In-medium effects on nuclei embedded in a nucleon gas in the framework of the Extended Thomas-Fermi theory

François Aymard

2$^{\text{nd}}$ year PhD student

PhD Director: Francesca Gulminelli

Laboratoire de Physique Corpusculaire of Caen, France

Collaboration:

A. Raduta (IFIN, Romania), J. Margueron (IPNL), P. Papakonstantinou (KIAS, Korea)

March 26$^{\text{th}}$ 2014

université de Caen Basse-Normandie

cnrs

lpc caen

Région Basse-Normandie

ENSICAEN ÉCOLE NATIONALE SUPÉRIEURE D'INGÉNIEURS DE CAEN & CENTRE DE RECHERCHE

- Stellar matter EoS: wide range of (ρ_b, y_p, T)
  $\Rightarrow$ wide variety of nuclei (A, δ_r) in any nucleon gas (ρ_g,δ_g)
  $\rightarrow$ Self-consistent mean-field theory

- Standard approach for $\rho_b < \rho_0$: Single Nucleus Approximation
  (*J.M. Lattimer and F. Douglas Swesty*, NPA 535, 331 (1991),
  H. Shen et al., NPA, 435 (1998))

- Improvement: extended NSE
  (*Ad. R. Raduta and F. Gulminelli*, PRC 82, 8065801 (2010),
  M. Hempel and J. Schaffner-Bielich, NPA 837, 210-254 (2010))
  + beyond SNA: statistical distribution of nuclei
    - $T = 0$: $\min[E_{WS}(A)/V_{WS}]$
    - Finite temperature: $P(A) \propto \exp\left[\beta(E_{WS}(A) - TS_{WS}(A))/V_{WS}\right]$

  − no in-medium effects: ideal clusters with vacuum energies

$\Rightarrow$ E_{WS} and its in-medium modifications

1. Formalism: modelling the energetics of the Wigner-Seitz cell
2. Results: in-medium modifications
3. Conclusions

1. Formalism: modelling the energetics of the Wigner-Seitz cell
2. Results: in-medium modifications
3. Conclusions

Skyrme energy density

$$\mathcal{H} = \mathcal{E}_{sky} = \mathcal{K} + \mathcal{H}_0 + \mathcal{H}_3 + \mathcal{H}_{eff} + \mathcal{H}_{fin} + \mathcal{H}_{so}$$

$$\mathcal{K} = \frac{\hbar^2}{2m}\tau \qquad \mathcal{H}_{eff} = C_{eff}\rho\tau + D_{eff}\rho_3\tau_3$$

$$\mathcal{H}_0 = C_0\rho^2 + D_0\rho_3^2 \qquad \mathcal{H}_{fin} = C_{fin}(\boldsymbol{\nabla}\rho)^2 + D_{fin}(\boldsymbol{\nabla}\rho_3)^2$$

$$\mathcal{H}_3 = (C_3\rho^2 + D_3\rho_3^2)\rho^\alpha \qquad \mathcal{H}_{so} = C_{so}\mathbf{J}\cdot\boldsymbol{\nabla}\rho + D_{so}\mathbf{J_3}\cdot\boldsymbol{\nabla}\rho_3$$

$$\rho(\mathbf{r}) = \rho_n(\mathbf{r}) + \rho_p(\mathbf{r}) \qquad \rho_3(\mathbf{r}) = \rho_n(\mathbf{r}) - \rho_p(\mathbf{r})$$

$$\tau(\mathbf{r}) = \tau_n(\mathbf{r}) + \tau_p(\mathbf{r}) \qquad \tau_3(\mathbf{r}) = \tau_n(\mathbf{r}) - \tau_p(\mathbf{r})$$

$$\mathbf{J}(\mathbf{r}) = \mathbf{J}_n(\mathbf{r}) + \mathbf{J}_p(\mathbf{r}) \qquad \mathbf{J_3}(\mathbf{r}) = \mathbf{J}_n(\mathbf{r}) - \mathbf{J}_p(\mathbf{r})$$

