Matching & Merging of Parton Showers and Matrix Elements

Frank Krauss

Institute for Particle Physics Phenomenology
Durham University

Fermilab, 14.11.2013

www.ippp.dur.ac.uk

Parton showers

Probabilistic treatment of emissions

- Sudakov form factor (**no-decay** probability)

$$\Delta^{(\mathcal{K})}_{ij,k}(t,t_0) = \exp\left[-\int\limits_{t_0}^{t}\frac{\mathrm{d}t}{t}\,\frac{\alpha_S}{2\pi}\int\mathrm{d}z\,\frac{\mathrm{d}\phi}{2\pi}\;\underbrace{\mathcal{K}_{ij,k}(t,z,\phi)}_{\text{splitting kernel for } (ij)\to ij \text{ (spectator } k)}\right]$$

- evolution parameter t defined by kinematics

generalised angle (HERWIG++) or transverse momentum (PYTHIA, SHERPA)

- will replace $\frac{\mathrm{d}t}{t}\mathrm{d}z\frac{\mathrm{d}\phi}{2\pi} \longrightarrow \mathrm{d}\Phi_1$
- scale choice for strong coupling: $\alpha_S(k_\perp^2)$

resums classes of higher logarithms

- regularisation through cut-off t_0

Emissions off a Born matrix element

- "compound" splitting kernels $\mathcal{K}_n$ and Sudakov form factors $\Delta_n^{(\mathcal{K})}$ for emission off n-particle final state:

$$\mathcal{K}_n(\Phi_1) = \frac{\alpha_S}{2\pi} \sum_{\text{all } \{ij,k\}} \mathcal{K}_{ij,k}(\Phi_{ij,k}), \quad \Delta_n^{(\mathcal{K})}(t, t_0) = \exp\left[-\int_{t_0}^{t} \mathrm{d}\Phi_1\, \mathcal{K}_n(\Phi_1) \right]$$

- consider first emission only off Born configuration

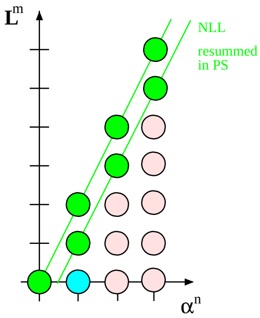
$$\mathrm{d}\sigma_B = \mathrm{d}\Phi_N\, \mathcal{B}_N(\Phi_N) \cdot \underbrace{\left\{ \Delta_N^{(\mathcal{K})}(\mu_N^2, t_0) + \int_{t_0}^{\mu_N^2} \mathrm{d}\Phi_1 \left[\mathcal{K}_N(\Phi_1) \Delta_N^{(\mathcal{K})}(\mu_N^2, t(\Phi_1)) \right] \right\}}_{\text{integrates to unity } \longrightarrow \text{ "unitarity" of parton shower}}$$

- further emissions by recursion with $\mu_N^2 \longrightarrow t$ of previous emission

NLO improvements: Matching

NLO matching: Basic idea

- parton shower resums logarithms
  fair description of collinear/soft emissions
  jet evolution (where the logs are large)
- matrix elements exact at given order
  fair description of hard/large-angle emissions
  jet production (where the logs are small)
- adjust ("match") terms:
  - cross section at NLO accuracy &
    correct hardest emission in PS to exactly reproduce ME at order α_S
    ($\mathcal{R}$-part of the NLO calculation)
    (this is relatively trivial)
  - maintain (N)LL-accuracy of parton shower
    (this is not so simple to see)



The POWHEG-trick: modifying the Sudakov form factor

(P. Nason, JHEP 0411 (2004) 040 & S. Frixione, P. Nason & C. Oleari, JHEP 0711 (2007) 070)

- reminder: $\mathcal{K}_{ij,k}$ reproduces process-independent behaviour of $\mathcal{R}_N/\mathcal{B}_N$ in soft/collinear regions of phase space

$$\mathrm{d}\Phi_1 \, \frac{\mathcal{R}_N(\Phi_{N+1})}{\mathcal{B}_N(\Phi_N)} \xrightarrow{\text{IR}} \mathrm{d}\Phi_1 \, \frac{\alpha_S}{2\pi} \mathcal{K}_{ij,k}(\Phi_1)$$

