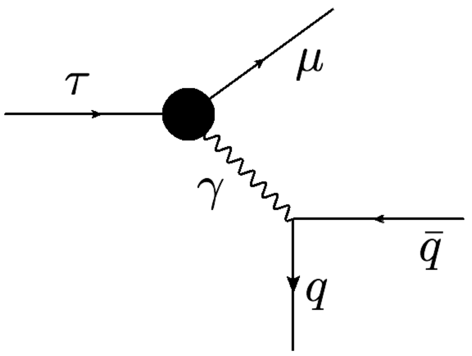
with the form factors:

$$\left\langle \pi^+\pi^- \left| m_u \bar{u}u + m_d \bar{d}d \right| 0 \right\rangle \equiv \Gamma_\pi(s)$$

$$\left\langle \pi^+\pi^- \left| m_s \bar{s}s \right| 0 \right\rangle \equiv \Delta_\pi(s) \qquad \left\langle \pi^+\pi^- \left| \theta_\mu^\mu \right| 0 \right\rangle \equiv \theta_\pi(s)$$

3.2.2 Constraints from $\tau \to \mu\pi\pi$

- Contribution from dipole diagrams



$$\boxed{\mathcal{L}_{eff} = c_L Q_{L\gamma} + c_R Q_{R\gamma} + h.c.}$$

with the dim-5 EM penguin operators :

$$\boxed{Q_{L\gamma,R\gamma} = \frac{e}{8\pi^2} m_\tau \left(\mu\sigma^{\alpha\beta} P_{L,R}\tau\right) F_{\alpha\beta}}$$

- $$\boxed{\frac{d\Gamma(\tau \to \ell\pi^+\pi^-)}{d\sqrt{s}} = \frac{\alpha^2 |F_V(s)|^2 \left(|c_L|^2 + |c_R|^2\right)}{768\pi^5 m_\tau} \frac{(s - 4m_\pi^2)^{3/2} (m_\tau^2 - s)^2 (s + 2m_\tau^2)}{s^2}}$$

with the vector form factor :

$$\boxed{C_{L,R} = f\left(Y_{\tau\mu}\right)}$$

$$\left\langle \pi^+(p_{\pi^+})\pi^-(p_{\pi^-}) \middle| \tfrac{1}{2}(\bar{u}\gamma^\alpha u - \bar{d}\gamma^\alpha d) \middle| 0 \right\rangle \equiv F_V(s)(p_{\pi^+} - p_{\pi^-})^\alpha$$

- Diagram only there in the case of $\tau^- \to \mu^-\pi^+\pi^-$ absent for $\tau^- \to \mu^-\pi^0\pi^0$

  ⇨ neutral mode more model independent

3.3.1 Determination of the form factors : $F_V(s)$

- Vector form factor

  - Precisely known from experimental measurement $e^+e^- \to \pi^+\pi^-$ and $\tau^- \to \pi^0\pi^-\nu_\tau$ (isospin rotation)

  - Theoretically: decay very well described by resonances
    Following properties of *analyticity* and *unitarity* of the FF

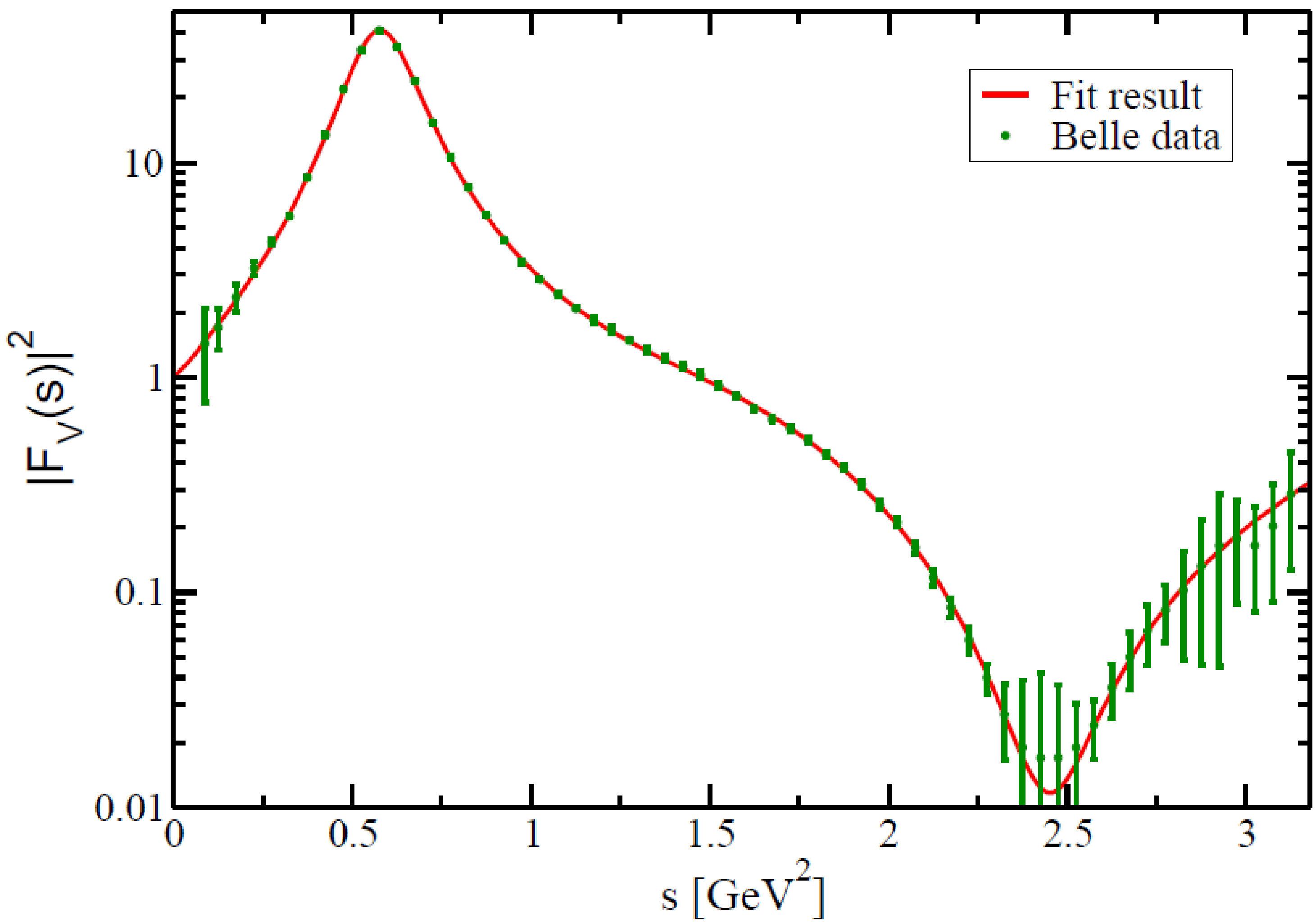
    ⇨ Dispersive parametrization for $F_V(s)$ to fit the Belle data on $\tau^- \to \pi^0\pi^-\nu_\tau$

Guerrero, Pich'98, Pich, Portolés'08
Gomez, Roig'13

$$\boxed{F_V(s) = \exp\left[\lambda'_+ \frac{s}{m_\pi^2} + \frac{1}{2}\left(\lambda''_V - \lambda'^2_V\right)\left(\frac{s}{m_\pi^2}\right)^2 + \frac{s^3}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'^3}\frac{\phi_V(s')}{(s'-s-i\varepsilon)}\right]}$$

Extracted from a model including
3 resonances ρ(770) , ρ'(1465)
and ρ''(1700) fitted to the data

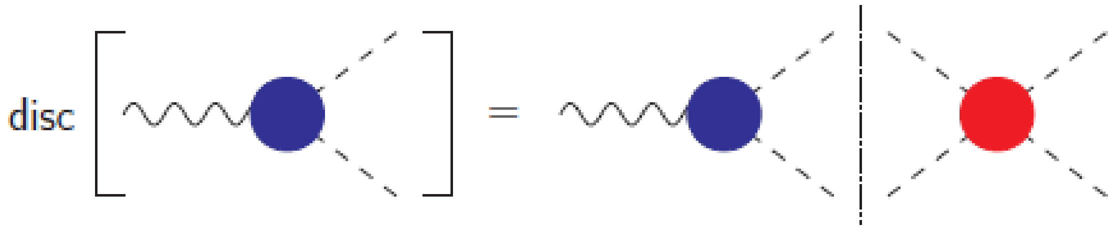
3.3.1 Determination of the form factors : $F_V(s)$



Very precise determination of $F_V(s)$ thanks to very precise measurements of Belle!

