

# Three-nucleon forces and shell structure of neutron-rich Ca isotopes

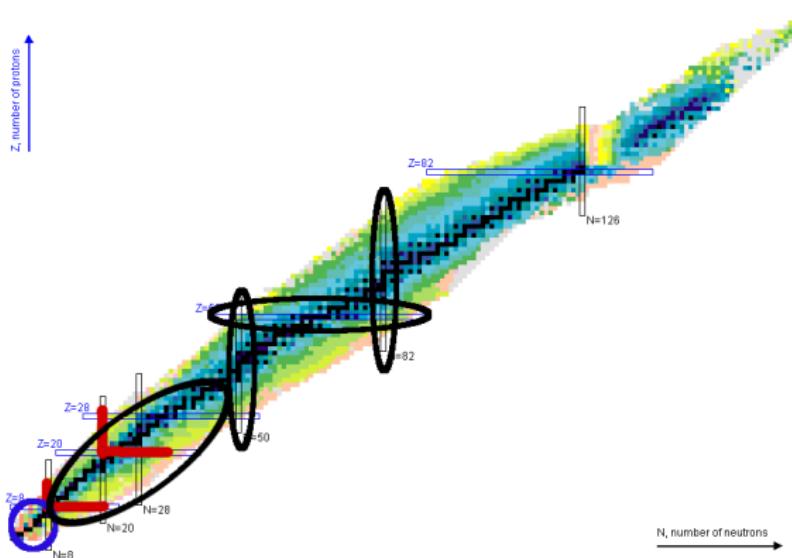
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# Nuclear Landscape



Big variety of nuclei in the nuclear chart,  $A \sim 2 \dots 300$

Systematic *ab initio* calculations only possible in the lightest nuclei

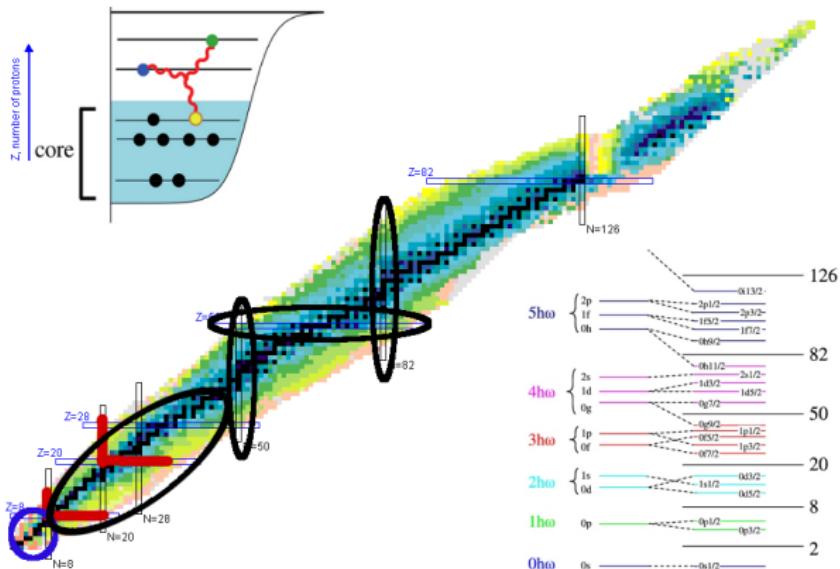
Hard many-body problem:  
approximate methods suited for different regions

## Interacting Shell Model:

Solve the problem choosing the (more) relevant degrees of freedom

Use realistic nucleon-nucleon (NN) and three-nucleon (3N) interactions

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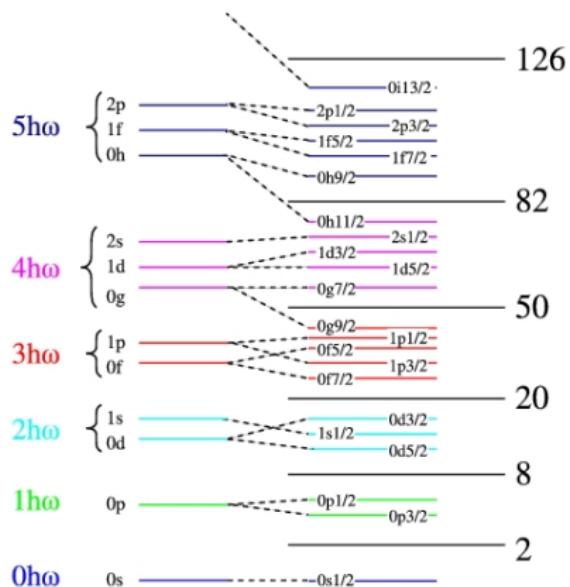
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# The Interacting Shell Model



