



Wir schaffen Wissen – heute für morgen

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Filter Design - Passive Power Filters

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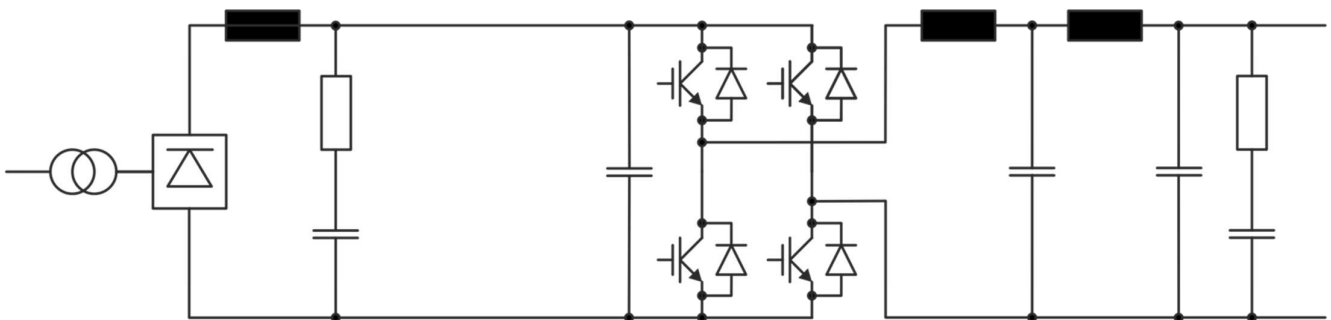
Suitable Filter Structures

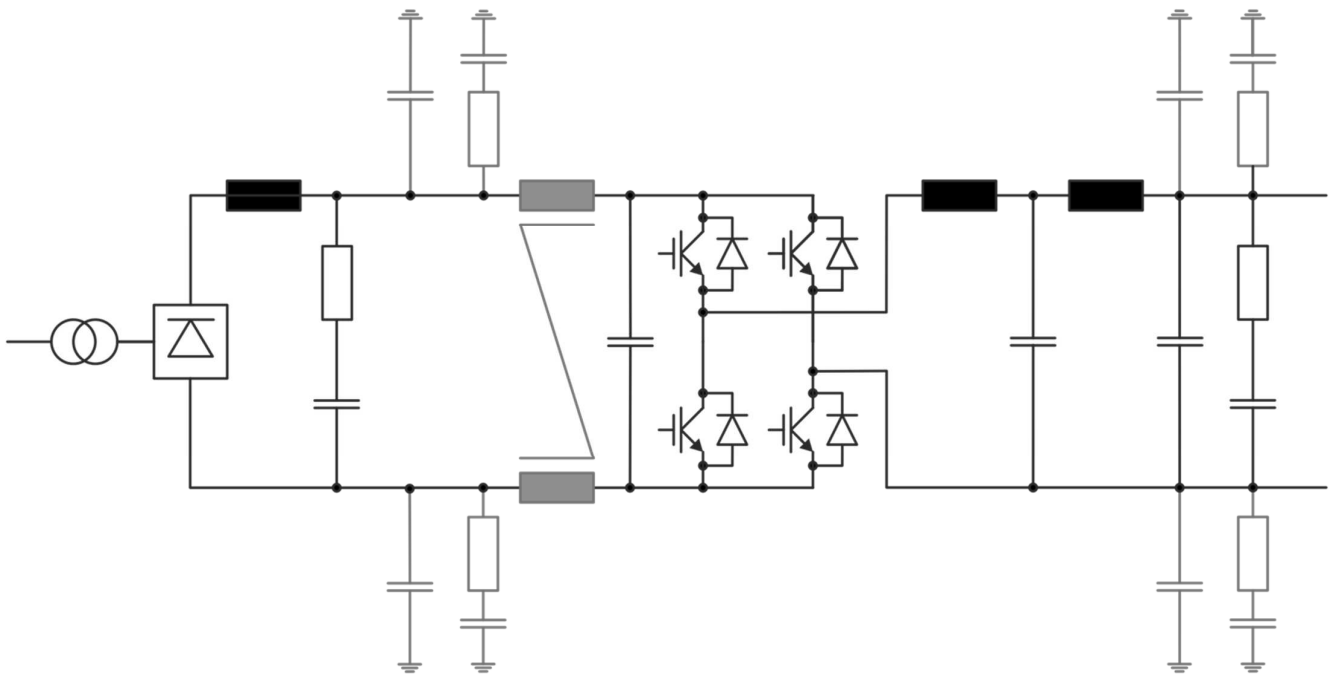
		<p>For signal filters simple RC are commonly used</p> <p>Attenuation only 20dB/decade Full current through R → Losses !</p>
		<p>With a LC structure we get 40dB/decade</p> <p>There is a high resonance</p>
		<p>Series damping in order to overcome the resonance problem</p> <p>Full current through R → Losses!</p>
		<p>Parallel damping in order to overcome the resonance problem</p> <p>Full voltage across R → Losses!</p>

Suitable Filter Structures

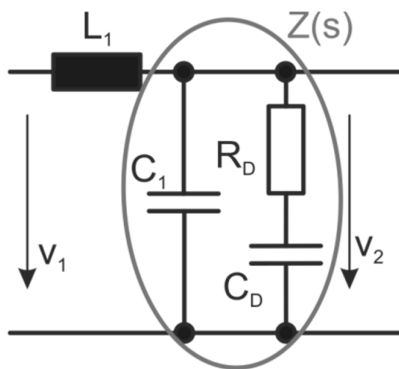
		<p>2nd order <u>lowpass</u> filter with parallel RC-damping</p> <p>Resonance can be damped Reasonable losses</p>
		<p>For better attenuation 2 LC stages are necessary</p> <p>Again high resonance</p>
		<p>RC damping after first stage</p> <p>Resonance ok High losses in RC damping due to high ripple currents</p>
		<p>4th order <u>lowpass</u> filter with parallel RC-damping after second stage</p> <p>Resonance can be damped Reasonable losses</p>