3.3.2 Determination of the form factors : $\Gamma_\pi(s)$, $\Delta_\pi(s)$, $\theta_\pi(s)$

- With one channel, in the energy region $\pi\pi \rightarrow \pi\pi$
  unitarity ⇒ the discontinuity of the form factor is known

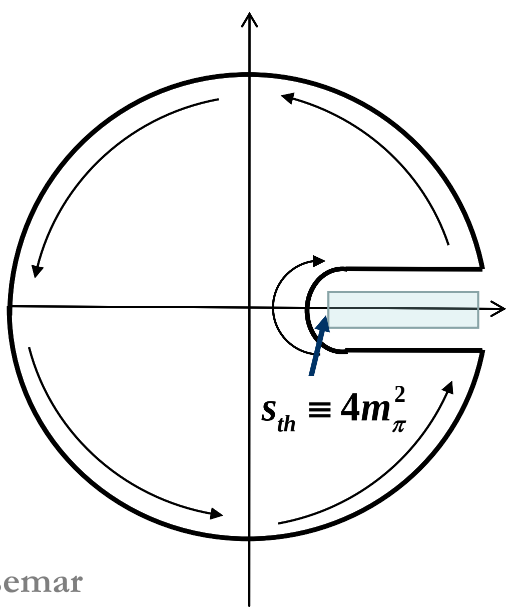


Phase of the FF is
$\pi\pi$ scattering phase
Known from experiment

$$\frac{1}{2i} disc\ F_I(s) = \mathrm{Im}\, F_I(s) = F_I(s)\ \sin\delta_I(s) e^{-i\delta_I(s)}$$

Watson's theorem

- Use analyticity to reconstruct the form factor in the entire space:



Omnès representation: $F_I(s) = P_I(s)\ \Omega_I(s)$

polynomial — Omnès function

$$\Omega_I(s) = \exp\left[\frac{s}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{s'}\frac{\delta_I(s')}{s'-s-i\varepsilon}\right]$$

$P_I(s)$ not known but determined from a matching to CHPT at low energy

3.3.2 Determination of the form factors : $\Gamma_\pi(s)$, $\Delta_\pi(s)$, $\theta_\pi(s)$

- $\tau \to \mu\pi\pi$ ⇨ $\boxed{4m_\pi^2 < s < (m_\tau - m_\mu)^2 \sim (1.77\ \mathrm{GeV})^2}$

  Two channels contribute $\pi\pi$ and $K\bar{K}$

Donoghue, Gasser, Leutwyler'90

Moussallam'99

- Generalisation of the previous method :

  Unitarity ⇨ $$\Gamma_m^\star(s) = \sum_n \{\delta_{mn} + 2\,i\,T_{mn}(s)\,\sigma_n(s)\}^\star\,\Gamma_n(s)$$

  Scattering matrix $\pi\pi \to \pi\pi$, $\pi\pi \to K\bar{K}$

  $K\bar{K} \to \pi\pi$, $K\bar{K} \to K\bar{K}$

- Solve the dispersive integral equations iteratively starting with Omnès functions

$$\boxed{\mathrm{Im}X_n^{(N+1)}(s) = \sum_{m=1}^{2} \mathrm{Re}\,\{T_{nm}^*\sigma_m(s)X_m^{(N)}\}} \longrightarrow \boxed{\mathrm{Re}X_n^{(N+1)}(s) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{s'-s}\mathrm{Im}X_n^{(N+1)}}$$

- According to *Muskhelishvili*, 2 sets of solutions $\{C_1(s), D_1(s)\}$, $\{C_2(s), D_2(s)\}$

  FFs linear combinations : $\boxed{\Gamma_n(s) = P_\Gamma(s)C_n(s) + Q_\Gamma(s)D_n(s)}$

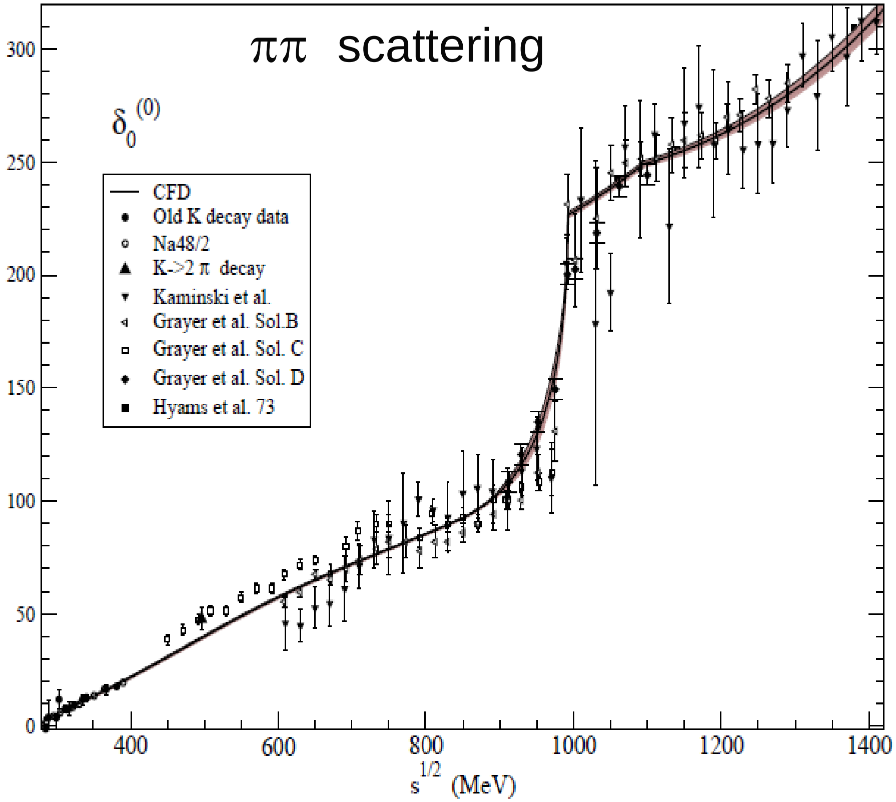
  Determined from a matching to ChPT

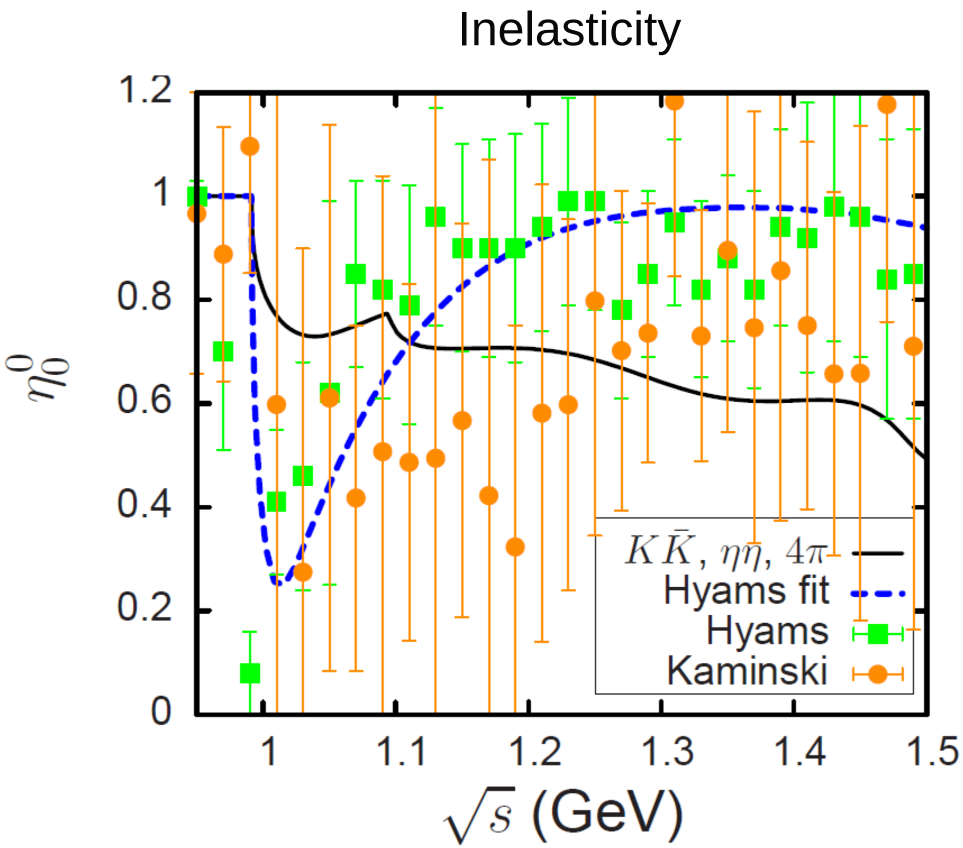
3.3.2 Determination of the form factors : $\Gamma_\pi(s)$, $\Delta_\pi(s)$, $\theta_\pi(s)$

