THE BASIS of VACUUM

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Physics of gases
Flow regimes
Definitions
The pumpdown process
Conductance
Conductance calculation

Physics of gases

Units and conversions

Pressure

From \ To	**Pa**	**mbar**	**torr**
Pa	1	0.01	7.5/1000
mbar	100	1	0.75
torr	133	1.33	1

$$x \cdot [\text{unit } 1] = x \cdot \, factor \cdot [\text{unit } 2]$$

Pumping speed
Volumetric flow
Conductance

From \ To	**l /s**	**cm^3/s**	**m^3/h**
l /s	1	1000	3.60
cm^3/s	0.001	1	0.0036
m^3/h	0.278	278	1

[Volume/time]

Flow or **Throughput**

From \ To	**Pa m^3/s**	**mbar l/s**	**torr l/s**
Pa m^3/s	1	10	7.5
mbar l/s	0.1	1	0.75
torr l/s	0.133	1.33	1

[Pressure ×Volume/time]

The physics of Gases

The ideal gas law

$$pV = n_m RT$$

p = pressure
V = volume
n_m = amount of gas (number of moles)
T = temperature
R = general gas constant [8,314 J/(mol K)]

$$pV = nk_B T$$

n = amount of gas (number of atoms or molecules)
k_B = Boltzmann constant = R/6.022x10^{23}

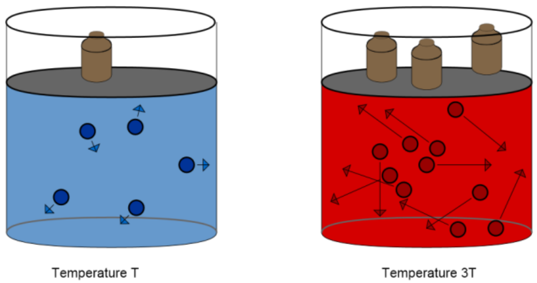
An **ideal gas** is composed of randomly-moving, non-interacting point particles.

1 dozen =

1 mole =

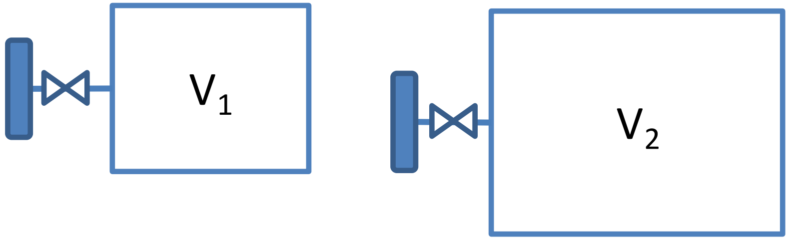
Useful forms of the ideal gas law

In a closed volume, increasing temperature from T_1 to T_2, pressure increases proportionally from p_1 to p_2



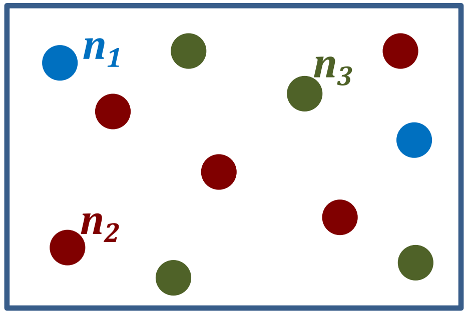
$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$

At constant temperature, the same number of molecules distribute in 2 volumes V_1 and V_2 at pressures p_1 and p_2 such that:



$$p_1 V_1 = p_2 V_2$$

Gas Mixtures and Partial Pressures



DEFINITION

Partial pressure is the pressure which a gas would exert if it occupied the volume of the mixture on its own

$$p_1 = \frac{n_1 RT}{V} \qquad p_2 = \frac{n_2 RT}{V} \qquad p_3 = \frac{n_3 RT}{V}$$

Dalton law

The total pressure exerted by a mixture of (non-reactive) gases is equal to the sum of the partial pressures of individual gases

$$p = p_1 + p_2 + p_3 + \ldots$$

$$\Rightarrow \quad p = \frac{RT}{V}\left(n_1 + n_2 + n_3 + \ldots\right)$$

Flow regimes

Mean free path λ and density

The average distance traveled by a molecule between collisions with other molecules

Vacuum range	Pressure [mbar]	n [molecules/cm³]	Mean free path
Ambient pressure	1013	2.5×10^{19}	68 nm
Low vacuum	300 – 1	$10^{19} - 10^{16}$	0.1 – 100 µm
Medium vacuum	$1 - 10^{-3}$	$10^{16} - 10^{13}$	0.1 – 100 mm
High vacuum	$10^{-3} - 10^{-7}$	$10^{13} - 10^{9}$	10 cm – 1 km
Ultra high vacuum	$10^{-7} - 10^{-12}$	$10^{9} - 10^{4}$	1 km – 10^{5} km
Extremely high vacuum	$<10^{-12}$	$<10^{4}$	$>10^{5}$ km

rarefaction ↓

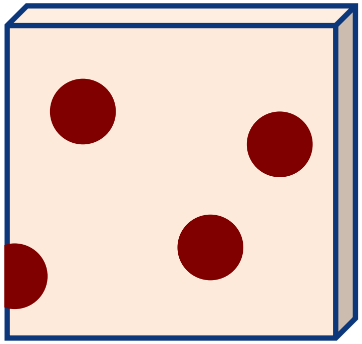
At 296K=23C

Mean Free Path λ

The average distance traveled by a molecule between collisions with other molecules

$$\lambda = \frac{1}{\sqrt{2}n\sigma}$$

n = density of molecules
σ = collisional cross section



For N_2:
$$d_{mol} = 0.31\text{nm} = 0.31\times10^{-9}\text{m}$$

In the hard sphere molecules approximation (d = diameter)

$$\sigma = \pi d_{mol}^{2}$$

Ideal gas law

$$n = \frac{p}{kT}$$

λ inversely proportional to p

$$p\cdot\lambda = \frac{kT}{\sqrt{2}\pi d_{mol}^{2}}$$

mean free path × pressure, room temperature, in [m×Pa] = [cm×mbar]

Gas	λ·p	Gas	λ·p
H_2	12×10^{-3}	CO_2	4×10^{-3}
N_2	6.4×10^{-3}	Ar	7×10^{-3}
He	19×10^{-3}	Ne	14×10^{-3}
CO	7×10^{-3}	Kr	5×10^{-3}

Mean free path and flow regime

The **flow dynamics** is characterized by the comparison of the mean free path λ to the dimension D of the vacuum vessel.

Knudsen number

$$\frac{\lambda}{D} = Kn$$

Collisions with wall (much) more frequent than with molecules

Free **Molecular** Flow	λ >D	K_n >1
Transitional (or intermediate) flow	D/100< λ< D	0.01< K_n< 1
Viscous (continuum) flow	λ< D/100	K_n< 0.01

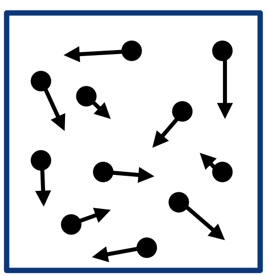
Collisions with molecules dominate

Applying the previous slide, we have a useful relation between pressure and dimension of the vessel to distinguish flow regimes:

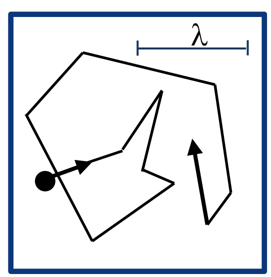
Molecular flow: p·D<0.064 [mbar · mm]

Viscous flow: p·D>6.4 [mbar · mm]

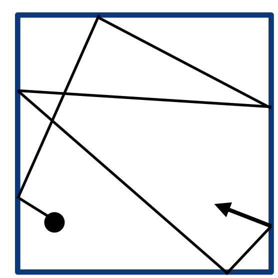
Room temperature, for N_2



molecular chaos



trajectory of a molecule



trajectory in a rarefied gas, λ>D

Definitions

Throughput
Pumping speed

Gas Flow: throughput …

French: ***débit***

Throughput=gas flow rate
Quantity of gas d(pV) crossing a plane along a duct in unit time dt.

$$pV = Nk_BT$$

From the ideal gas law written per unit time, we see that this is a energy flow rate.

$$Q = \frac{d(pV)}{dt} = \frac{dN}{dt}(k_BT) = \frac{dN_{mol}}{dt}RT$$

$\frac{dN}{dt}$: particle flow rate

k_BT: packet of energy

$Pa\cdot m^3\cdot s^{-1} = N\cdot m\cdot s^{-1} = J\cdot s^{-1} = W$

count money=count dwarfs

➡ This is equivalent to a particle flow rate only at constant temperature.