Basis states 3D Harmonic Oscillator  
Configuration space is separated into

- Outer orbits:  
orbits that are always empty
- Valence space: the space in which we explicitly solve the problem
- Inner core:  
orbits that are always filled

$$H|\Psi\rangle = E|\Psi\rangle \rightarrow H_{\text{eff}}|\Psi\rangle_{\text{eff}} = E|\Psi\rangle_{\text{eff}}$$

Full diagonalization code ANTOINE Caurier et al. RMP77 427(2005)

# Forces and Currents in Chiral EFT

Chiral EFT: low energy approach to QCD for nuclear structure energies

Approximate chiral symmetry of QCD: pions pseudo-Goldstone bosons

Short-range couplings are fitted to experiment once

Systematic expansion of nuclear forces

	2N force	3N force	4N force
LO	X H	—	—
NLO	X H H X H H	—	—
N <sup>3</sup> LO	H H	H H X X	—
N <sup>3</sup> LO	X H H X H H ...	H H H H H H ...	H H H H H H ...

NN forces up to N<sup>3</sup>LO

3N forces up to N<sup>2</sup>LO

NN fitted to:

- NN scattering
- $\pi$ -N scattering

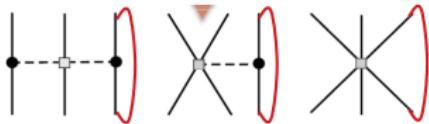
3N fitted to:

- $^3\text{H}$  Binding Energy
- $^4\text{He}$  radius

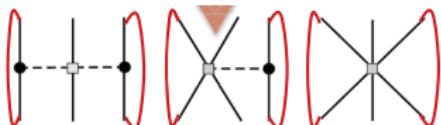
# 3N Forces

Treatment of 3N forces:

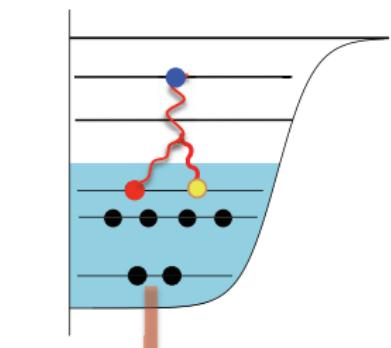
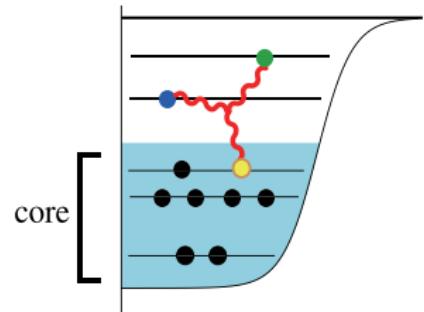
normal-ordered 2B: 2 valence, 1 core particle  
 ⇒ (effective) Two-body Matrix Elements (TBME)



normal-ordered 1B: 1 valence, 2 core particles  
 ⇒ (effective) Single particle energies (SPE)



