Matching & Merging of Parton Showers and Matrix Elements

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Fermilab, 14.11.2013
Parton showers
Probabilistic treatment of emissions

- Sudakov form factor (no-decay probability)

\[ \Delta_{ij,k}(t, t_0) = \exp \left[ - \int_{t_0}^{t} \frac{dt}{t} \frac{\alpha_S}{2\pi} \int dz \frac{d\phi}{2\pi} K_{ij,k}(t, z, \phi) \right] \]

- evolution parameter \( t \) defined by kinematics
  
  generalised angle (HERWIG++) or transverse momentum (PYTHIA, SHERPA)

- will replace \( \frac{dt}{t} \frac{dz}{2\pi} \frac{d\phi}{2\pi} \rightarrow d\Phi_1 \)

- scale choice for strong coupling: \( \alpha_S(k^2_\perp) \)
  
  resums classes of higher logarithms

- regularisation through cut-off \( t_0 \)
Emissions off a Born matrix element

- "compound" splitting kernels $\mathcal{K}_n$ and Sudakov form factors $\Delta_n^{(\mathcal{K})}$ for emission off $n$-particle final state:

$$\mathcal{K}_n(\Phi_1) = \frac{\alpha_s}{2\pi} \sum_{\text{all } \{ij,k\}} \mathcal{K}_{ij,k}(\Phi_{ij,k}), \quad \Delta_n^{(\mathcal{K})}(t, t_0) = \exp\left[-\int_{t_0}^{t} d\Phi_1 \mathcal{K}_n(\Phi_1)\right]$$

- consider first emission only off Born configuration

$$d\sigma_B = d\Phi_N B_N(\Phi_N)$$

$$\cdot \left\{ \Delta_N^{(\mathcal{K})}(\mu_N^2, t_0) + \int_{t_0}^{\mu_N^2} d\Phi_1 \left[ \mathcal{K}_N(\Phi_1)\Delta_N^{(\mathcal{K})}(\mu_N^2, t(\Phi_1)) \right] \right\}$$

integrates to unity $\rightarrow$ "unitarity" of parton shower

- further emissions by recursion with $\mu_N^2 \rightarrow t$ of previous emission
NLO improvements: Matching
NLO matching: Basic idea

- Parton shower resums logarithms
  - fair description of collinear/soft emissions
  - jet evolution
    (where the logs are large)

- Matrix elements exact at given order
  - fair description of hard/large-angle emissions
  - jet production
    (where the logs are small)

- Adjust ("match") terms:
  - Cross section at **NLO accuracy** &
    - correct hardest emission in PS to exactly
    - reproduce ME at order $\alpha_S$
      - ($R$-part of the NLO calculation)
    (this is relatively trivial)

- Maintain **(N)LL-accuracy** of parton shower
  (this is not so simple to see)
The POWHEG-trick: modifying the Sudakov form factor


- reminder: $\mathcal{K}_{ij,k}$ reproduces process-independent behaviour of $\mathcal{R}_N/\mathcal{B}_N$ in soft/collinear regions of phase space

$$d\Phi_1 \frac{\mathcal{R}_N(\Phi_{N+1})}{\mathcal{B}_N(\Phi_N)} \xrightarrow{\text{IR}} d\Phi_1 \frac{\alpha_S}{2\pi} \mathcal{K}_{ij,k}(\Phi_1)$$

- define modified Sudakov form factor (as in ME correction)

$$\Delta^{(\mathcal{R}/\mathcal{B})}_N(\mu^2_N, t_0) = \exp \left[-\int_{t_0}^{\mu^2_N} d\Phi_1 \frac{\mathcal{R}_N(\Phi_{N+1})}{\mathcal{B}_N(\Phi_N)} \right]$$

- assumes factorisation of phase space: $\Phi_{N+1} = \Phi_N \otimes \Phi_1$

- typically will adjust scale of $\alpha_S$ to parton shower scale
Local $K$-factors


- start from Born configuration $\Phi_N$ with NLO weight:

\[
d\sigma_{\text{NLO}}^{(N)} = d\Phi_N \bar{B}(\Phi_N)
= d\Phi_N \left\{ B_N(\Phi_N) + \mathcal{V}_N(\Phi_N) + B_N(\Phi_N) \otimes S \overline{\mathcal{V}}_N(\Phi_N) \right\}
+ \int d\Phi_1 \left[ \mathcal{R}_N(\Phi_N \otimes \Phi_1) - B_N(\Phi_N) \otimes dS(\Phi_1) \right]
\]