When flow doesn't change with time (we say steady state), Q has the same value at every position along the pipework: mass is conserved.

$$Q_{in} = Q_{out}$$

The same flow subsists in different locations of a continuous unbranched isothermal pipework

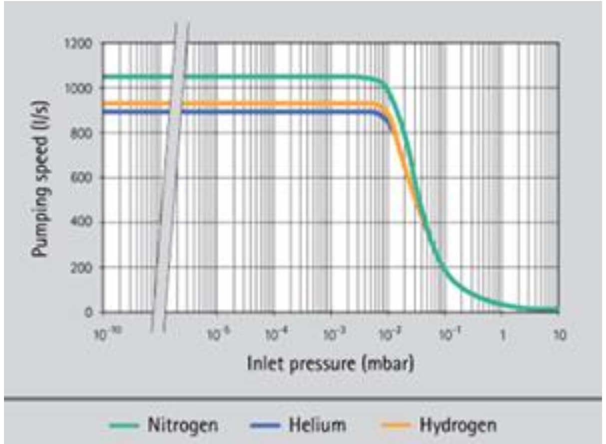
…and pumping speed

We usually call $\dot{V}$ the volumetric flow rate (**débit volumétrique**)

in particular, at the entrance of a pump, we call it pumping speed, *S*.
Substituting this in the definition of throughput, we obtain:

$$Q = p \times S$$

"The quantity of gas flowing is the product of pressure and the volumetric flow rate at that pressure"

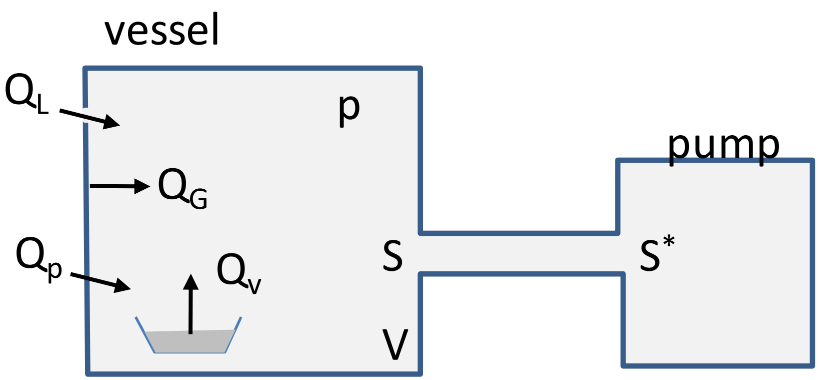


Pumping speed is usually depending on pressure, so *S=S(p)*

Pumping speed is different for different gas species

The pumpdown process

The pumping process

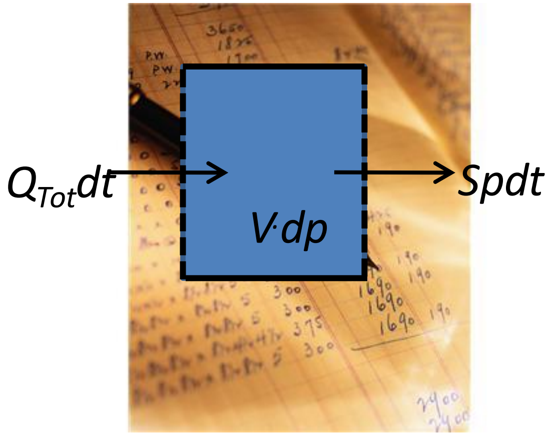


*S** pumping speed of the pump
S pumping speed at the vessel, of volume *V*
p pressure in the vessel
Q_{tot} = sum of all gas loads entering the vessel: outgassing, leaks and permeation, process gas…

$$Q_{Tot} = Q_G + Q_L + Q_p + \text{etc}$$

BASIC EQUATION OF PUMPING – or the Continuity Equation

$$V \cdot dp = Q_{Tot} \cdot dt - S \cdot p \cdot dt$$

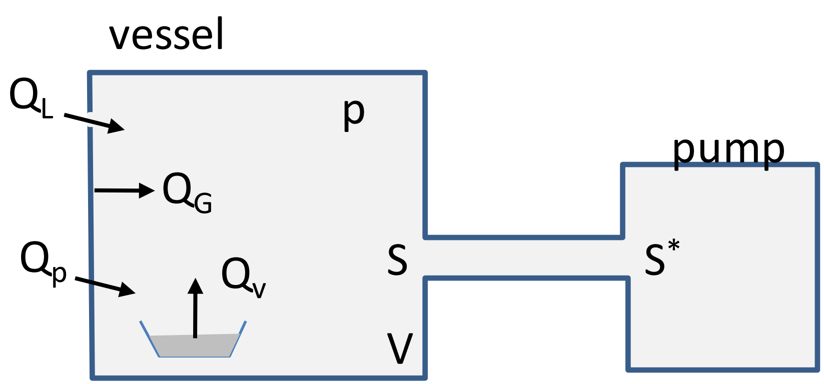


$Q_{tot} \cdot dt$ Amount of gas getting into the free volume in a time dt at pressure p and speed S

$S \cdot p \cdot dt$ Gas pumped away in a time dt at pressure p and speed S

$V \cdot dp$ Change of amount of free gas in the volume V

The pumping process



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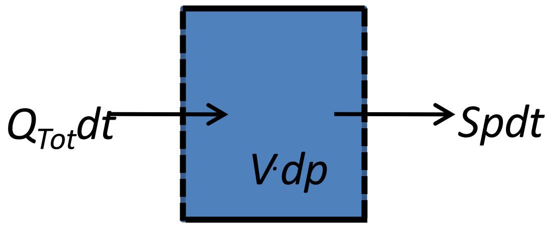


The difference between the quantity of gas entering the volume and the one leaving it in a small interval of time *dt* is equal to the net **change** in the quantity of gas in the volume *V*, d(pV)=*V·dp*

To go deeper:

For conserved quantities:
«Everything which enters a volume ***minus*** everything which leaves it, ***equals*** the net increase in the quantity in the volume»

NB: T=constant, i.e. **isothermal conditions**, or pV is not the conserved quantity

The continuity equation

$$V \cdot dp = Q_{Tot} \cdot dt - S \cdot p \cdot dt$$

Usually, the quantity of gas in a pumped volume **decreases** with time, so we can turn the equation to better express this reduction:

$$-V\left(\frac{dp}{dt}\right) = S \cdot p - Q_{Tot}$$

The rate of change of the amount of gas in a chamber is the difference between the rate of its removal and the influx rate.

What does *dp* mean? Or *dt*?
dp is the **change** in pressure, not the absolute, measured pressure.
dt is a **small time interval** in which the change occurs.

Notice that it is not always easy to apply the continuity equation to know how pressure reduces with time:
Q_{Tot} is time dependent, and also depends on the previous history of the system. S is dependent on pressure and also on gas species…

Example: continuity equation

After 3h pumping, pressure is presently $5\cdot10^{-7}$mbar. The pumping speed of the turbomolecular pump, including reduction by the conductance connecting it to the vessel, is 10ls^{-1}.

The vessel is a tube, 1m long and 400mm in diameter. Chronometer in the hand, you notice that 40s later pressure has decreased to $4\cdot10^{-7}$mbar.

What is the rate of change of pressure in this moment?

In absence of leaks, can you evaluate the total outgassing rate of the vacuum chamber in this moment?

$p=5\cdot10^{-7}$ mbar

$dt=40\text{s}$, $dp=1\cdot10^{-7}$ mbar $dp/dt=2.5\cdot10^{-9}\text{mbar/s}$

$S_{eff}=10$ l/s

$d=400\text{mm}$, $L=1000\text{mm}$

$$V=\pi\left(\frac{d}{2}\right)^2\cdot L=125.7\text{ liters}$$

$$Q_{tot}=Q_{outg}=S\cdot p+V\frac{dp}{dt}$$

$$A=\pi\left(2\cdot\left(\frac{d}{2}\right)^2+d\cdot L\right)=1.5\,\text{m}^2$$

$$Q_{outg}=\varphi\cdot A$$

$$\varphi=Q_{outg}/A=3.5\cdot10^{-10}\text{ mbar}\cdot\text{l}\cdot\text{s}^{-1}\cdot\text{cm}^{-2}$$

$$Q_{outg}=(5\cdot10^{-6}+3\cdot10^{-7})\text{ mbar}\cdot\text{l}\cdot\text{s}^{-1}=5.3\cdot10^{-7}\text{ mbar}\cdot\text{l}\cdot\text{s}^{-1}$$

Pumpdown: initial phase

$$V\left(\frac{dp}{dt}\right) = \cancel{Q_{Tot}} - S \cdot p \rightarrow Q_{pumped}$$

Initially, the pumpdown process is dominated by evacuation of the free gas in the volume. Let's write $Q_{tot}=0$ and let's call $p_{initial}(t)$ the pressure decrease curve in the initial phase of pumpdown.

Gas quantity present in the volume (p□V) decreases while gas is evacuated by the pump.

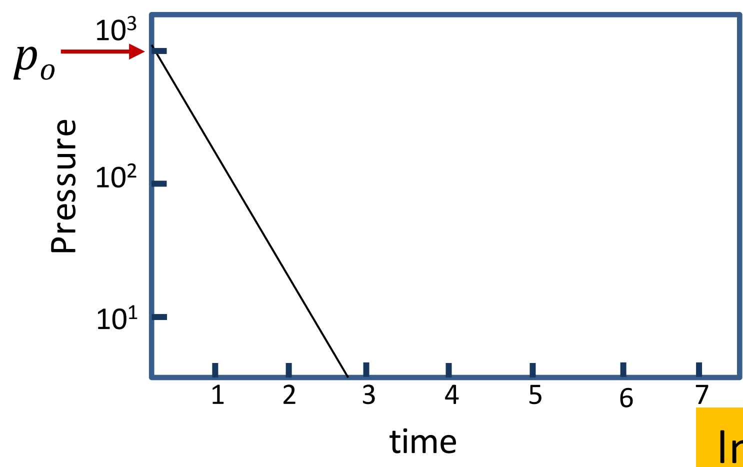
$$V\frac{dp}{dt} = -Sp$$

Volume depletion

$$\ln p_{initial} = \ln p_o - \frac{S}{V}t$$

$$p_{initial} = p_o e^{-\frac{S}{V}t} = p_o e^{-t/\tau}$$

$$\tau = \frac{V}{S}$$ Characteristic time or time constant



ln p versus t

Remember maths! A function which changes with a rate proportional to the function itself is an exponential…

Initial pumpdown time

$$p = p_0 \exp\left\{-\left(\frac{S}{V}\right)t\right\}$$

$$t = \frac{V}{S}\ln\left({p_0}/{p}\right)$$

To make **time** appear alone, let's rearrange by taking the natural logarithm on both sides:

We obtain the time to lower the pressure from the initial value p_0 to some value p