SLY4: *E. Chabanat et al.*, Nucl. Phys. **A** 635 (1998) 231

Extended Thomas-Fermi model

$$\tau_q(\mathbf{r}) = \frac{3}{5}(3\pi^2)^{2/3}\rho_q(\mathbf{r})^{5/3} + O(\hbar^2) \qquad (LDA)$$

$$\mathbf{J}_{0\boldsymbol{q}}(\mathbf{r}) = \mathbf{0} + O(\hbar^2) \qquad (LDA)$$

M. Brack, C. Guet, H. B. Hakansson, Phys.Rep. 123 (1985) 275

Extended Thomas-Fermi model

$$\tau_q(\mathbf{r}) = \frac{3}{5}(3\pi^2)^{2/3}\rho_q(\mathbf{r})^{5/3} + \frac{1}{36}\frac{(\boldsymbol{\nabla}\rho_q)^2}{\rho_q} + \frac{1}{3}\Delta\rho_q + \frac{1}{6}\frac{\boldsymbol{\nabla}\rho_q\boldsymbol{\nabla}f_q}{f_q} + \frac{1}{6}\rho_q\frac{\Delta f_q}{f_q} - \frac{1}{12}\rho_q\left(\frac{\boldsymbol{\nabla}f_q}{f_q}\right)^2 + \frac{1}{2}\left(\frac{2m}{\hbar^2}\right)^2\rho_q\left(\frac{W_q}{f_q}\right)^2 + O(\hbar^4)$$

$$\mathbf{J}_{0\boldsymbol{q}}(\mathbf{r}) = \mathbf{0} - \frac{2m}{\hbar^2}\frac{\rho_q}{f_q}\mathbf{W}_{\boldsymbol{q}} + \mathbf{O}(\hbar^4)$$

with $f_q(\mathbf{r}) = 1 + \frac{2m}{\hbar^2}\left(C_{eff}\,\rho \pm D_{eff}\,\rho_3\right)$ and $\mathbf{W}_{\boldsymbol{q}}(\mathbf{r}) = C_{so}\boldsymbol{\nabla}\rho \pm D_{so}\boldsymbol{\nabla}\rho_3$

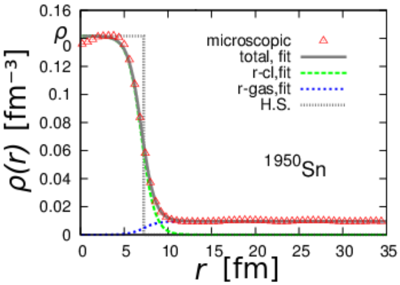
M. Brack, C. Guet, H. B. Hakansson, Phys.Rep. 123 (1985) 275

Analytical model of the optimal density profile

$$\rho_q(r) = \rho_q^{cl}(r) + \rho_q^{gas}(r)$$

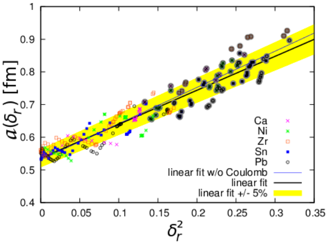
$$= \rho_{0q} F_q(r) + \rho_{gq}\left[1 - F_q(r)\right] \quad \text{with} \quad F_q(r) = \frac{1}{1 + e^{(r-R_q)/a_q}}$$

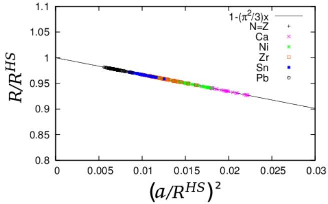
- $\delta_e = \frac{(N_e - Z_e)/A_e + 3Q/(8a_c)\, Z_e^2/A_e^{5/3}}{1 + 9J_0/(4Q)\, A_e^{-1/3}}$
- $\delta_r = (\rho_{0n} - \rho_{0p})/\rho_0$
- $\delta_r = \left(1 - \frac{\rho_g}{\rho_0(\delta_r)}\right)\delta_e + \frac{\rho_g}{\rho_0(\delta_r)}\delta_g$
- $\rho_0(\delta_r) = \rho_0^{\delta_r=0}\left[1 - \frac{3L}{K_\infty + K_{sym}\delta_r^2}\delta_r^2\right]$
- $a_q(\delta_r) = \alpha_q \delta_r^2 + \beta_q$
- $R_q(\delta_r) = R_q^{HS}\left[1 - \pi^2/3\left(a_q/R_q^{HS}\right)^2\right]$
- $R_q^{HS} = \left[3/(4\pi)\, A/\rho_{0q}\right]^{1/3}$

