Andreas Schmitt

Institut für Theoretische Physik
Technische Universität Wien
1040 Vienna, Austria

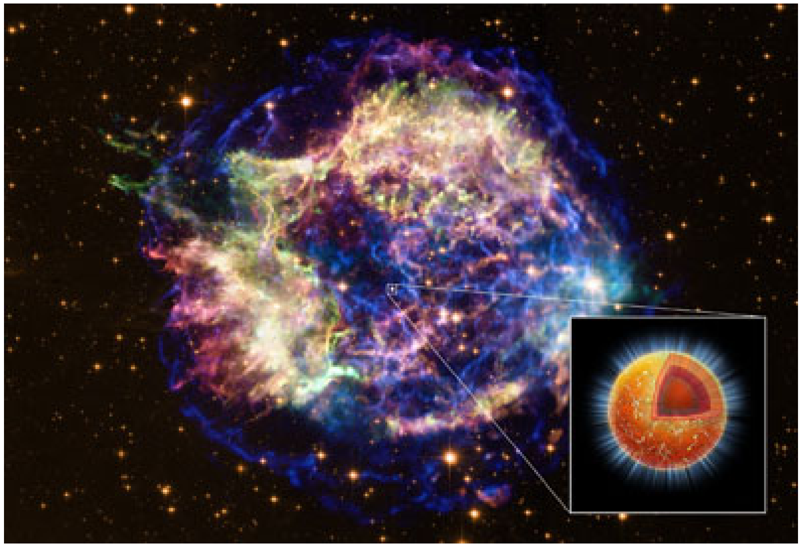
Sound modes and the two-stream instability in relativistic superfluids

M.G. Alford, S.K. Mallavarapu, A. Schmitt, S. Stetina, PRD 87, 065001 (2013)
M.G. Alford, S.K. Mallavarapu, A. Schmitt, S. Stetina, arXiv:1310.5953 [hep-ph]
A. Schmitt, PRD 89, 065024 (2014)

- two-fluid picture of a superfluid
- role reversal in first and second sound
- two-stream instability

• Superfluid hydrodynamics: relevance for compact stars

- r-mode instability
- pulsar glitches
- precession
- asteroseismology
- superfluid turbulence (?)



• Superfluidity in dense matter

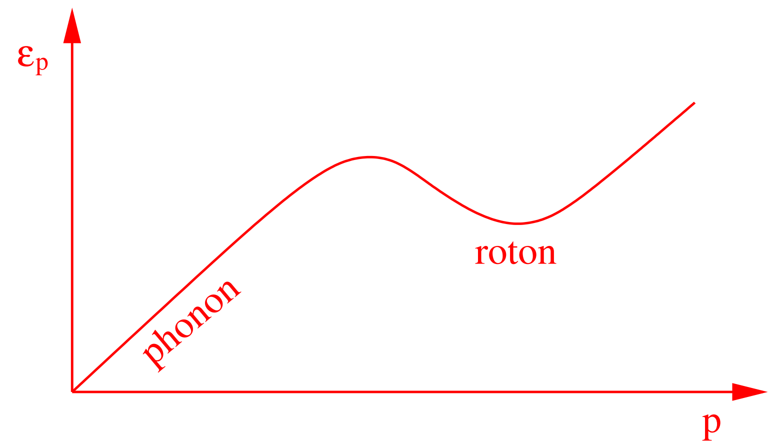
Nuclear matter	Quark matter
neutrons ($T_c \lesssim 10\,\mathrm{keV}$)	color-flavor locked phase ($T_c \sim 10\,\mathrm{MeV}$)
hyperons	color-spin locked phase ($T_c \sim 10\,\mathrm{keV}$)

• Two-fluid picture of a superfluid (liquid helium)

London, Tisza (1938); Landau (1941)

relativistic: Khalatnikov, Lebedev (1982); Carter (1989)

- "superfluid": condensate, carries no entropy
- "normal fluid": excitations (Goldstone mode), carries entropy



Hydrodynamic eqs. ⇒ **two sound modes**

1st sound	**2nd sound**
in-phase oscillation	out-of-phase oscillation
(primarily) density wave	(primarily) entropy wave

• Goals

How does the (covariant) two-fluid picture
arise from a microscopic field theory?