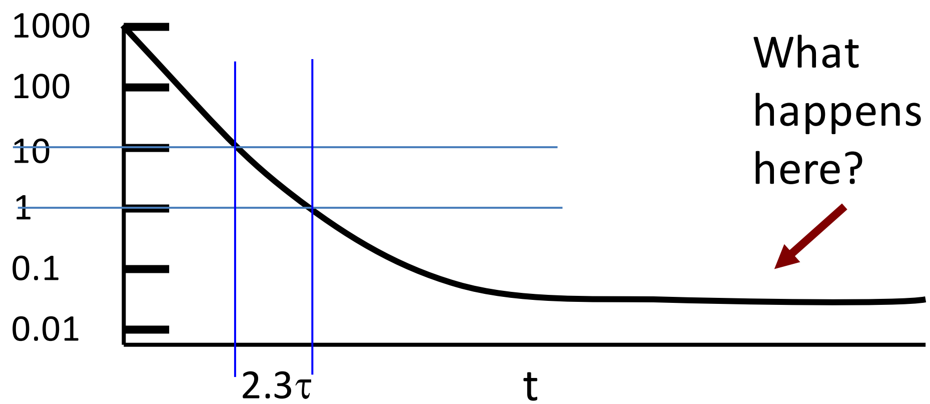
Example:
A *50l* volume is pumped down with
$S=1\ l\cdot s^{-1}$, starting from *1000*mbar to *1*mbar.
What is the value of the time constant τ?
How much time does it take per decade pressure lost?
How much time in total?

$$\ln(10) = 2.3$$

$$\ln(p_o / p) = \ln(100) = 2 \cdot \ln(10)$$

$$\ln(a^x) = x\ln(a)$$

$\tau=V/S=50s$
Time per decade=$\tau\cdot$ln(10)=115s
Total time=3·115s=345s



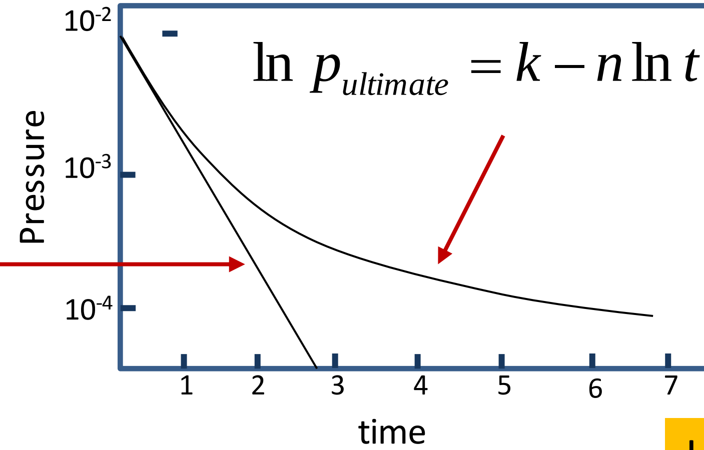
Pumpdown: effect of outgassing on p(t)

Below 1 Pa=10^{-2}mbar, (~roughing), the curve $p(t)$ starts to deviate from the "free volume" straight line. We cannot neglect outgassing Q_{out} anymore.

$$V\frac{dp}{dt} = Q_{tot} - S \cdot p$$

Volume depletion

$$\ln p_{initial} = \ln p_o - \frac{S}{V}t$$



ln p versus t

$$p_{initial} = p_o e^{-\frac{S}{V}t}$$

$$p_{ultimate} = kt^{-n} = \frac{k}{t^n}$$

with 0.5□n□1.2

n ~1 for metallic unbaked surfaces

~0.5 for elastomers, (for baked metallic surfaces)

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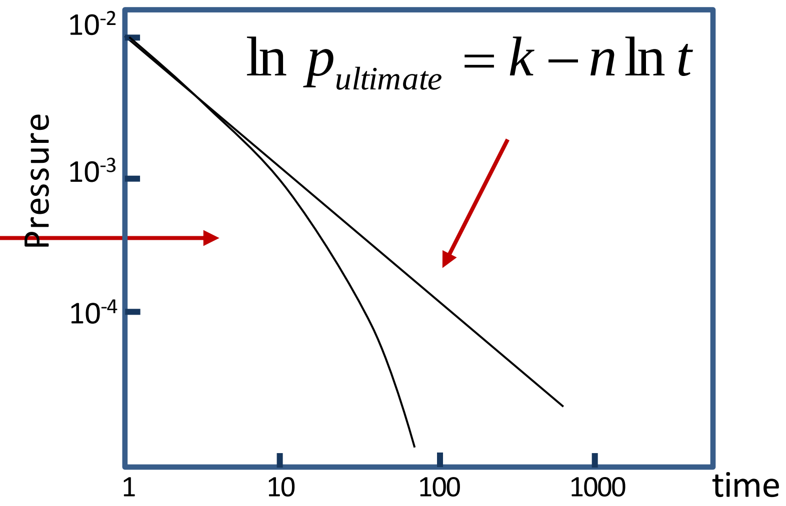

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ln(p) versus ln(t)

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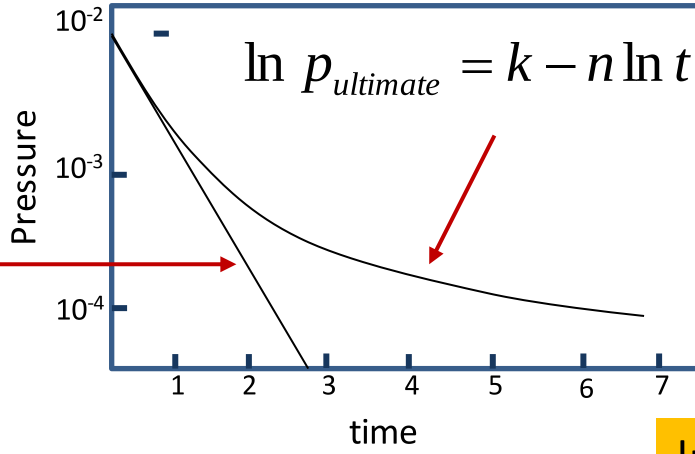

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with 0.5□n□1.2

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~0.5 for elastomers

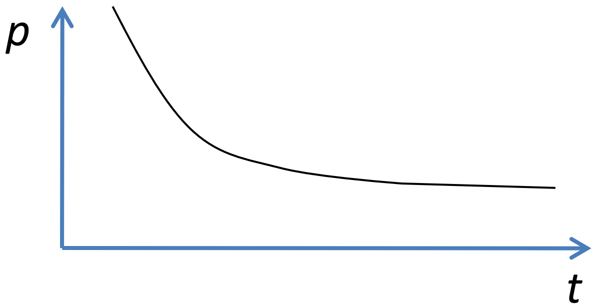
Eventually, the pressure flattens down and becomes "constant" on the time scale of observation. On a long time scale, it continues to decrease slowly.

Pumpdown: when outgassing dominates

$$-V\left(\frac{dp}{dt}\right) = Sp - Q_{out}$$

When pressure ceases to fall and becomes constant on the time scale of observation, $dp/dt=0$.
The equation becomes:

$$p_{ultimate} = \frac{Q_{out}}{S}$$



We write $p_{ultimate}$ because this pressure won't decrease on the time scale of observation (ex. 1h)

Actually, it decreases, because Q_{out} decreases, but this process is much slower.
The walls of the vessel get progressively emptied from their initial gas contents and gas release to the free volume decreases.

$$p_{ultimate}(t) = \frac{k}{t^n} \quad \Rightarrow \quad Q_{out}(t) = A\frac{\varphi}{t^n} = A\frac{\varphi_{1h}}{t^n}$$

For unbaked metals of standard rugosity, $n=1,\quad \varphi_{1h} \approx 3\cdot 10^5 \frac{\text{mbar}\cdot\text{l}}{\text{m}^2} \quad Q(t) = \frac{3\cdot 10^{-5}}{t[h]}\left[\frac{\text{mbar}\cdot\text{l}}{\text{s}\cdot\text{cm}^2}\right]$

Example: pumpdown with outgassing

$$p = \frac{Q_{out}}{S}$$

$$Q_{out} = A\frac{\varphi}{t}$$

From the pressure and applied pumping speed, you can evaluate the outgassing rate and see if it is conform to expectation

Example:

A stainless steel vacuum vessel of 1m length and 250mm diameter is evacuated by one pumping group of an effective pumping speed 50l/s. After 5h pumping, pressure has stabilized at $\sim 1 \cdot 10^{-6}$mbar.

Do you consider that everything is running fine?

$p \cdot S = 5 \cdot 10^{-7}$mbar·l/s

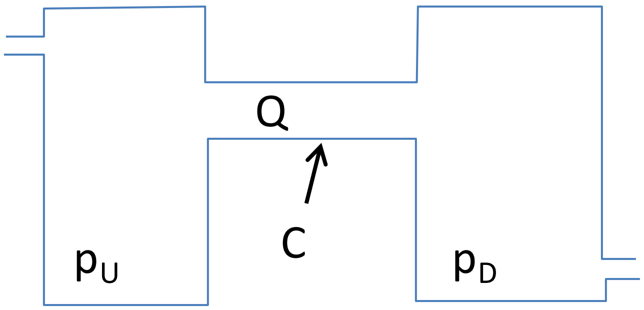
$A = 7850\ cm^2$

$t = 5hr = 18{,}000s$ $\varphi = 3 \cdot 10^{-9}$mbar·l/cm^2

$$Q_{out} \approx \frac{8000 \times 3 \times 10^{-9}}{18{,}000}\left[\frac{\text{mbar} \cdot \text{l}}{\text{s}}\right] = \frac{4}{3} \cdot 10^{-9}\left[\frac{\text{mbar} \cdot \text{l}}{\text{s}}\right]$$

Conductance

Conductance

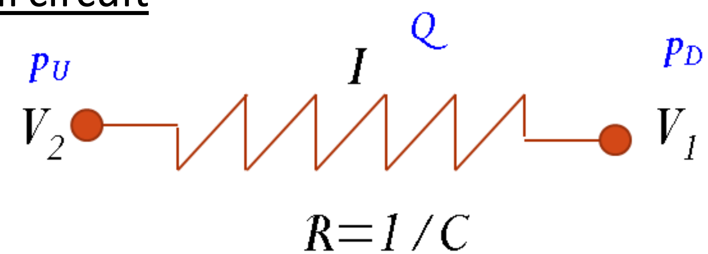


$$Q = C(p_U - p_D)$$

This equation relates throughput (**fr.** *débit*) to the ***difference*** between upstream and downstream pressure.
It is the DEFINITION of conductance.