- by construction: exactly reproduce cross section at NLO accuracy
- note: second term vanishes if $\mathcal{R}_N \equiv B_N \otimes dS$

(relevant for MC@NLO)
NLO accuracy in radiation pattern


- generate emissions with $\Delta_N^{(R/B)}(\mu^2_N, t_0)$:

$$d\sigma_N^{(NLO)} = d\Phi_N \bar{B}(\Phi_N) \times \left\{ \Delta_N^{(R/B)}(\mu^2_N, t_0) + \int_{t_0}^{\mu^2_N} d\Phi_1 \frac{R_N(\Phi_N \otimes \Phi_1)}{B_N(\Phi_N)} \Delta_N^{(R/B)}(\mu^2_N, k^2_\perp(\Phi_1)) \right\}$$

integrating to yield 1 - “unitarity of parton shower”

- radiation pattern like in ME correction
- pitfall, again: choice of upper scale $\mu^2_N$
- apart from logs: which configurations enhanced by local $K$-factor

(this is vanilla POWHEG)

(K-factor for inclusive production of $X$ adequate for $X+$ jet at large $p_\perp$?)
Improved POWHEG

- split real-emission ME as

\[ R = R \left( \frac{h^2}{p_\perp^2 + h^2} + \frac{p_\perp^2}{p_\perp^2 + h^2} \right) \]

- can “tune” \( h \) to mimick NNLO - or maybe resummation result
- differential event rate up to first emission

\[
\frac{d\sigma}{d\Phi_B} = \Phi^{(S)}_B \left[ \Delta^{(R^{(S)}/B)}(s, t_0) + \int_{t_0}^{s} d\Phi_1 \frac{R^{(S)}}{B} \Delta^{(R^{(S)}/B)}(s, k_\perp^2) \right] + \Phi^{(F)}_R (\Phi_R)
\]
Resummation in MC@NLO

(S. Frixione & B. Webber, JHEP 0602 (2002) 029)


- divide \( \mathcal{R}_N \) in soft ("S") and hard ("H") part:

\[
\mathcal{R}_N = \mathcal{R}_N^{(S)} + \mathcal{R}_N^{(H)} = \mathcal{B}_N \otimes dS_1 + \mathcal{H}_N
\]

- identify subtraction terms and shower kernels \( dS_1 \equiv \sum_{\{ij,k\}} K_{ij,k} \)

(modify \( K \) in 1\textsuperscript{st} emission to account for colour)

\[
d\sigma_N = d\Phi_N \underbrace{\tilde{B}_N(\Phi_N)}_{\mathcal{B}+\tilde{\mathcal{B}}} \left[ \Delta_N^{(K)}(\mu_N^2, t_0) + \int_{t_0} \right. d\Phi_1 K_{ij,k}(\Phi_1) \Delta_N^{(K)}(\mu_N^2, k_2^2)]
\]

+ \( d\Phi_{N+1} \mathcal{H}_N \)

- effect: only resummed parts modified with local \( K \)-factor
Aside: impact of full colour


- evaluate effect of full colour treatment, MC@NLO without $H$-part vs. parton shower with $B \rightarrow \bar{B}$
- take $t\bar{t}$ production (red = full colour, blue = “PS” colours)

Matching & Merging of Parton Showers and Matrix Elements
MC@NLO for light jets: $R_{32}$ & forward energy flow

Parton showers

MC@NLO

Matching & Merging of Parton Showers and Matrix Elements

Conclusion

MC@NLO for light jets: jet vetoes

Multijet merging @ leading order
Multijet merging: basic idea

- Parton shower resums logarithms
- Fair description of collinear/soft emissions
- Jet evolution
  (where the logs are large)

- Matrix elements exact at given order
- Fair description of hard/large-angle emissions
- Jet production
  (where the logs are small)

- Combine ("merge") both:
  - Result: “towers” of MEs with increasing number of jets evolved with PS

  - Multijet cross sections at Born accuracy
  - Maintain (N)LL accuracy of parton shower