M.G. Alford, S.K. Mallavarapu, A. Schmitt, S. Stetina, PRD 87, 065001 (2013)

this talk:

Compute sound modes in a relativistic superfluid
(and in the presence of a superflow)

M.G. Alford, S.K. Mallavarapu, A. Schmitt, S. Stetina, arXiv:1310.5953 [hep-ph]
A. Schmitt, PRD 89, 065024 (2014)

- **Microscopic calculation (page 1/2)**

- starting point:
  complex scalar field φ

$$\mathcal{L} = \partial_\mu \varphi^* \partial^\mu \varphi - m^2 |\varphi|^2 - \lambda |\varphi|^4$$

- Bose condensate $\langle \varphi \rangle = \rho\, e^{i\psi}$ → superfluid velocity $v^\mu = \dfrac{\partial^\mu \psi}{\mu}$

- effective action density in the 2PI formalism (CJT)

$$\Gamma[\rho, S] = -U(\rho) - \frac{1}{2}\mathrm{Tr}\ln S^{-1} - \frac{1}{2}\mathrm{Tr}[S_0^{-1}(\rho) S - 1] - V_2[\rho, S]$$

- $V_2[\rho, S]$: two-loop two-particle irreducible (2PI) diagrams

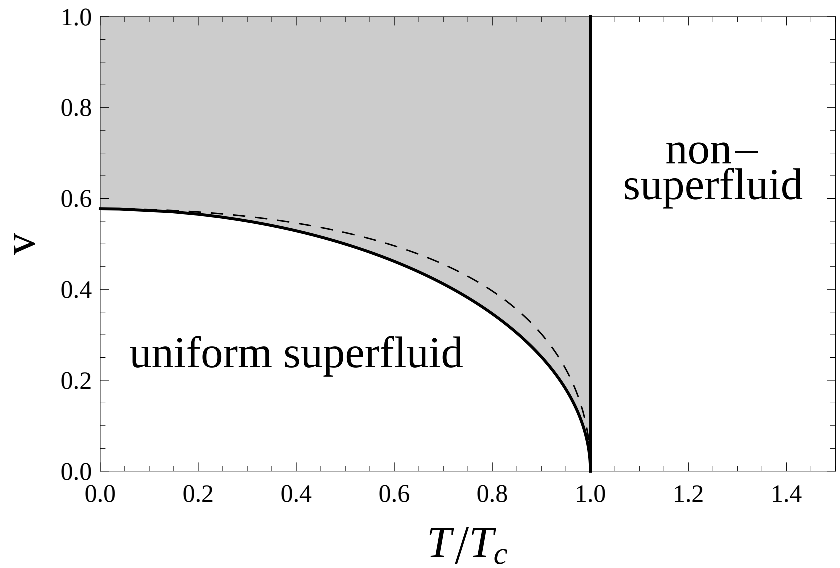
• Microscopic calculation (page 2/2)

- minimize w.r.t. condensate ρ and solve Dyson-Schwinger equation (Hartree approximation; impose Goldstone theorem by hand)
- restrict to weak coupling
  $\rightarrow$ no dependence on renormalization scale
- consider uniform superflow $\mathbf{v}$
- neglect dissipation, compute sound modes in linear regime
  $\rightarrow$ thermodynamics with $(\mu, T, \mathbf{v})$

• Results I: critical velocity

- instability at $v = v_c(T)$:
  negative energies in Goldstone dispersion $\epsilon_{\mathbf{k}}(\mathbf{v}) < 0$

- generalization of Landau's original argument $\epsilon_{\mathbf{k}} - \mathbf{k} \cdot \mathbf{v} < 0$

