On the gravitational signal from tidally deformed neutron stars in coalescing binaries

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A. Maselli, L. G., V. Ferrari, PRD 88, 104040 (2013)

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In this talk we do not consider the frequencies of the GW signal
emitted during the merger (see *Shibata, PRL '05; Oechslin & Janka, PRL '07; Stergioulas et al., MNRAS '11; Bauswein et al., PRL '11, PRD '12, arXiv:1403.5301; Takami at al., arXiv:1403.5672*)

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In recent years an huge effort has been carried on to determine R from EM observations (e.g., *Steiner et al., ApJ ’10; Bhattacharyya et al. MNRAS ’09; Ozel et al., PRD ’09,’10; Guver et al., ApJ ’10; Guillot et al., ApJ ’11* and many others)

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Progress has been made, but these measure are still, to some extent, model dependent.
I think that we can get solid evidence from EM observations supporting a value of R, but only GWs - which are not affected by the matter and energy surrounding the NS - could allow us to really *measure* the NS radius.

2) The NS tidal deformability λ

Tidal deformations of NSs can be described in terms of a set of parameters, the Love numbers, relating the mass multipole moments of the deformed star with the multipole moments of the external tidal field.
Most of the information on the stellar deformation is encoded in the quadrupole Love number k_2, i.e. in the tidal deformability λ:

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$$h_{PN}(x) = \mathcal{A}(x) e^{\mathrm{i}[\Psi_{PP}(x)+\Psi_T(x)]}$$

$$x = (\pi m f)^{2/3} \qquad m + m_1 + m_2 \qquad \nu = m_1 m_2/m^2 \qquad \mathcal{M} = m\nu^{3/5}$$

$$\mathcal{A}(x) = \sqrt{\frac{5}{24}}\frac{\mathcal{M}^{5/6}}{\pi^{2/3}d} f^{-7/6}\left[1 + \beta_1 x + \beta_2 x^2 + \ldots\right]$$

$$\Psi_{PP}(x) = 2\pi f t_c - \phi_c + \frac{3}{128\nu x^{5/2}}\left[1 + \alpha_2 x + \alpha_3 x^{3/2} + \alpha_4 x^2 + \ldots\right]$$

$$\Psi_T(x) = -\frac{117\tilde{\lambda}}{8\nu m^5} x^{5/2}\left[1 + \tilde{\alpha}_2 x + \tilde{\alpha}_3 x^{3/2} + \ldots\right] \qquad \tilde{\lambda} = \frac{m_1 + 12 m_2}{26 m_2}$$