- define modified Sudakov form factor (as in ME correction)

$$\Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2, t_0) = \exp\left[-\int\limits_{t_0}^{\mu_N^2} \mathrm{d}\Phi_1 \, \frac{\mathcal{R}_N(\Phi_{N+1})}{\mathcal{B}_N(\Phi_N)}\right] ,$$

- assumes factorisation of phase space: $\Phi_{N+1} = \Phi_N \otimes \Phi_1$
- typically will adjust scale of α_S to parton shower scale

Local K-factors

(P. Nason, JHEP 0411 (2004) 040 & S. Frixione, P. Nason & C. Oleari, JHEP 0711 (2007) 070)

- start from Born configuration Φ_N with NLO weight:

("local K-factor")

$$\mathrm{d}\sigma_N^{(\mathrm{NLO})} = \mathrm{d}\Phi_N\,\bar{\mathcal{B}}(\Phi_N)$$

$$= \mathrm{d}\Phi_N \left\{ \mathcal{B}_N(\Phi_N) + \underbrace{\mathcal{V}_N(\Phi_N) + \mathcal{B}_N(\Phi_N)\otimes\mathcal{S}}_{\tilde{\mathcal{V}}_N(\Phi_N)} \right.$$

$$\left. + \int \mathrm{d}\Phi_1\,[\mathcal{R}_N(\Phi_N\otimes\Phi_1) - \mathcal{B}_N(\Phi_N)\otimes\mathrm{d}\mathcal{S}(\Phi_1)] \right\}$$

- by construction: exactly reproduce cross section at NLO accuracy
- note: second term vanishes if $\mathcal{R}_N \equiv \mathcal{B}_N \otimes \mathrm{d}\mathcal{S}$

(relevant for MC@NLO)

NLO accuracy in radiation pattern

(P. Nason, JHEP 0411 (2004) 040 & S. Frixione, P. Nason & C. Oleari, JHEP 0711 (2007) 070)

- generate emissions with $\Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2,\, t_0)$:

$$\mathrm{d}\sigma_N^{(\mathrm{NLO})} = \mathrm{d}\Phi_N\, \bar{\mathcal{B}}(\Phi_N)$$

$$\times \underbrace{\left\{ \Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2,\, t_0) + \int\limits_{t_0}^{\mu_N^2} \mathrm{d}\Phi_1 \frac{\mathcal{R}_N(\Phi_N \otimes \Phi_1)}{\mathcal{B}_N(\Phi_N)} \Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2,\, k_\perp^2(\Phi_1)) \right\}}_{\text{integrating to yield 1 - “unitarity of parton shower”}}$$

- radiation pattern like in ME correction
- pitfall, again: choice of upper scale μ_N^2 (this is vanilla POWHEG!)
- apart from logs: which configurations enhanced by local K-factor

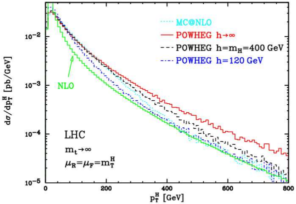
(K-factor for inclusive production of X adequate for $X+$ jet at large $p_\perp$?)

Improved POWHEG

(S. Alioli, P. Nason, C. Oleari, & E. Re, JHEP 0904 (2009) 002)

- split real-emission ME as

$$\mathcal{R} = \mathcal{R}\left(\underbrace{\frac{h^2}{p_\perp^2 + h^2}}_{\mathcal{R}^{(S)}} + \underbrace{\frac{p_\perp^2}{p_\perp^2 + h^2}}_{\mathcal{R}^{(F)}}\right)$$