H-Bridge





2nd order filter - Transferfunction



$$Z(s) = \frac{1}{C_1 s + \frac{1}{R_D + \frac{1}{C_D s}}} = \frac{R_D C_D s + 1}{C_1 R_D C_D s^2 + (C_1 + C_D) s}$$

$$G(s) = \frac{v_2(s)}{v_1(s)} = \frac{Z(s)}{L_1 s + Z(s)} = \frac{R_D C_D s + 1}{L_1 C_1 R_D C_D s^3 + L_1 (C_1 + C_D) s^2 + R_D C_D s + 1} \quad (1)$$

$$G(s) = \frac{k_1 s + 1}{k_3 s^3 + k_2 s^2 + k_1 s + 1}$$

1st order PD
3rd order PT

with

$$k_1 = R_D C_D$$

$$k_2 = L_1 (C_1 + C_D)$$

$$k_3 = L_1 C_1 R_D C_D$$

3rd order PT in its normalized form:

$$G_{PT}(s) = \frac{1}{(1 + a_1 \frac{s}{\omega_0}) \cdot (1 + a_2 \frac{s}{\omega_0} + b_2 \frac{s^2}{\omega_0^2})} \quad (2)$$

Method	a ₁	a ₂	b ₂
Butterworth	1.0000	1.0000	1.0000
Bessel	0.7560	0.9996	0.4772
Critical damping	0.5098	1.0197	0.2599

By expanding (2) and comparing the coefficients with (1) we get:

$$G_{PT}(s) = \frac{1}{\frac{a_1 b_2}{\omega_0^3} s^3 + \frac{(a_1 a_2 + b_2)}{\omega_0^2} s^2 + \frac{(a_1 + a_2)}{\omega_0} s + 1}$$

$$k_1 = R_D C_D = \frac{a_1 + a_2}{\omega_0} \quad (3a)$$

$$k_2 = L_1 (C_1 + C_D) = \frac{a_1 a_2 + b_2}{\omega_0^2} \quad (3b)$$

$$k_3 = L_1 C_1 R_D C_D = \frac{a_1 b_2}{\omega_0^3} \quad (3c)$$

The 3 independent equations (3a....3c) contain 5 unknowns (L_1 , C_1 , R_D , C_D and ω_0). Therefore we have the choice to select 2 of them and the remaining 3 depend on that selection.

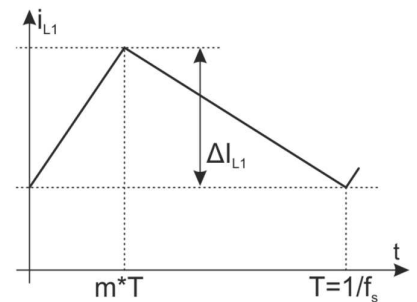
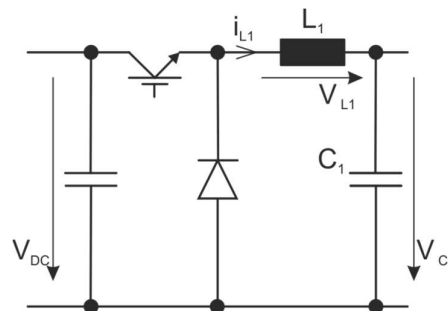
For a given frequency ω_B well in the blocking area ($\omega_B \gg \omega_0$) we can define the desired attenuation G_B . In the blocking area the highest order terms of both the numerator and denominator in equation (1) dominate, therefore (1) can be simplified to:

$$G_B = \frac{R_D C_D s}{L_1 C_1 R_D C_D s^3} = \frac{a_1 + a_2}{\omega_0} s = \frac{a_1 + a_2}{a_1 b_2} \cdot \frac{\omega_0^2}{s^2} = \frac{a_1 + a_2}{a_1 b_2} \cdot \frac{\omega_0^2}{\omega_B^2}$$

$(j)^2 = -1$

$$\omega_0 = \omega_B \cdot \sqrt{\frac{|G_B| \cdot a_1 b_2}{a_1 + a_2}} \quad (4)$$

For cost reasons L_1 should be as small as possible, but a too small inductance will result in an excessive ripple current!



The DC-voltage across C_1 is $m \cdot V_{DC}$. When the IGBT is on, the current in L_1 increases and the peak-peak ripple current ΔI_{L1} can be calculated:

$$V_{L1} = L_1 \cdot \frac{di_{L1}}{dt} = V_{DC} - V_{C1} = V_{DC} \cdot (1 - m)$$

Maximum 0.25
for $m = 0.5$

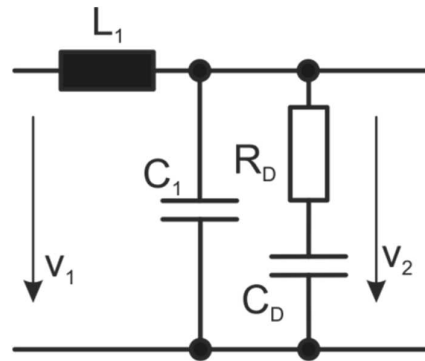
$$\Delta I_{L1} = m \cdot T \cdot \frac{di_{L1}}{dt} = m \cdot \frac{1}{f_s} \cdot \frac{V_{DC} \cdot (1 - m)}{L_1} = \frac{V_{DC} \cdot (1 - m) \cdot m}{f_s \cdot L_1}$$

$$L_1 = \frac{V_{DC} \cdot 0.25}{f_s \cdot \Delta I_{L1}} \quad (5)$$

Alternative approach to determine L₁:

$$I_{L1_ripple_pp} = \frac{v_{1_ripple_pp}}{2 \cdot \pi \cdot f_{ripple} \cdot L_1}$$

$$L_1 = \frac{v_{1_ripple_pp}}{2 \cdot \pi \cdot f_{ripple} \cdot I_{L1_ripple_pp}} \quad (6)$$