The quantity of gas which is flowing across a given pressure difference depends on the ***ease of flow*** , described by CONDUCTANCE.
Its reciprocal *1/C* is a resistance to flow; i.e., the opposition the system exerts to gas flow.

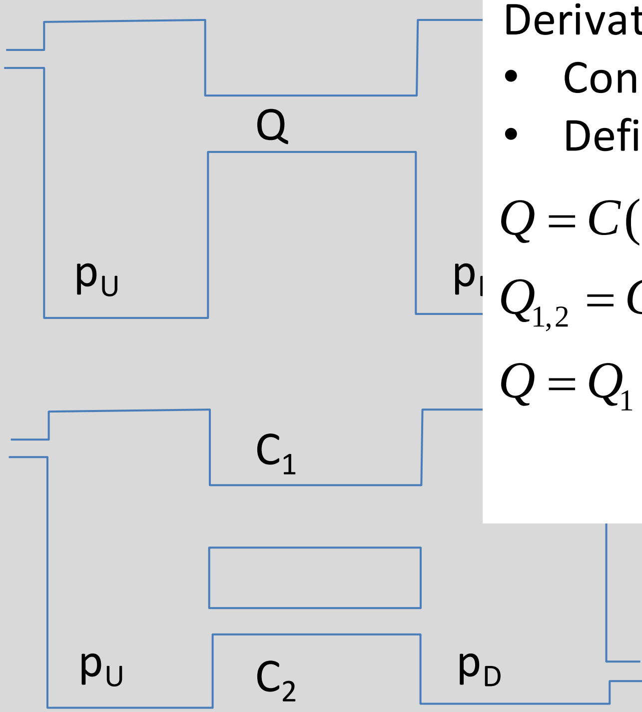
<u>Useful (but approximate) analogy to electrical circuit</u>



Driving force: voltage drop or pressure difference
Flowing quantity: electrical charge or molecules
Flow: electrical current or throughput

Combining conductances

In parallel



$$C = C_1 + C_2 + \text{etc...}$$

Derivation:

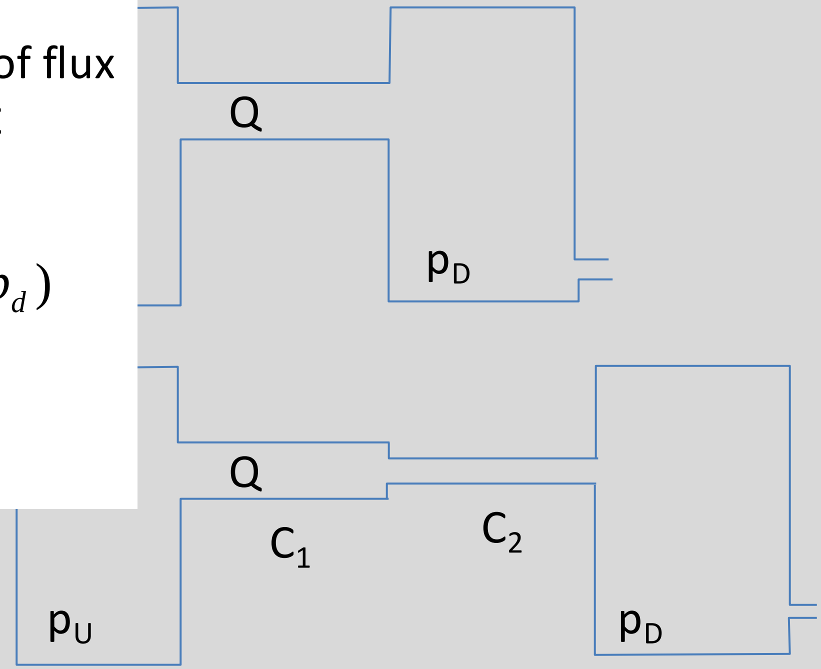
- Conservation of flux
- Definition of C

$$Q = C(p_u - p_d)$$

$$Q_{1,2} = C_{1,2}(p_u - p_d)$$

$$Q = Q_1 + Q_2$$

In series



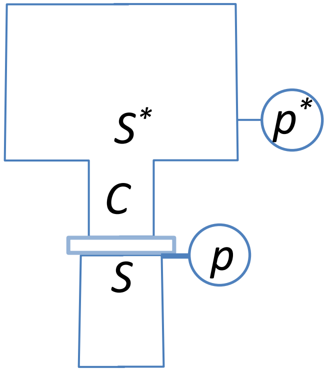
$$R = R_1 + R_2 + etc...$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \text{etc...}$$

Approximate: it overestimate resistance, by neglecting beaming and entrance resistance cancelling

Reduction of pumping speed by connecting pipe

Application of the **combination of conductances in series**.

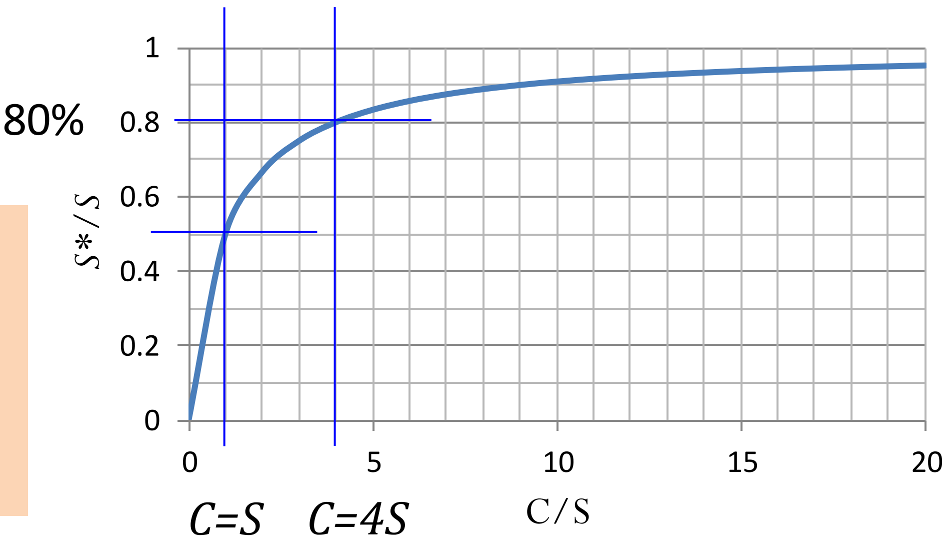


$$Q = C(p^* - p) = S^* \cdot p^* = S \cdot p$$

$$\frac{1}{S^*} = \frac{1}{S} + \frac{1}{C} \quad \text{or} \quad S^* = \frac{S \cdot C}{S + C}$$

*S** effective pumping speed at the chamber
S real pumping speed
p* pressure in the chamber
p pressure at the entrance of the pump
C connecting conductance
Q throughput through the pipe and into the pump

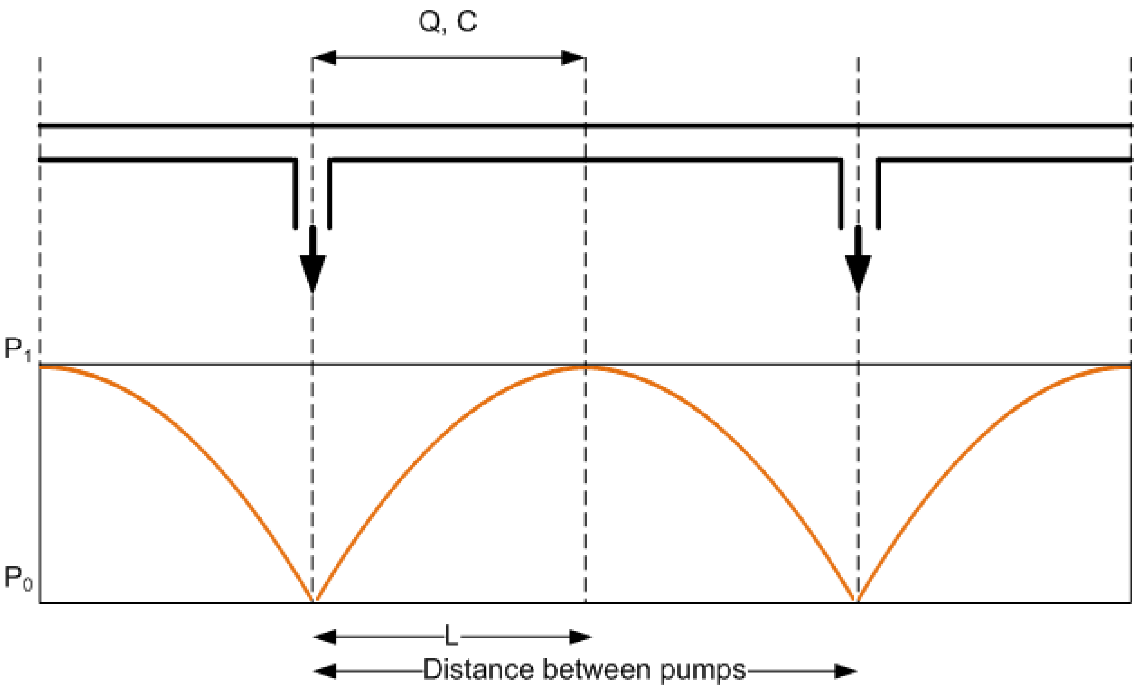
*S** is less than *S*!
Lets plot *S*/S* reduction of pumping speed, against *C/S*, ratio between conductance and pumping speed



Ex: If the conductance is equal to the pump's speed, we only get 1/2 of it at the vessel. To get 80% pumping speed at the vessel, we need a conductance 4x larger than the pumping speed.