- Inputs : Several inputs ⇒ solve the Roy-Steiner equations

Ananthanarayan et al'01, Colangelo et al'01

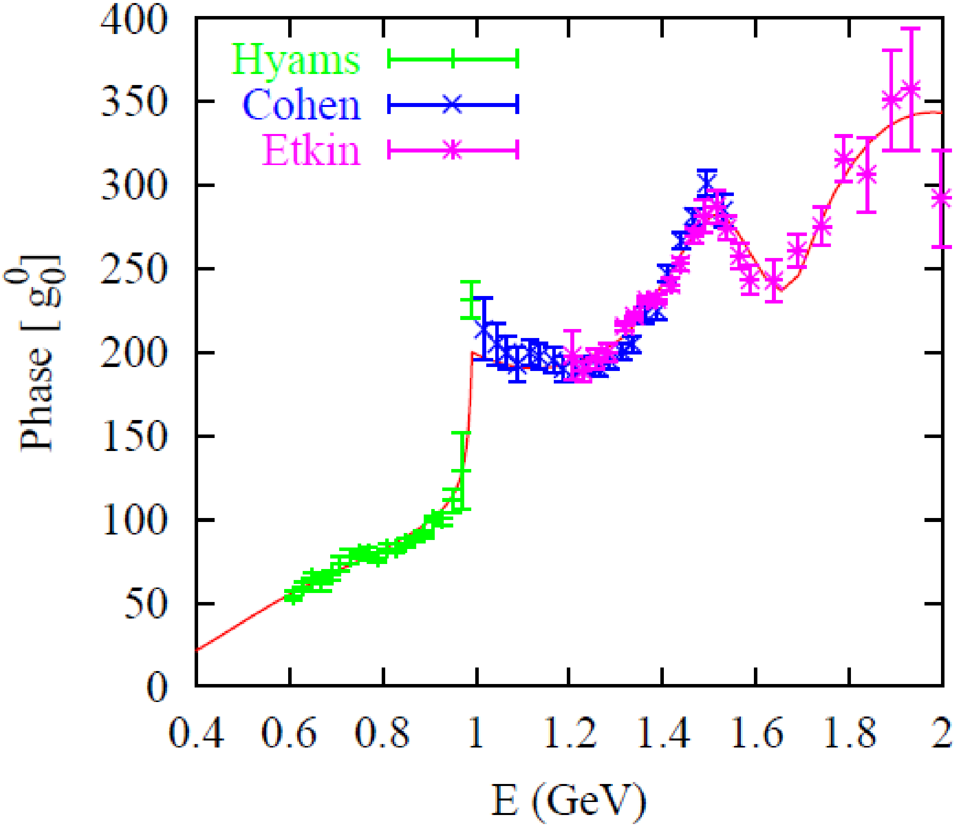
Buettiker, Descotes, Moussallam '02

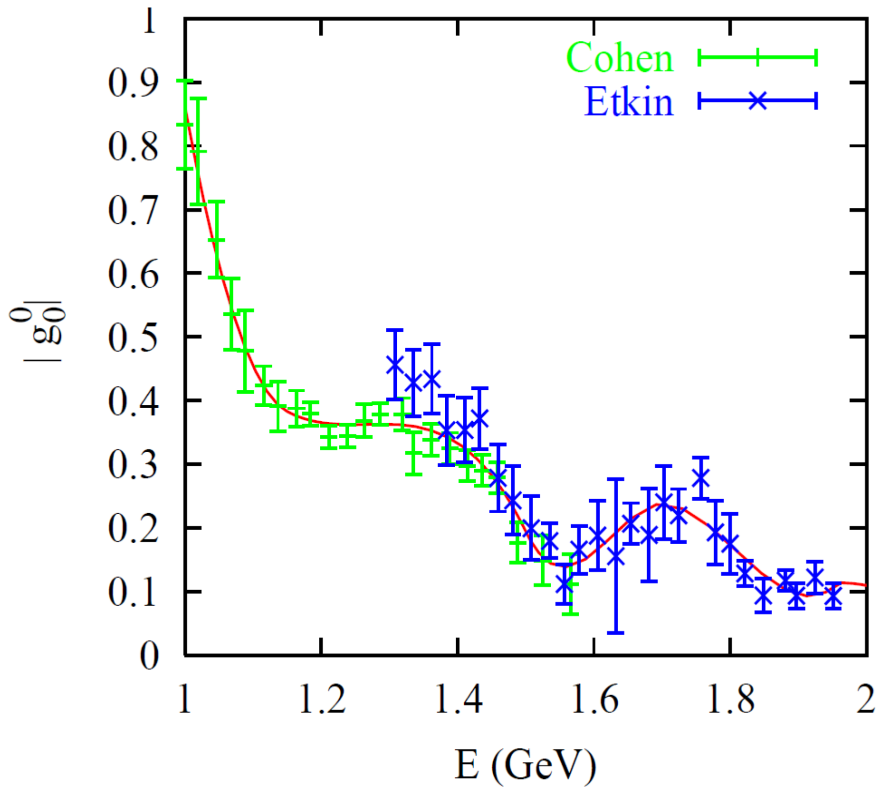




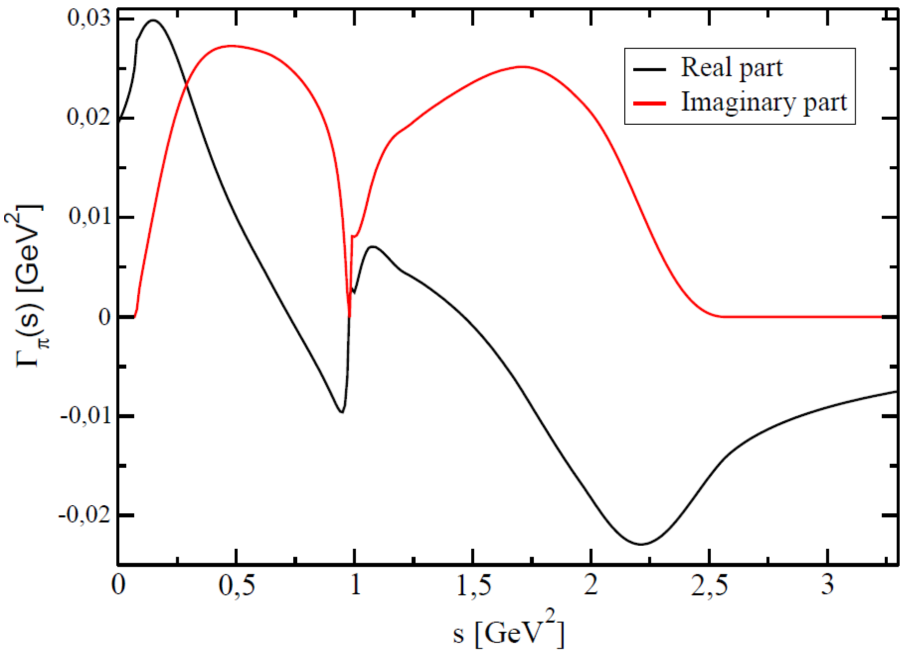
3.3.2 Determination of the form factors : $\Gamma_\pi(s)$, $\Delta_\pi(s)$, $\theta_\pi(s)$