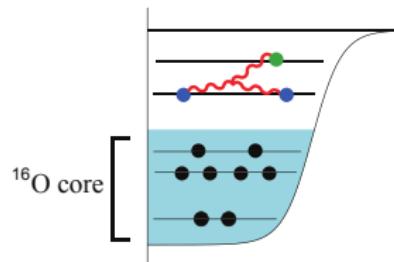
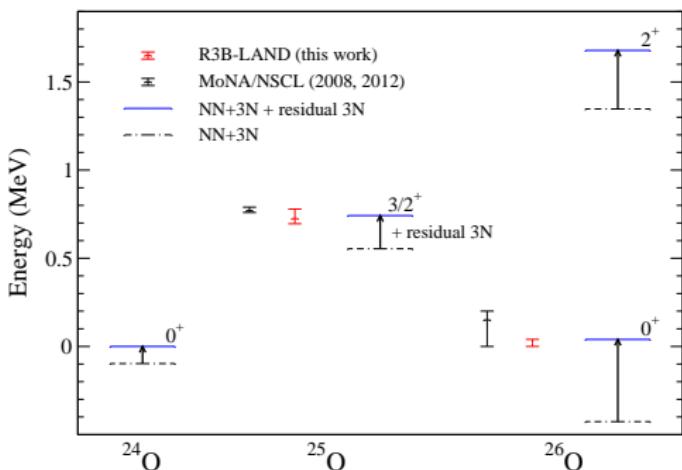
residual 3B:  
 ⇒ Estimated to be suppressed by  $N_{\text{valence}}/N_{\text{core}}$



# Residual 3N Forces

Results with normal-ordered  
two-body part of 3N forces

In extreme neutron-rich oxygen isotopes,  
3N forces between 3 valence neutrons  
can give a relevant contribution



Repulsive residual 3N contributions

Small compared to normal-ordered  
3N force, but increase with  $N$

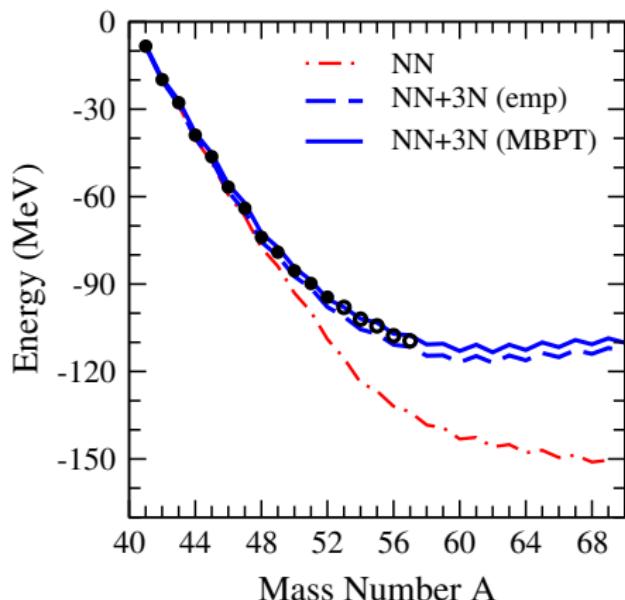
Very good agreement with  
resonances in  $^{25}\text{O}$  and  $^{26}\text{O}$

Caesar, Simonis et al  
PRC88 034313 (2013)

Challenge: include continuum

# Ca isotopes: Masses

Ca isotopes: explore nuclear shell evolution  $N = 20, 28, 32?, 34?$



Ca measured from  $^{40}\text{Ca}$  core

3N forces repulsive contribution,  
chiral NN-only forces too attractive

Flat behavior towards  $^{60}\text{Ca}$  does not  
allow clear prediction of the dripline

Sensitivity to SPEs

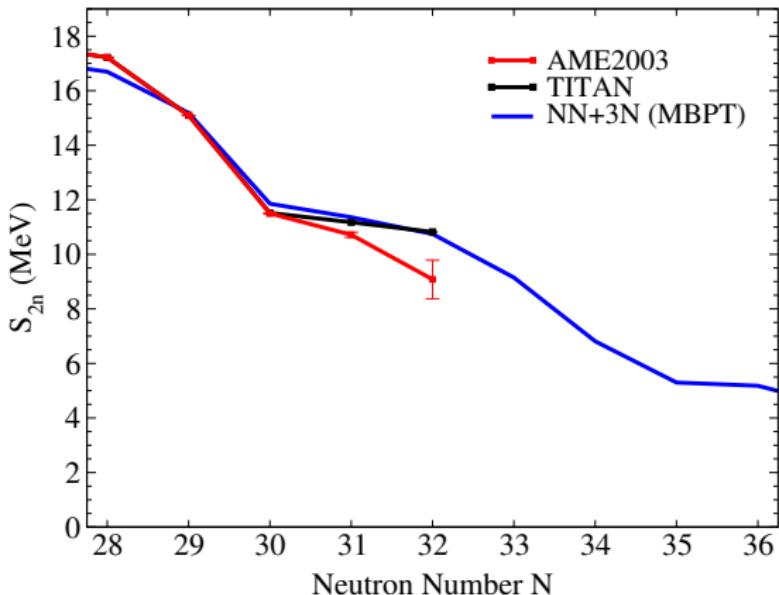
MBPT (calculated from NN+3N forces)