2nd order filter – Calculating filter elements

Substitute (3a) in (3c) and we receive:

Selection: L₁ and ω₀

$$C_1 = \frac{a_1 b_2}{L_1 \omega_0^2 (a_1 + a_2)} \quad (7a)$$

Selection: C₁ and ω₀

$$L_1 = \frac{a_1 b_2}{C_1 \omega_0^2 (a_1 + a_2)} \quad (7b)$$

Selection: L₁ and C₁

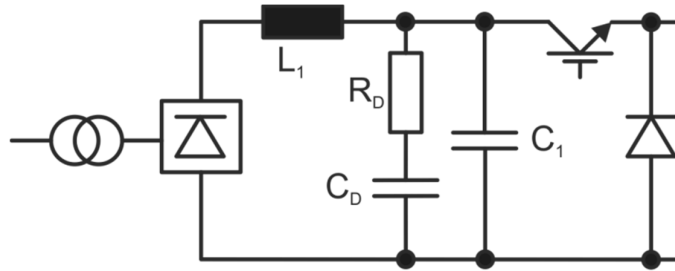
$$\omega_0 = \sqrt{\frac{a_1 b_2}{L_1 C_1 (a_1 + a_2)}} \quad (7c)$$

Solve (3b) for C_D:

$$C_D = \frac{a_1 a_2 + b_2}{L_1 \omega_0^2} - C_1 \quad (8)$$

Solve (3a) for R_D:

$$R_D = \frac{a_1 + a_2}{C_D \omega_0} \quad (9)$$



- DC-link voltage: 200V
- DC-link current: 500A
- $\Delta I_{L1} \leq 50\text{App}$.
- C_1 must be $\geq 22\text{mF}$ (because of high ripple current)

Design a 2nd order filter for all three given optimization methods and compare the results.

Select L_1 to meet the ripple current requirement:

$$L_1 = \frac{v_{1_ripple_pp}}{2 \cdot \pi \cdot f_{ripple} \cdot I_{L1_ripple_pp}} \quad (6)$$

$$L_1 = \frac{(200 \cdot 0.13)V_{pp}}{2 \cdot \pi \cdot 300s^{-1} \cdot 50App} = 300\mu H$$

Select C_1 :

$$C_1 = 22mF$$

Calculate the remaining filter elements

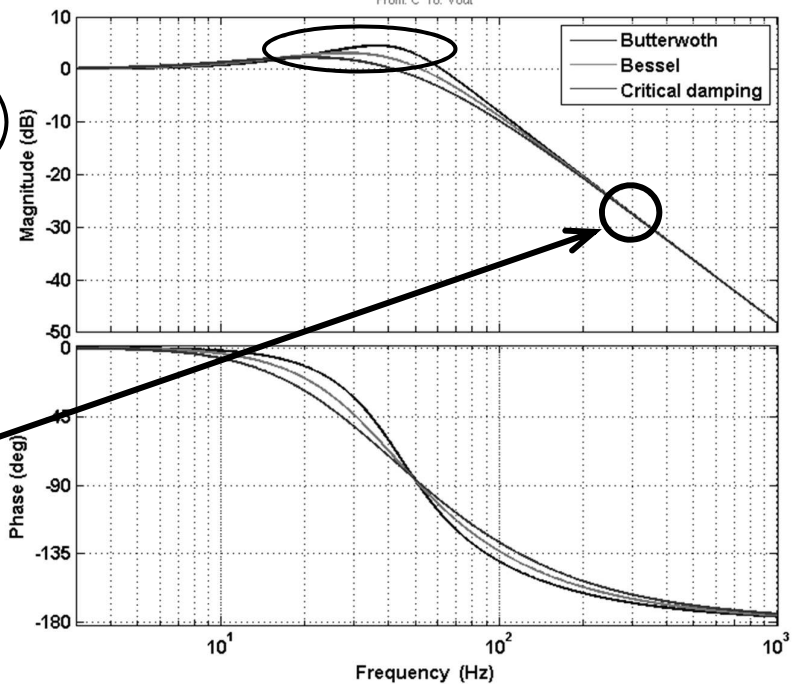
(7c, 8, 9)

Results:

	Butterworth	Bessel	Critical damping
	$a_1 = 1.0000$ $a_2 = 1.0000$ $b_2 = 1.0000$	$a_1 = 0.7560$ $a_2 = 0.9996$ $b_2 = 0.4772$	$a_1 = 0.5098$ $a_2 = 1.0197$ $b_2 = 0.2599$
ω_0 (f_0)	275 s^{-1} (44 Hz)	177 s^{-1} (28 Hz)	115 s^{-1} (18 Hz)
L_1	300 μH	300 μH	300 μH
C_1	22 mF \rightarrow	22 mF \rightarrow	22 mF \rightarrow
C_D	66 mF \leftarrow x3	110 mF \leftarrow x5	176 mF \leftarrow x8
R_D	0.11 Ω	0.09 Ω	0.08 Ω

Bode Plot 2nd order lowpass filter

From: C To: Vout



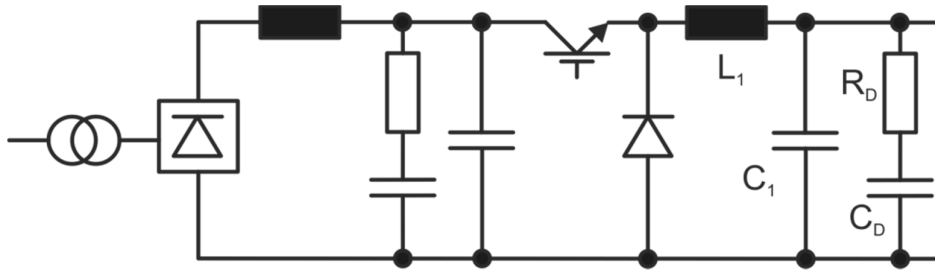
Maximum amplitude of resonance:

- Butterworth 4.5 dB
- Bessel 3.1 dB
- Critical damping 2.3 dB

Frequency, for -3 dB attenuation:

- Butterworth 74 Hz
- Bessel 67 Hz
- Critical damping 59 Hz

Attenuation: -28dB @ 300Hz



- DC-link voltage: 120V
- $f_s = 20\text{kHz}$
- $I_{\text{Out_max}} = 500\text{A}$
- $\Delta I_{L1} \leq 50\text{App.}$
- Attenuation: $G_B = 250$ @ $\omega_B = 2 \cdot \pi \cdot 20\text{kHz}$

Same premises as
for example 3

Design a 2nd order filter for all three given optimization methods and compare the results.