(S. Catani, F. Krauss, R. Kuhn, B. Webber, JHEP 0111 (2001) 063,
Separating jet evolution and jet production

- separate regions of jet production and jet evolution with jet measure $Q_J$
  ("truncated showering" if not identical with evolution parameter)
- matrix elements populate hard regime
- parton showers populate soft domain
First emission(s), again

\[ d\sigma = d\Phi_N B_N \left[ \Delta^{(K)}_N (\mu^2_N, t_0) + \int_{t_0}^{\mu^2_N} d\Phi_1 \mathcal{K}_N \Delta^{(K)}_N (\mu^2_N, t_{N+1}) \Theta(Q_J - Q_{N+1}) \right] \]

\[ + d\Phi_{N+1} B_{N+1} \Delta^{(K)}_N (\mu^2_{N+1}, t_{N+1}) \Theta(Q_{N+1} - Q_J) \]

• note: \( N + 1 \)-contribution includes also \( N + 2, N + 3, \ldots \)
  
  (no Sudakov suppression below \( t_{n+1} \), see further slides for iterated expression)

• potential occurrence of different shower start scales: \( \mu_{N,N+1}, \ldots \)

• “unitarity violation” in square bracket: \( B_N \mathcal{K}_N \rightarrow B_{N+1} \)

(cured with UMEPS formalism, L. Lonnblad & S. Prestel, JHEP 1302 (2013) 094 &
Di-photons @ ATLAS: $m_{\gamma\gamma}$, $p_{\perp,\gamma\gamma}$, and $\Delta\phi_{\gamma\gamma}$ in showers

(Improved measurement of $m_{\gamma\gamma}$, $p_{\perp,\gamma\gamma}$, and $\Delta\phi_{\gamma\gamma}$ in ATLAS showers using Monte Carlo simulations.)

Data compared to PYTHIA and SHERPA Monte Carlo predictions, showing good agreement at lower $m_{\gamma\gamma}$ values. Differences observed at higher $m_{\gamma\gamma}$, with ATLAS data showing slightly higher rates than MC simulations.

$\sigma_{\gamma\gamma} = 4.9 \text{ fb}$

$\int \text{Data 2011, } 1.2 \text{ (MRST2007)} \times \text{PYTHIA MC11c} 1.2 \text{ (CTEQ6L1)} \times \text{SHERPA MC11c}$

ATLAS = 7 TeV

$7 \times 10^3 \text{ pb/GeV}$

$\Delta\phi_{\gamma\gamma}$ distribution shows a peak at $\Delta\phi_{\gamma\gamma} = 0$, with some scattering at higher values.

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(arXiv:1211.1913 [hep-ex])
Aside: Comparison with higher order calculations

\( \Gamma = 7 \text{ TeV} \)
\[
\begin{align*}
\text{ATLAS} & \quad \text{\textcopyright{2011, ICFAT}} \quad \text{fb}^{-1}
\end{align*}
\]

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\( \int \) Data 2011, DIPHOX+GAMMA2MC (CT10), NNLO (MSTW2008)

\( \text{ATLAS} = 7 \text{ TeV s} \)
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\( \int \) Data 2011, DIPHOX+GAMMA2MC (CT10), NNLO (MSTW2008)

\( \text{arXiv:1211.1913 [hep-ex]} \)
Aside': restoring unitarity with UMEPS

- as indicated, MEPS@LO formalism breaks unitarity: inclusive $n$-jet cross sections not exactly maintained due to mismatch of kernels in actual emission term and Sudakov form factor
- can be cured by adding/subtracting shower and ME-like terms
- low merging cut possible
Multijet merging @ next-to leading order
Multijet-merging at NLO: MEPS@NLO

