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# Outline

- Introduction
- Magnetic Components in Power Converters
- Basic Electromagnetic Theory for Magnetic Component Design
- Dimensioning with Magnetostatics & Electrostatics
- Transformer & Inductor Dimensional Analysis
- Optimal Design Methodology of Magnetic Components

# Introduction

#### **Main presentation Objectives**

- Present design problematic of power converter magnetic component for the Power Electronics Designer
- How to:
  - specify a magnetic component
  - evaluate size & performance from specifications with simple dimensional analysis
- Review of:
  - simple dimensioning models of magnetic components
  - basic design methodology (with analytical & FEA tools)
  - optimal design methodology of magnetic components
- Introduction to integrated optimal dimensioning of power converters

# MAGNETIC COMPONENTS IN POWER CONVERTERS

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#### **Magnetic Components functionalities**

- Storage
- Filtering
- Galvanic Isolation-Voltage/Current adaptation
- Couplers







#### **Inductor Functionalities**

- Electrical Energy storage: SMES, indirect-link converters
- Adaptation of converter I/O sources: DC or AC current & voltage filters, Bouncers ...
- Phase control of power flow through HF resonant LC stage

#### **Inductors in Power Converters**

- DC polarized inductor
- AC reactance

#### **Transformer Functionalities**

- High Voltage or Low Voltage adaptation
- Galvanic Isolation
- Transformers in Power Converters
- Voltage or Current transformers
- Low or Medium Frequency
- Medium or High Voltage
- Pulse Transformers

#### **Influence of Reactive Components on Power Converter Performance & Size**

Converter Efficiency

• Converter Power Density: W/m<sup>3</sup> & W/kg

• Converter Control Bandwith: Filter Time constants

### Requirements of Power Converters Magnetic Components

• Equivalent circuit model of magnetic components is essential for the power electronics designer

But....

- Feasibility, size & performance specifications of magnetic components based on electrical equivalent circuit specifications only must be evaluated in the early steps of the converter design procedure to avoid unexpected surprises or poor performance
- Early interactivity between converter designers & magnetic component designers is mandatory
- In any discipline even genial designers cannot make miracles & violate laws of physics (it's why systematic practice of simple dimensional analysis is mandatory)

#### **Requirements of Magnetic Components**

- Energy (J)
- Apparent Power (VA)
- Frequency
- Saturation limits
- Electrical isolation
- Grounding
- Size, Volume & mass
- Efficiency
- Temperature Rise & Ambient temperature
- Operation Cycle
- Lifetime estimation

### Specifications of Magnetic Components in Power Converters

# Magnetic component specifications for technical proposal to manufacturer

- Deliver simplified electrical circuit of system
- Identify connections & grounding system
- Specify operation with simulated instantaneous I/O Voltage and current shapes
- Define ambient temperature & temperature rise
- Specify electrical parameters (resistance, inductance capacitance) with absolute precision in %
- Specify relative tolerance between components in case of series production
- Specify magnetic saturation limit in terms of max current & voltage
- Specify Electrical isolation (grounded core)
- Specify estimated lifetime in terms of hours of operation or total number of pulses for pulsed applications
- Specify max current to withstand (duration amplitude & waveshape)
- Impose functional tests before delivery
- ....

# **BASIC ELECTROMAGNETIC THEORY FOR MAGNETIC COMPONENT DESIGN**

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Magnetic components Modelling & Dimensioning methodology based on simplified formulations of Maxwell equations

- Magnetostatics for magnetic circuit dimensioning
  - Determination of Core & coil dimensions
  - Choice of current & flux density according to material limits
  - Dimensioning model based on Ampere law, Magnetic flux conservation & stored magnetic energy
- Magnetodynamics for AC supplied component dimensioning
  - Voltage vs frequency design adaptation
  - Evaluation of high frequency copper loss associated to skin & proximity effects
  - Dimensioning model based on Faraday law
- Electrostatics for isolation
  - Insulation dimensioning: estimation of voltage gradients & conservative insulation distances
  - Dimensioning model based on Gauss law, voltage gradients & electrical energy stored