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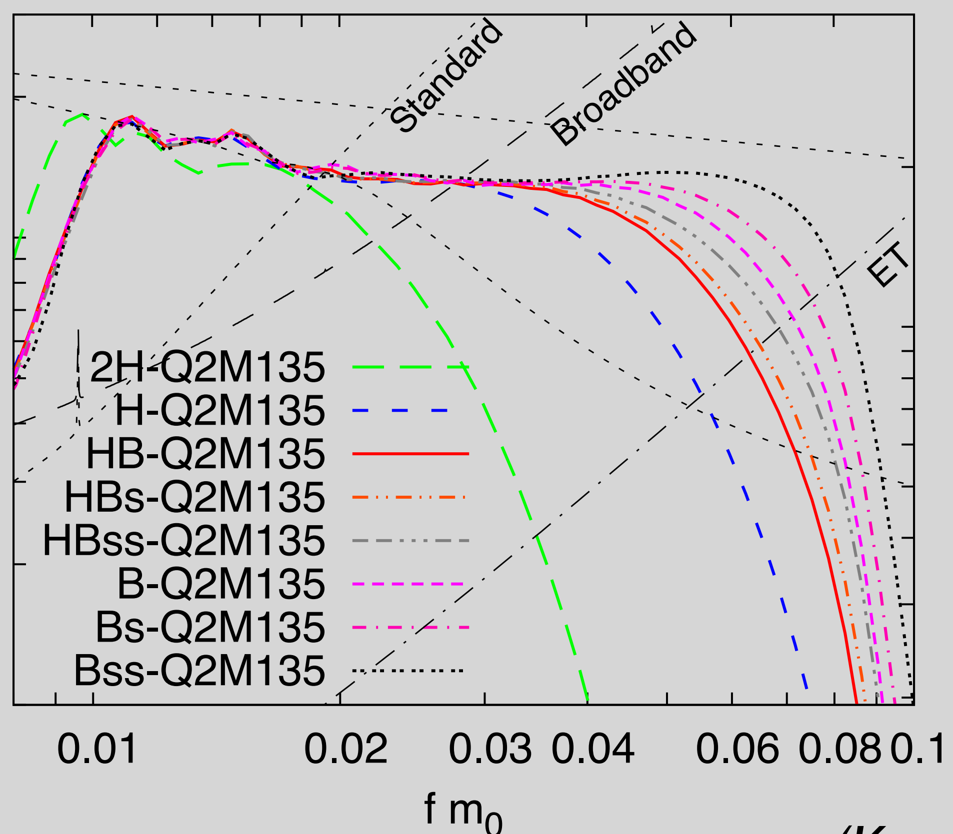
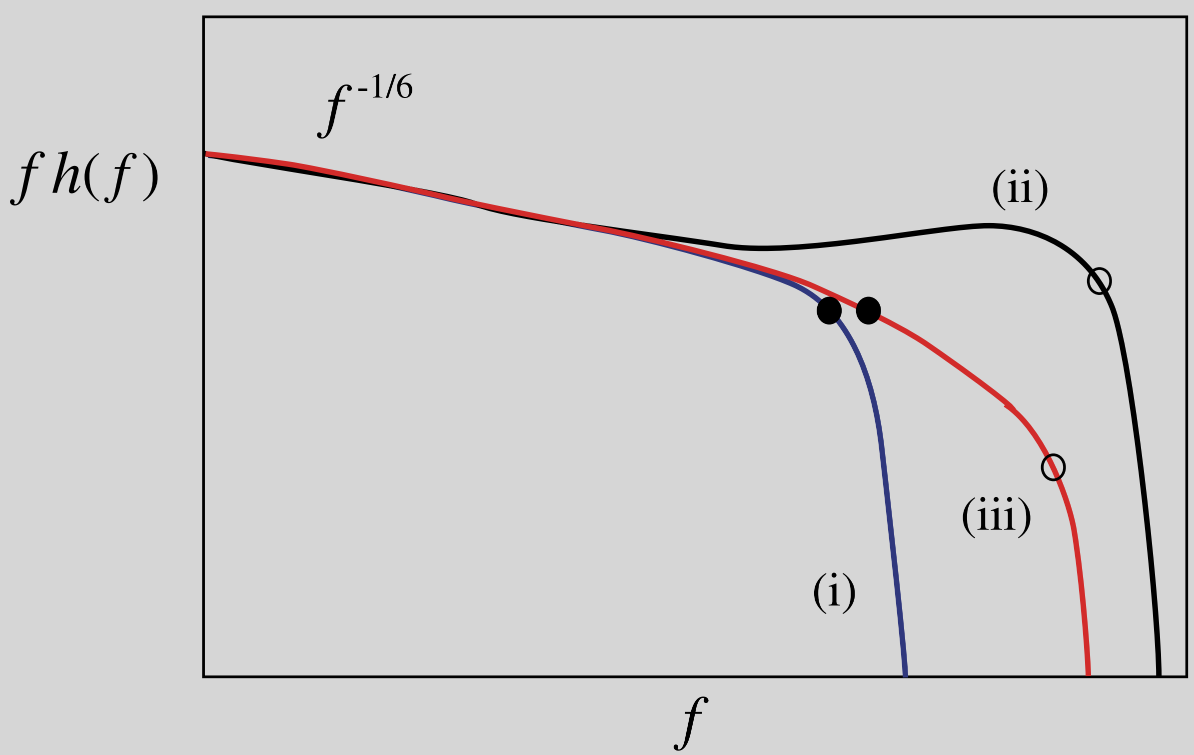
3) The frequency cut-off f_{cut} (only for BH-NS binaries)

In a BH-NS binary coalescence, the NS can be tidally disrupted
before plunging into the BH, and form a disk around it
(which may become the engine for a short gamma-ray burst!).
This occurs only under certain conditions
(Kyutoku et al., PRD '10, '11; Pannarale et al., PRD '11; Shibata et al., PRD '12):
large NS deformability, large BH spin, small ratio m_{NS}/m_{BH}.
In this case, the GW amplitude has a sharp decrease at a cut-off frequency f_{cut}.





(Kyutoku et al., PRD '10, '11)

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- Relation between deformability and radius

Recently discovered I-Love-Q universal relations *(Yagi & Yunes, Science '13, PRD '13)* among momentum of inertia I, deformability λ and rotation quadrupole Q.
A similar universal relation can be found between λ and C *(Maselli et al., PRD '13a)*:

$$C=0.371 - 0.0391 \ln(\lambda/m^5) + 0.001056 (\ln(\lambda /m^5))^2$$

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- Relation between cut-off frequency and radius

The frequency cut-off is a characteristic feature of the waveform which can be in principle measured and used to determine R. This was first proposed in *Vallisneri, PRL '00*. More recently, in *Kyutoku et al., PRD '10* a relation between f_{cut} and C was extracted from fully relativistic simulations:

$$\ln(f_{cut}\, m) = 3.87 \ln C + 4.03$$

Looking to the best strategy to extract information on the NS EoS, we tried to answer two questions *(Maselli et al., PRD '13b)*:

a) Is it possible (and useful) to include both λ and f_{cut} in the data analysis?

b) In the literature, either R or λ is used to discriminate among possible EoSs. Which of these quantities is the most effective?

a) Is it possible (and useful) to include both λ and f_{cut} in the analysis?

We have modified the gravitational template $h_{PN}(x)$, which includes λ, to also include f_{cut}:

$$h(f) = \begin{cases} h_{PN} & f < f_{cut} \\ h_{PN}\Theta(f, f_{cut}) & f_{cut} \leq f \leq 2f_{cut} \\ 0 & f > 2f_{cut} \end{cases}$$

where $h_{PN} = \mathcal{A}e^{\mathrm{i}[\Psi_{PP}+\Psi_T]}$ and $\Theta(f,f_{cut})$ is a function reproducing the sharp decrease in amplitude corresponding to tidal disruption.