- Inputs : $\pi\pi \to K\bar{K}$

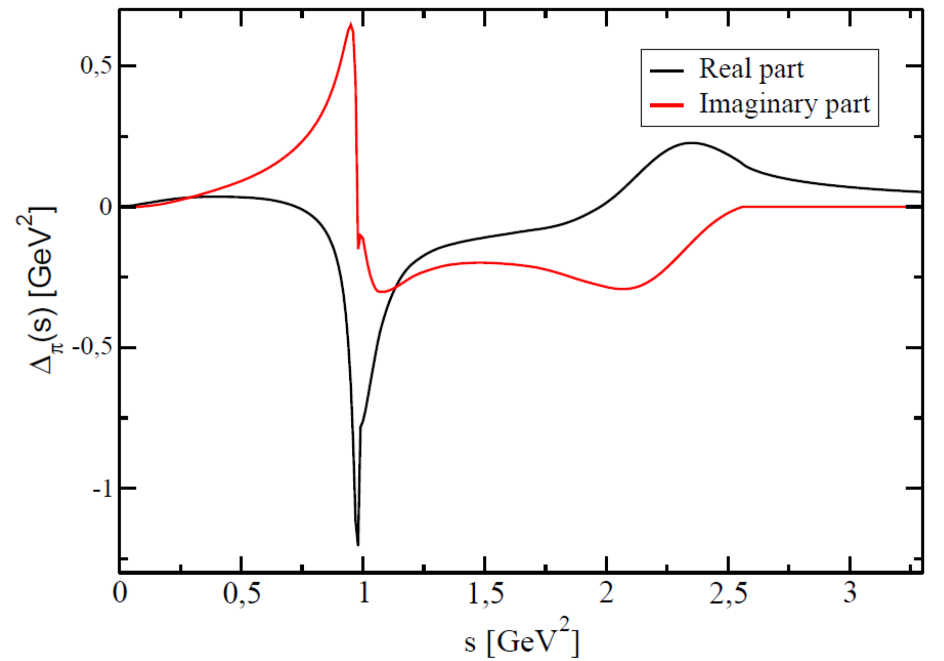




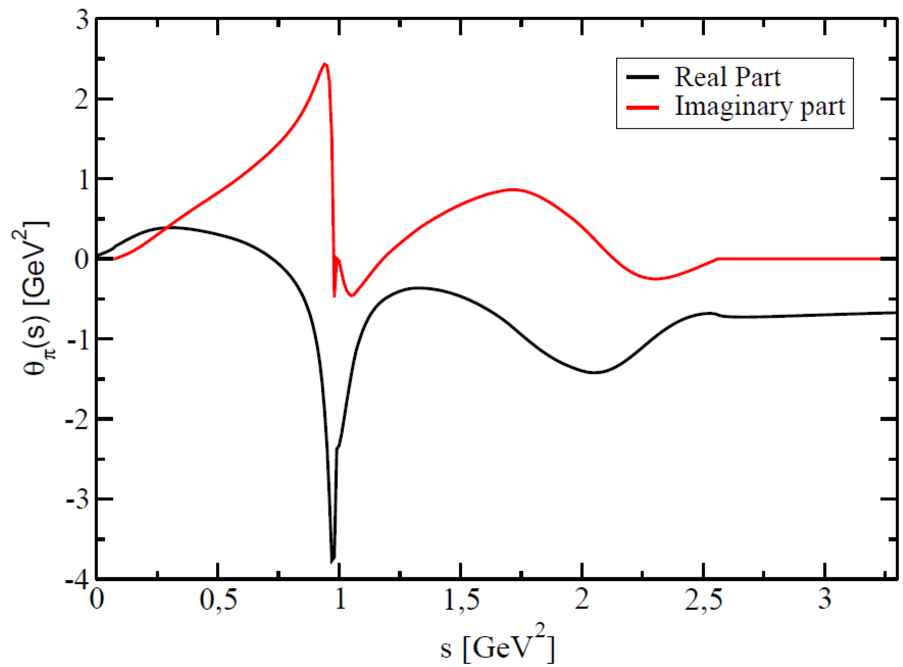
- A large number of theoretical analyses *Descotes-Genon et al'01, Kaminsky et al'01, Garcia-Martin et al'09, Colangelo et al.'11* and all agree
- 3 inputs: $\delta_\pi(s)$, $\delta_K(s)$, η from *B. Moussallam* ⇒ *reconstruct T matrix*



Real part
Imaginary part
0,03
0,02
0,01
0
-0,01
-0,02
0
0,5
1
1,5
2
2,5
3
$\Gamma_\pi(s)$ [GeV^2]
s [GeV^2]

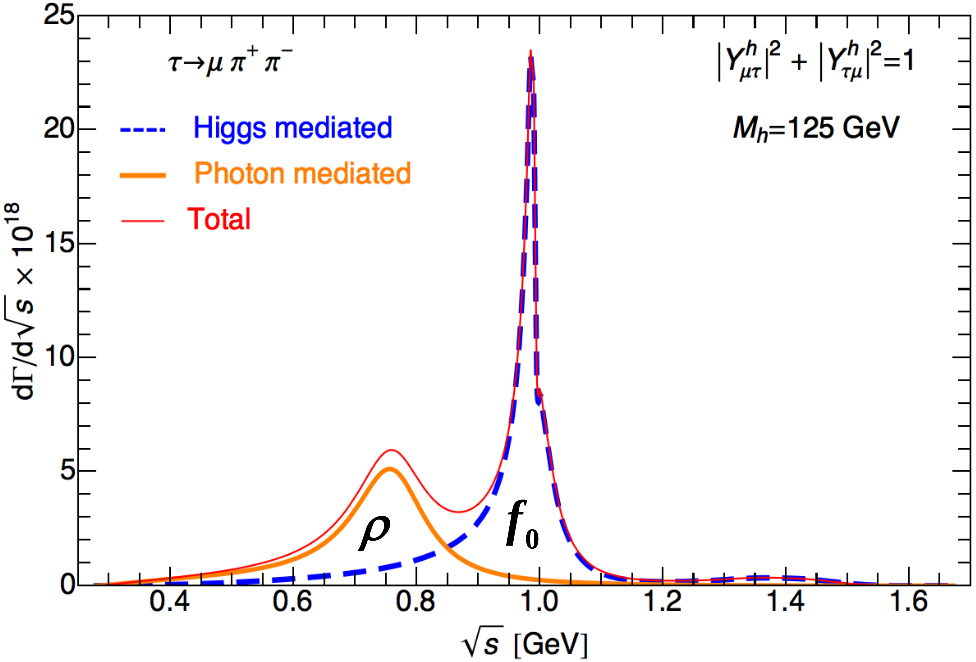


Real part
Imaginary part
0,5
0
-0,5
-1
0
0,5
1
1,5
2
2,5
3
$\Delta_\pi(s)$ [GeV^2]
s [GeV^2]



Real Part
Imaginary part
3
2
1
0
-1
-2
-3
-4
0
0,5
1
1,5
2
2,5
3
$\theta_\pi(s)$ [GeV^2]
s [GeV^2]