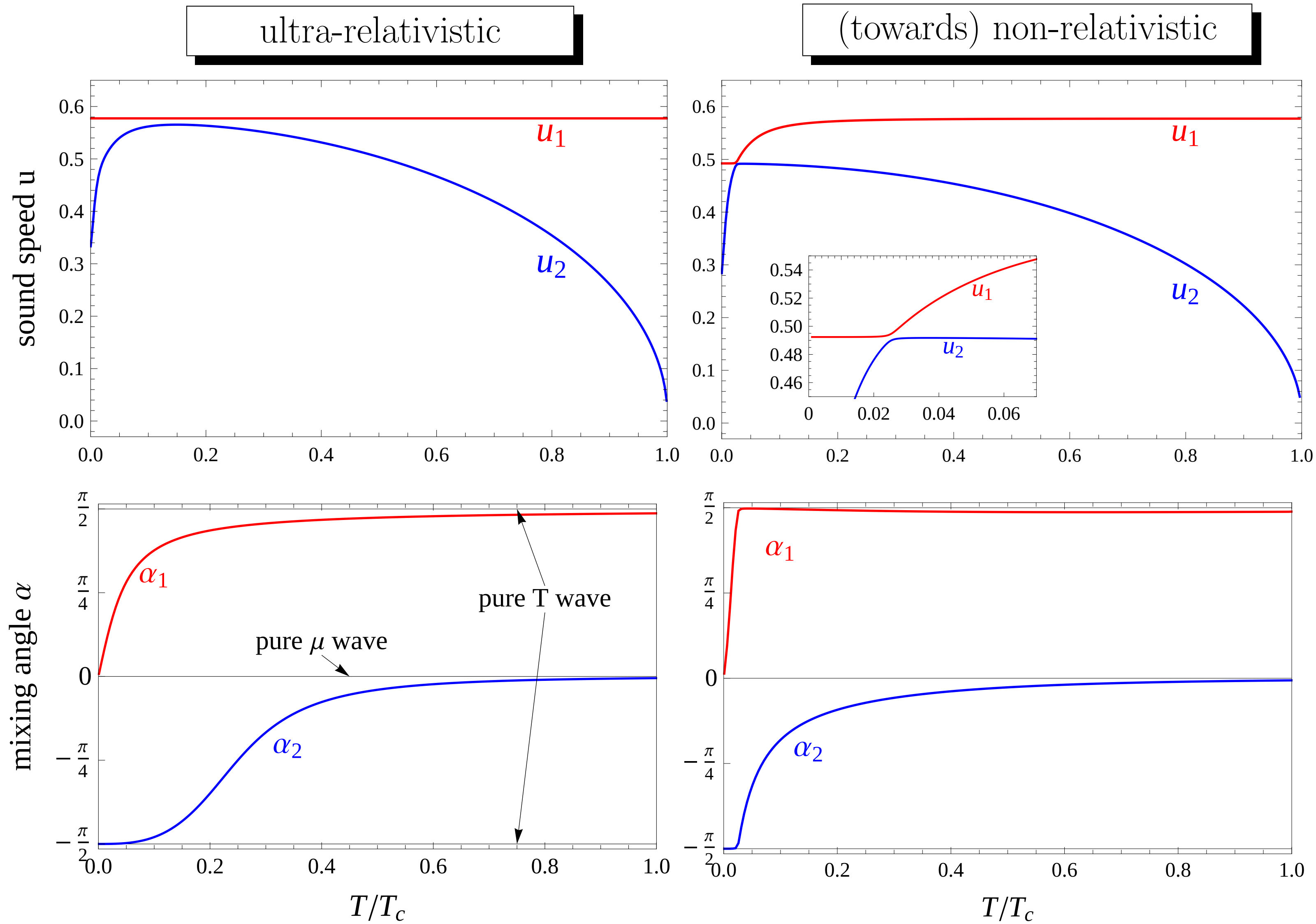


- dashed line: without backreaction of condensate

- shaded region: dissipation, turbulence?