#### **Maxwell equations in material media**

• General formulation in any medium (vacuum, Matter)

$$rot \vec{B} = \mu_o (\vec{J} + \varepsilon_o \frac{\partial \vec{E}}{\partial t}) \qquad div \vec{B} = 0 \qquad \vec{J} = \sigma \vec{E}$$
$$rot \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad div \vec{E} = \frac{\rho}{\varepsilon_o}$$

ŤE	Electric Fied	₿	Magnetic Field	Ĵ	Total Current Density $\vec{J} = \vec{J_f} + \vec{J_l}$
$\mathcal{E}_o$ Vacuur	n Permittivity	$\mu_o$	Vacuum Permeability	ρ	Total volume charge $\rho = \rho_f + \rho_l$ density
$\varepsilon_{o} = 8.85.$	$10^{-12} C / Vm$	$\mu_o$ =	$= 4.\pi.10^{-7} Vs / Am$	σ	Electrical Conductivity

In engineering, introduction of Fields H & D related to free Current & charge density only, more adapted to «continuous medium» modelling of materials (B(H) & D(E))

rot $\vec{H} = \vec{J}_f + \frac{\partial D}{2}$	$\vec{B} = \mu_r \mu_o \ \vec{H} = \mu \ \vec{H}$
$div\vec{B} = 0  \overrightarrow{\partial B}  \overrightarrow{\partial B}$	$\vec{D} = \varepsilon_r \varepsilon_o \vec{E} = \varepsilon \vec{E}$
$rot E = -\frac{1}{\partial t}$ $div \vec{D} = \rho_f$	$\vec{J} = \sigma \vec{E}$

Ē	Electric Field	H	Magnetic Field	$\vec{J}_f$ Free Current Density
			(created by $\vec{J} = \vec{J_f}$ )	$\vec{J} = \vec{J_f} + \vec{J_l}$
$ _{\overrightarrow{D}}$	Electric	$\vec{\mathbf{R}}$	Magnetic Field or	$\rho_f$ Volumic free Charge
	Displacement Field	D	Induction	$ \rho = \rho_f + \rho_l $ Density
3	Permittivity	μ	Permeability	$\sigma$ Electrical Conductivity

Magnetic components Modelling & Design tools derived from simplified formulations of Maxwell equations. For many technical examples, low-frequency approximations of Maxwell equations are applicable



Displacement currents negligeable with respect to conduction currents at low frequency & voltage: for f=10<sup>5</sup> Hz, E= 10<sup>5</sup> V/m,  $I_d \approx 10^{-7} A/mm^2 <<$  Conduction currents  $\approx 1$  to 10A/mm<sup>2</sup>

Magnetostatics used to design Magnetic circuits & compute inductance of equivalent circuits

• Ampere's Law  $\oint_{L(S)} \vec{H} \cdot d\vec{l} = \oiint_{S} \vec{J} \cdot d\vec{s} \qquad \oint_{L(S)} \vec{H} \cdot d\vec{l} = n.I$ Ampere-turn magnetomotive force FMM = n.I

*Kirchhoff's* Mesh rule in Magnetic circuits  $\sum_{n} FMM_{i} - \sum_{n} H_{i} \cdot l_{i} = 0$ 

- Magnetic flux conservation  $\varphi = \iint_{S} \vec{B} \cdot d\vec{s}$ *Kirchhoff's* Node rule in Magnetic circuits  $div\vec{B} = 0 \implies \sum_{n} \varphi_{i} = 0$
- Magnetic material characteristic  $B = \mu \cdot H$
- Volumic density of stored magnetic energy (J/m<sup>3</sup>)  $W_{magvol} = \frac{1}{2} \frac{B^2}{U}$

Magnetodynamics used to design magnetic circuits of Transformers & Inductors

• Faraday-Lenz law  $\oint_{S} rot \vec{E} \cdot d\vec{s} = \oint_{S} -\frac{\partial B}{\partial t} \cdot d\vec{s}$ 

Electromotive force Emf 
$$e = \oint_{L(S)} \vec{E} \cdot d\vec{l} = -\frac{d\varphi}{dt}$$

Magnetodynamics used for dimensioning of AC supplied magnetic components & evaluation of high frequency copper loss associated to skin & proximity effects

Electrostatics used to design Isolation system of High Voltage Transformers & compute capacitance of equivalent circuits