We have considered one of the stellar models employed in *Kyutoku et al., PRD '10* (where the relation $f_{cut}(C)$ was derived), H2, the one showing tidal disruption; values of the distance d=100,500,1000,2000 Mpc.

Since the farthest of these systems are at cosmological distance, we also have included the cosmological redshift factor in the template.

a) Is it possible (and useful) to include both λ and f_{cut} in the analysis?

To give a (rough) estimate of the errors, we have computed the Fisher matrix corresponding to the variable set $\theta^a = (t_c, \phi_c, \ln \mathcal{M}, \ln \nu, \tilde{\lambda})$ replacing in the template h(f) the function $f_{cut}(C(\lambda))$ given by the two fits

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d (Mpc)	Δλ/λ (without f_{cut})	Δλ/λ (with f_{cut})
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500	0.067	0.061
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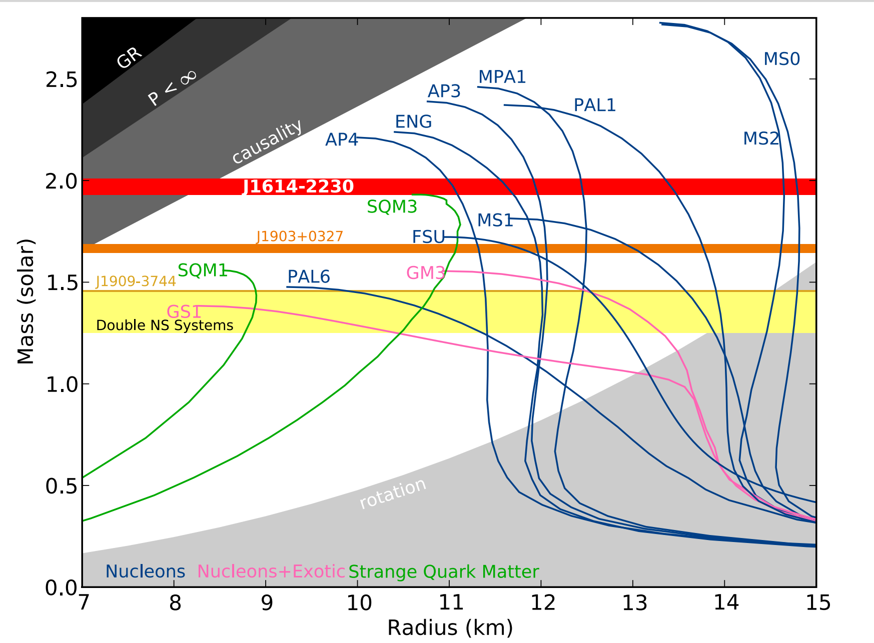
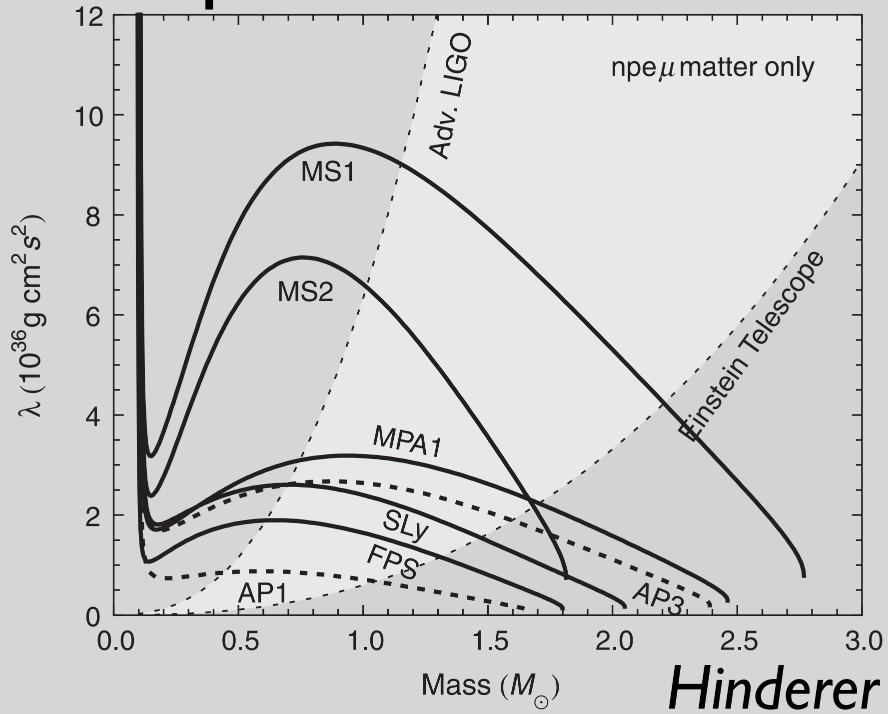
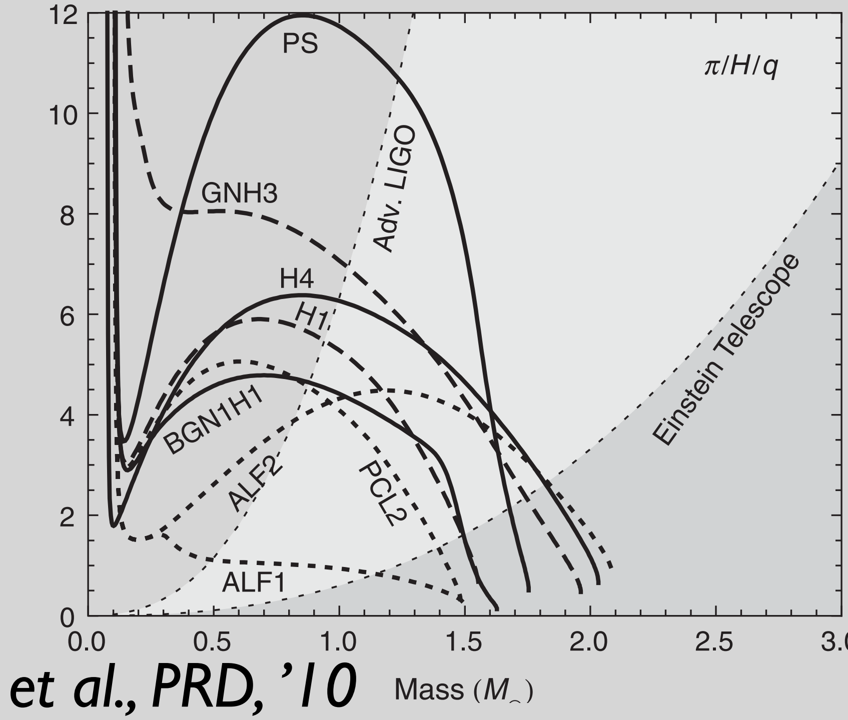
For those sources, it may be useful to include f_{cut} in the analysis.

