Requirements for Fringe fields model for HL-LHC

We ideally need the simplest quadrupole fringe fields model that:

- Give accurate prediction of the DA (imply symplecticity) in the region of the phase space under interest (but not necessarily accurate single orbits since they shows anyway strong sensitivity to the initial conditions)

- Allow simple analytical derivation of the effect using first order Hamiltonian perturbation theory.

References

$$\Delta x = \left(\tfrac{1}{12}x^3 + \tfrac{1}{4}xy^2\right)k_0,$$
$$\Delta p_x = \left[\tfrac{1}{2}xyp_y - \tfrac{1}{4}p_x\left(x^2+y^2\right)\right]k_0,$$
$$k_0 = \frac{B'}{B\rho} \text{ at the center of the magnet.}$$

or in a local rotated frame

$$\exp\left(:\alpha\frac{y^3 p_x}{1+\delta}:\right)x = x - \frac{\alpha y^3}{1+\delta},$$
$$\exp\left(:\alpha\frac{y^3 p_x}{1+\delta}:\right)p_y = p_y + \frac{3\alpha}{1+\delta}y^2 p_x,$$
$$\exp\left(:\alpha\frac{y^3 p_x}{1+\delta}:\right)\tau = \tau + \frac{\alpha y^3 p_x}{(1+\delta)^2}\frac{\mathrm{d}\delta}{\mathrm{d}p_\tau},$$
$$\alpha = \frac{q\hat{B}}{6p_0}a_1 \equiv \frac{1}{6B\rho}\frac{\mathrm{d}\hat{B}_y}{\mathrm{d}x} \equiv \frac{B'}{6B\rho} \equiv \frac{k_0}{6}.$$

Review Symplectic methods (including fringe fields) and symplectification methods:

- E. Forest, Geometric Integration for Particle Accelerators

Are Leading Order Hard Edge Fringe Fields, enough? (E. Forest, J. Milutinovic,

Can we use Yoshida hierarchy to reduce the number of steps of a 2nd order integrator?

$$\mathcal{S}_4(\Delta t) = \mathcal{S}_2(x_1\Delta t)\mathcal{S}_2(x_2\Delta t)\mathcal{S}_2(x_1\Delta t),$$
$$\text{where} \quad x_1 = \frac{1}{2-2^{1/3}} \text{ and } x_2 = \frac{-2^{1/3}}{2-2^{1/3}}.$$

Dipoles:

- K. L. Brown, Technical Report No. 75, SLAC.

Quadrupoles:

- E. Forest, J. Milutinovic, Leading Order Hard Edge Fringe Fields Effects Exact in (1 + delta) and Consistent with Maxwell's equations for Rectilinear Magnets
- G.E. Lee-Whiting, Third-order aberrations of a magnetic quadrupole lens

Can we make symplectic maps where the first derivatives of the fields are explicit?