- Gauss Theorem  $\oint_V div \vec{D} \cdot dv = \oint_V \rho \cdot dv = Q = C.V$
- Electrical field & voltage gradient

$$\vec{E} = -\overrightarrow{grad V}$$

• Volumic density of stored electrostatic energy (J/m<sup>3</sup>)

$$W_{elvol} = \frac{1}{2}\varepsilon.E^2$$

# DIMENSIONING WITH MAGNETOSTATICS & ELECTROSTATICS

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 For simple structures and small airgaps, simple analytical dimensioning model can be derived from resolution of Magnetostatic problems



 For structures with large airgaps, Inductance must be computed by FEA Magnetostatic solvers



Inductor with ungapped Core

- Magnetic Flux concentrated in material with permeability  $\mu >> \mu_o$
- Ampere's law  $ni = H \cdot l_m = \frac{B}{\mu} \cdot l_m$
- Magnetic flux in core

$$\varphi = B \cdot A_e$$

- I<sub>m</sub> core average length
- A<sub>e</sub> Core section
- H & B uniform on  $A_e$

$$ni = H \cdot l_m = \frac{B}{\mu} \cdot l_m = \frac{\varphi}{A_e \cdot \mu} \cdot l_m = \varphi \cdot \frac{1}{\mu} \cdot \frac{l_m}{A_e}$$

$$ni = H \cdot l_m = \frac{B}{\mu} \cdot l_m = \frac{\varphi}{A_e \cdot \mu} \cdot l_m = \varphi \cdot \frac{1}{\mu} \cdot \frac{l_m}{A_e} = \mathcal{R} \cdot \varphi$$

- Magnetomotive force FMM = ni
- Magnetic flux circulating in core  $\varphi$

 $\mathcal{R}_{\cdot}$ 

Core reluctance

$$=\frac{1}{\mu}\cdot\frac{l_m}{A_e}$$

• Magnetic equivalent circuit



$$\begin{array}{c} & & & \\ & & & & \\ & & & \\ & & & \\ & & & \\ & & & & \\ & & & \\ & & & & \\ & & & &$$

• Inductance

$$L = \frac{\Phi}{i}$$

 $\Phi$  total magnetic flux in coil

$$L = \frac{\Phi}{i} = \frac{n \cdot \varphi}{i} = \frac{n}{i} \cdot \frac{ni}{\mathcal{R}} = \frac{n^2}{\mathcal{R}}$$
  
uctance  $L = \frac{n^2}{\mathcal{R}}$ 

- L vs core reluctance
- Inductor Dimensioning model

$$L = n^2 \cdot \frac{\mu \cdot A_e}{\ell_m}$$

n

 $I_m A_e$ 

- Inductance depends on
  - Coil turn number
  - Core dimensions
  - Material permeability  $\mu$



Magnetic energy stored in inductor with ungapped core

$$W_{mag} = \frac{1}{2}Li^{2} = \frac{1}{2}\Phi i = \frac{1}{2}n\varphi \cdot i = \frac{1}{2}ni.BA_{e} = \frac{1}{2}Hl_{m} \cdot BA_{e} = \frac{1}{2}\frac{B^{2}}{\mu} \cdot l_{m}A_{e}$$

• Core volume  $V_e = l_m A_e$ 

Volumic density of stored magnetic energy (J/m<sup>3</sup>)

- proportional to  $B^2$
- but also inversely proportional to core material permeability!

$$W_{mag} = \frac{1}{2} \frac{B^2}{\mu} \cdot V_e = \frac{1}{2} L i^2$$



#### Inductor with gapped core

- H & B distribution non uniform in air gap
  - What is flux circulation section in airgap?
- What is airgap reluctance
- Ampere's law & Flux conservation cannot be applied to derive **simple & precise** dimensioning model
- <sup>P</sup> For precise inductance computation FEA is mandatory
- But for small airgaps & large core sections & length, analytical dimensioning is acceptable

Simplified assumption for inductor with gapped core:

• flux circulation section in airgap = core section  $A_e$ 



Distribution of magnetic energy in gapped inductor

$$W_{mag} = \frac{1}{2} \frac{B^2}{\mu_o} \cdot V_e + \frac{1}{2} \frac{B^2}{\mu} \cdot V_{noyau} \approx \frac{1}{2} \frac{B^2}{\mu_o} \cdot V_e$$