- can "tune" h to mimick NNLO - or maybe resummation result
- differential event rate up to first emission

$$\begin{aligned} \mathrm{d}\sigma &= \mathrm{d}\Phi_B \bar{\mathcal{B}}^{(\mathrm{R}^{(S)})} \left[\Delta^{(\mathcal{R}^{(S)}/\mathcal{B})}(s,\, t_0) + \int\limits_{t_0}^{s} \mathrm{d}\Phi_1 \frac{\mathcal{R}^{(S)}}{\mathcal{B}} \Delta^{(\mathcal{R}^{(S)}/\mathcal{B})}(s,\, k_\perp^2)\right] \\ &\quad + \mathrm{d}\Phi_R\, \mathcal{R}^{(F)}(\Phi_R) \end{aligned}$$

Resummation in MC@NLO

(S. Frixione & B. Webber, JHEP 0602 (2002) 029)

(S. Hoeche, F. Krauss, M. Schoenherr, & F. Siegert, JHEP 1209 (2012) 049)

- divide $\mathcal{R}_N$ in soft ("S") and hard ("H") part:

$$\mathcal{R}_N = \mathcal{R}_N^{(S)} + \mathcal{R}_N^{(H)} = \mathcal{B}_N \otimes \mathrm{d}\mathcal{S}_1 + \mathcal{H}_N$$

- identify subtraction terms and shower kernels $\mathrm{d}\mathcal{S}_1 \equiv \sum_{\{ij,k\}} \mathcal{K}_{ij,k}$

(modify $\mathcal{K}$ in 1st emission to account for colour)

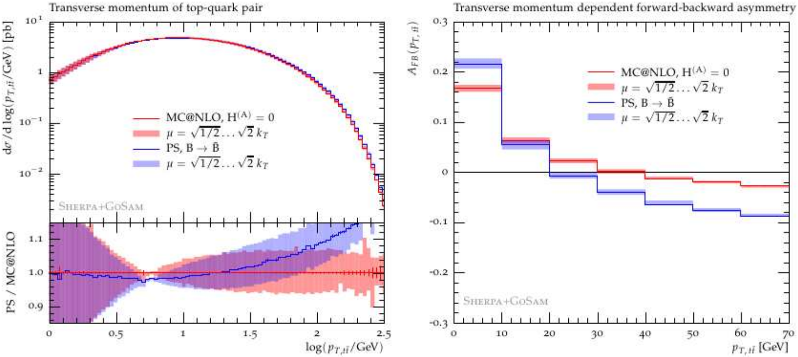
$$\mathrm{d}\sigma_N = \mathrm{d}\Phi_N \underbrace{\tilde{\mathcal{B}}_N(\Phi_N)}_{\mathcal{B}+\tilde{\mathcal{V}}} \left[\Delta_N^{(\mathcal{K})}(\mu_N^2, t_0) + \int_{t_0}^{\mu_N^2} \mathrm{d}\Phi_1 \, \mathcal{K}_{ij,k}(\Phi_1) \, \Delta_N^{(\mathcal{K})}(\mu_N^2, k_\perp^2) \right] + \mathrm{d}\Phi_{N+1} \, \mathcal{H}_N$$

- effect: only resummed parts modified with local K-factor

Aside: impact of full colour

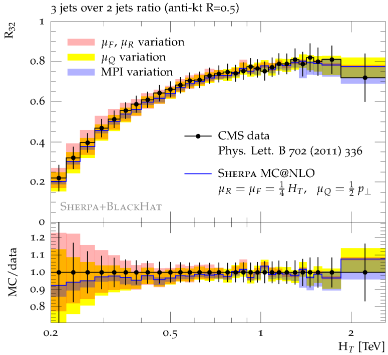
(S. Hoeche, J. Huang, G. Luisoni, M. Schoenherr, & J. Winter, arXiv:1306.2703 [hep-ph])

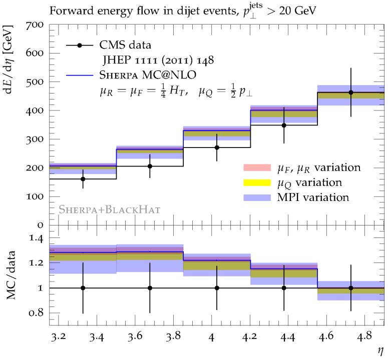
- evaluate effect of full colour treatment, MC@NLO without **H**-part vs. parton shower with $\mathcal{B} \longrightarrow \tilde{\mathcal{B}}$
- take $t\bar{t}$ production (red = full colour, blue = "PS" colours)