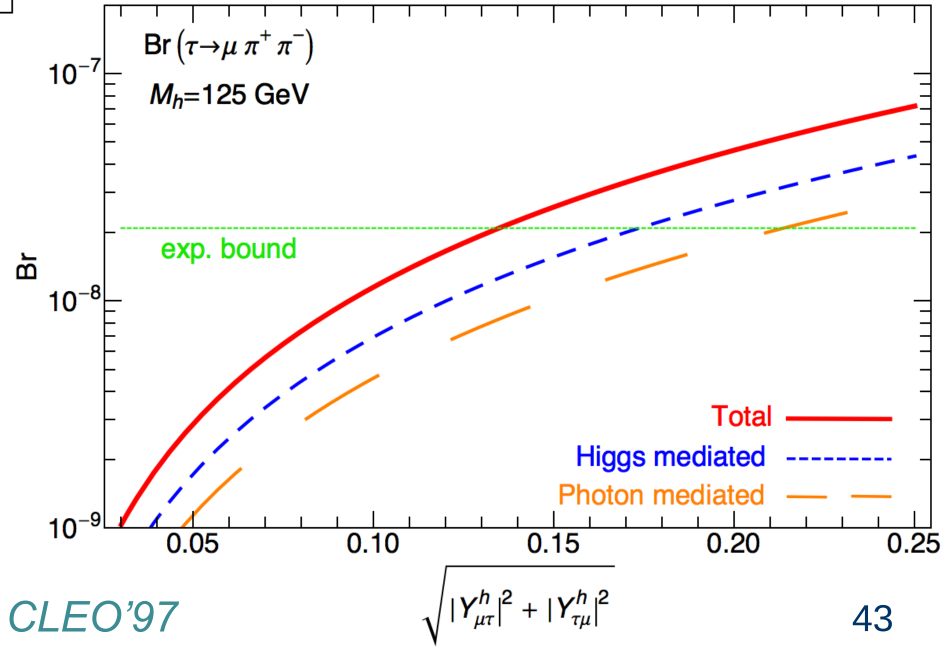
3.4 Results



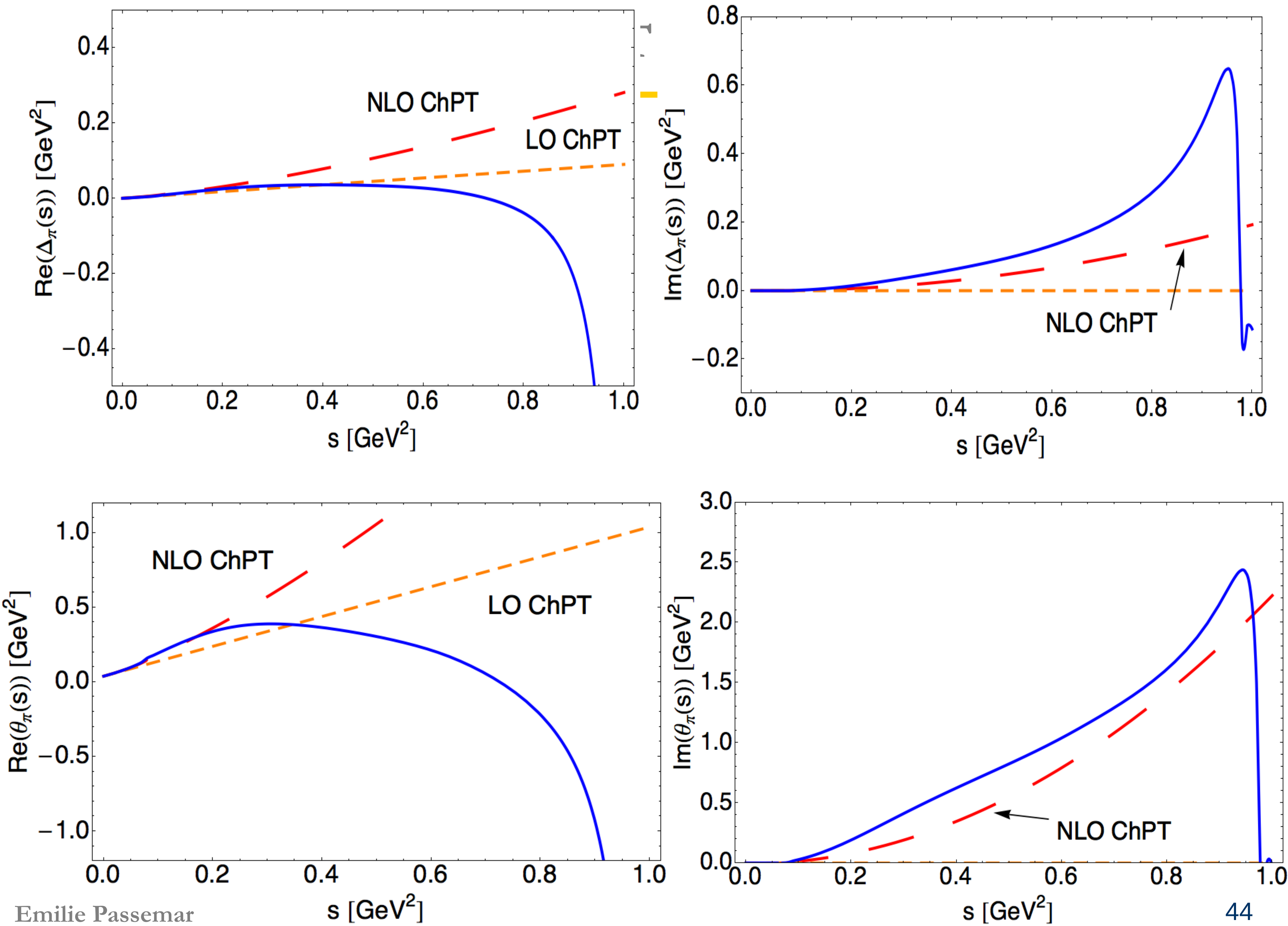
Dominated by

- $\rho(770)$ (photon mediated)
- $f_0(980)$ (Higgs mediated)

Channel	BR 90% CL	$\sqrt{\lvert Y^h_{ij}\rvert^2 + \lvert Y^h_{ji}\rvert^2}$
$\tau \to \mu\gamma$	$< 4.4 \times 10^{-8}$	< 0.016
$\tau \to 3\mu$	$< 2.1 \times 10^{-8}$	$\lesssim 0.25$
$\tau \to \mu\pi^+\pi^-$	$< 2.1 \times 10^{-8}$	< 0.13
$\tau \to \mu\rho$	$< 1.2 \times 10^{-8}$	< 0.13
$\tau \to \mu\pi^0\pi^{0(*)}$	$< 1.4 \times 10^{-5}$	< 6.3

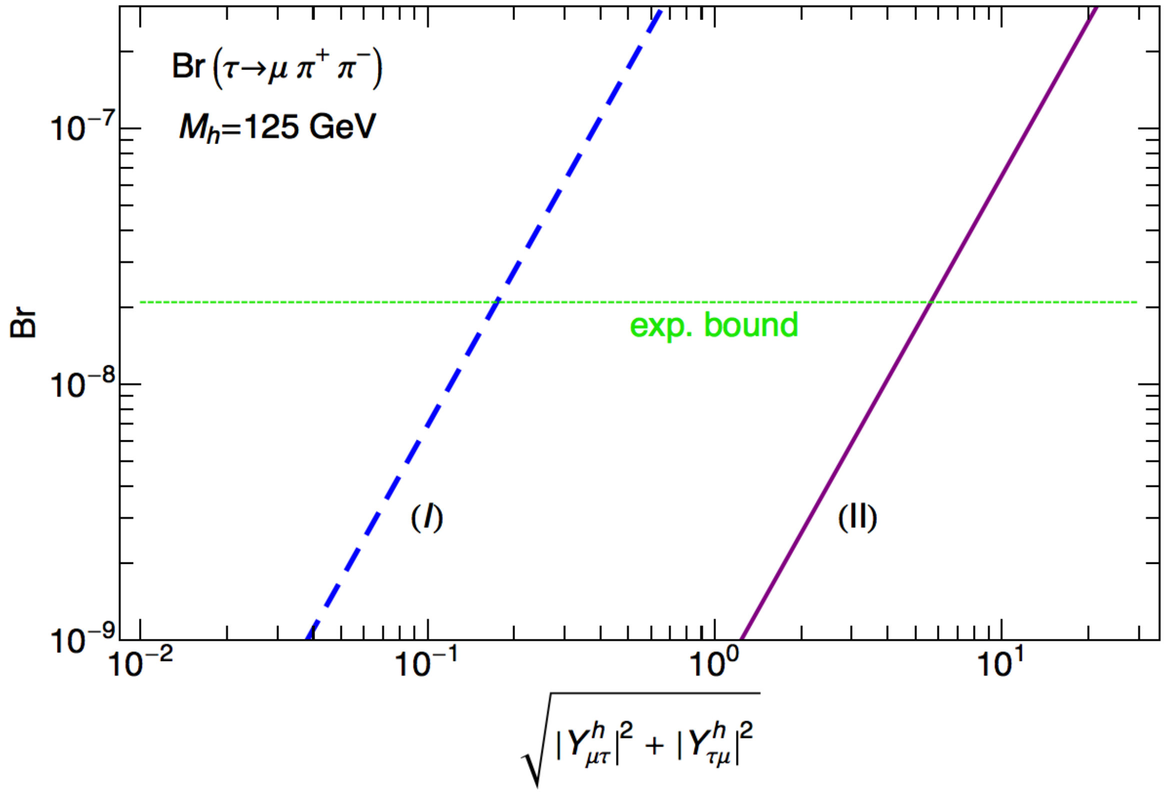


Belle'08'11'12 except for * *CLEO'97*



NLO ChPT
LO ChPT
$\mathrm{Re}(\Delta_\pi(s))$ [GeV^2]
s [GeV^2]
NLO ChPT
$\mathrm{Im}(\Delta_\pi(s))$ [GeV^2]
s [GeV^2]
NLO ChPT
LO ChPT
$\mathrm{Re}(\theta_\pi(s))$ [GeV^2]
s [GeV^2]
NLO ChPT
$\mathrm{Im}(\theta_\pi(s))$ [GeV^2]
s [GeV^2]

3.5 Comparison with ChPT



- Rigorous treatment of hadronic part ⇒ bound reduced by one order of magnitude! ⇒ Very *robust bounds*!