• for fixed *B*, Volumic density of magnetic energy (J/m<sup>3</sup>)  $\mu > 1000.\mu$ much lower in high permeability core than in airgap

- Because in inductor specs ( L &  $i_{max}$ ) impose magnetic energy stored in inductor volume minimization is performed by use of:
  - a high permeability core to concentrate flux created by coil MMF
     & to maximize flux density B airgap
  - to store the magnetic energy in airgap with a high volumic density

Analytical modelling is less precise but much efficient to analyze physical behaviour & to understand sensitivity of design variables

# HF Transformer Electrical Equivalent Circuit



Determination of Transformer Electrical Equivalent circuit elements

inductances with Magnetostatics, capacitances with Electrostatics

 2D (or 3D) FEA identification in Magnetostatics & Electrostatics preferred to simplified analytical computation (better precision)
 FEA "simulated experiment" technique

# FEA Identification of Equivalent Circuit Inductances



Magnetic Flux Density Distribution under short-circuit operation

#### Magnetostatics 2D FEA tool

- Direct identification of transformer inductances
- 2 FEA identification tests:
- Open Secondary (no load operation)
- Short-circuit operation

Magnetizing inductance derived from FEA computation of magnetic energy stored during no-load operation Total leakage inductance derived from FEA computation of magnetic energy stored during short-circuit operation

# FEA Identification of Equivalent Circuit Capacitances



#### Electrostatics 2D FEA tool

- direct identification of capacitors
   C<sub>1</sub>, C<sub>2</sub>, C<sub>12</sub>
- 3 FEA identification tests:
- Secondary short circuited & grounded,
- Primary short circuited & grounded,
- Primary & secondary supplied with V<sub>2n</sub> linear voltage distribution

C<sub>1</sub>, C<sub>2</sub>, C<sub>12</sub> derived from electrical energies  $W_{2cc}$ ,  $W_{1cc}$  &  $W_{122}$  computed by electrostatic FEA

# FEA Identification of Equivalent Circuit Capacitances



Distribution of Voltage in Pulse Transformer Tank for the 3 identification tests rated voltage operation

$V_1 = V_{1n}$	V <sub>2</sub> =0	$W_{2cc}$
$V_1 = 0$	$V_2 = V_{2n}$	$W_{1cc}$
$V_1 = V_{2n}$	$V_2 = V_{2n}$	$W_{122}$



Both modeling tools are necessary in the design process

# An efficient Design Methodology must take advantage of both analytical modelling & FEA "simulated experiment" techniques

Both modeling tools are necessary in the design process

# **TRANSFORMER & INDUCTOR DIMENSIONAL ANALYSIS**

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#### Total apparent power of n winding transformer (VA)



$$S = \sum_{k=1}^{n} V_k \cdot I_k$$

- n total number of windings
- V<sub>k</sub> Rated RMS voltage of winding k
- I<sub>k</sub> Rated RMS current in winding k

Instantaneous voltage on winding k ٠

$$v_k = n_k \cdot \frac{\mathrm{d}\varphi}{\mathrm{d}t}$$

- Instantaneous flux in core section Φ
- n<sub>k</sub> Turn number of winding k
- RMS voltage of winding k (for any voltage waveform) (V) ٠

$$V_k = n_k \cdot K_v \cdot A_e \cdot B \cdot f$$

- $A_e$  Transformer core section (m<sup>2</sup>)
- f Operation frequency (Hz)
- B Max core flux density (T)
- K<sub>v</sub> Form factor of voltage waveform v<sub>k</sub> (ex:  $K_v = \frac{2\pi}{\sqrt{2}}$  for sinewave)

$$S = \sum_{k=1}^{n} n_k \cdot K_v \cdot A_e \cdot B \cdot f \cdot I_k = K_v \cdot A_e \cdot B \cdot f \sum_{k=1}^{n} n_k \cdot I_k$$
$$\sum_{k=1}^{n} n_k \cdot I_k = J \cdot \sum_{k=1}^{n} n_k \cdot s_k = J \cdot S_{Cu} = K_u \cdot J \cdot W_a$$