MC@NLO for light jets: R_{32} & forward energy flow

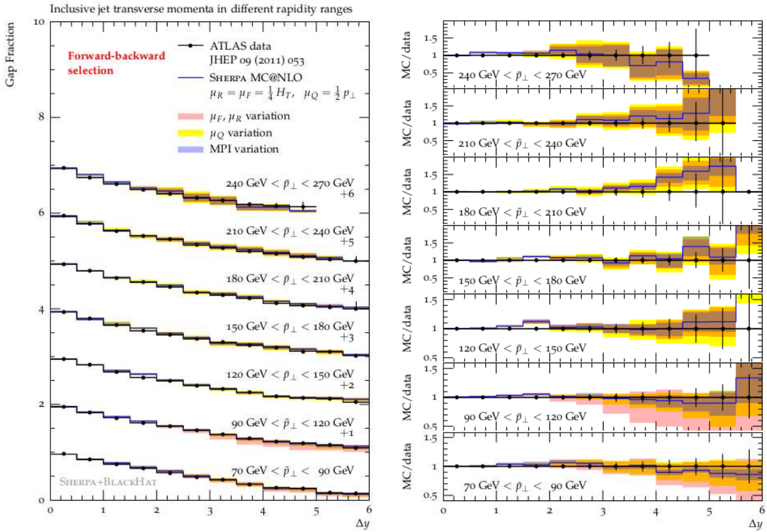
(S. Hoeche & M. Schoenherr, Phys. Rev. D86 (2012) 094042)





MC@NLO for light jets: jet vetoes

(S. Hoeche & M. Schoenherr, Phys. Rev. D86 (2012) 094042)

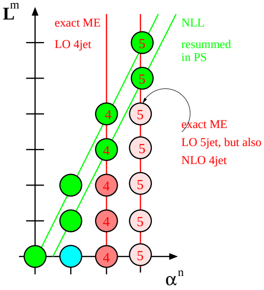


Multijet merging @ leading order

Multijet merging: basic idea

(S. Catani, F. Krauss, R. Kuhn, B. Webber, JHEP 0111 (2001) 063,
L. Lonnblad, JHEP 0205 (2002) 046, & F. Krauss, JHEP 0208 (2002) 015)

- parton shower resums logarithms
  fair description of collinear/soft emissions
  jet evolution (where the logs are large)
- matrix elements exact at given order
  fair description of hard/large-angle emissions
  jet production (where the logs are small)
- combine ("merge") both:
  result: "towers" of MEs with increasing number of jets evolved with PS
  - multijet cross sections at Born accuracy
  - maintain (N)LL accuracy of parton shower

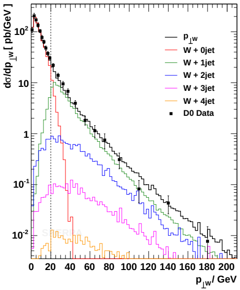


Separating jet evolution and jet production

- separate regions of jet production and jet evolution with jet measure Q_J

  ("truncated showering" if not identical with evolution parameter)

- matrix elements populate hard regime
- parton showers populate soft domain



First emission(s), again

(S. Hoeche, F. Krauss, S. Schumann, F. Siegert, JHEP 0905 (2009) 053)

$$\mathrm{d}\sigma = \mathrm{d}\Phi_N\,\mathcal{B}_N\left[\Delta_N^{(\mathcal{K})}(\mu_N^2,\,t_0)+\int\limits_{t_0}^{\mu_N^2}\mathrm{d}\Phi_1\,\mathcal{K}_N\Delta_N^{(\mathcal{K})}(\mu_N^2,\,t_{N+1})\Theta(Q_J-Q_{N+1})\right]$$

$$+\mathrm{d}\Phi_{N+1}\,\mathcal{B}_{N+1}\,\Delta_N^{(\mathcal{K})}(\mu_{N+1}^2,\,t_{N+1})\Theta(Q_{N+1}-Q_J)$$

- note: $N+1$-contribution includes also $N+2$, $N+3$, ...