- similar phase diagram for holographic superfluid I. Amado, D. Arean, A. Jimenez-Alba, K. Landsteiner, L. Melgar and I. S. Landea, JHEP 1402, 063 (2014)

• Results II: sound speeds and mixing angle

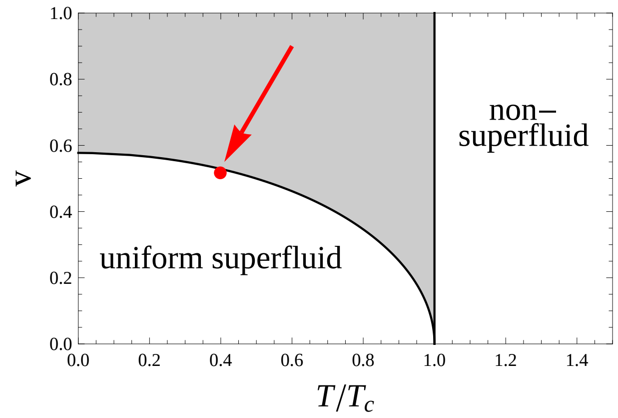


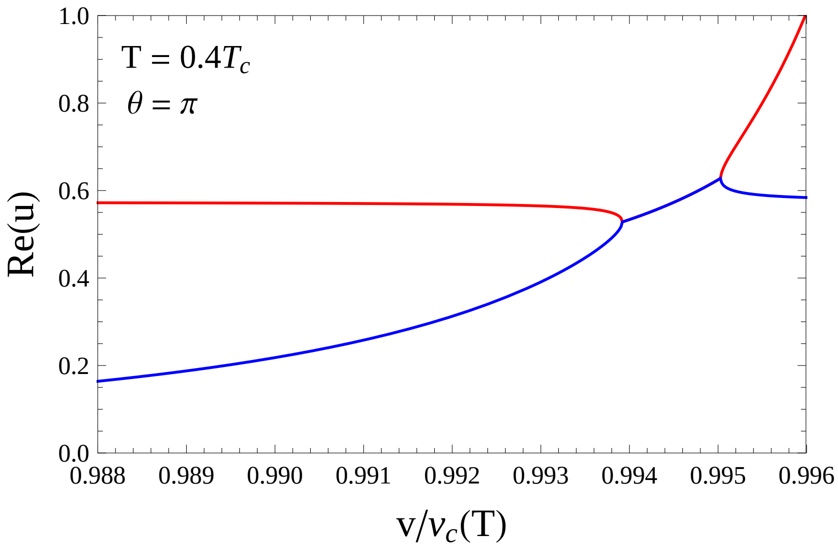
$$\alpha = \arctan \frac{\delta T}{\delta \mu}$$

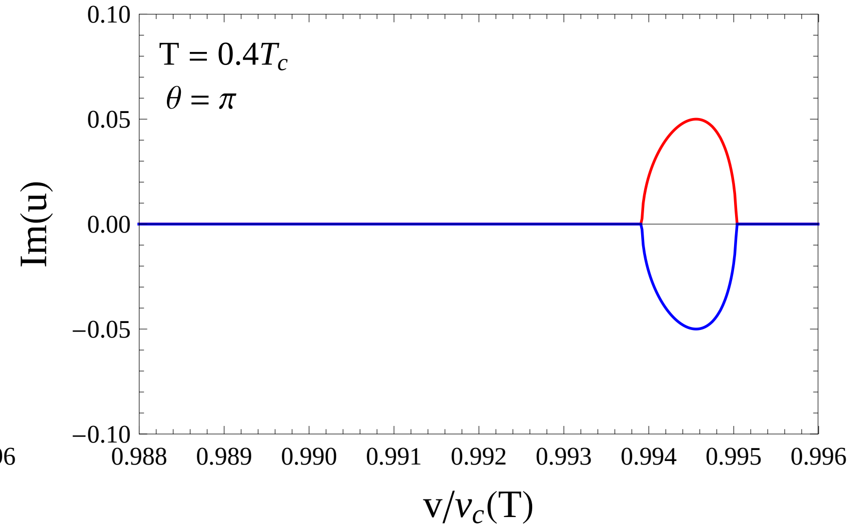
role reversal in first and second sound!