Select L_1 to meet the ripple current requirement:

$$L_1 = \frac{V_{DC} \cdot 0.25}{f_s \cdot \Delta I_{L1}} = \frac{120\text{V} \cdot 0.25}{20\text{kHz} \cdot 50\text{A}} = 30\mu\text{H} \quad (5)$$

Select ω_0 to meet the attenuation requirement:

$$\omega_0 = \omega_B \cdot \sqrt{\frac{|G_B| \cdot a_1 b_2}{a_1 + a_2}} \quad (4)$$

Calculate the remaining filter elements

(7a, 8, 9)

Results:

	Butterworth	Bessel	Critical damping
	$a_1 = 1.0000$ $a_2 = 1.0000$ $b_2 = 1.0000$	$a_1 = 0.7560$ $a_2 = 0.9996$ $b_2 = 0.4772$	$a_1 = 0.5098$ $a_2 = 1.0197$ $b_2 = 0.2599$
ω_B	$1.26 \cdot 10^5 \text{ s}^{-1}$ (20kHz)		
G_B	0.004 (-48dB)		
ω_0	$5.62 \cdot 10^3 \text{ s}^{-1}$ (894 Hz)	$3.60 \cdot 10^3 \text{ s}^{-1}$ (573 Hz)	$2.34 \cdot 10^3 \text{ s}^{-1}$ (372 Hz)
L_1	30 μH	30 μH	30 μH
C_1	528 μF	528 μF	528 μF
C_D	1'580 μF \leftarrow x3	2'640 μF \leftarrow x5	4'220 μF \leftarrow x8
R_D	0.22 Ω	0.18 Ω	0.15 Ω

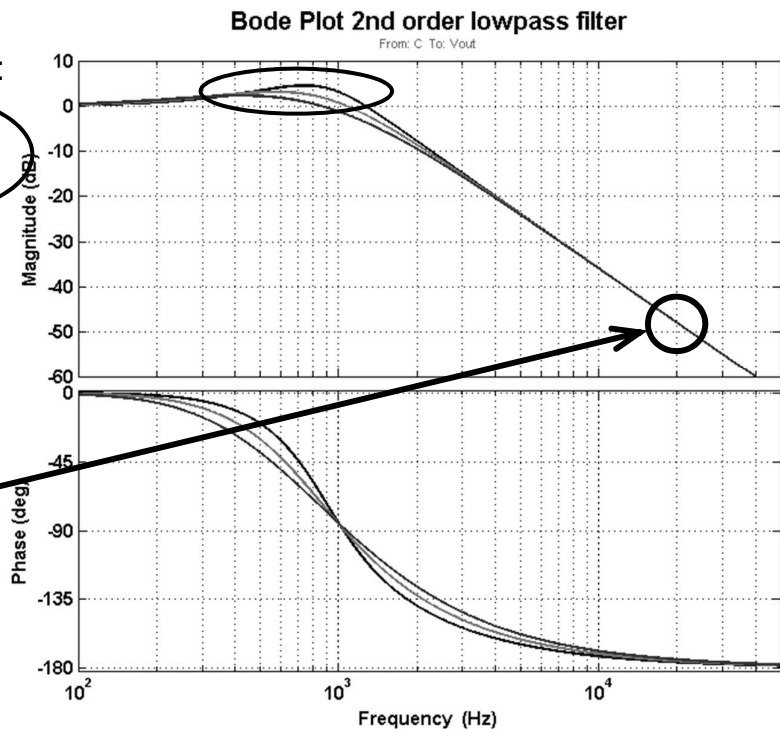
Maximum amplitude of resonance:

Butterworth 4.5 dB
Bessel 3.1 dB
Critical damping 2.3 dB

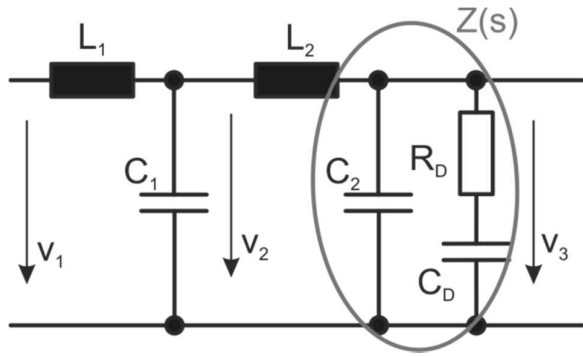
Frequency, for -3 dB attenuation:

Butterworth 1.5 kHz
Bessel 1.4 kHz
Critical damping 1.2 kHz

Attenuation: -48dB @ 20kHz



4th order filter - Transferfunction



$$Z(s) = \frac{1}{C_2 s + \frac{1}{R_D + \frac{1}{C_D s}}} = \frac{R_D C_D s + 1}{C_2 R_D C_D s^2 + (C_2 + C_D) s}$$

$$G(s) = \frac{v_3(s)}{v_1(s)} = \frac{\frac{1}{C_1 s + \frac{1}{L_2 s + Z(s)}}}{L_1 s + \frac{1}{C_1 s + \frac{1}{L_2 s + Z(s)}}} \cdot \frac{Z(s)}{L_2 s + Z(s)}$$

$\rightarrow G_1(s) = \frac{v_2(s)}{v_1(s)}$
 $\rightarrow G_2(s) = \frac{v_3(s)}{v_2(s)}$