(no Sudakov suppression below t_{n+1}, see further slides for iterated expression)

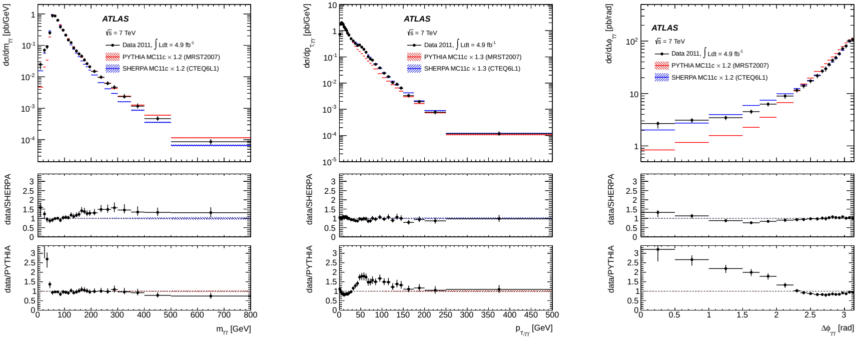
- potential occurence of different shower start scales: $\mu_{N,N+1,\ldots}$
- "unitarity violation" in square bracket: $\mathcal{B}_N\mathcal{K}_N \longrightarrow \mathcal{B}_{N+1}$

(cured with UMEPS formalism, L. Lonnblad & S. Prestel, JHEP 1302 (2013) 094 &

S. Platzer, arXiv:1211.5467 [hep-ph] & arXiv:1307.0774 [hep-ph])

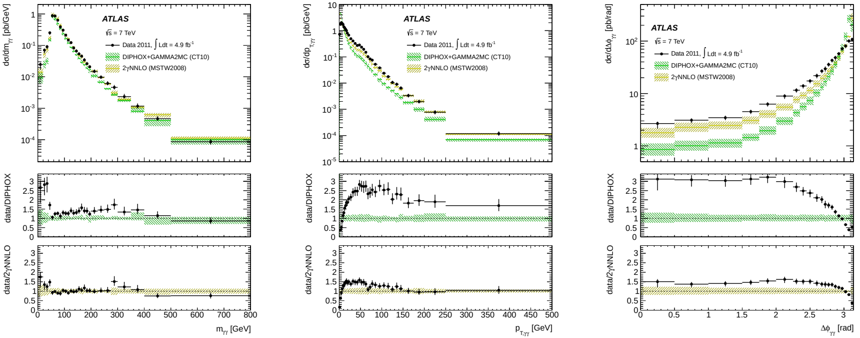
Di-photons @ ATLAS: $m_{\gamma\gamma}$, $p_{\perp,\gamma\gamma}$, and $\Delta\phi_{\gamma\gamma}$ in showers

(arXiv:1211.1913 [hep-ex])



Aside: Comparison with higher order calculations

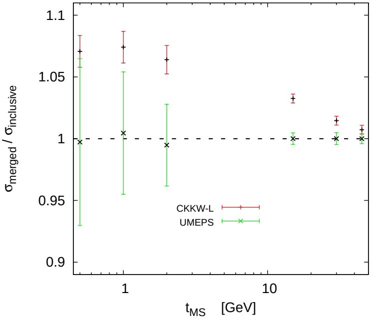
(arXiv:1211.1913 [hep-ex])



Aside': restoring unitarity with UMEPS

(L. Lonnblad, S. Prestel, JHEP1302 (2013) 094)

- as indicated, MEPS@LO formalism breaks unitarity: inclusive n-jet cross sections not exactly maintained due to mismatch of kernels in actual emission term and Sudakov form factor
- can be cured by adding/subtracting shower and ME-like terms
- low merging cut possible