Empirical (from GXPF1 interaction)

Estimate of the uncertainty

# Two-Neutron separation energies

Compare  $S_{2n} = -[B(N, Z) - B(N - 2, Z)]$  with experiment



Precision measurements  
with TITAN changed AME  
2003  $\sim 1.74$  MeV in  $^{52}\text{Ca}$

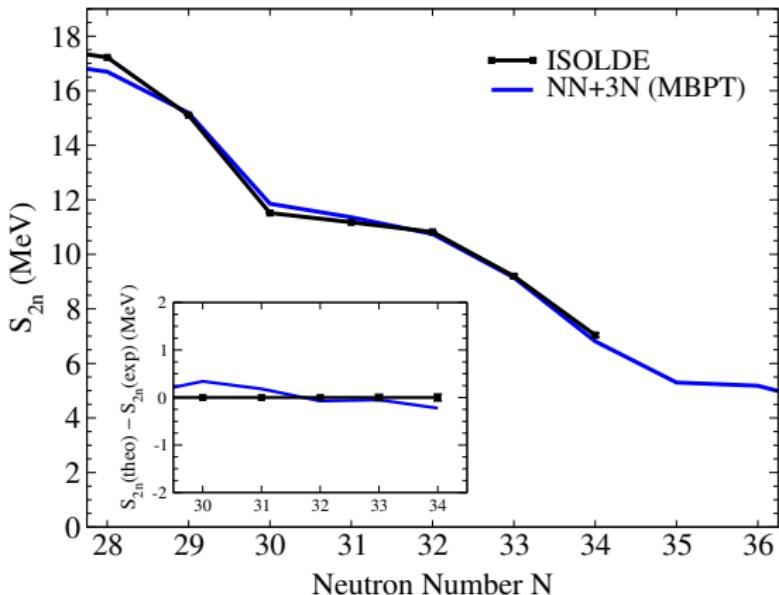
More flat behavior in  
 $^{50}\text{Ca}-^{52}\text{Ca}$

3N forces needed in  
theoretical calculation

Gallant et al.  
PRL 109 032506 (2012)

# $^{54}\text{Ca}$ and $N = 32$ shell closure

Compare  $S_{2n} = -[B(N, Z) - B(N - 2, Z)]$  with experiment



Very recently  $^{53,54}\text{Ca}$   
measured at ISOLDE

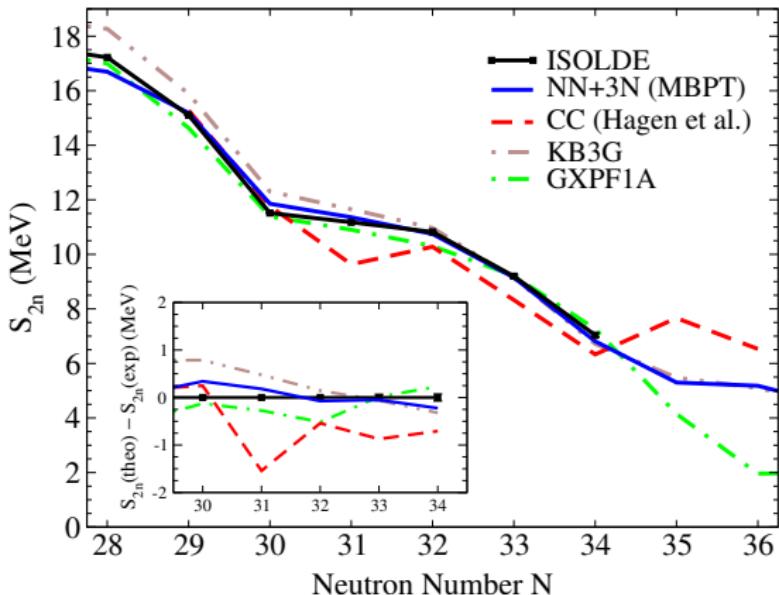
Excellent agreement with  
theoretical prediction