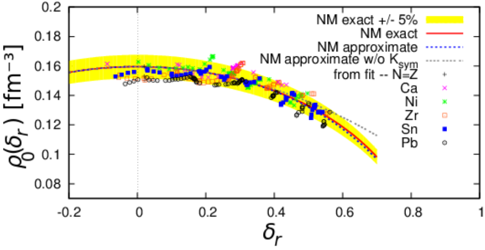


P. Papakonstantinou et al.,
Phys. Rev. **C 88** 045805 (2013)

Validity of the parameters model



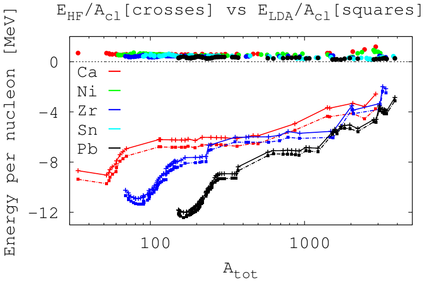


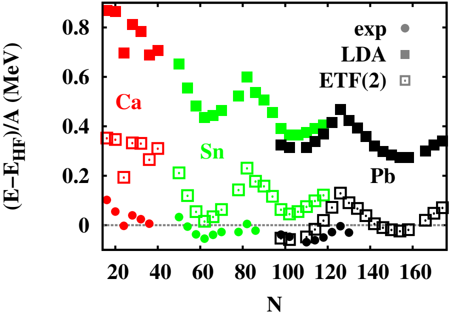


P. Papakonstantinou et al., Phys. Rev. **C 88** 045805 (2013)

Comparison with Hartree-Fock calculations

$$E_{WS}^{ETF} = \int_{V_{WS}} \mathcal{E}_{Sky}^{ETF}(\rho_q, \nabla\rho_q,)d\mathbf{r}$$





F. Aymard,

Proceeding JRJC (2014)

F. Aymard, J. Margueron, F. Gulminelli,

to be submitted

Wigner-Seitz cell energetics

- $$E_{WS}^{ETF} = \int_0^{R_{WS}} \mathcal{E}_{Sky}^{ETF}(\rho_q, \nabla\rho_q,)\mathrm{d}^3r$$
$$= \int_0^{R_{HS}} \mathcal{E}_{Sky}^{ETF}(\rho_q, \nabla\rho_q,)\mathrm{d}^3r + \int_{R_{HS}}^{R_{WS}} \mathcal{E}_{Sky}^{ETF}(\rho_q, \nabla\rho_q,)\mathrm{d}^3r$$
$$= \mathcal{E}_{Sky}^{ETF}(\rho_{0q})V_{HS} + \mathcal{E}_{Sky}^{ETF}(\rho_{gq})(V_{WS} - V_{HS}) + E_{S,m}$$

- $$E_{B,m} = \left[\mathcal{E}_{Sky}^{ETF}(\rho_{0q}) - \mathcal{E}_{Sky}^{ETF}(\rho_{gq})\right]\frac{A}{\rho_0}$$

- $$E_{S,m} = \int_0^{R_{WS}} \mathcal{E}_{Sky}^{ETF}(\rho_q, \nabla\rho_q,)\mathrm{d}r - E_{B,m} - \mathcal{E}_{Sky}^{ETF}(\rho_{gq})V_{WS}$$