b) In the literature, either R or λ is used to discriminate among possible EoSs. Which of these quantities is the most effective?



Demorest et al., Nature, '10





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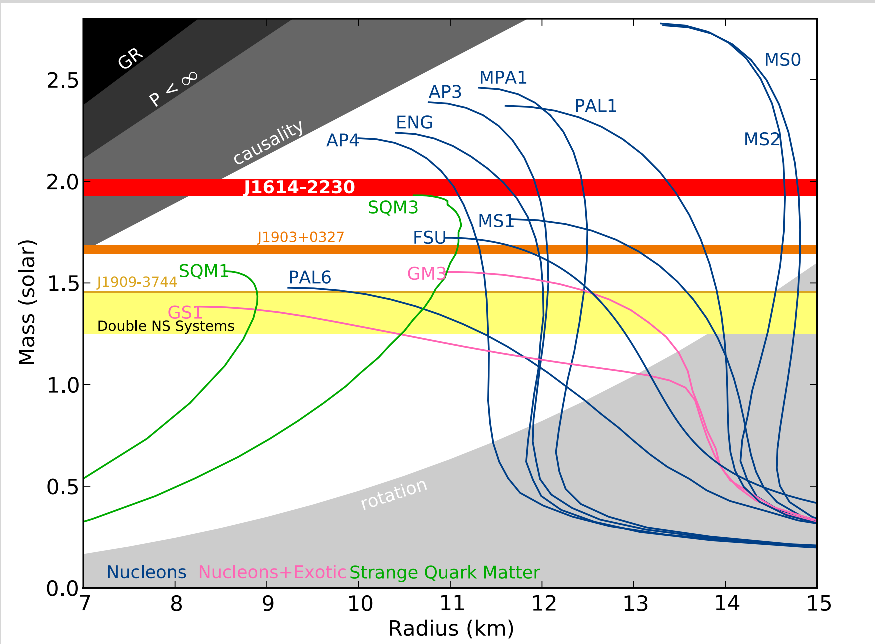