$S_{2n}$  evolution:  
 $^{52}\text{Ca}-^{54}\text{Ca}$  similar to  
 $^{48}\text{Ca}-^{50}\text{Ca}$ , point to  
 $N = 32$  shell closure

Wienholtz et al.  
Nature 498 346 (2013)

# Two-neutron separation energies

Compare  $S_{2n} = -[B(N, Z) - B(N - 2, Z)]$  with experiment



Phenomenology  
masses/gaps as input,  
differ markedly beyond  $^{54}\text{Ca}$

Coupled-Cluster calculations  
good agreement with  
adjusted 3N forces

Wienholtz et al.  
Nature 498 346 (2013)

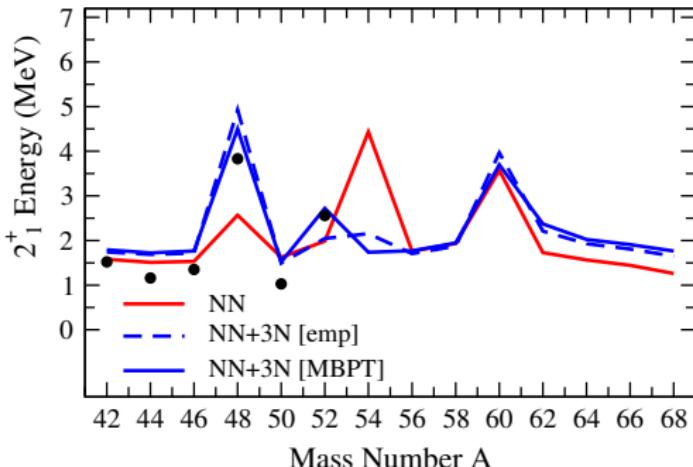
# Shell closures and $2_1^+$ energies

$2_1^+$  energies characterize shell closures

Correct closure at  $N = 28$   
when 3N forces are included

Holt et al. JPG39 085111(2012)

Holt, JM, Schwenk,  
JPG40 075105 (2013)



- 3N forces enhance closure at  $N = 32$
- 3N forces reduce strong closure at  $N = 34$  (1.7-2.2 MeV)

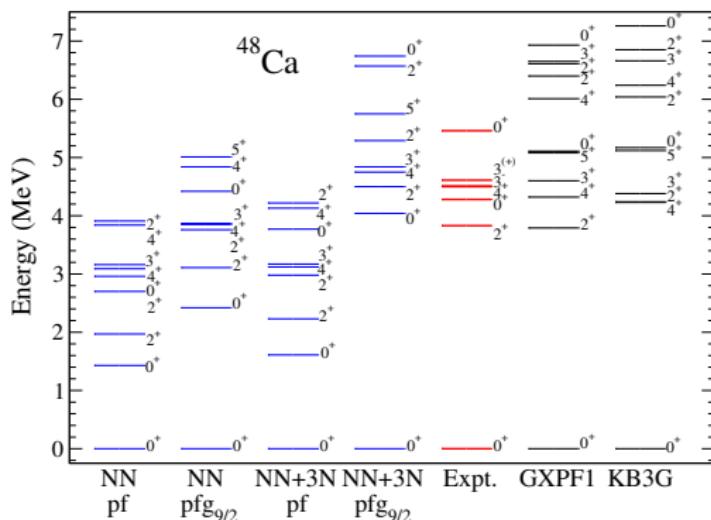
Measured at 2.04 MeV, suggest  $N = 34$  shell closure

Steppenbeck et al. Nature 502 207(2013)