Wigner-Seitz cell energetics

- $$E_{WS}^{ETF} = \int_0^{R_{WS}} \mathcal{E}_{Sky}^{ETF}(\rho_q, \nabla\rho_q,)\mathrm{d}^3 r$$
$$= \int_0^{R_{HS}} \mathcal{E}_{Sky}^{ETF}(\rho_q, \nabla\rho_q,)\mathrm{d}^3 r + \int_{R_{HS}}^{R_{WS}} \mathcal{E}_{Sky}^{ETF}(\rho_q, \nabla\rho_q,)\mathrm{d}^3 r$$
$$= \mathcal{E}_{Sky}^{ETF}(\rho_{0q}) V_{HS} + \mathcal{E}_{Sky}^{ETF}(\rho_{gq})(V_{WS} - V_{HS}) + E_{S,m}$$

- $$E_{B,m} = \left[\mathcal{E}_{Sky}^{ETF}(\rho_{0q}) - \mathcal{E}_{Sky}^{ETF}(\rho_{gq})\right] \frac{A}{\rho_0}$$

- $$E_{S,m} = \int_0^{R_{WS}} \mathcal{E}_{Sky}^{ETF}(\rho_q, \nabla\rho_q,)\mathrm{d}r - E_{B,m} - \mathcal{E}_{Sky}^{ETF}(\rho_{gq}) V_{WS}$$

Wigner-Seitz cell energetics

- $$E_{WS}^{ETF} = \int_0^{R_{WS}} \mathcal{E}_{Sky}^{ETF}(\rho_q, \nabla\rho_q,)\mathrm{d}^3r$$
$$= \int_0^{R_{HS}} \mathcal{E}_{Sky}^{ETF}(\rho_q, \nabla\rho_q,)\mathrm{d}^3r + \int_{R_{HS}}^{R_{WS}} \mathcal{E}_{Sky}^{ETF}(\rho_q, \nabla\rho_q,)\mathrm{d}^3r$$
$$= \mathcal{E}_{Sky}^{ETF}(\rho_{0q})V_{HS} + \mathcal{E}_{Sky}^{ETF}(\rho_{gq})(V_{WS} - V_{HS}) + E_{S,m}$$

- $$E_{B,m} = \left[\mathcal{E}_{Sky}^{ETF}(\rho_{0q}) - \mathcal{E}_{Sky}^{ETF}(\rho_{gq})\right] \frac{A}{\rho_0}$$

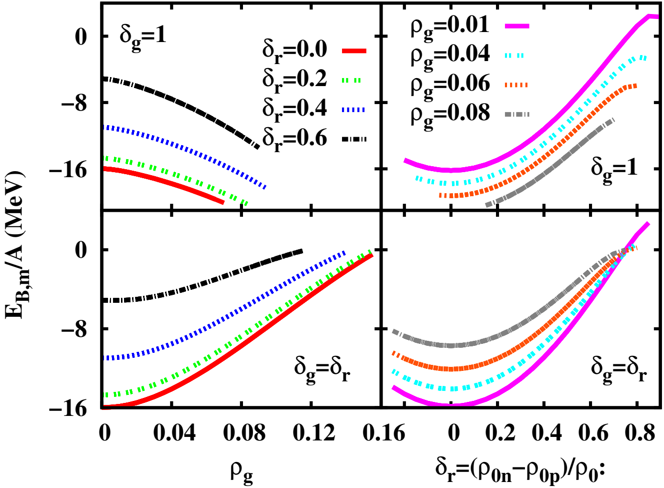
- $$E_{S,m} = \int_0^{R_{WS}} \mathcal{E}_{Sky}^{ETF}(\rho_q, \nabla\rho_q,)\mathrm{d}r - E_{B,m} - \mathcal{E}_{Sky}^{ETF}(\rho_{gq})V_{WS}$$