- ChPT, EFT only valid at low energy for $\boxed{\mathbf{p} \ll \Lambda = 4\pi f_\pi \sim 1\ \mathbf{GeV}}$
  ⇒ *not valid up to* $E = \left(m_\tau - m_\mu\right)$!

4. Conclusion and Outlook

4.1 Conclusion

- Hadronic τ-decays very interesting to study, very rich phenomenology
  - Need precise measurements
  - Theoretically in the intermediate regime $\boxed{0 < s < m_\tau^2 \sim (1.77\ \mathrm{GeV})^2}$ ChPT not valid anymore!
    - ➢ Inclusive : *perturbative tools (OPE…)*
    - ➢ Exclusive : *non perturbative tools (FFs using RChT, matched to ChPT…)*

- Excellent probe of the SM and New Physics. Here I presented 2 examples
  - Extraction of V_{us} : Possibility to determine it with inclusive and exclusive decays
    - ➢ Inclusive τ decays $\Rightarrow$ $\boxed{|V_{us}| = 0.2173 \pm 0.0020_{\mathrm{exp}} \pm 0.0010_{\mathrm{th}}}$

      Error dominated by experiment $\Rightarrow$ Potentially the more precise extraction of V_{us}
    - ➢ Use information on the ffs from $\tau \to K\pi\nu_\tau$ + Kaon Brs

      $\boxed{|V_{us}| = 0.2173 \pm 0.0022}$ $\Rightarrow$ $\boxed{|V_{us}| = 0.2211 \pm 0.0025}$

    Difference between inclusive/exclusive modes:
    Data normalization, unmeasured modes? New Physics?

4.1 Conclusion

- Hadronic τ-decays very interesting to study, very rich phenomenology
  - Need precise measurements
  - Theoretically in the intermediate regime ChPT not valid anymore! $\boxed{0 < s < m_\tau^2 \sim (1.77\ \mathrm{GeV})^2}$
    - ➢ Inclusive : *perturbative tools (OPE…)*
    - ➢ Exclusive : *non perturbative tools (FFs using RChT, matched to ChPT…)*

- Excellent probe of the SM and New Physics. Here I presented 2 examples
  - LFV mode $\tau \to \mu\pi\pi$ for constraining LFV couplings of the Higgs
    Very interesting and important :
    - ➢ The more model-independent (tree level exchange of Higgs)
    - ➢ Same process can be studied at LHC and at the flavour factories with totally different experimental and theoretical conditions
    - ➢ Very little hadronic uncertainties: form factors determined using dispersion relations + ChPT ⇨ *Robust bounds!*

4.2 Outlook

- High precision era in τ:
  - more precise data with LHC-B, Belle II, Tau-Charm?
  - theoretically: ffs parametrizations, EM, IB corrections

- For the 2 examples I gave :
  - V_{us} : new measurements for the strange Brs are needed!
  - In the LFV mode $\tau \to \mu\pi\pi$, the more model independent process is

    $\boxed{\tau^- \to \mu^- \pi^0 \pi^0}$: no loop induced process

    but the only experimental bound from *CLEO* and weak ~10^{-5}

    ⇨ need to be remeasured

- I hope this week in Krakow will allow us to make some progress towards a better understanding of hadronic τ decays!

6. Back-up

1.1 Test of New Physics : Vus

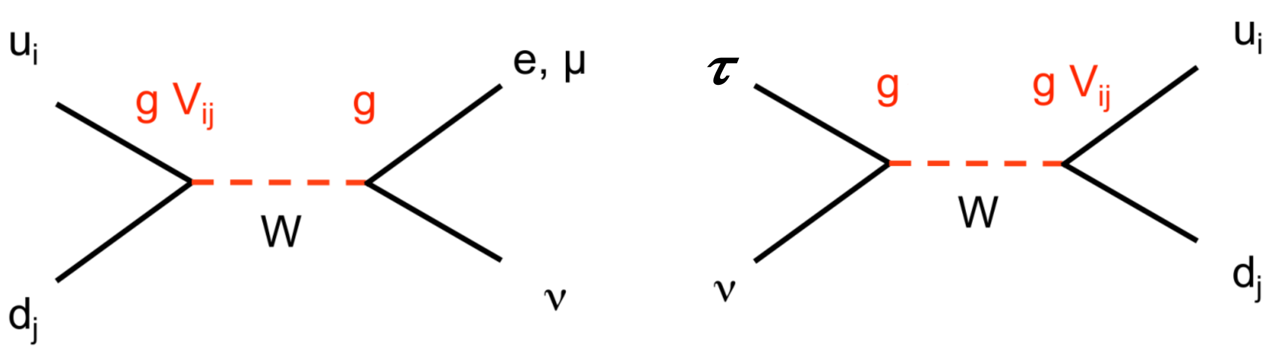
- Extraction of the Cabibbo-Kobayashi-Maskawa matrix element *V_{us}*
  - Fundamental parameter of the Standard Model
    Check unitarity of the first row of the CKM matrix:
    ⇨ *Cabibbo Universality*

$$\boxed{|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \stackrel{?}{=} 1}$$

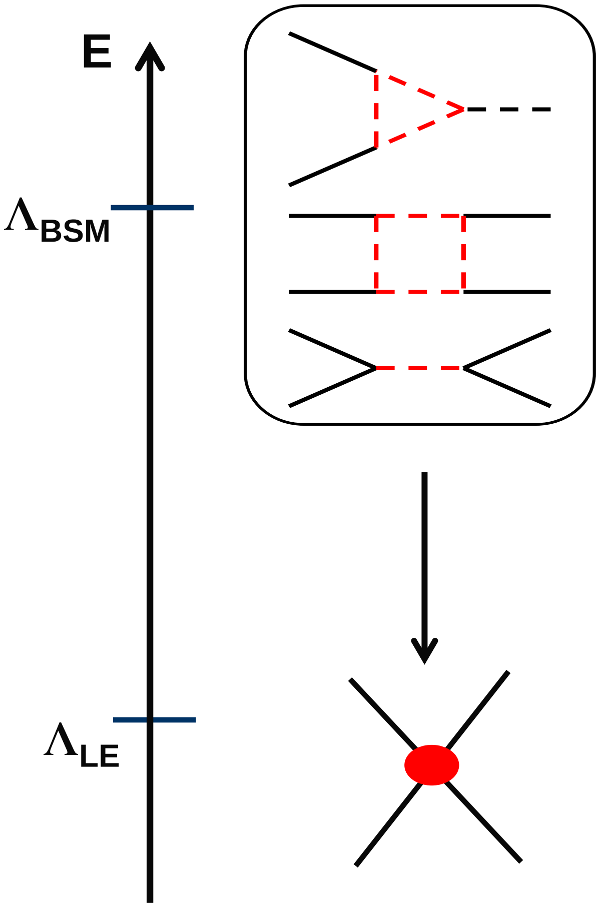
Negligible
(B decays)

  - Input in UT analysis

- Look for *new physics*
  - In the Standard Model : W exchange ⇨ only V-A structure



1.2 New Physics: Flavour factories & LHC



High energy: → if Λ_{BSM} ~ TeV $\Rightarrow$ sensitive to new resonances, direct discovery

Low energy: if $\Lambda_{LE} << \Lambda_{BSM}$ $\Rightarrow$ EFT approach sensitive to scale + flavour structure of couplings