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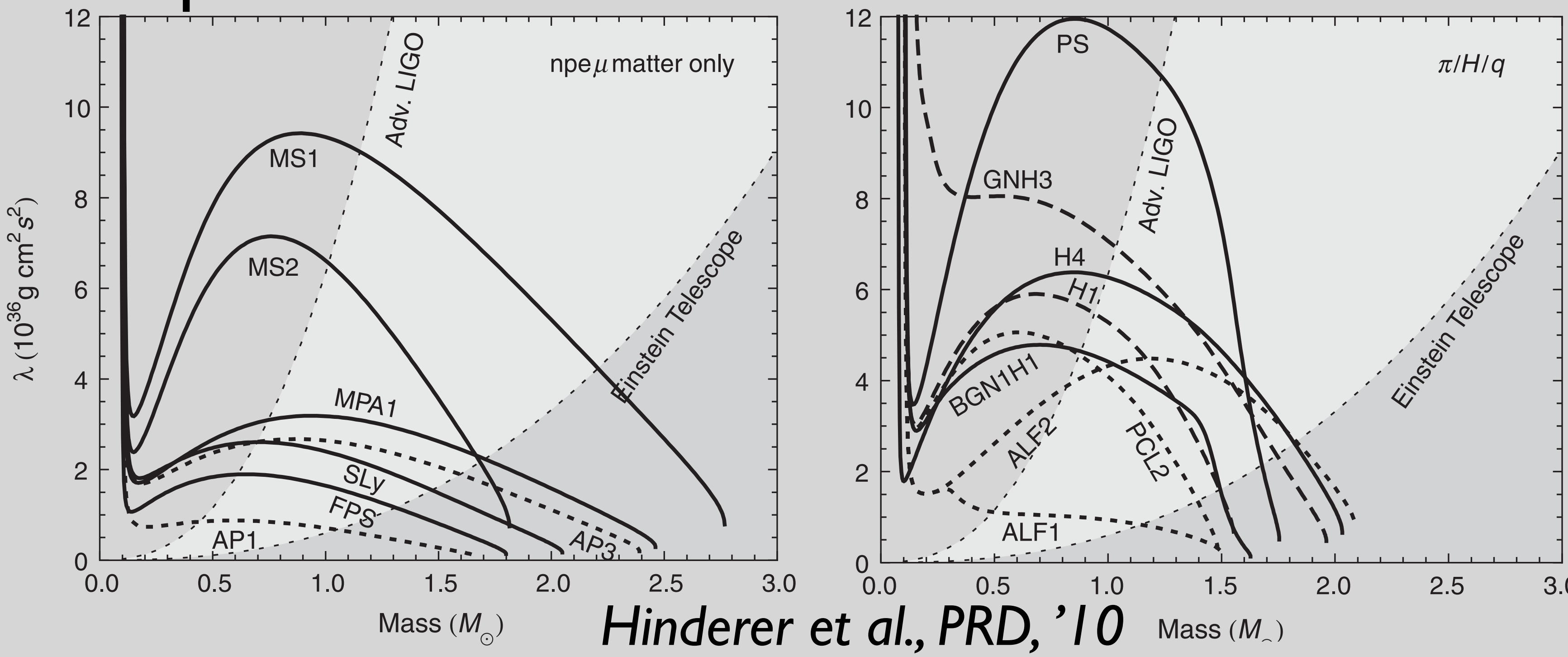

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If we extract this information from the GW waveform, since it depends on λ (and on C=M/R only through the C(λ) relation) we expect the tidal deformability to be most suitable.

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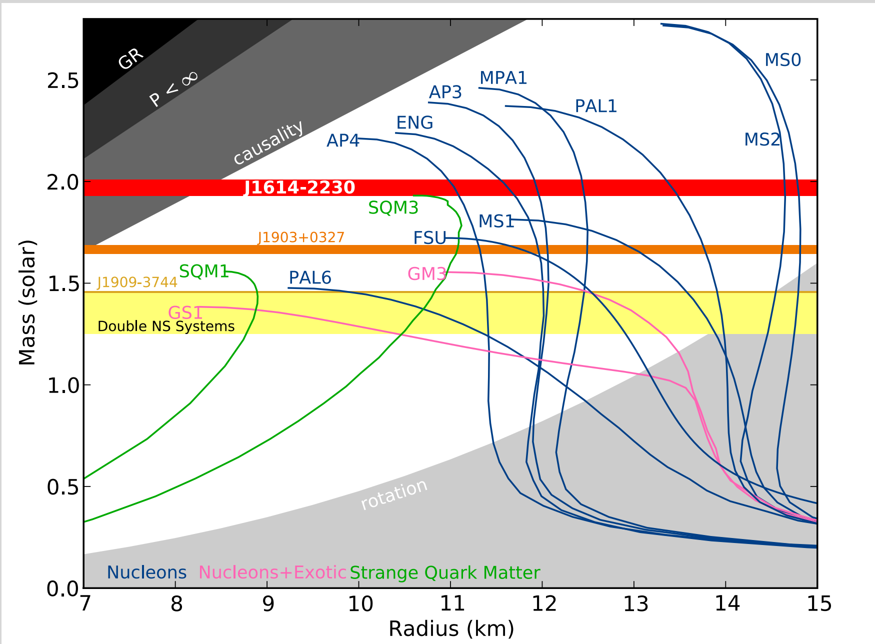

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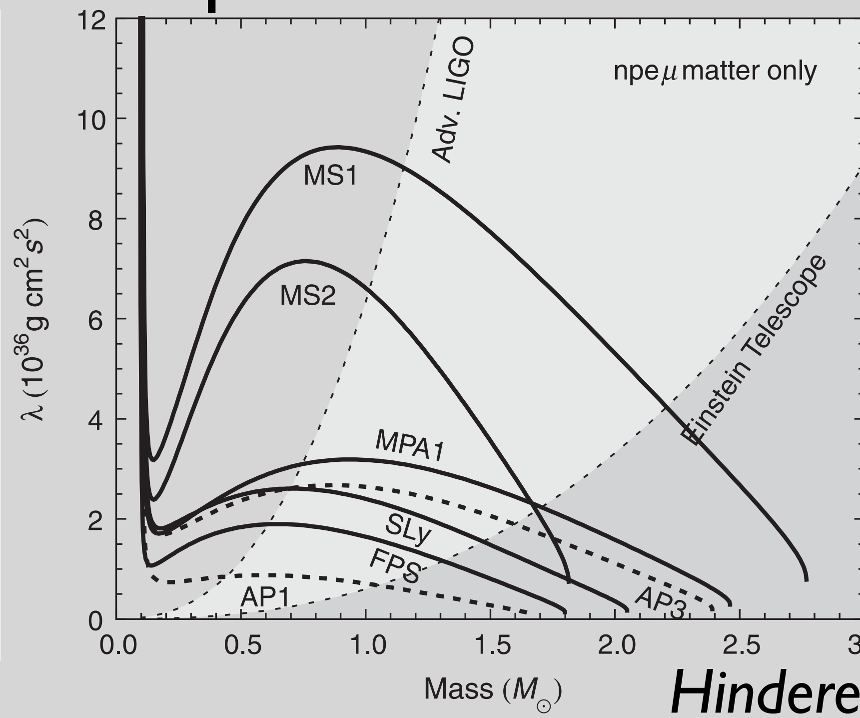