With

Same RMS current density J (A/m<sup>2</sup>)in all windings to impose uniform distribution of copper loss volumic density & avoid hot spot

 $s_k$  conductor section of winding k  $S_{Cu}$  Total Copper section of transformer winding  $K_u$  Copper filling factor in core window  $W_a$  effective section of core window

Then

$$S = \sum_{k=1}^{n} n_k \cdot K_v \cdot A_e \cdot B \cdot f \cdot I_k = K_v \cdot A_e \cdot B \cdot f \cdot K_u \cdot J \cdot W_a$$

Total apparent power of transformer (VA)

$$S = K_u \cdot K_v \cdot f \cdot B \cdot J \cdot A_e \cdot W_a$$

A simple transformer power dimensioning model derived from specifications (RMS rated voltage & current in each winding)

- K<sub>u</sub> Copper filling factor in core window
- $K_v$  Form factor of supply voltage waveform  $v_k$
- f Operation frequency (Hz)
- B Max core flux density (T)
- *J* RMS current density (A/m<sup>2</sup>)
- A<sub>e</sub> Transformer core section (m<sup>2</sup>)
- $W_a$  Effectice section of core window (m<sup>2</sup>)

Transformer Dimensioning Power (VA)



**Converter Supply Specs** 

VA/m<sup>3</sup> & VA /kg improved at medium frequency ... for fixed B.J

### **Transformer Dimensional Analysis**

For fixed  $B_J$  and f all linear dimensions multiplied by k



In terms of dimensional analysis

Magnetic Energy stored in inductor (J)



$$W_{mag} = \frac{1}{2}LI^2$$

Inductor specifications

- L Inductance (H)
- I Maximum Supply Current (A)

Magnetic Energy stored in inductor (J)

$$W_{mag} = \frac{1}{2} \cdot L \cdot I^2 = \frac{1}{2} \cdot \Phi \cdot I \qquad L =$$

Φ

i

- I Maximum Supply Current (A)
- Φ Total magnetic flux in coil

$$\Phi = \mathbf{n} \cdot \boldsymbol{\varphi} = \mathbf{n} \cdot \mathbf{B} \cdot \mathbf{A}_{e}$$

- $\varphi$  Magnetic flux circulating in core
- A<sub>e</sub> core section (m<sup>2</sup>)
- B Max core flux density (T)

Magnetic Energy stored in inductor (J)

$$W_{mag} = \frac{1}{2} \cdot (n \cdot B \cdot A_e) \cdot J \cdot s = \frac{1}{2} \cdot B \cdot A_e \cdot J \cdot S_{cu}$$

s Coil conductor section

S<sub>Cu</sub> Total Copper section of inductor winding

$$W_{mag} = \frac{1}{2} \cdot K_{u} \cdot B \cdot J \cdot W_{a} \cdot A_{e}$$

- J RMS current density  $(A/m^2)$
- B Max core flux density (T)
- K<sub>u</sub> Copper filling factor in core window
- W<sub>a</sub> effective section of core window

Inductor Dimensioning Energy (J)



### **Inductor Dimensional Analysis**

For fixed B.J and f all linear dimensions multiplied by k



### Dimensional Analysis of Component Losses

Transformer & Inductor Copper losses

$$P_J = \sum_{k=1}^n R_k \cdot I_k^2 = \sum_{k=1}^n \rho \cdot L_k \cdot J^2 \cdot s_k = \rho \cdot J^2 \cdot \sum_{k=1}^n L_k \cdot s_k = \rho \cdot V_{Cu} \cdot J^2$$

- J RMS current density  $(A/m^2)$
- $s_k$  Conductor section of winding k
- L<sub>k</sub> Total conductor length of winding k
- V<sub>cu</sub> Total Copper volume of windings
- K<sub>u</sub> Copper filling factor in core window
- W<sub>a</sub> Effective section of transformer window
- ρ Copper resistivity

In terms of dimensional analysis CopperLosses  $\propto J^2 [L]^3$ 

*If Skin & Proximity effect cannot be neglected Copper losses also influenced by frequency* 