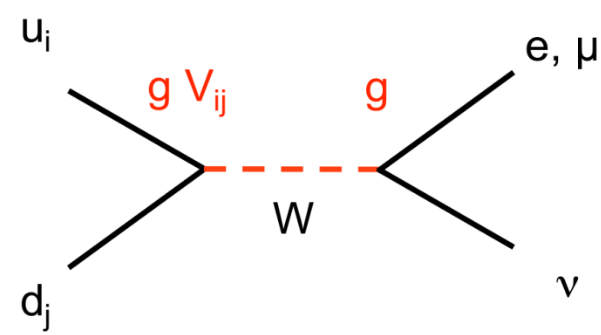
$$\boxed{\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \ldots}$$

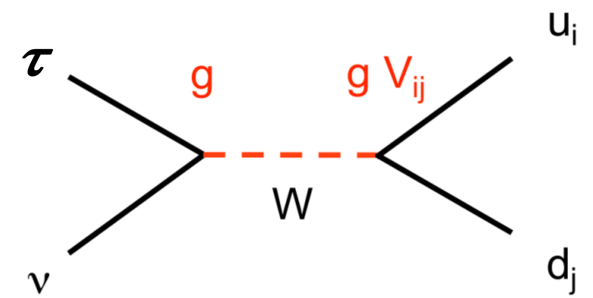
$\Rightarrow$ Reconstruct the underlying dynamics

1.3 Paths to V_{ud} and V_{us}

- From kaon, pion, baryon and nuclear decays

V_{ud}	$0^+ \to 0^+$ $\pi^\pm \to \pi^0 e \nu_e$	$n \to p e \nu_e$	$\pi \to \ell\, \nu_\ell$
V_{us}	$K \to \pi \ell \nu_\ell$	$\Lambda \to p e \nu_e$	$K \to \ell\, \nu_\ell$





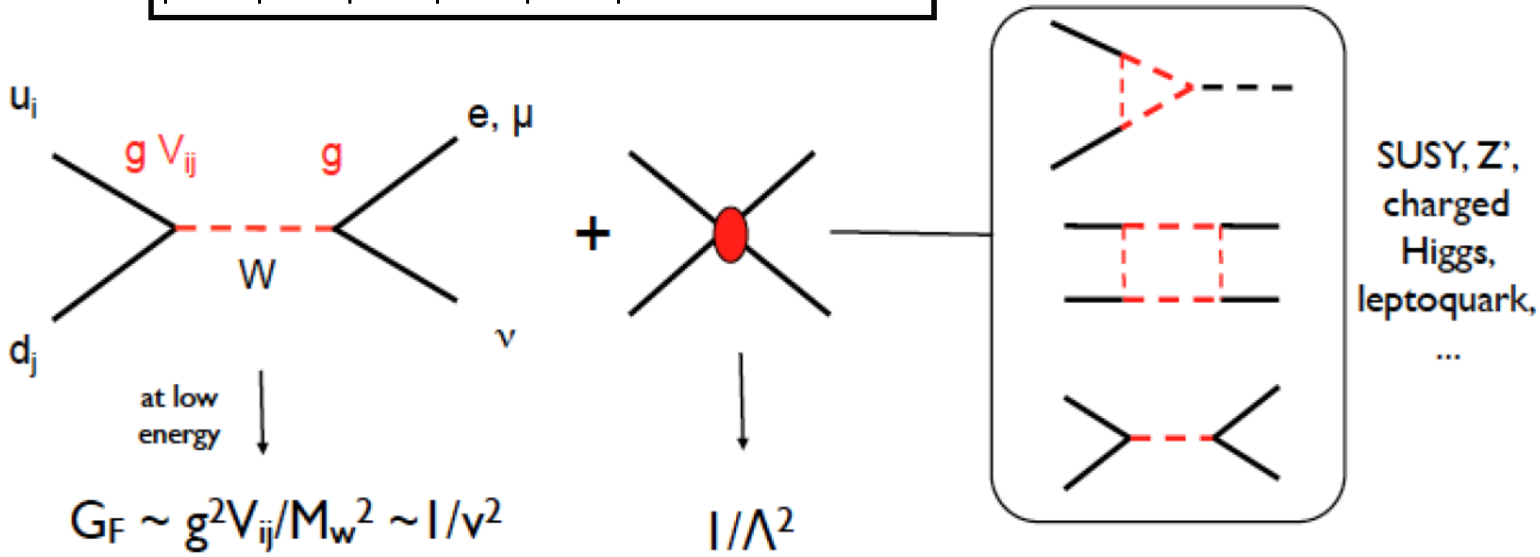
- From τ decays

V_{ud}			$\tau \to \pi \nu_\tau$	$\tau \to h_{NS} \nu_\tau$
V_{us}	$\tau \to K \pi \nu_\tau$		$\tau \to K \nu_\tau$	$\tau \to h_S \nu_\tau$ (inclusive)

1.1 Test of New Physics : Vus

- BSM: sensitive to tree-level and loop effects of a large class of models

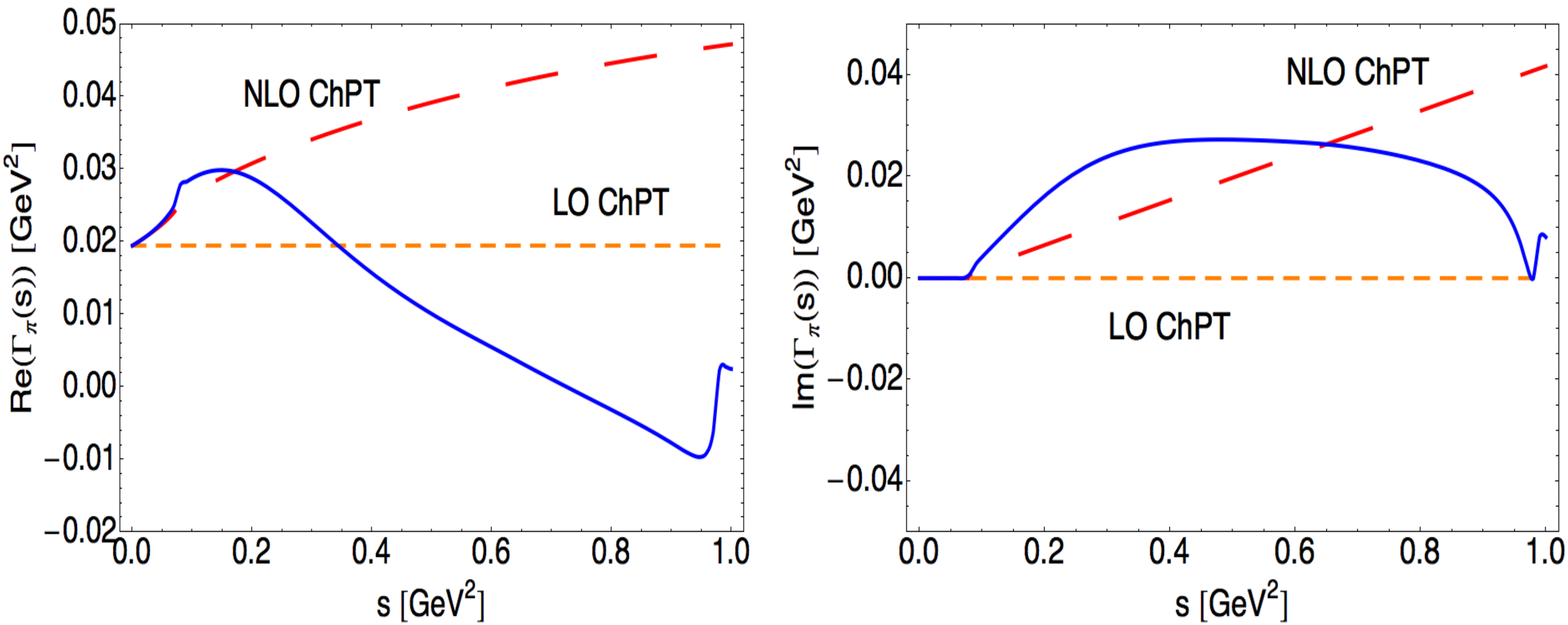
$$\left|V_{ud}\right|^2 + \left|V_{us}\right|^2 + \left|V_{ub}\right|^2 = 1 + \Delta_{CKM}$$