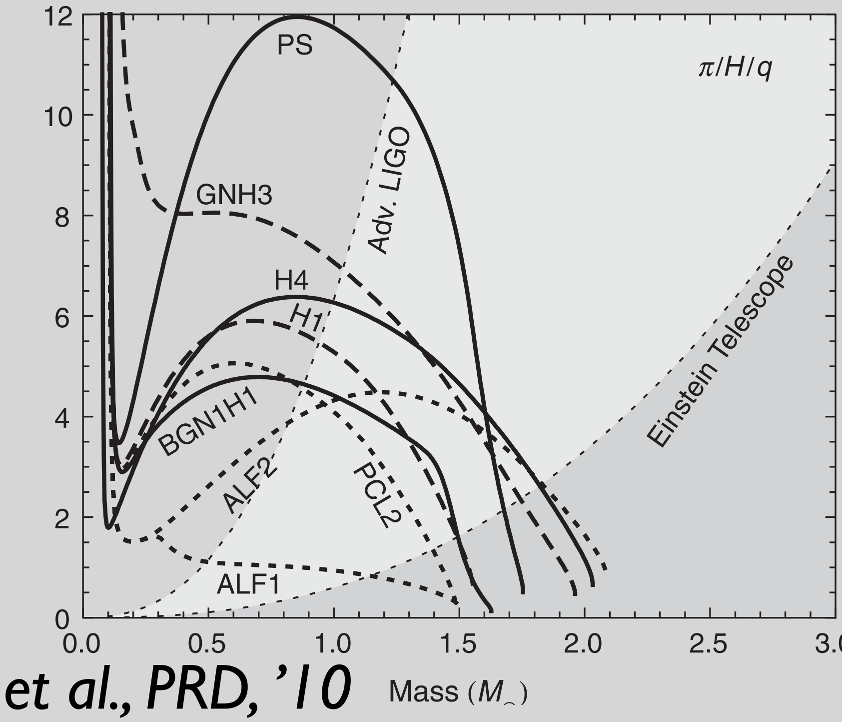

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To be more quantitative, we considered 7 different EoSs (not including f_{cut}), NS-NS binaries and NS-BH binaries with mass ratio 2,4, with various masses, at distance 20 Mpc - 2 Gpc (templates suitably redshifted). We computed the tidal deformability λ for these systems and, using the Fisher matrix approach (for Advanced LIGO/Virgo and for ET) we evaluated the corresponding errors.

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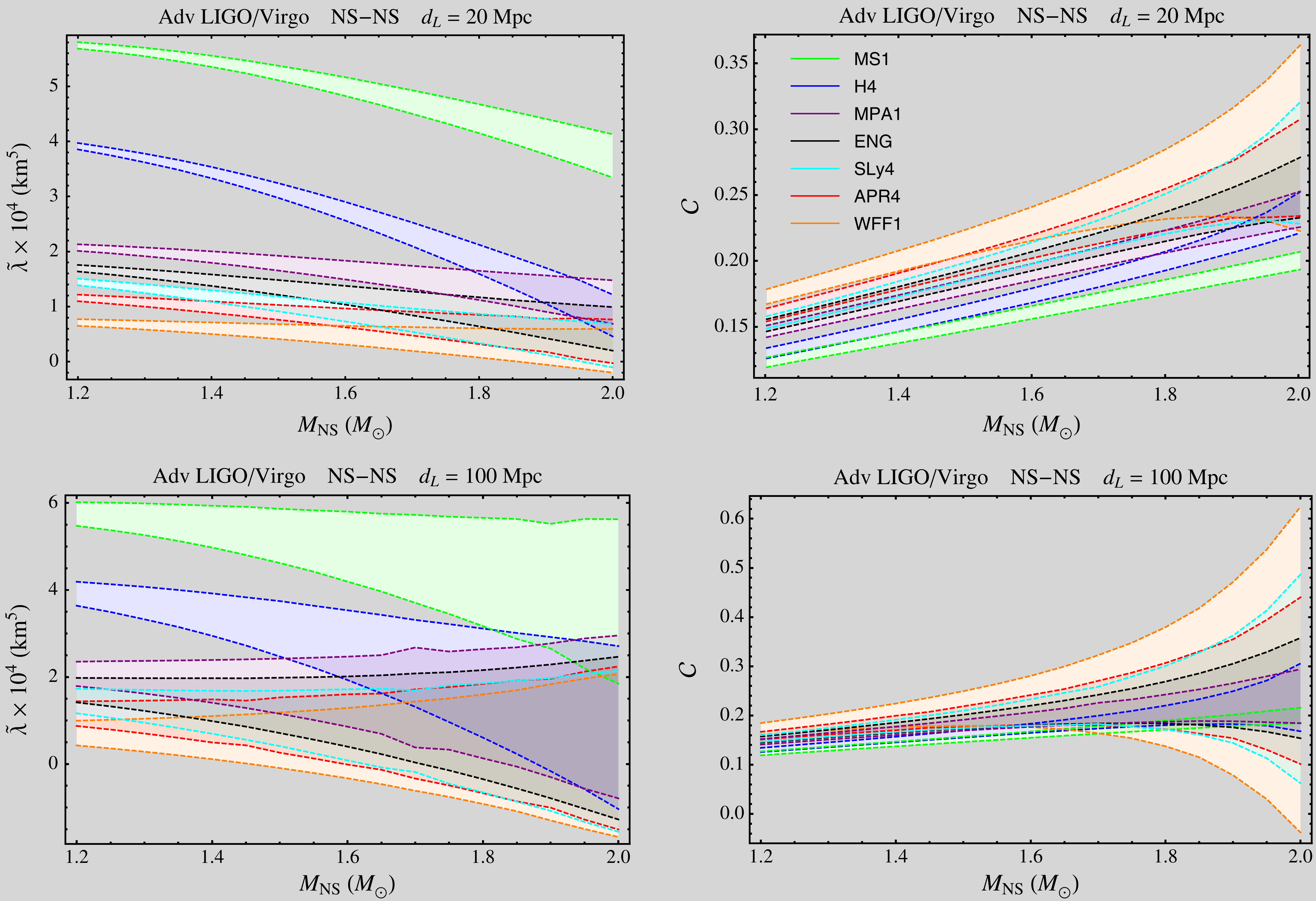