### Dimensional Analysis of Component Losses

Simplified Transformer Magnetic losses (Steinmetz)

$$P_{mag} = V_e \cdot C_m \cdot f^x \cdot B^y = P_{magv} \cdot V_e$$

BCore flux density (T)C<sub>m</sub>, x, yLoss coefficient of materialV<sub>e</sub>Core volume (m³)

In terms of dimensional analysis  $MagLosses \propto f^x B^y [L]^3$ 

## Dimensional Analysis of Temperature Rise

Simplified Heat Transfer at external surface

$$\Delta T = \frac{P_J + P_{mag}}{h.S_{ext}} \propto \frac{[L]^3}{[L]^2}$$

```
In terms of dimensional analysis TempRise = \frac{Losses}{h.S_{ext}} \propto [L]
For fixed B.J and f
```

# Correction of Scaling Laws

- Max Temperature Rise is always constrained by thermal limits of Insulation & Magnetic materials
- For high power components of large dimensions, dimensional analysis demonstrates that correction mechanisms must be adopted to limit Max Temperature rise within acceptable limits:
  - Heat transfer coefficient h can be improved by use of forced convection, or cooling fluids
  - Heatsinks and cooling fins can increase external dissipation surface  ${\rm S}_{\rm ext}$
  - J & B are decreased according to  $B.J \propto [L]^{-1}$
- Consequently  $S \propto [L]^C = W_{mag} \propto [L]^C$  with C < 4
- Use of active cooling fluids is necessary to maintain acceptable values of W/m<sup>3</sup> J/m<sup>3</sup> , W/kg & J/kg

# OPTIMAL DESIGN METHODOLOGY OF MAGNETIC COMPONENTS

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# Magnetic Component Design Problem

#### The design problem is an optimization problem

- More design variables than constraint equations leads to an infinite number of feasible solutions. Optimal solution in terms of performance must be selected in this space
- Determination of optimal dimensions of a given device structure to maximize a cost/ performance objective function according to application & production constraints



### Optimal Design Methodology of Magnetic Components

#### Design Problem formulation

 Optimal Design problem formulation: selection of design variables, constraints, objectives & dimensioning models & their couplings (electromagnetic, electrical circuits, thermal, mechanical modeling)

#### Design variables (or optimization state variables)

 Design variables controllable by designer: physical dimensions, current densities, etc... Design variables can be continuous or discrete but continuous variables improve efficiency of optimization methods. Design variables are bounded, with max & min values

#### Optimal Design Methodology use "inverse problem" resolution method

 Iterative optimization procedure is trying virtual prototypes & evaluating their performance with explicit analytical dimensioning models or by testing them using FEA "simulated experiment"

### Optimal Design Methodology of Magnetic Components

#### Constraints functions of design variables

 Constraints must be respected for design feasibility according to specifications & physical limits of materials or bounds on the validity of dimensioning models or performance (efficiency, temp rise,...). The constraint functions usually incorporated into objective functions using Lagrange multipliers

#### Objective functions of design variables

 An objective is a numerical value to be maximized or minimized: weight, volume, cost. Single or multi-objective optimization methods are available. A suitable formulation of design objective function is preferable to simple weighting of different objective function (example: transformer input apparent power to weight ratio for rated load)

#### Dimensionning Models

 Dimensionning Models used to evaluate the objective & constraint functions of the design variables of optimization problem

# Generic Dimensioning Model

Dimensioning model used to derive performance & constraints imposed by specifications of the application & material operating limits



# Generic Dimensioning Model

Determination of Electrical Equivalent Circuit (inductances & Capacitances) can be performed analytically or directly identified from FEA simulated experiment approach



# Optimal Design Methodology

Non-linear constrained optimization procedure is used to solve the inverse design problem iteratively



### Integrated design or Global optimization of Power Converters

- Multi-disciplinary design optimization (MDO) can be extended to Global Design Optimization of Power Converters including optimal dimensioning of power stage, magnetic components, ... even control strategies & tuning
- MDO allows designers to incorporate all their design dimensioning tools simultaneously in a single CAD environment, to talk together, to share & improve their global expertise
- Global optimal solution usually better than the design found by optimizing each component sequentially, since it can exploit their interaction in terms of global performance to find optimal compromises.

# Integrated design or Global optimization



with Global Optimization Approach