- **Results III: two-stream instability**

  - compute sound speed close to Landau's critical velocity







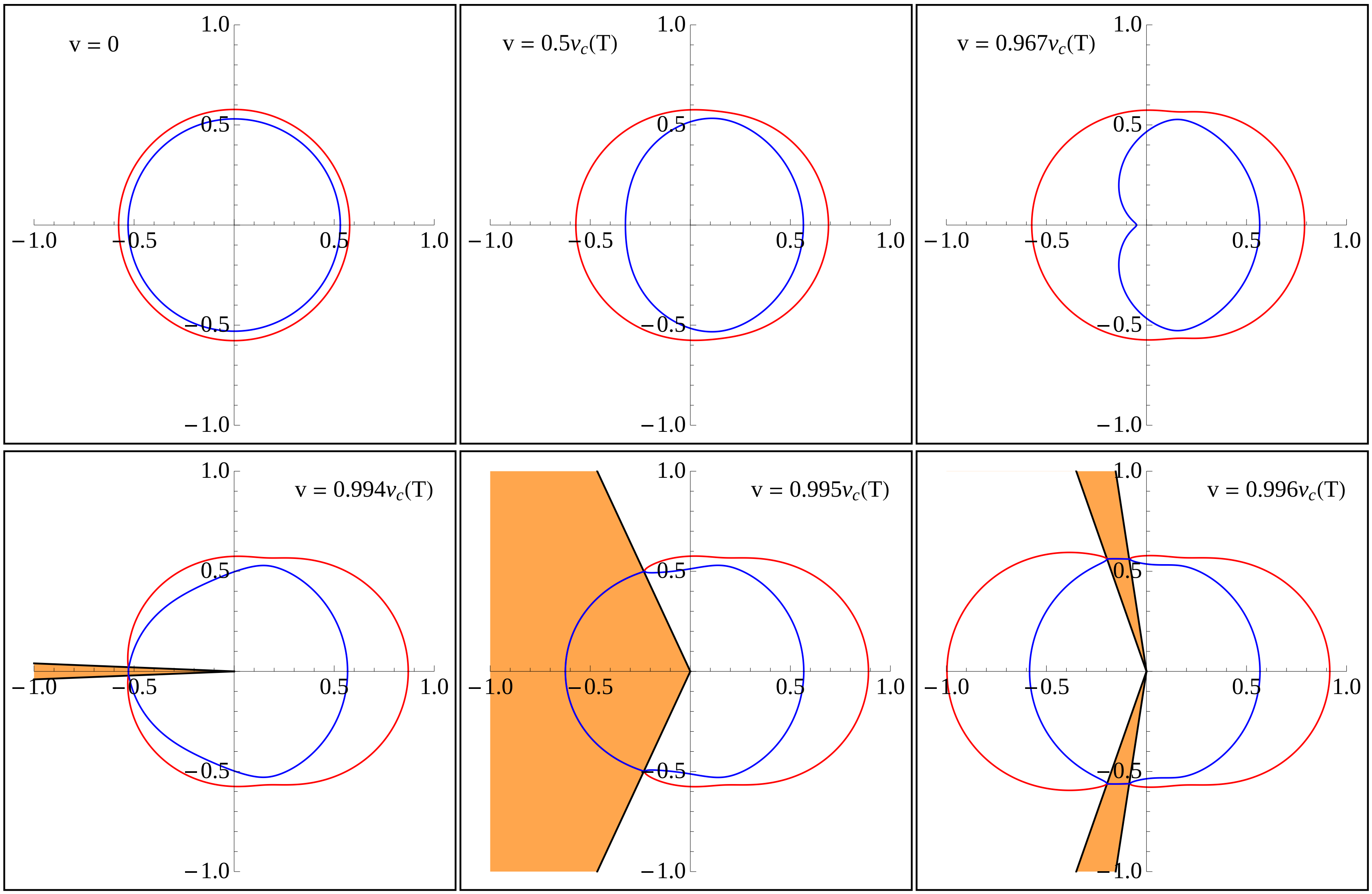
  - complex sound speeds → one mode damped, one mode explodes

    plasma physics: O. Buneman, Phys.Rev. 115, 503 (1959); D.T. Farley, PRL 10, 279 (1963)

    general two-fluid system: L. Samuelsson, C. S. Lopez-Monsalvo, N. Andersson, G. L. Comer, Gen. Rel. Grav. 42, 413 (2010)