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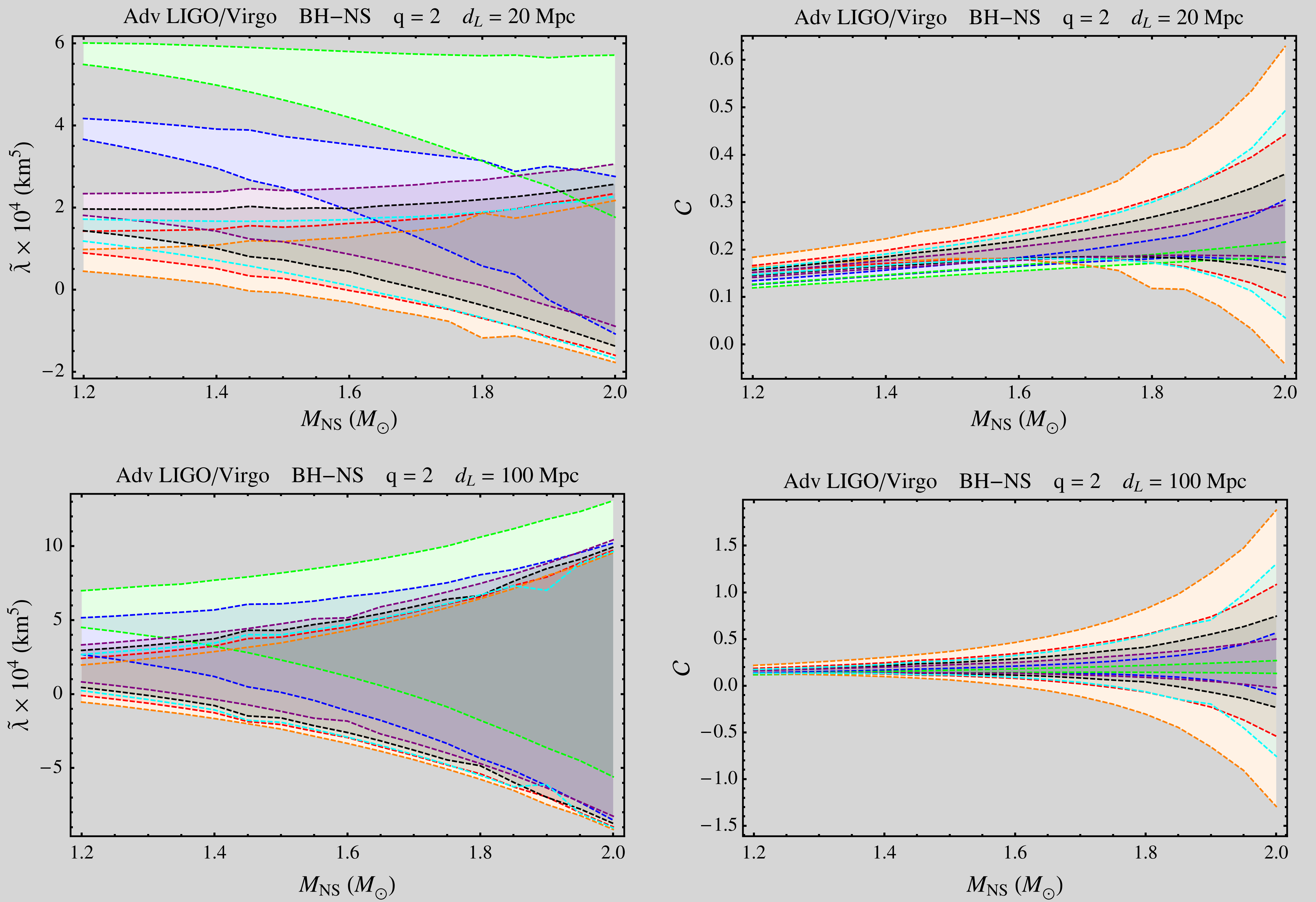