Multijet merging @ next-to leading order

Multijet-merging at NLO: MePs@Nlo

- basic idea like at LO: towers of MEs with increasing jet multi (but this time at NLO)
- combine them into one sample, remove overlap/double-counting

maintain NLO and (N)LL accuracy of ME and PS

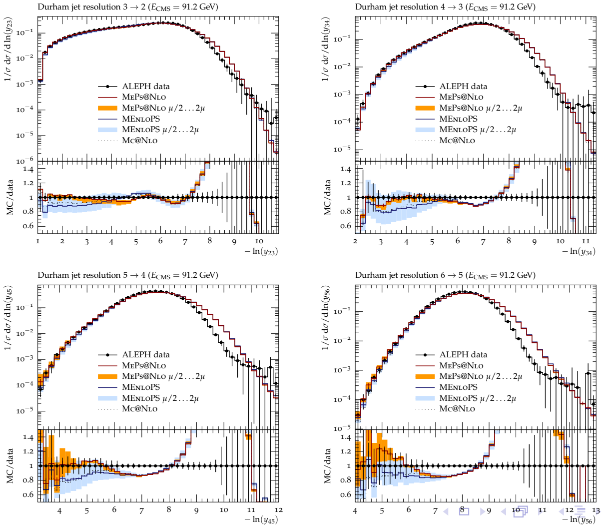
- this effectively translates into a merging of MC@NLO simulations and can be further supplemented with LO simulations for even higher final state multiplicities

First emission(s), once more

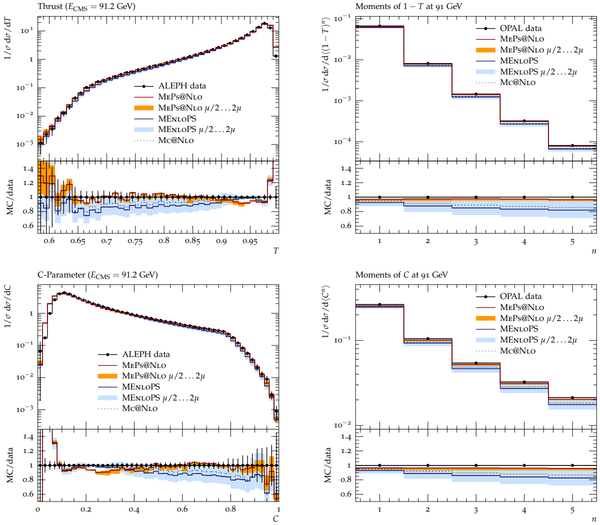
$$
\begin{aligned}
\mathrm{d}\sigma \;=\;& \mathrm{d}\Phi_N\,\tilde{\mathcal{B}}_N\left[\Delta_N^{(\mathcal{K})}(\mu_N^2,t_0)+\int_{t_0}^{\mu_N^2}\mathrm{d}\Phi_1\,\mathcal{K}_N\Delta_N^{(\mathcal{K})}(\mu_N^2,t_{N+1})\Theta(Q_J-Q_{N+1})\right]\\
&+\mathrm{d}\Phi_{N+1}\,\mathcal{H}_N\Delta_N^{(\mathcal{K})}(\mu_N^2,t_{N+1})\Theta(Q_J-Q_{N+1})\\
&+\mathrm{d}\Phi_{N+1}\,\tilde{\mathcal{B}}_{N+1}\left(1+\frac{\mathcal{B}_{N+1}}{\tilde{\mathcal{B}}_{N+1}}\int_{t_{N+1}}^{\mu_N^2}\mathrm{d}\Phi_1\,\mathcal{K}_N\right)\Theta(Q_{N+1}-Q_J)\\
&\quad\cdot\Delta_N^{(\mathcal{K})}(\mu_N^2,t_{N+1})\cdot\left[\Delta_{N+1}^{(\mathcal{K})}(t_{N+1},t_0)+\int_{t_0}^{t_{N+1}}\mathrm{d}\Phi_1\,\mathcal{K}_{N+1}\Delta_{N+1}^{(\mathcal{K})}(t_{N+1},t_{N+2})\right]\\
&+\mathrm{d}\Phi_{N+2}\,\mathcal{H}_{N+1}\Delta_N^{(\mathcal{K})}(\mu_N^2,t_{N+1})\Delta_{N+1}^{(\mathcal{K})}(t_{N+1},t_{N+2})\Theta(Q_{N+1}-Q_J)+\ldots
\end{aligned}
$$