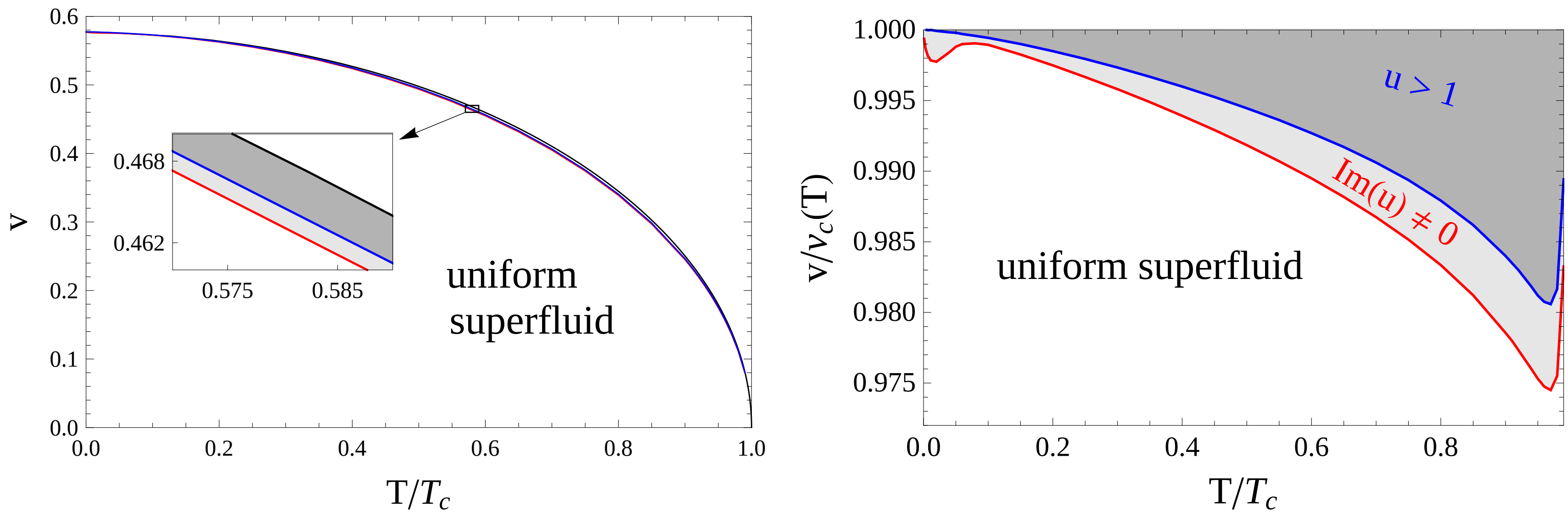
    relevance for superfluids: N. Andersson, G. L. Comer, R. Prix, MNRAS 354, 101 (2004)

• All directions



(superflow pointing to the right)

• Instability window in phase diagram



- tiny window for weak coupling $\lambda = 0.05$
  (varying λ shows that the window grows with λ)
- region with $u > 1$: problem in the formalism?
  (Hartree? enforced Goldstone theorem?)
- very small T: qualitatively different angular structure of instability

• Outlook

- role reversal of sound modes:
  - predictions for ^{4}He or ultracold gases?
  - relevance for compact stars?
- two-stream instability:
  - instability more prominent at strong coupling?
    holographic approach: C.P.Herzog and A.Yarom, PRD 80, 106002 (2009); I.Amado, D.Arean, A.Jimenez-Alba, K.Landsteiner, L.Melgar, I.S.Landea, JHEP 1402, 063 (2014)
  - time evolution of instability
    I. Hawke, G. L. Comer and N. Andersson, Class. Quant. Grav. 30, 145007 (2013)
  - relevance for compact stars, e.g., pulsar glitches
    N. Andersson, G. L. Comer, R. Prix, MNRAS 354, 101 (2004)
- start from fermionic theory
  D. Müller, A. Schmitt, work in progress
- behavior beyond critical velocity