Pressure profile in long tube with localized pumps

A tube under vacuum will outgas uniformly over its whole internal surface. Pumps are installed along the tube. Due to the limited (i.e., not infinite) conductance of the tube, pressure will have maxima at equal distance from two pumps and minima above the pumps.



PRESSURE PROFILE

From the continuity equation, it is obvious that:

$$P_0 = \frac{2Q}{S}$$

It is not difficult to show also that

$$P_1 - P_0 = \frac{Q}{2C}$$

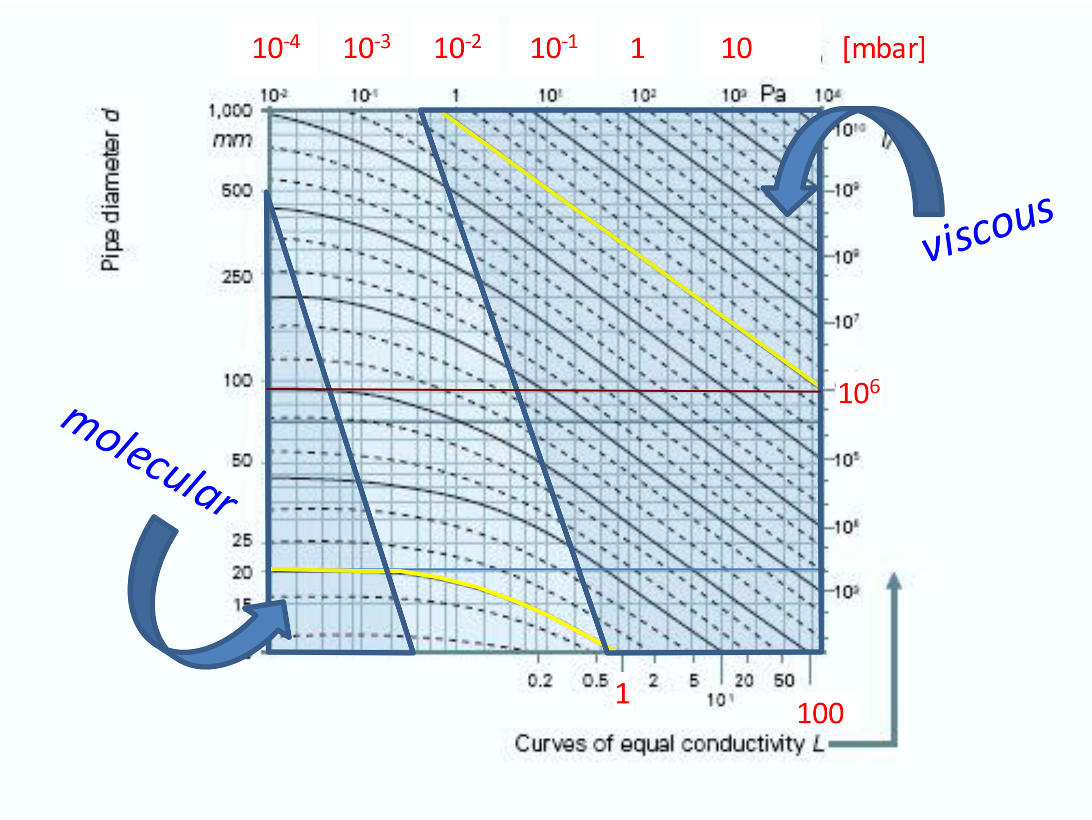
Q: outgassing flux [mbar.l.s-1]
S: pumping speed [l/s]
C: tube conductance [l/s]

Notice differences in pressure measurement in long tubes, depending on their distance to the pump!

Conductance calculation

Conductance in viscous flow
Conductance in molecular flow

Conductances and flow regimes



On the figure: the equal conductivity curves on a p,d graph
- read for example: **10^6 l/s** and **1 l/s**

Let's anticipate, that the conductances of pipes will differ in different flow regimes:

- In **continuum flow**, they are proportional to mean pressure *p*.
- In **molecular flow** they are not a function of pressure.
- In Knudsen flow, a transition between the two types of flow, conductances vary with Knudsen number.
- At the same diameter, conductance in continuum regime is much larger than in molecular regime

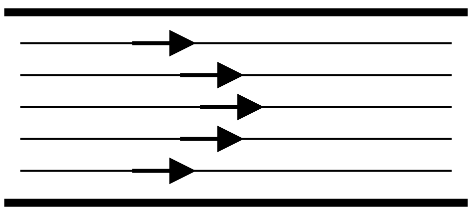
Above p·D>6.4 mbar·mm, flow is viscous
Below p·D<0.064 mbar·mm, flow is molecular

Continuum (viscous) flow through pipes

Viscous gas flow may be:

LAMINAR:

gas flows smoothly in stream lines, parallel to the duct walls.



Inertia forces compared to viscous forces

Viscous forces are stabilizing

Re<2000

TURBULENT :

In most cases treated in vacuum technology, we are in laminar conditions

Reynolds number Re compares the two effects

$$\mathrm{Re} = \frac{\rho u D_h}{\eta}$$

ρ = density of the fluid

η = viscosity

u = flow velocity

D_h = hydraulic diameter

$$D_h = \frac{4A}{B}$$

A = cross sectional area

B = wetted perimeter

Viscous laminar flow - Poiseuille

This is valid only for **long tubes**, i.e., rarely in vacuum... But it helps understanding this matter.

$$Q = \frac{\pi D^4}{128\,\eta L}\left(\frac{p_1 + p_2}{2}\right)(p_1 - p_2)$$

Circular cross section, long tube
Diameter D
Between pressures p_1 and p_2
With η viscosity of the fluid

$$C = \frac{\pi D^4}{128\,\eta L}\,\bar{p}$$

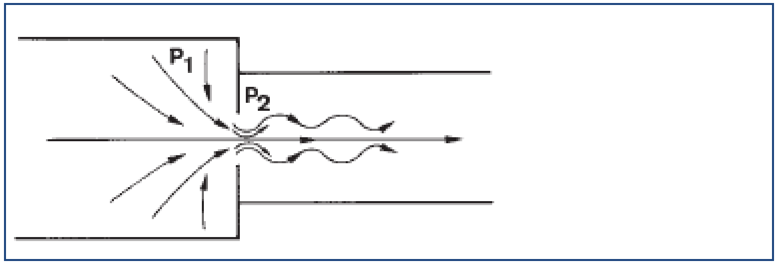
$\bar{p}$ is the average pressure between entrance and exit pressure

NB:

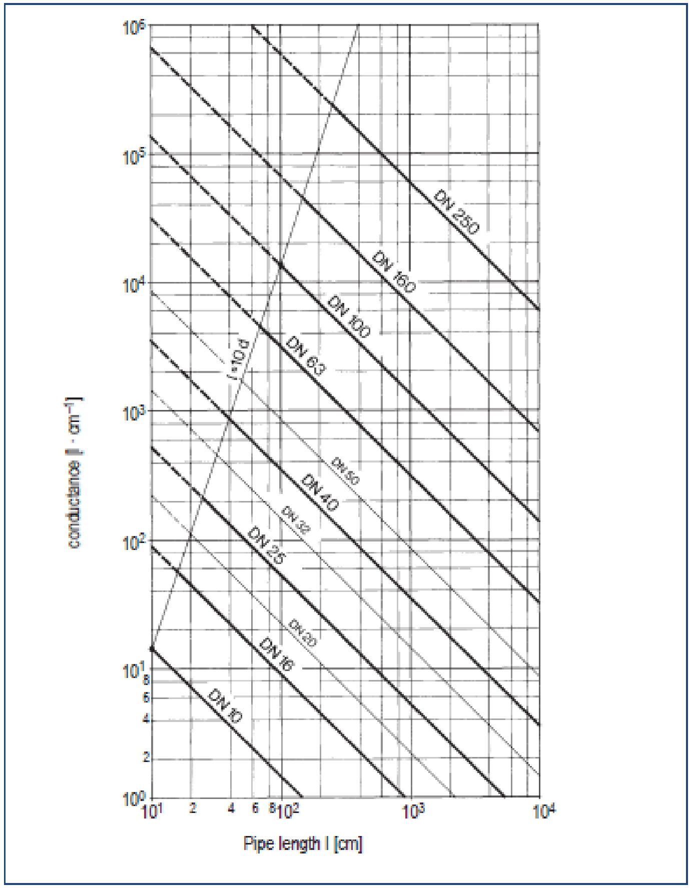
- **In viscous flow, conductance is proportional to pressure**! Already seen 2 slides ago
- The effect of diameter D on C is with D^4, so doubling D means a factor 16 in conductance

Viscous regime: Short tubes conductance

The difficulty here is that the velocity profile changes at the aperture, with a minimum cross section at the aperture, then with oscillations of contraction/expansion before reaching fully developed flow.



Nomograms may be used.

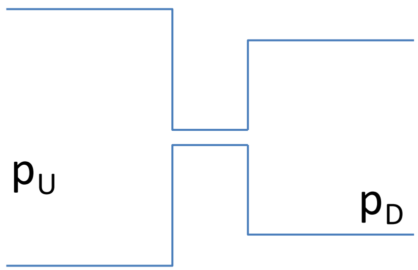


Particular case of practical use in vacuum: choked (or sonic) flow.