$2_1^+$  energy in  $^{54}\text{Ca}$  complemented with mass measurements in  $^{55,56}\text{Ca}$   
confirm magic number  $N = 34$  and test theoretical calculations

# $^{48}\text{Ca}$ spectrum

Challenge: doubly-magic nucleus  $^{48}\text{Ca}$ ,  
 $N = 28$  shell closure not reproduced with NN forces

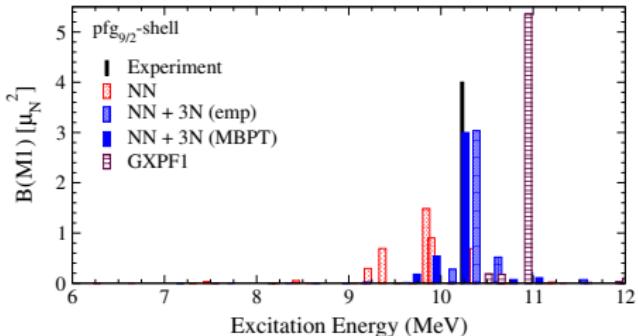


Spectra in reasonable overall agreement with experiment

$2_1^+$  state well reproduced,  
 energy slightly too high

$0_1^+$  state too low  
 especially compared to  
 phenomenological interactions

# B(M1) and B(E2) Transitions



B(M1) strength in  $^{48}\text{Ca}$   
too fragmented with NN forces

NN+3N good agreement with  
experiment strength and energy

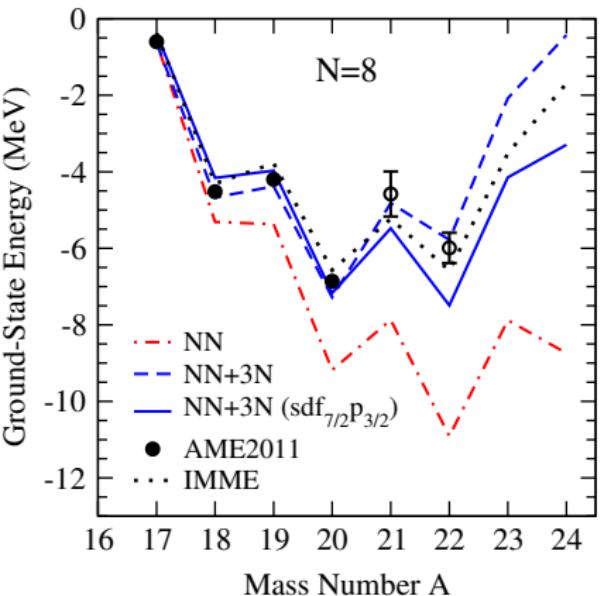
Comparable to phenomenology

B(E2)s in reasonable  
agreement with experiment

Similar quality as  
phenomenological  
interactions

Isotope	Transition	GXF1A	MBPT	EXP.
$^{46}\text{Ca}$	$2^+ \rightarrow 0^+$	9.2	13.3	$25.4 \pm 4.5$ $36.4 \pm 2.6$
$^{46}\text{Ca}$	$6^+ \rightarrow 4^+$	3.6	4.8	$5.38 \pm 0.29$
$^{47}\text{Ca}$	$3/2^- \rightarrow 7/2^-$	3.6	1.0	$4.0 \pm 0.2$
$^{48}\text{Ca}$	$2^+ \rightarrow 0^+$	11.9	10.3	$19 \pm 6.4$
$^{49}\text{Ca}$	$7/2^- \rightarrow 3/2^-$	4.0	0.22	$0.53 \pm 0.21$
$^{50}\text{Ca}$	$2^+ \rightarrow 0^+$	9.1	11.2	$7.4 \pm 0.2$

# Proton dripline at $N = 8$



Holt, JM, Schwenk PRL110 022502 (2013)