MEPS@NLO: example results for $e^- e^+ \to$ hadrons

(T. Gehrmann et al., JHEP 1301 (2013) 144)

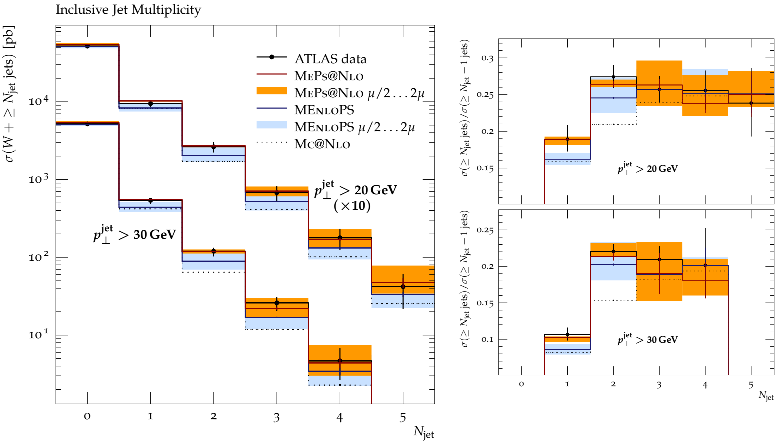


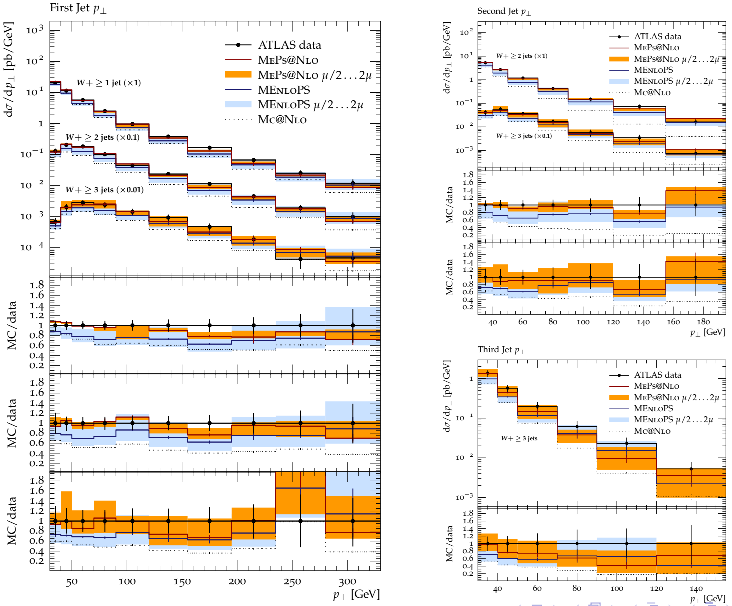
MEPS@NLO: example results for $e^-e^+ \to$ hadrons

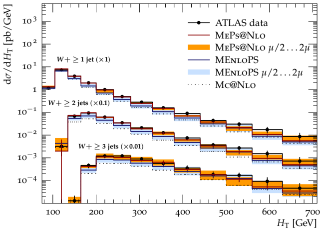


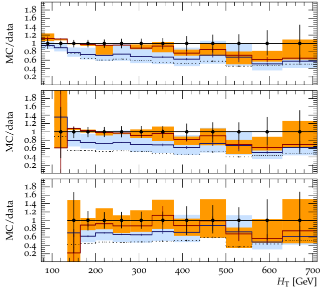
Example: MEPS@NLO for W+jets

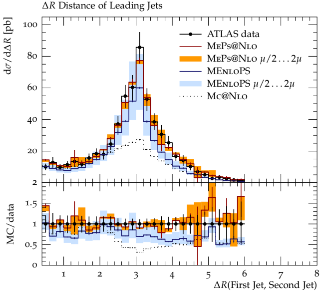
(S. Hoeche, F. Krauss, M. Schoenherr & F. Siegert, JHEP 1304 (2013) 027)