4th order filter - Transferfunction

$$G(s) = \frac{R_D C_D s + 1}{L_1 L_2 C_1 C_2 R_D C_D s^5 + (C_2 + C_D) L_1 L_2 C_1 s^4 + [L_1 C_1 R_D C_D + (L_1 + L_2) C_2 R_D C_D] s^3 + \dots}$$

$$\dots \frac{\dots + [L_1 C_1 + (L_1 + L_2)(C_2 + C_D)] s^2 + R_D C_D s + 1}{\dots} \quad (10)$$

$$G(s) = \frac{k_1 s + 1}{k_5 s^5 + k_4 s^4 + k_3 s^3 + k_2 s^2 + k_1 s + 1}$$

1st order PD
5th order PT

with

$$k_1 = R_D C_D$$

$$k_2 = L_1 (C_1 + C_2 + C_D) + L_2 (C_2 + C_D)$$

$$k_3 = R_D C_D (L_1 C_1 + L_2 C_2 + L_1 C_2)$$

$$k_4 = L_1 L_2 C_1 (C_2 + C_D)$$

$$k_5 = L_1 L_2 C_1 C_2 C_D R_D$$

5th order PT in its normalized form:

$$G_{PT}(s) = \frac{1}{(1 + a_1 \frac{s}{\omega_0}) \cdot (1 + a_2 \frac{s}{\omega_0} + b_2 \frac{s^2}{\omega_0^2}) \cdot (1 + a_3 \frac{s}{\omega_0} + b_3 \frac{s^2}{\omega_0^2})} \quad (11)$$

Optimisation methods:

	a ₁	a ₂	b ₂	a ₃	b ₃
Butterworth	1.0000	1.6180	1.0000	0.6180	1.0000
Bessel	0.6656	1.1402	0.4128	0.6216	0.3245
Critical damping	0.3856	0.7712	0.1487	0.7712	0.1487

By expanding (11) and comparing the coefficients with (10) we get:

$$G_{PT}(s) = \frac{1}{\frac{a_1 b_2 b_3}{\omega_0^5} s^5 + \frac{(b_2 b_3 + a_1 a_2 b_3 + a_1 a_3 b_2)}{\omega_0^4} s^4 + \frac{(a_2 b_3 + a_3 b_2 + a_1 b_3 + a_1 a_2 a_3 + a_1 b_2)}{\omega_0^3} s^3 + \dots + \frac{(b_3 + a_2 a_3 + b_2 + a_1 a_3 + a_1 a_2)}{\omega_0^2} s^2 + \frac{(a_1 + a_2 + a_3)}{\omega_0} s + 1}$$

$$k_1 = R_D C_D = \frac{a_1 + a_2 + a_3}{\omega_0} \quad (12a)$$

$$k_2 = L_1(C_1 + C_2 + C_D) + L_2(C_2 + C_D) = \frac{b_3 + a_2 a_3 + b_2 + a_1 a_3 + a_1 a_2}{\omega_0^2} \quad (12b)$$

$$k_3 = R_D C_D (L_1 C_1 + L_2 C_2 + L_1 C_2) = \frac{a_2 b_3 + a_3 b_2 + a_1 b_3 + a_1 a_2 a_3 + a_1 b_2}{\omega_0^3} \quad (12c)$$

$$k_4 = L_1 L_2 C_1 (C_2 + C_D) = \frac{b_2 b_3 + a_1 a_2 b_3 + a_1 a_3 b_2}{\omega_0^4} \quad (12d)$$

$$k_5 = L_1 L_2 C_1 C_2 C_D R_D = \frac{a_1 b_2 b_3}{\omega_0^5} \quad (12e)$$

The 5 independent equations (12a...12e) contain 7 unknowns ($L_1, C_1, L_2, C_2, R_D, C_D$ and ω_0). Therefore we have the choice to select 2 of them (ω_0 and L_1) the remaining 5 depend on that selection.

For a given frequency ω_B well in the blocking area ($\omega_B \gg \omega_0$) we can define the desired attenuation G_B . In the blocking area the highest order terms of both the numerator and denominator in equation (10) dominate, therefore (10) can be simplified to:

$$G_B = \frac{R_D C_D s}{L_1 L_2 C_1 C_2 R_D C_D s^5} = \frac{\frac{a_1 + a_2 + a_3}{\omega_0} s}{\frac{a_1 b_2 b_3}{\omega_0^5} s^5} = \frac{a_1 + a_2 + a_3}{a_1 b_2 b_3} \cdot \frac{\omega_0^4}{s^4} = \frac{a_1 + a_2 + a_3}{a_1 b_2 b_3} \cdot \frac{\omega_0^4}{\omega_B^4}$$

$\downarrow \quad \uparrow$
 $(j)^4 = +1$

$$\omega_0 = \omega_B \cdot \sqrt[4]{\frac{G_B \cdot a_1 b_2 b_3}{a_1 + a_2 + a_3}} \quad (13)$$

Select L_1 according to ripple current requirements with (5) or (6)