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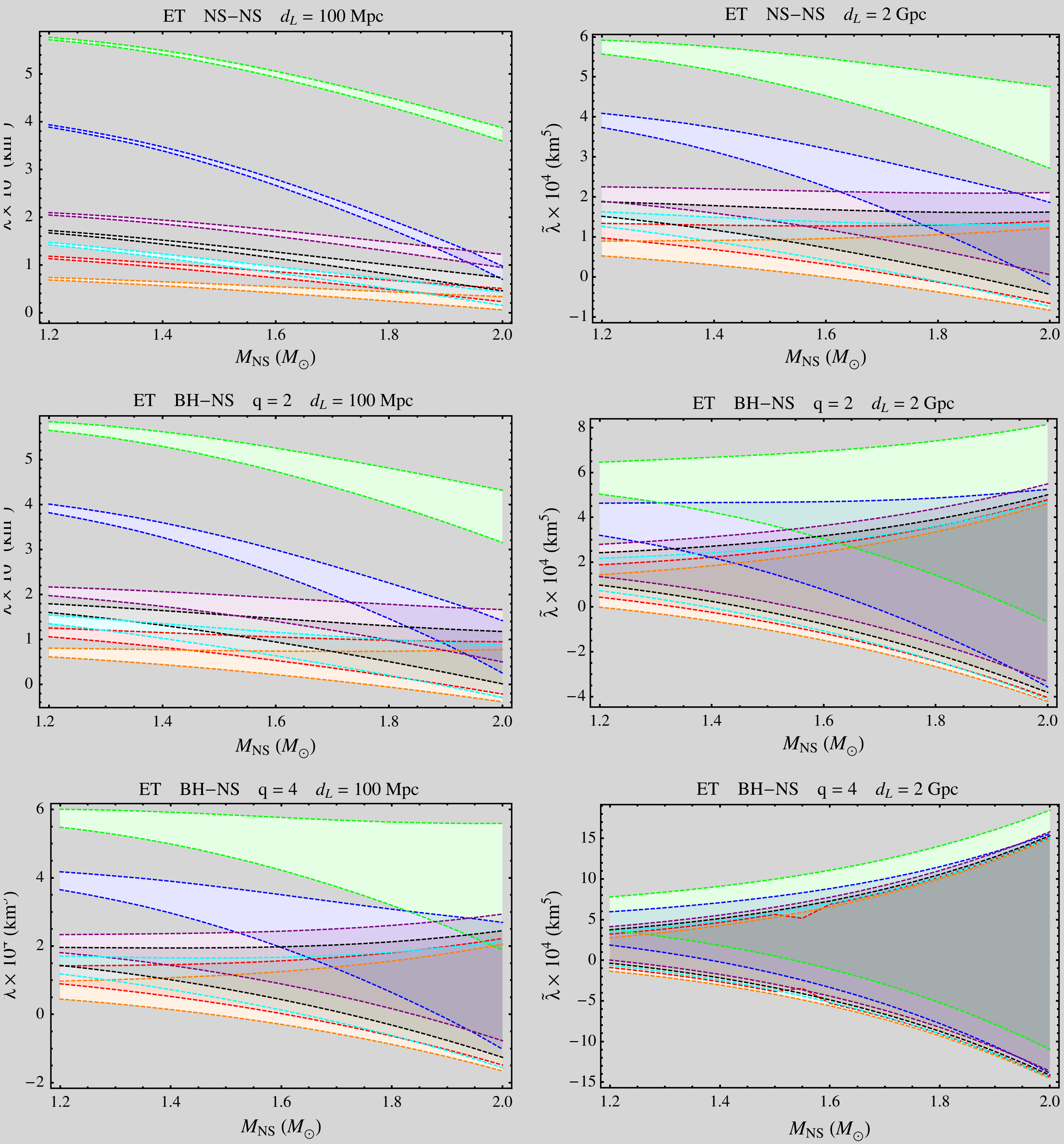

Conclusions

- The gravitational signal from NSs in coalescing binaries, emitted in the latest stages of the inspiral, carries the imprint of the NS EoS.
- The main tool to extract this information is the NS tidal deformability. The frequency cut-off can improve this analysis in particular cases only: for BH-NS binaries, when the NS is disrupted, for systems at cosmological distances (1-2 Gpc) detected by third generation interferometers such as ET.
- In order to discriminate between different possible EoSs using GWs, we should focus on the tidal deformability rather than on the NS radius. This is easier with NS-NS binaries than with BH-NS binaries; and it is easier if the NS mass is smaller. Other promising approach based on the oscillating frequencies of the merger was not discussed here.
- We can expect that second generation detectors will at most tell us whether the EoS is soft or stiff; with third generation detectors, instead, we should be able to exclude most of the EoSs currently considered in the literature.