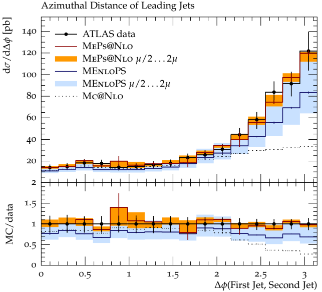



First Jet $p_\perp$
$d\sigma/dp_\perp$ [pb/GeV]
$W+\geq 1$ jet ($\times 1$)
$W+\geq 2$ jets ($\times 0.1$)
$W+\geq 3$ jets ($\times 0.01$)
ATLAS data
MEPS@NLO
MEPS@NLO $\mu/2\ldots2\mu$
MENLOPS
MENLOPS $\mu/2\ldots2\mu$
MC@NLO
MC/data
$p_\perp$ [GeV]
Second Jet $p_\perp$
$W+\geq 2$ jets ($\times 1$)
$W+\geq 3$ jets ($\times 0.1$)
Third Jet $p_\perp$
$W+\geq 3$ jets


ATLAS data
MEPS@NLO
MEPS@NLO μ/2...2μ
MENLOPS
MENLOPS μ/2...2μ
MC@NLO
W + ≥ 1 jet (×1)
W + ≥ 2 jets (×0.1)
W + ≥ 3 jets (×0.01)
dσ/dH_T [pb/GeV]
H_T [GeV]

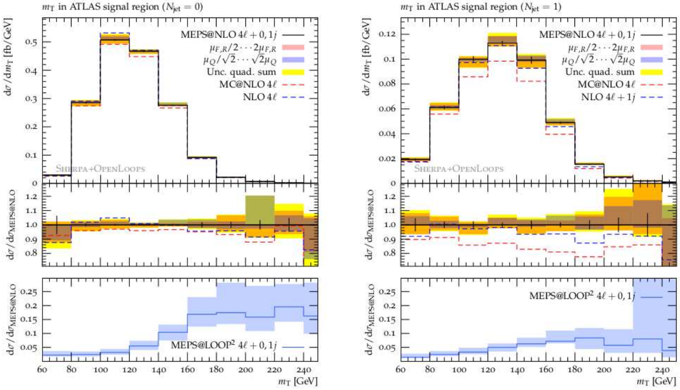

MC/data
MC/data
MC/data
H_T [GeV]


ΔR Distance of Leading Jets
ATLAS data
MEPS@NLO
MEPS@NLO μ/2...2μ
MENLOPS
MENLOPS μ/2...2μ
MC@NLO
dσ/dΔR [pb]
MC/data
ΔR(First Jet, Second Jet)


Azimuthal Distance of Leading Jets
ATLAS data
MEPS@NLO
MEPS@NLO μ/2...2μ
MENLOPS
MENLOPS μ/2...2μ
MC@NLO
dσ/dΔφ [pb]
MC/data
Δφ(First Jet, Second Jet)

Example: MEPS@NLO for W^+W^-+jets

(F. Cascioli et al., arXiv:1309.0500)

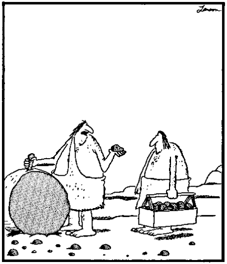


summary and

concluding remarks

Summary

- systematic improvement of event generators by including higher orders has been at the core of QCD theory and developments in the past decade:
  - multijet merging ("CKKW", "MLM")
  - NLO matching ("MC@NLO", "POWHEG")
  - MENLOPS – combination of matching and merging
  - multijet merging at NLO (MEPS@NLO, "FxFx")



"So what's this? I asked for a *hammer*! A hammer! *This* is a crescent wrench! ... Well, maybe it's a hammer. ... Damn these stone tools."

(first 3 methods well understood and used in experiments)

- multijet merging at NLO under scrutiny
- complete automation of NLO calculations ≈ done
  time to optimise the impact of this gargantuan task