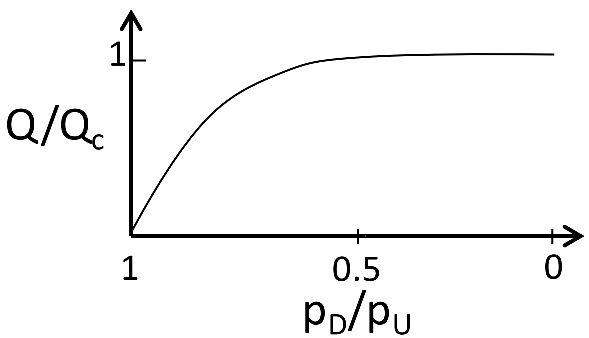
Choked flow

An effect of compressibility of a gas…

French: ***flux sonique***



Keep p_U constant and lower progressively p_D… At some value of p_D/p_U, you attain *choked (or blocked) flow*.



In choked flow, throughput is not depending on downstream pressure p_D any longer, but only on upstream pressure p_U.
This happens when flow velocity in the restriction is equal to sound velocity. For air, it occurs when $p_D<0.53\ p_U$.

Even if pressure p_D is lowered, throughput remains constant.

$$Q_c = p_U \cdot A \cdot K(\text{gas}, T)$$

$$S_c = 20 \cdot A[\text{l/s}],\ \ A[\text{cm}^2]$$

Remember…

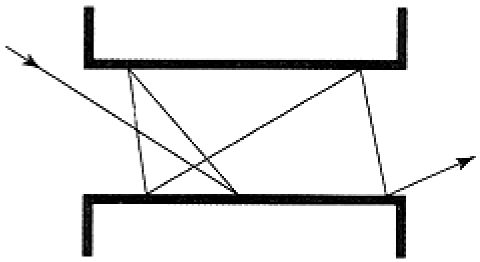
$$Q = S \times p$$

A choked aperture has constant pumping speed.

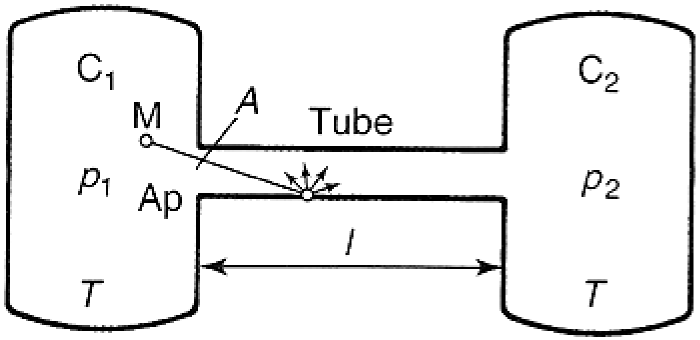
Perfect flow control! Controlled soft venting! …and more

Molecular Flow: Introduction

- **Kn>1**: the mean free path of the particles is large compared with the size of the container
- The molecules-wall collisions dominate the gas behavior



- **Randomizing effect of molecule wall collisions**: there are no favored directions, the probability of emerging in any direction is the same, not related to incident direction

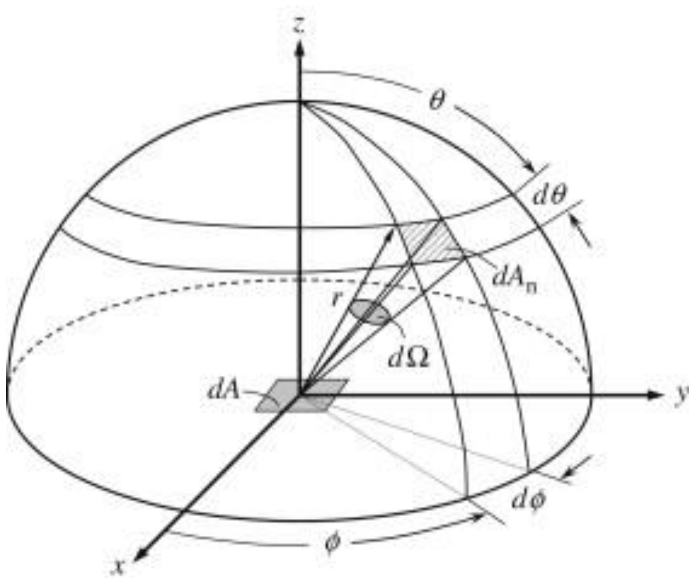


In molecular flow, a molecule is not "sucked" away, it falls on the pump entrance as the result of random motion.

Important to learn about the rate of molecular impingements on a elementary surface

Impact rate

- **Impact Rate J**: Number of molecular impacts on a surface per unit time and unit surface



The molecules impacting on a surface dS are those having velocity vectors comprised between v and v+dv and direction θ. Integrated over the solid angle:

$$J = n\frac{\bar{v}}{4}$$

$$\left[\frac{\text{molecules}}{\text{cm}^3}\cdot\frac{\text{cm}}{\text{s}}\right]$$

$$\bar{v} = \sqrt{\frac{8kT}{\pi m}}$$

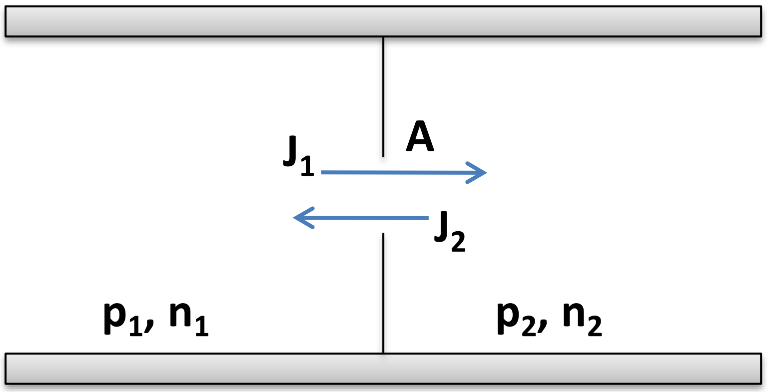
average or mean speed in Maxwell-Boltzmann distribution

$$J = \frac{n\bar{v}}{4} = \frac{p}{\sqrt{2\pi m k_B T}}$$

n = density of molecules

Conductance of an aperture

Aperture of area A in a thin wall separating two regions of pressures p_1 and p_2



$$\frac{dN}{dt} = (J_1 - J_2)A$$

Net flux= difference between currents in either direction

$$Q = kT\left(\frac{dN}{dt}\right)$$

Ideal gas law: definition of flow

$$Q = C(p_1 - p_2)$$

Definition of conductance

$$Q = \sqrt{\frac{RT}{2\pi m}}A(p_1 - p_2)$$

$$C_A = A\sqrt{\frac{RT}{2\pi m}} \propto \sqrt{\frac{T}{m}}$$

For air at 20°C

$$C_A = 11{,}8 \cdot A \left[l \cdot s^{-1}\right]$$

with A in cm^2

Conductance of standard flanges

$$C_A = 11{,}8 \cdot A\left[l \cdot s^{-1}\right]$$

with A in cm^2

Conductance of (infinitely thin) flange of standard opening

DN	C_A Conductance (l/s)
16	24
25	58
40	148
63	368
100	927
150	2085
200	3707

Conductance of standard flanges for N_2 at 296K

How to transform from N_2 to other gases?

MEMO

Small mass ↔ large conductance

$$C_A^{air} = 11.8 \cdot \pi \cdot r^2$$

$$C_A^{air} = C_A^{gas} \cdot \sqrt{\frac{M_{air}}{M_{gas}}}$$

NB

Usually, we put a ~500l/s pump on a DN150. The flange's conductance is >4x the pump's pumping speed.

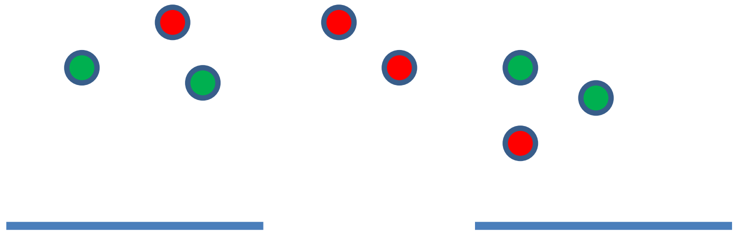
Maximal pumping speed

From the preceding, we can see that the maximal pumping speed of a high vacuum pump cannot be larger than the conductance of its aperture.

The maximum throughput a pump can feature is when all molecules crossing its aperture don't return into the vacuum vessel.

$$C_{N_2} = 9.3\,D^2 \;\; [\mathrm{l/s}]$$

maximal pumping speed for nitrogen, for a pump of aperture diameter D (in cm).



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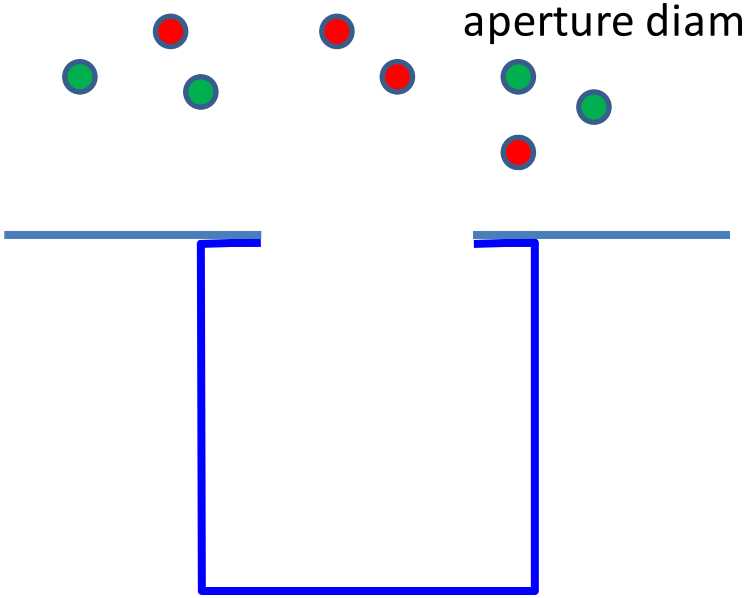


High vacuum pumps do not "suck" molecules into them. They rather prevent (a certain number of) molecules having crossed **by chance** their aperture to travel back to the vacuum vessel.