1 Formalism: modelling the energetics of the Wigner-Seitz cell

2 **Results: in-medium modifications**

3 Conclusions

Bulk binding energy shift



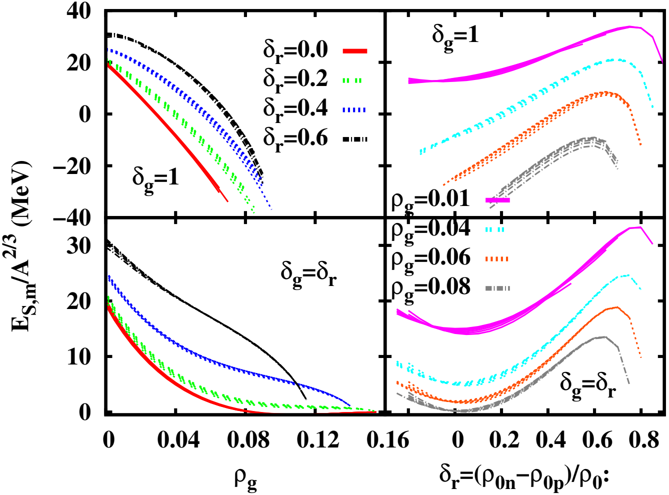
pure neutron gas:

Homogeneous asymmetry

isospin asymmetry in the bulk cluster

F. Aymard, J. Margueron, F. Gulminelli, to be submitted

Modification of the surface energy



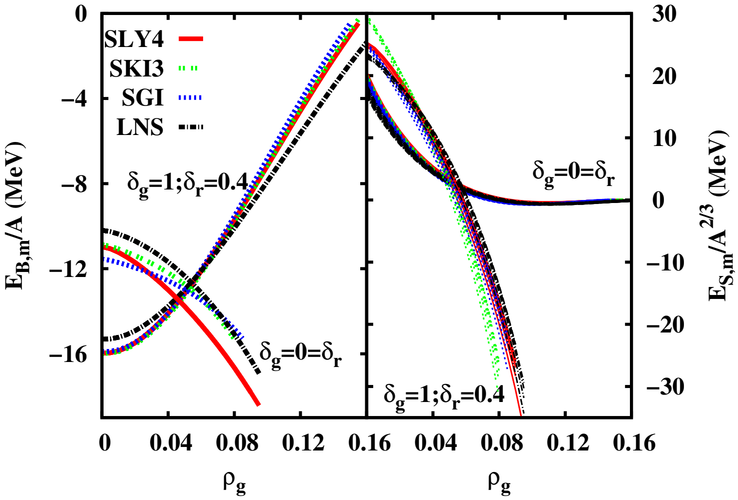
pure neutron gas:

Homogeneous asymmetry

isospin asymmetry in the bulk cluster

F. Aymard, J. Margueron, F. Gulminelli, to be submitted

Dependence on the effective interaction



F. Aymard, J. Margueron, F. Gulminelli, to be submitted

1. Formalism: modelling the energetics of the Wigner-Seitz cell
2. Results: in-medium modifications
3. Conclusions

Summary and perspectives

⇒ In-medium modifications to the cluster energies

- Semi-classical expansion (ETF): quasi-analytical model reproducing HF energy calculations with accuracy $\lesssim 200$ keV$/A$
- Consistent treatment of nuclei and unbound nucleons with a non-artificial excluded volume
- Results:
  - Bulk effect: binding energy shift
  - Surface effect: Interaction at the cluster-gas interface
  - Very different effects depending on the proton fraction
  - Same qualitative behaviour using different Skyrme interactions

⋆ Tabulation of the surface in-medium energies as $\mathcal{T}(A, \delta_r, \rho_g, \delta_g)$

Summary and perspectives

$\Rightarrow$ In-medium modifications to the cluster energies

- Semi-classical expansion (ETF): quasi-analytical model reproducing HF energy calculations with accuracy $\lesssim 200$ keV$/A$
- Consistent treatment of nuclei and unbound nucleons with a non-artificial excluded volume
- Results:
  - Bulk effect: binding energy shift
  - Surface effect: Interaction at the cluster-gas interface
  - Very different effects depending on the proton fraction
  - Same qualitative behaviour using different Skyrme interactions

$\star$ Tabulation of the surface in-medium energies as $\mathcal{T}(A, \delta_r, \rho_g, \delta_g)$