BSM effects : $\boxed{\Delta_{CKM} \sim \left(v/\Lambda\right)^2}$

- Look for new physics by comparing the extraction of V_{us} from different processes: helicity suppressed $K_{\mu 2}$, helicity allowed K_{l3}, hadronic τ decays

2.4 Comparison with ChPT

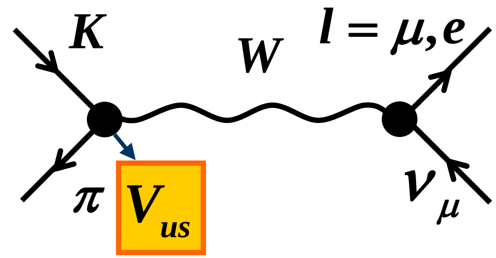


1.1 Test of New Physics : Vus

- Studying τ and K_{l3} decays ⇨ indirect searches of new physics, several possible high-precision tests:
  - ➢ Extraction of V_{us}

$$\left(K \to \pi l \nu_l\right)$$

$$\left(l = e, \mu\right)$$



$$\boxed{\Gamma_{K^{+/0}l3} = N \left| f_+(0) \mathrm{V}_{us} \right|^2 I^l_{K^{+/0}}}$$

with

$$\boxed{I^l_{K^{+/0}} = \int dt \frac{1}{m^8_{K^{+/0}}} \lambda^{3/2} F\left(t, \overline{f}_+(t), \overline{f}_0(t)\right)}$$

⇨ Knowledge of the two form factors:

$$\boxed{\left\langle \pi(p_\pi) \right| \bar{\mathrm{s}} \gamma_\mu \mathrm{u} \left| \mathrm{K}(\mathrm{p_K}) \right\rangle = \left[\left(p_K + p_\pi\right)_\mu - \frac{\Delta_{K\pi}}{t} \left(p_K - p_\pi\right)_\mu \right] f_+(t) + \frac{\Delta_{K\pi}}{t} \left(p_K - p_\pi\right)_\mu f_0(t)}$$

vector scalar

$$t = q^2 = (p_\mu + p_{\nu_\mu})^2 = (p_K - p_\pi)^2, \quad \overline{f}_{0,+}(t) = \frac{f_{0,+}(t)}{f_+(0)}$$

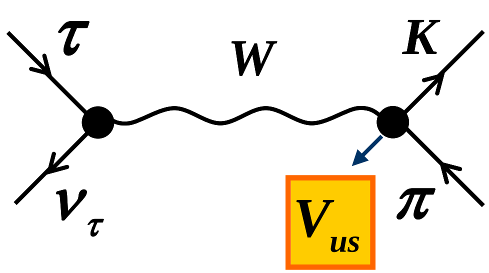
1.2 Determination of the Kπ form factors

- $\bar{f}_+(t)$ accessible in K_{e3} and $K_{\mu 3}$ decays

- $\bar{f}_0(t)$ only accessible in $K_{\mu 3}$ (suppressed by m_l^2/M_K^2) + correlations
  ⇨ difficult to measure

- Data from *Belle* and *BaBar* on $\tau \to K\pi\nu_\tau$ decays (*Belle II*, *Tau-Charm* soon!)
  ⇨ Use them to constrain the form factors and especially $\bar{f}_0$

- $\tau \to K\pi\nu_\tau$ decays



Hadronic matrix element: Crossed channel

$$\boxed{\langle \mathbf{K}\pi | \,\bar{\mathbf{s}}\gamma_\mu \mathbf{u}\, | \mathbf{0} \rangle = \left[(p_K - p_\pi)_\mu + \frac{\Delta_{K\pi}}{s}(p_K + p_\pi)_\mu \right] f_+(s) - \frac{\Delta_{K\pi}}{s}(p_K + p_\pi)_\mu \, f_0(s)}$$

vector

scalar

with $s = q^2 = (p_K + p_\pi)^2$

3.1 Kπ form factors from $\tau \to K\pi\nu_\tau$ and K_{l3} decays

- Fit to the $\tau \to K\pi\nu_\tau$ decay data
  - from *Belle [Epifanov et al'08]* (*BaBar?*)

$$N_{events} \propto N_{tot}\, b_w \, \frac{1}{\Gamma_{K\pi}} \frac{d\Gamma_{K\pi}}{d\sqrt{s}} \quad \Rightarrow \quad \chi^2_\tau = \sum_{bins} \left(\frac{N_{events} - N_\tau}{\sigma_{N_\tau}} \right)^2$$

with

Number of events/bin

bin width

$$\frac{d\Gamma_{K\pi}}{d\sqrt{s}} = \frac{G_F^2 m_\tau^3}{32\pi^3 s} C_K^2 S_{EW} \left| f_+(0) V_{us} \right|^2 \left(1 - \frac{s}{m_\tau^2}\right)^2 \left[\left(1 + \frac{2s}{m_\tau^2}\right) q^3_{K\pi}(s) \left| \bar{f}_+(s) \right|^2 + \frac{3\Delta^2_{K\pi}}{4s} q_{K\pi}(s) \left| \bar{f}_0(s) \right|^2 \right]$$

- Possible combination with K_{l3} decay data fits *Flavianet Kaon WG'10*

$$\chi^2 = \chi^2_\tau + \begin{pmatrix} \lambda_+ - \lambda_+^{K_{l3}} \\ \ln C - \ln C^{K_{l3}} \end{pmatrix}^T V^{-1} \begin{pmatrix} \lambda_+ - \lambda_+^{K_{l3}} \\ \ln C - \ln C^{K_{l3}} \end{pmatrix}$$

+ sum-rules

Results for the Kπ form factors

	$\tau \to K\pi\nu_\tau$ & $K_{\ell 3}$ Belle	$\tau \to K\pi\nu_\tau$ & $K_{\ell 3}$ 2^{nd} generation B factory (projected)
$\ln C$	0.20352 ± 0.00890	0.19880 ± 0.00498
$\lambda'_0 \times 10^3$	13.824 ± 0.824	13.703 ± 0.521
$\tilde{m}_{K^*}$ [MeV]	943.59 ± 0.58	943.76 ± 0.06
$\tilde{\Gamma}_{K^*}$ [MeV]	67.064 ± 0.846	67.290 ± 0.088
$\tilde{m}_{K^{*\prime}}$ [MeV]	1392.2 ± 57.6	1361.7 ± 6.3
$\tilde{\Gamma}_{K^{*\prime}}$ [MeV]	296.67 ± 160.28	254.62 ± 17.45
β	-0.0404 ± 0.0206	-0.0338 ± 0.0023
$\lambda'_+ \times 10^3$	25.621 ± 0.405	25.601 ± 0.277
$\lambda''_+ \times 10^3$	1.2221 ± 0.0183	1.2150 ± 0.0090
$\chi^2/d.o.f$	$60.2/68$	$28.1/71$

Results for the π π vector form factor

$$\tilde{F}_V(s) = \frac{\tilde{M}_\rho^2 + \left(\alpha' e^{i\phi'} + \alpha'' e^{i\phi''}\right) s}{\tilde{M}_\rho^2 - s + \kappa_\rho \, \mathrm{Re}\left[A_\pi(s) + \frac{1}{2} A_K(s)\right] - i\tilde{M}_\rho \tilde{\Gamma}_\rho(s)} - \frac{\alpha' e^{i\phi'} s}{D(\tilde{M}_{\rho'}, \tilde{\Gamma}_{\rho'})} - \frac{\alpha'' e^{i\phi''} s}{D(\tilde{M}_{\rho''}, \tilde{\Gamma}_{\rho''})}$$

$\lambda'_V \times 10^3$	36.7 ± 0.2
$\lambda''_V \times 10^3$	3.12 ± 0.04
$\tilde{M}_\rho$[MeV]	833.9 ± 0.6
$\tilde{\Gamma}_\rho$[MeV]	198 ± 1
$\tilde{M}_{\rho'}$[MeV]	1497 ± 7
$\tilde{\Gamma}_{\rho'}$[MeV]	785 ± 51
$\tilde{M}_{\rho''}$[MeV]	1685 ± 30
$\tilde{\Gamma}_{\rho''}$[MeV]	800 ± 31
α'	0.173 ± 0.009
ϕ'	-0.98 ± 0.11
α''	0.23 ± 0.01
ϕ''	2.20 ± 0.05
$\chi^2/d.o.f$	$38/52$