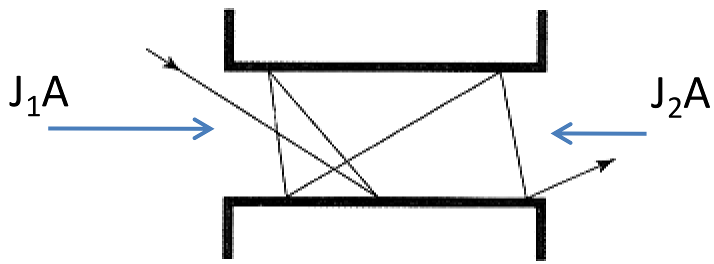
If the pumping mechanism were 100% efficient, the pumping sped would be equal to the conductance of the entrance.

Pumping speed is highest when backstreaming into the vessel is lowest.

Transmission probability α

If N molecules arrive at the entrance plane of a duct, a number Nα reach the exit plane, with α<1. A number $N(1\text{-}\alpha)$ return to the entrance. Remember that $N=J$□A with A area and J impingement rate of molecules at the aperture of the duct.

Of the J_1A molecules entering the duct, $\alpha \cdot J_1A$ reach its exit



Of the J_2A molecules entering the duct from downstream, α J_2A reach its entrance plane

Net flux from entrance to exit

$$\alpha(J_2 - J_1)A$$

Net throughput from entrance to exit

$$Q = \alpha(J_2 - J_1)A \cdot kT$$

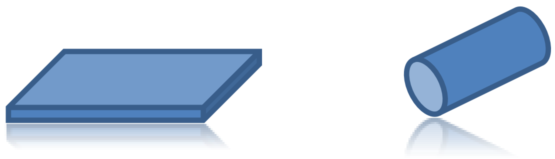
Substituting the expression $J = p/\sqrt{2\pi mkT}$ we make p appear.

$$Q = \alpha(p_2 - p_1)A \cdot \sqrt{\frac{R_o T}{2\pi M}} = \alpha C_A (p_2 - p_1)$$

$$C = \alpha C_A$$

The conductance of a component is the conductance of its aperture times a transmission probability

Transmission probability calculations



$$C = \alpha C_A$$

The concept of transmission probability was introduced by **Clausing** (1932). The values he has calculated by integrals (sometimes called **Clausing factor**) are still widely used for circular tubes and are accurate to ~ percent. **Nomograms often use his values.**

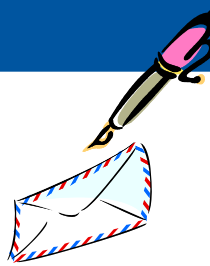
Cole (1976) refined the same methods, his values are very accurate.

Dushman (1992) has proposed an analytical formula (next slide) easy to apply, but not completely accurate. We use it for quick estimations.

Livesey (1998-2004) collected all transmission probability data, analyzed them and obtained **analytical formulas** (2004), very useful for computer calculation. For exotic forms, we have proposed his values, which are fairly simple and accurate to <1% over a wide L/D range.

One of the best ways to calculate transmission probabilities is the statistical **Monte-Carlo** method, which can be applied with success to complicated shapes which escape from a simple geometrical treatment.

Dushman transmission probability for cylindrical tubes



to be used for “back of the envelope” estimations

$$C = \frac{C_A C_L}{C_A + C_L} = \frac{12.4 D^3 / L}{1 + 4D/3L} \quad D, L \text{ in [cm]}$$

$$\alpha = \frac{C_L}{C_A} = \frac{C_L}{C_A + C_L}$$

$$\alpha = \frac{1}{1 + \frac{3}{4} \cdot \frac{L}{D}}$$

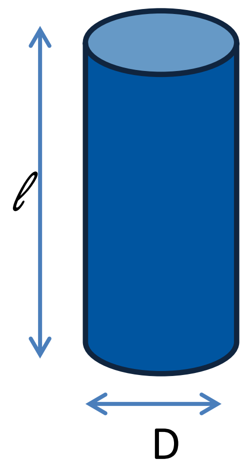
Transmission probability of a cylindrical tube, of diameter D and length L, or of form factor L/D.

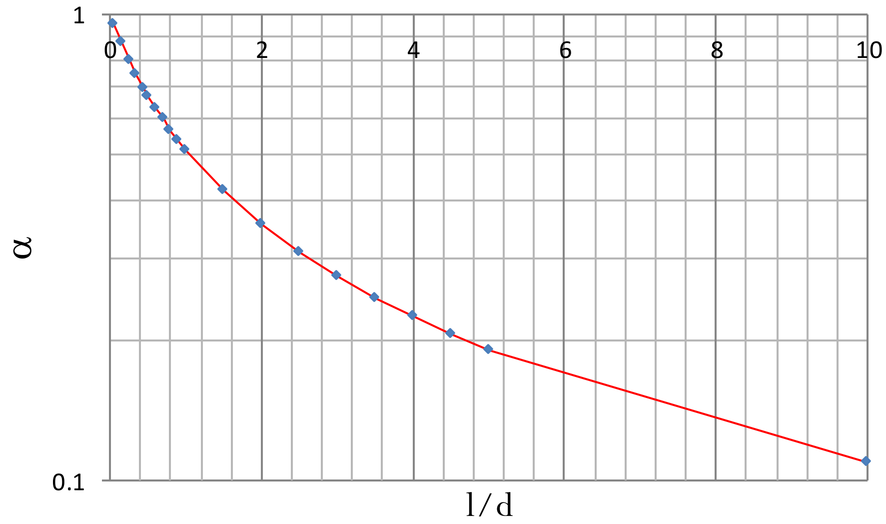
Dushman 1992

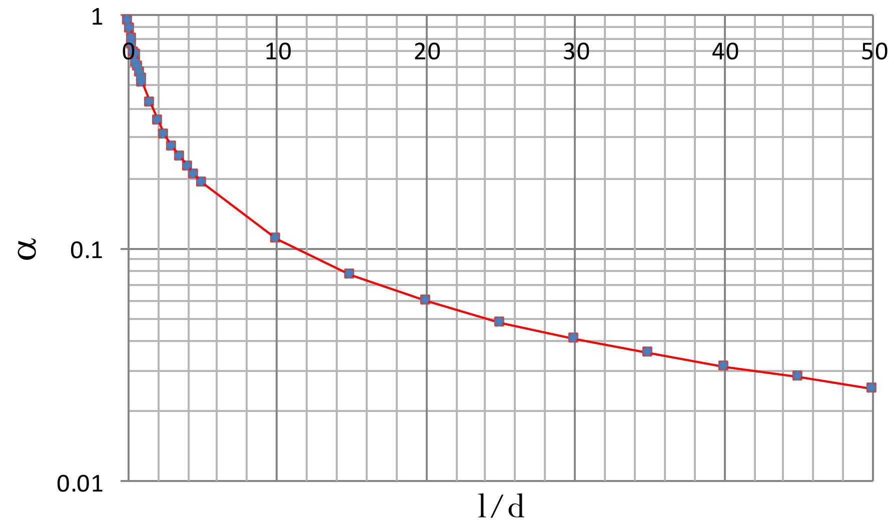
$C_A = 9.3 D^2 \; ls^{-1} \quad D \text{ in [cm]}$	Aperture
$C_L = 12.4 \frac{D^3}{L} \; ls^{-1} \quad D, L \text{ in [cm]}$	Long tube

Cylindrical ducts

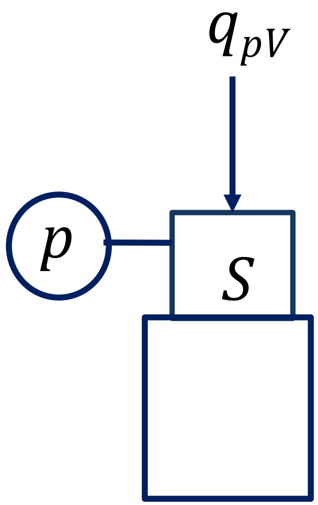
l/D	α [Cole]
0.05	0.952399
0.15	0.869928
0.25	0.801271
0.35	0.74341
0.45	0.694044
0.5	0.671984
0.6	0.632228
0.7	0.597364
0.8	0.566507
0.9	0.538975
1	0.514231
1.5	0.420055
2	0.356572
2.5	0.310525
3	0.275438
3.5	0.247735
4	0.225263
4.5	0.206641
5	0.190941
10	0.109304
15	0.076912
20	0.059422
25	0.048448
30	0.040913
35	0.035415
40	0.031225
45	0.027925
50	0.025258
500	0.002646







Pumping speed measurement



Introduce a known throughput q_{pV} above the pump and measure pressure p at the inlet

$$S = \frac{q_{pV}}{p - p_o} \qquad p_o = \text{Pressure when } q_{pV} = 0$$

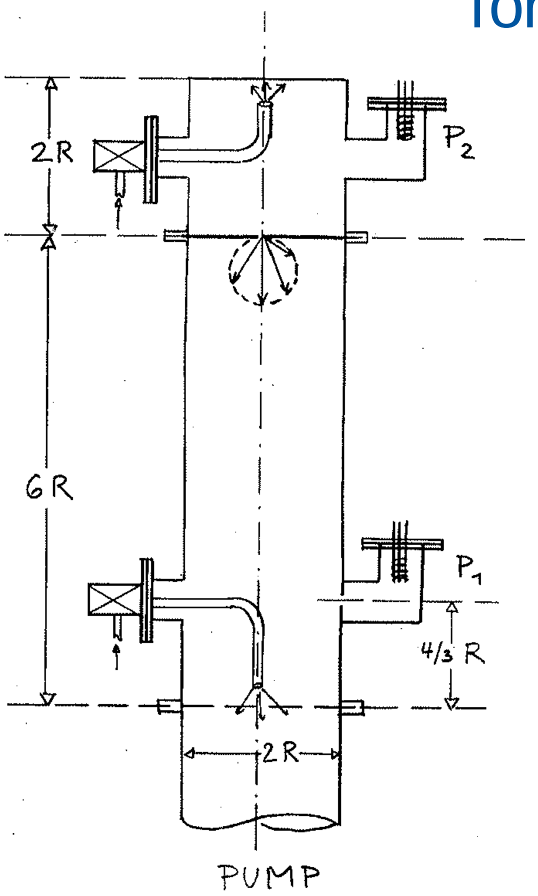
Gas flow q_{pV} may be measured by a precise flowmeter or in a double-dome with known conductance between the domes

Measure pressure p in front of the pump is problematic: directional gas flow!
Pressure is defined in an enclosed system, at equilibrium.