By solving the equation system (12a.....12e) we get:

$$L_2 = \frac{L_1}{\frac{(k_3 k_4 - k_2 k_5)(k_1 k_2 - k_3)}{(k_1 k_4 - k_5)^2} - 1} \quad (14a)$$

$$C_2 = \frac{k_5 (k_1 k_2 - k_3)}{k_1 (k_1 k_4 - k_5) (L_1 + L_2)} \quad (14b)$$

$$C_1 = \frac{k_5}{k_1 L_1 L_2 C_2} \quad (14c)$$

$$R_D = \frac{k_1 k_5}{C_2 (k_1 k_4 - k_5)} \quad (14d)$$

$$C_D = \frac{k_1}{R_D} \quad (14e)$$

with

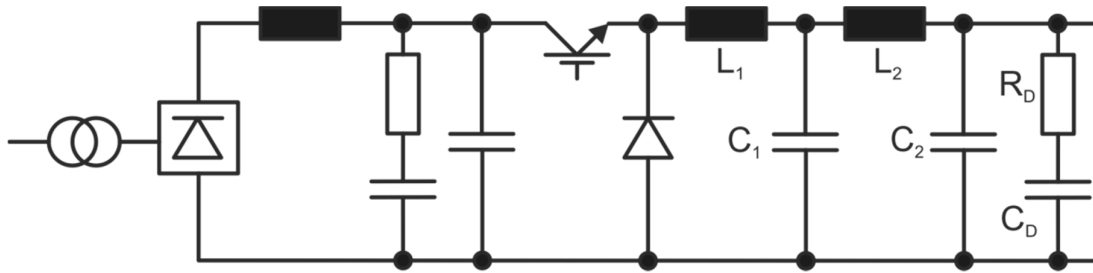
$$k_1 = \frac{a_1 + a_2 + a_3}{\omega_0}$$

$$k_2 = \frac{b_3 + a_2 a_3 + b_2 + a_1 a_3 + a_1 a_2}{\omega_0^2}$$

$$k_3 = \frac{a_2 b_3 + a_3 b_2 + a_1 b_3 + a_1 a_2 a_3 + a_1 b_2}{\omega_0^3}$$

$$k_4 = \frac{b_2 b_3 + a_1 a_2 b_3 + a_1 a_3 b_2}{\omega_0^4}$$

$$k_5 = \frac{a_1 b_2 b_3}{\omega_0^5}$$



- DC-link voltage: 120V
- $f_s = 20\text{kHz}$
- $I_{\text{Out_max}} = 500\text{A}$
- $\Delta I_{L1} \leq 50\text{App.}$
- Attenuation: $G_B = 250$ @ $\omega_B = 2 \cdot \pi \cdot 20\text{kHz}$

Same premises as for example 2

Design a 4th order filter for all three given optimization methods and compare the results.

Select L_1 to meet the ripple current requirement:

$$L_1 = \frac{V_{DC} \cdot 0.25}{f_s \cdot \Delta I_{L1}} = \frac{120\text{V} \cdot 0.25}{20\text{kHz} \cdot 50\text{A}} = 30\mu\text{H} \quad (5)$$

Select ω_0 to meet the attenuation requirement:

$$\omega_0 = \omega_B \cdot \sqrt[4]{\frac{G_B \cdot a_1 b_2 b_3}{a_1 + a_2 + a_3}} \quad (13)$$

Calculate the remaining filter elements

(14a....e)

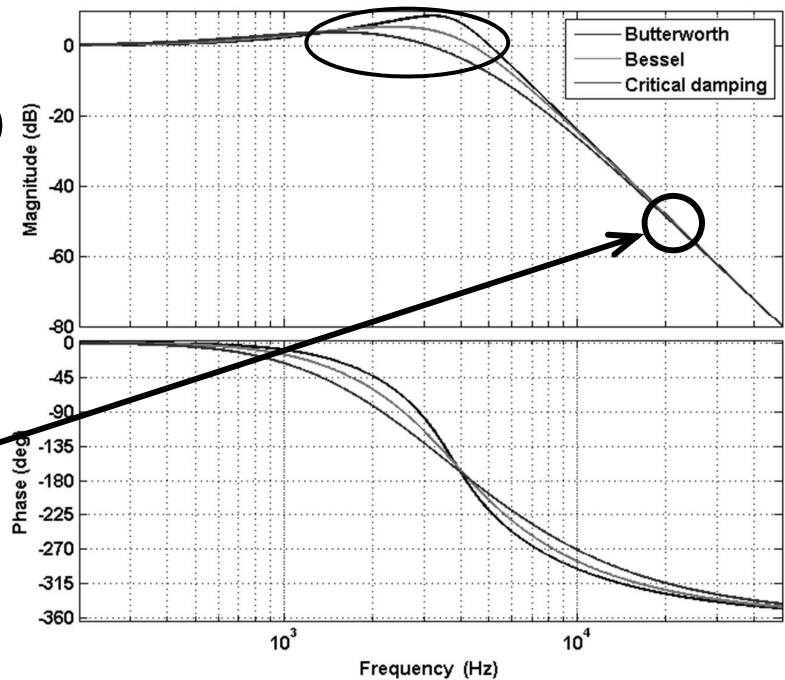
4th order filter – Example 3

Results:

	Butterworth	Bessel	Critical damping
	$a_1 = 1.0000$ $a_2 = 1.6180$ $b_2 = 1.0000$ $a_3 = 0.6180$ $b_3 = 1.0000$	$a_1 = 0.6656$ $a_2 = 1.1402$ $b_2 = 0.4128$ $a_3 = 0.6216$ $b_3 = 0.3245$	$a_1 = 0.3856$ $a_2 = 0.7712$ $b_2 = 0.1487$ $a_3 = 0.7712$ $b_3 = 0.1487$
ω_B	$1.26 \cdot 10^5 \text{ s}^{-1}$ (20kHz)		
G_B	0.004 (-48dB)		
ω_0	$2.36 \cdot 10^4 \text{ s}^{-1}$ (3.8 kHz)	$1.38 \cdot 10^4 \text{ s}^{-1}$ (2.2 kHz)	$8.15 \cdot 10^3 \text{ s}^{-1}$ (1.3 kHz)
L_1	30 μH	30 μH	30 μH
L_2	57 μH	31 μH	17 μH
C_1	74 μF	90 μF	124 μF
C_2	7.9 μF	12 μF	16 μF
C_D	75 μF	168 μF	382 μF
R_D	1.83 Ω	1.04 Ω	0.62 Ω

4th order filter – Example 3

Bode Diagram 4th order lopass filter
From: C To: Vout



Maximum amplitude of resonance:

Butterworth 8.6 dB
Bessel 5.4 dB
Critical damping 3.8 dB

Frequency, for -3 dB attenuation:

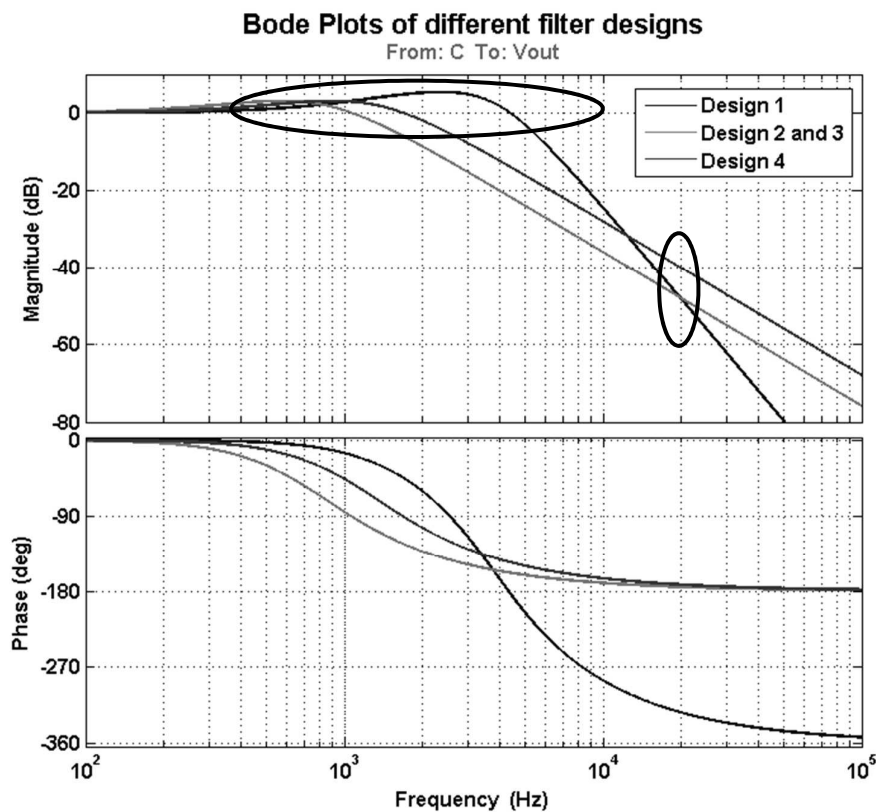
Butterworth 5.5 kHz
Bessel 5.0 kHz
Critical damping 3.9 kHz

Attenuation: -48dB @ 20kHz

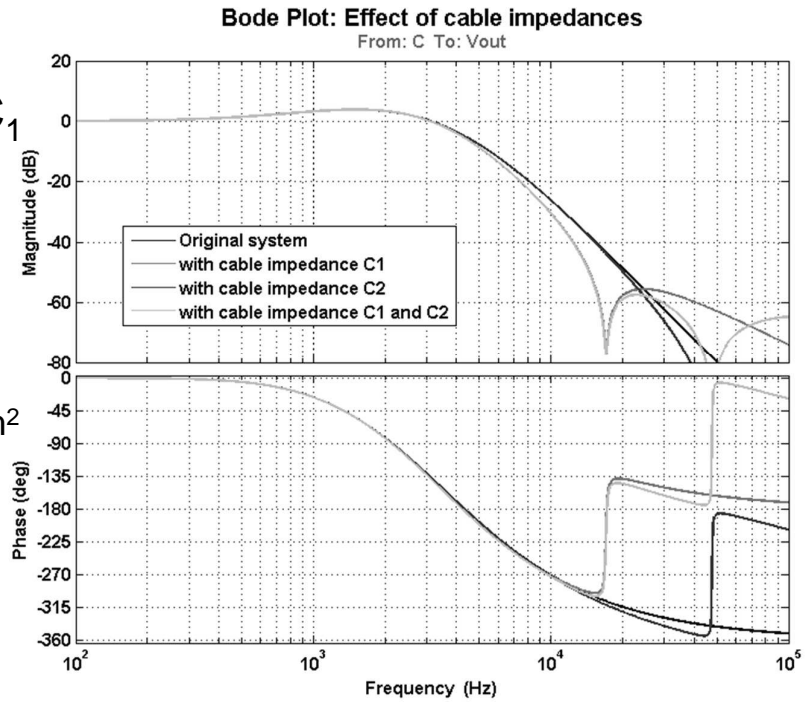
Comparison of different filter designs

Design 1 4 th order Acc. to example 3	Design 2 2 nd order Acc. to example 2	Design 3 2 nd order Acc. to example 2 but $L_1 = 100\mu\text{H}$	Design 4 2 nd order Acc. to example 2 but $L_1 = 100\mu\text{H}$ and $G_B = 0.01$
<u>Optimisation:</u> Bessel	<u>Optimisation:</u> Bessel	<u>Optimisation:</u> Bessel	<u>Optimisation:</u> Bessel
$L_1 = 30 \mu\text{H}$ $L_2 = 31 \mu\text{H}$	$L_1 = 30 \mu\text{H}$	$L_1 = 100 \mu\text{H}$	$L_1 = 100 \mu\text{H}$ +64%
$C_1 = 90 \mu\text{F}$ $C_2 = 12 \mu\text{F}$ $C_D = 168 \mu\text{F}$	$C_1 = 528 \mu\text{F}$	$C_1 = 158 \mu\text{F}$	$C_1 = 63 \mu\text{F}$ +42%
$R_D = 1.04 \Omega$	$R_D = 0.18 \Omega$	$R_D = 0.62 \Omega$	$R_D = 0.98 \Omega$
$f_0 = 2'200 \text{ Hz}$	$f_0 = 573 \text{ Hz}$	$f_0 = 573 \text{ Hz}$	$f_0 = 907 \text{ Hz}$

Comparison of different filter designs



- Effect of a 0.5m long wire 16mm² (wiring of C₁ and C₂)
 - Skin Effect
 - Skin depth in Cu @ 20kHz: 0.5mm
 - Reduces the effective cross section to 6.3 mm²
 - Wire resistance @ 20kHz: 1.4mΩ
 - Wire inductance is approx. 0.5μH



To be avoided !