- basic idea like at LO: towers of MEs with increasing jet multi (but this time at NLO)
- combine them into one sample, remove overlap/double-counting
  maintain NLO and (N)LL accuracy of ME and PS
- this effectively translates into a merging of MC@NLO simulations and can be further supplemented with LO simulations for even higher final state multiplicities
First emission(s), once more

\[ d\sigma = d\Phi_N \tilde{B}_N \left[ \Delta_N^{(K)}(\mu_N^2, t_0) + \int_{t_0}^{\mu_N^2} d\Phi_1 \mathcal{K}_N \Delta_N^{(K)}(\mu_N^2, t_{N+1}) \Theta(Q_J - Q_{N+1}) \right] \]

\[ + d\Phi_{N+1} \mathcal{H}_N \Delta_N^{(K)}(\mu_N^2, t_{N+1}) \Theta(Q_J - Q_{N+1}) \]

\[ + d\Phi_{N+1} \tilde{B}_{N+1} \left( 1 + \frac{\mathcal{B}_{N+1}}{\tilde{B}_{N+1}} \int_{t_{N+1}}^{\mu_N^2} d\Phi_1 \mathcal{K}_N \right) \Theta(Q_{N+1} - Q_J) \]

\[ \cdot \Delta_N^{(K)}(\mu_N^2, t_{N+1}) \cdot \left[ \Delta_{N+1}^{(K)}(t_{N+1}, t_0) + \int_{t_0}^{t_{N+1}} d\Phi_1 \mathcal{K}_{N+1} \Delta_{N+1}^{(K)}(t_{N+1}, t_{N+2}) \right] \]

\[ + d\Phi_{N+2} \mathcal{H}_{N+1} \Delta_N^{(K)}(\mu_N^2, t_{N+1}) \Delta_{N+1}^{(K)}(t_{N+1}, t_{N+2}) \Theta(Q_{N+1} - Q_J) + \ldots \]
MEPS@NLO: example results for $e^- e^+ \rightarrow$ hadrons

(T. Gehrmann et al., JHEP 1301 (2013) 144)
MEPS@NLO: example results for $e^-e^+ \rightarrow \text{hadrons}$

Thrust ($E_{CMS} = 91.2$ GeV)

Moments of $1 - T$ at 91 GeV

C-Parameter ($E_{CMS} = 91.2$ GeV)

Moments of C at 91 GeV

ALEPH data

MePS@NLO

MePS@NLO $\mu/2 \ldots 2\mu$

MC@NLO

MEPS@NLO

MEPS@NLO $\mu/2 \ldots 2\mu$

Moments of $1 - T$ at 91 GeV

Moments of C at 91 GeV

Matchings $\langle C \rangle$

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Example: MEPS@NLO for $W+\text{jets}$


Inclusive Jet Multiplicity

- ATLAS data
- MEPS@NLO
- MEPS@NLO $\mu/2\ldots2\mu$
- MELOPS
- MELOPS $\mu/2\ldots2\mu$
- MC@NLO

$p_{T}^{\text{jet}} > 20\text{ GeV}$

$p_{T}^{\text{jet}} > 30\text{ GeV}$

$\sigma(W+\geq N_{\text{jet}} \text{jets}) [\text{pb}]$

$\sigma(\geq N_{\text{jet}} \text{jets})/\sigma(\geq N_{\text{jet}} - 1 \text{jets})$

0.1 0.15 0.2 0.25 0.3

0

$N_{\text{jet}}$

$0 1 2 3 4 5$
$W + \geq 1 \text{ jet} (\times 1)$

$W + \geq 2 \text{ jets} (\times 0.1)$

$W + \geq 3 \text{ jets} (\times 0.01)$

$W + \geq 2 \text{ jets}$

$W + \geq 3 \text{ jets}$

First Jet $p_T$

Second Jet $p_T$

Third Jet $p_T$

MC/data

MC/ATLAS data

MC/data

MC/ATLAS data

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W + ≥ 1 jet (×1)
W + ≥ 2 jets (×0.1)
W + ≥ 3 jets (×0.01)

AtLAs data

Matching & Merging of Parton Showers and Matrix Elements
Example: MEPS@NLO for $W^+W^- + \text{jets}$

(F. Cascioli et al., arXiv:1309.0500)
summary and concluding remarks
Summary

- systematic improvement of event generators by including higher orders has been at the core of QCD theory and developments in the past decade:
  - multijet merging ("CKKW", "MLM")
  - NLO matching ("MC@NLO", "POWHEG")
  - ME@NLO – combination of matching and merging
  - multijet merging at NLO (ME@NLO, "FxTx")

(first 3 methods well understood and used in experiments)

- multijet merging at NLO under scrutiny
- complete automation of NLO calculations ≈ done
  time to optimise the impact of this gargantuan task