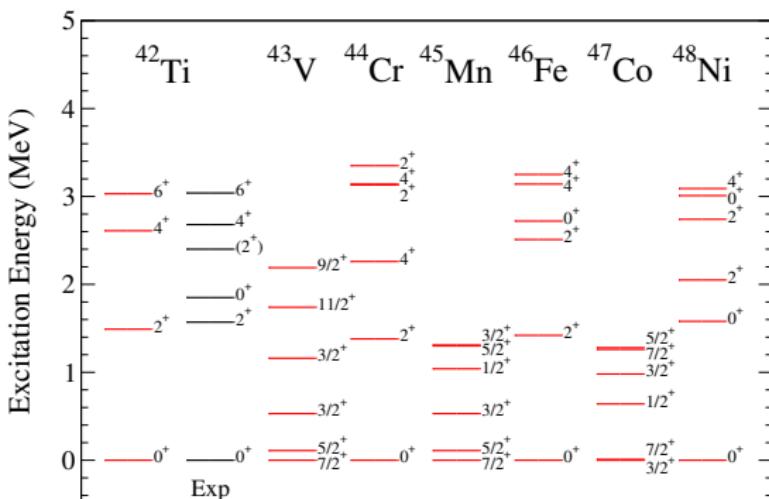
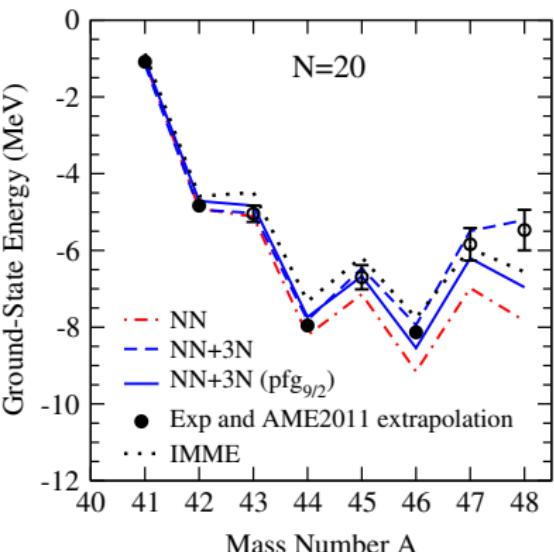
Compare NN+3N theory to  
isobaric mass-multiplet formula (IMME)  
 $E(A, T, T_z) = E(A, T, -T_z) + 2b(A, T)T_z$

Isospin-symmetry breaking terms  
predicted by chiral EFT  
Coulomb included in calculations

Proton dripline not certain  
predicted at  $^{20}\text{Mg}$  or  $^{22}\text{Si}$ :  
 $S_{2p} = -0.12$  (Theory) /  $+0.01$  (IMME)

Excitation spectra for  $N=8$  isotones  
predicted:  $^{20}\text{Mg}$ ,  $^{21}\text{Al}$ ,  $^{22}\text{Si}$

# Masses and spectra of $N = 20$ isotones



Holt, JM, Schwenk PRL110 022502 (2013)

Dripline robustly predicted at  $^{46}\text{Fe}$

Good description of  $^{48}\text{Ni}$ :  $S_{2p} = -1.02$  (Th) vs  $-1.28(6)$  (Exp) Pomorski (2012)

Spectra: ideal ground for spectroscopic studies



## Summary and Outlook

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Shell Model based on chiral EFT (NN+3N forces)

Good agreement with experimental shell evolution and spectroscopy:

- Oxygen dripline, unbound  $^{25,26}\text{O}$  reproduced with residual 3N forces
- Predicted neutron rich Ca  $S_{2n}$ 's with NN+3N forces agree with recent measurements of  $^{51,52}\text{Ca}$  (TRIUMF) and  $^{53,54}\text{Ca}$  (ISOLTRAP)
- Shell structure: prominent closure at  $N = 32$
- Ca spectroscopy: spectra, electromagnetic strengths
- Dripline and spectra of proton-rich  $N = 8, 20$  isotones predicted

### Outlook:

Heavier isotope and isotope chains: include T=0 (pn) TBMEs

Explore uncertainties in the theoretical calculation

# Collaborators

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ISOLTRAP Collaboration  
(F. Wienholtz, K. Blaum...)



TITAN Collaboration  
(A. Gallant, J. Dilling...)