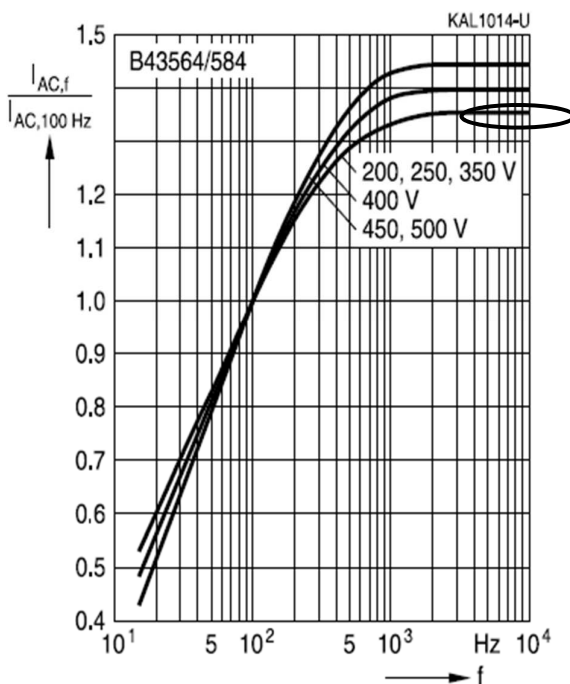


The useful life time of an electrolytic capacitor depends very much on the ripple current and the ambient temperature.

C_R	Case dimensions	ESR_{typ}	Z_{max}	$I_{AC,max}$	$I_{AC,B}$	$I_{AC,R(B)}$	Ordering code (composition see below)
100 Hz	$d \times l$	100 Hz	10 kHz	100 Hz	100 Hz	100 Hz	
20 °C	mm	20 °C	20 °C	40 °C	85 °C	85 °C	
μF		$m\Omega$	$m\Omega$	A	A	A	
$V_R = 200 V DC$							
3300	51.6 × 80.7	40	48	21	7.9	15.3	B435*4E2338M0##
4700	51.6 × 105.7	29	35	27	10.1	17.6	B435*4E2478M0##
4700	64.3 × 80.7	29	35	27	10.0	18.6	B435*4F2478M0##
6800	64.3 × 105.7	21	25	34	12.6	22.0	B435*4E2688M0##
8200	76.9 × 105.7	17	20	41	15.2	26.8	B435*4E2828M0##
10000	76.9 × 105.7	14	17	47	17.4	32.8	B435*4E2109M0##
15000	76.9 × 143.2	8	10	57	25.6	43.6	B435*4E2159M0##
22000	91.0 × 144.5	5	6	80	35.9	63.6	B435*4E2229M0##

- Nominal ripple current at
 - nominal frequency (100Hz) and
 - nominal capacitor temperature (85°C).
 - 17.4A in our example

Frequency factor of permissible ripple current I_{AC} versus frequency f



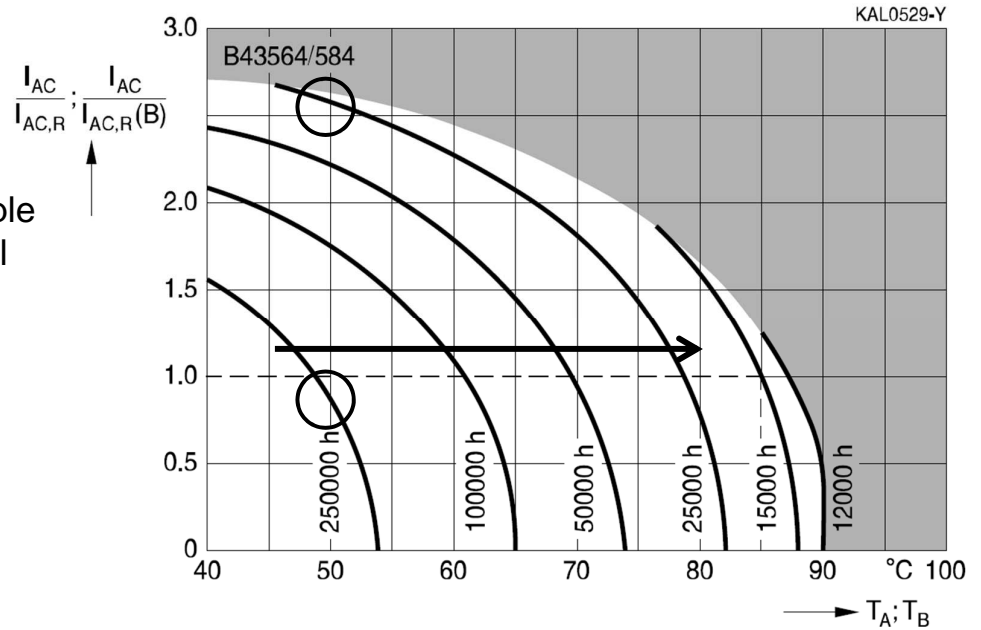
Apply frequency factor:

For 10kHz a current factor of 1.35 is applicable
 → 23.5A @ 10kHz

Determine the allowed ripple current for a desired useful life and T_a :

For 25'000h (3 years)
and $T_a = 50^\circ\text{C}$,
 $\rightarrow 2.6 * 23.5\text{A} = 61\text{A}$

For 250'000h (30years)
and $T_a = 50^\circ\text{C}$
 $\rightarrow 0.85 * 23.5\text{A} = 20\text{A}$



The useful life time dramatically decreases at higher ambient temperatures!

Thank you for your attention

References

- U. Tietze, Ch. Schenk; Halbleiter-Schaltungs-Technik, 12. Auflage, Pages 815ff
- Epcos, Datasheet, Capacitors with screw terminals Type B43564, B43584, November 2012

Questions?