To avoid perturbation of the isotropic (i.e. uniform) Maxwellian distribution by the pump inlet, one would need an infinitely large volume.
Solution: ***Fisher-Mommsen*** dome (double and "isotropic")

Fisher-Mommsen dome

for pumping speed measurement



$$S = \frac{q_{pV}}{p_1} = C(\frac{p_2}{p_1} - 1)$$

Thermodynamically, *p* is defined only in an enclosed system at equilibrium.
To approach isotropy, we need a “infinite volume” dome, and gauges far from the pump. The gauge should then measure a isotropic, Maxwellian gas distribution.

The inlet *C* and pump *S* should have a negligible effect on the distribution.

The Fisher Mommsen dome has **geometry**, **dimensions** and **position of the gauge** such that measured pressures are identical to those measurable in the ideal case.

E. Fischer, H. Mommsen, Vacuum 17, 309 (1967)

Examples

From the table of transmission probabilities, molecular flow transmission probability for a pipe whose length is equal to its diameter is 0.51. This means that only about ½ of the molecules that enter it, pass through. What fraction will get through for a pipe with L/D=5?

α=0.19

The molecular flow transmission probability of a component with entrance area 4cm^2 is 0.36. Calculate its conductance for nitrogen at (a) 295K, (b) 600K.

From C_A=11.8A l/s (with A in cm^2) for nitrogen at 295K, and $C=\alpha C_A$, we get C=17l/s.

The effect of temperature is with √T, conductance being proportional to √T. We must divide by √295 and multiply by √600, to obtain C=24 l/s.

Examples

A component has a molecular flow conductance of 500l·s^{-1} for nitrogen. What will its conductance be for (a)hydrogen, (b)carbon dioxide?

We have to **multiply** by the square root of the molar mass of nitrogen (N_2, M=28) and **divide** by the square root of the molar mass of hydrogen (H_2, M=2) or carbon dioxide (CO_2, M=44).
In the first case, this gives a factor 3.74, in the second, a factor 0.8.
So the conductance for hydrogen will be 3.74 times larger, the one for carbon dioxide 0.8 times larger. We get 1870l·s^{-1} for hydrogen and 399l·s^{-1} for carbon dioxide.

Notice that the square root factor "damps " the effect of mass, but nevertheless this factor is ~4 for hydrogen if compared to air….

By what factor will the molecular flow conductance of a long pipe be increased, if its diameter is doubled?

With the simplified formula of Dushman for long tubes in molecular flow, we see that the effect of D is with D^3.
So doubling D will increase conductance by a factor 8 !!

Examples

A vessel of volume 3m^3 has to be evacuated from 1000 mbar to 1 mbar in 20 min. What pumping speed (in m^3 per hour) is required?

HINT: $\ln(10) = 2.3, \quad \ln(10^a) = a\ln(10)$

$$t = \frac{V}{S}\ln\left(\frac{p_0}{p}\right)$$

We apply the formula for the rough vacuum regime (which neglects outgassing), to obtain: $t=6.9V/S$. So $S=6.9V/t=$62m^3/h
Notice that 3m^3 is the typical volume of a Booster sector, of a SPS arc sector. With half this pumping speed, time is doubled...

How long will it take for a vessel of volume 80 l connected to a pump of speed 5 l/s to be pumped from 1000 mbar to 10 mbar? What is the time per decade?

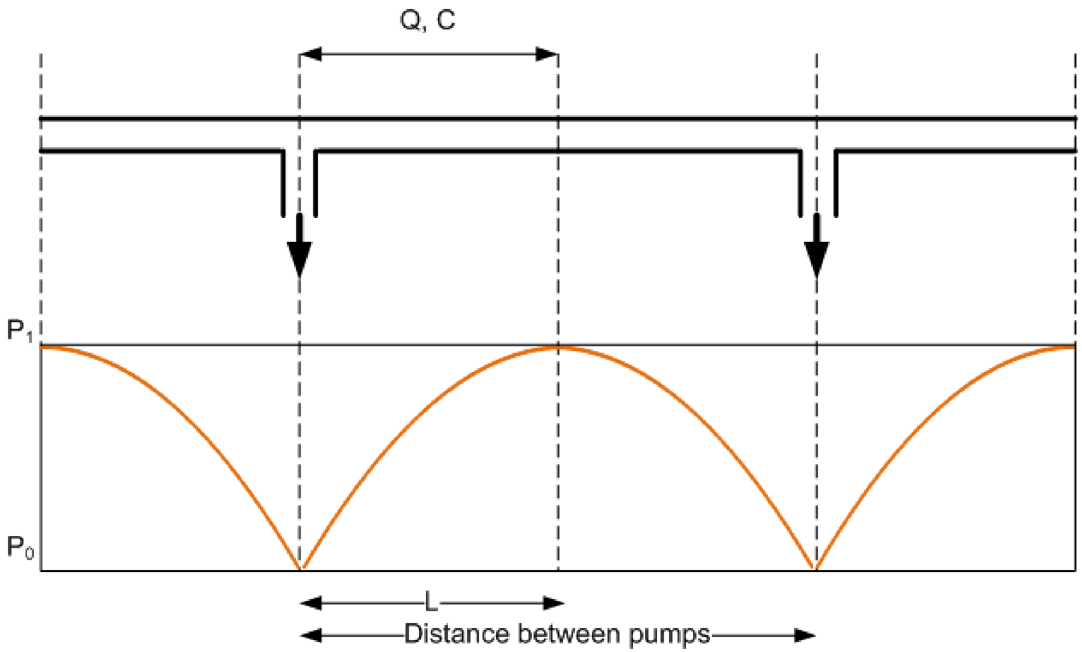
As above: $\ln(p_o/p)=4.6$, so $t=4.6V/S=4.6 \cdot 16 \approx$80s

Time per decade: $5 \cdot 2.3$s=12s

Example: pressure profile

In a SPS transfer line, the diameter of the pipe is ~60mm, the distance between pumps ~60m. The effective pumping speed of each ionic pump is ~15 l/s, including a correction for the connecting element. Let's assume an outgassing rate of $3 \cdot 10^{-11}$ mbar·l·s^{-1}·cm^{-2}, typical after 100h pumping time.

Calculate the maximum and minimum pressure measured at a pressure gauge in a segment between 2 pumps.



Formulas to use:

$$C_L = 12.4 \frac{D^3}{L} \; ls^{-1} \quad D, L \text{ in [cm]}$$

$$P_{\min} = \frac{2Q}{S}$$

$$P_{\max} - P_{\min} = \frac{Q}{2C_L}$$

Example: pressure profile

$$C_L = 12.4 \frac{D^3}{L}\ ls^{-1} \quad D, L \text{ in [cm]}$$

$$P_{min} = \frac{2Q}{S}$$

$$P_{max} - P_{min} = \frac{Q}{2C_L}$$

Input data

$L := 30m \quad D := 60mm \quad S_{eff} := 20\frac{1}{s} \quad q := 3 \cdot 10^{-11} mbar \cdot l \cdot s^{-1} \cdot cm^{-2}$

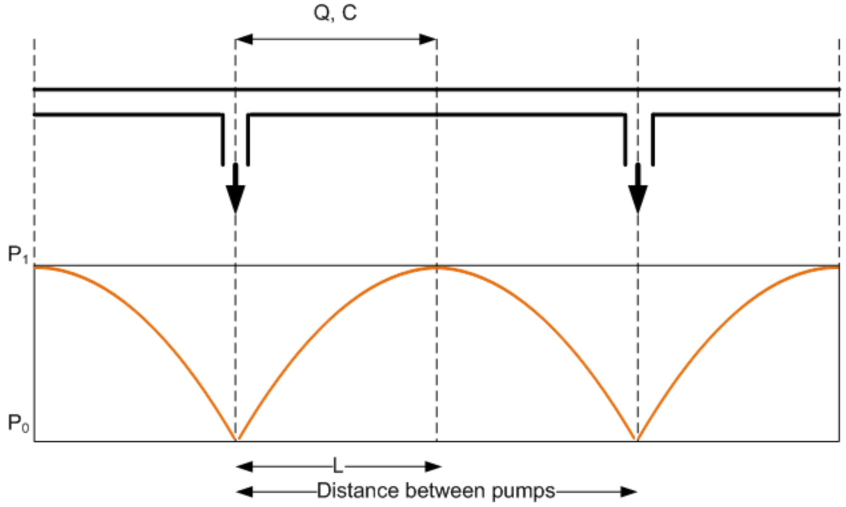
Calculation of C

$$C_L := 12.4 \cdot \frac{6^3}{3000} \frac{1}{s} = 0.89 \frac{1}{s}$$

Calculation of Q

$$Surf := \pi \cdot D \cdot L = 5.655\, m^2$$

$$Q := q \cdot Surf = 1.696 \times 10^{-6} mbar \cdot \frac{1}{s}$$



Calculation of P_{min} and P_{max}

$$P_{min} := \frac{2Q}{S_{eff}} = 2.3 \times 10^{-7} mbar$$

$$P_{max} := P_{min} + \frac{Q}{2C_L} = 1.2 \times 10^{-6} mbar$$