LOCV Calculations for Maximum mass of Neutron stars

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Introduction

-Equation of state

EOS of dense matter

-density and temperature dependence

-Microscopic EOS

BHF, FHNC, …..

Lowest Order Constrained Variational Method

L O C V

1977-79: Symmetrical Nuclear Matter , Reid68 Jop(1979) Bishop,Irvine,Modarres

80's :Asymmetrical Nuclear Matter, Reid68,
Δ -Reid MM jop(1982)
Neutron Matter MM jop(1985) MM Cjop(1989)

90's: UV14,AV14, 3-body cluster energy, T#0
MM PRC(98) MM PRC(97) HRM,MM JOPG(98)24

00's: b-stable matter, He3 ,electron gas T=0, T#0
HRM PRC(2000), HRM,MM EJOPB(2004), MM,HRM EjopB(2003)
Reid93, ELOCV,
MM,HRM PTP (2004) HRM,MM EJOPB(2004) HRM NPA(2008,2009)

10's: finite Nuclei, Lattice, RLOCV
MM NPA(2011,2012) HRM(2010,2013,2014)

Three Body Forces?

THE LOCV FORMALISM

trial wave functions for many-body interacting system

$$\Psi(1....A) = F(1...A)\Phi(1....A)$$

Φ : uncorrelated ground state wave function
F : A-body correlation operator which is assumed to have the Jastrow type

$$F = S\prod_{i \succ j} f(ij)$$

$$f(ij) = \sum_{\alpha, p=1}^{3} f_{\alpha}^{p}(ij;\mathrm{T})O_{\alpha}^{p}(ij)$$

$$\alpha = \{J, L, S, T, T_z\}$$

for uncoupled channels, p=1
for coupled channels, p=2,3

$$O_{\alpha}^{p=1,3} = 1, (\frac{2}{3} + \frac{1}{6} S_{12}), (\frac{1}{3} - \frac{1}{6} S_{12})$$

The general form of AV18 potential

$$V(12) = \sum_{p=1}^{18} V^{p}(r_{12}) O_{12}^{p}$$

cluster expansion for the expectation value of our Hamiltonian

$$E[f] = \frac{1}{A} \frac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle} = E_1 + E_{MB} \cong E_1 + E_2$$

$$E_1 = \sum_{i=n,p} \frac{\hbar^2}{2m\rho\pi^2}\frac{\rho_i}{\rho}\int_0^{\infty} k^4 n_i(k)\,dk$$

$$n_i(k) = \begin{cases} \dfrac{1}{\exp[(\varepsilon_i(k)-\mu)\beta]+1} & T>0 \\ \Theta(k_F - |k|) & T=0 \end{cases}$$

$$\varepsilon_i(k) = \frac{\hbar^2 k^2}{2m^*}$$

$$E_2 = \frac{1}{2A}\sum_{ij}\langle ij|\, W(12)\, |ij - ji\rangle$$

$$W(12) = -\frac{\hbar^2}{2m}[f(12),[\nabla_{12}^2, f(12)]] + f(12)\,V(12)f(12)$$

After some tedious algebra:

$\gamma_{\alpha}^{ij}(r)$ are the matrix elements of different terms of the AV18 potential taken with respect to the plane-waves

$$E_2 = \frac{2}{\pi^4 \rho}\frac{\hbar^2}{2m}\sum_{JLSTT_z}\left|\left\langle \frac{1}{2}\tau_{z1}\frac{1}{2}\tau_{z2}|TT_z\right\rangle\right|^2 (2J+1)\frac{1}{2}\{1-(-1)^{L+S+T}\}$$

$$\times\int_0^{\infty} dr\{[f_{\alpha}^{(1)'^2} a_{\alpha}^{(1)^2}(r\mathrm{T}) + \frac{2m}{\hbar^2}\gamma_{\alpha}^{11}(r) a_{\alpha}^{(1)^2}(r\mathrm{T}) + \gamma_{\alpha}^{12}(r) c_{\alpha}^{(1)^2}(r\mathrm{T}) f_{\alpha}^{(1)^2}]$$

$$+[f_{\alpha}^{(2)'^2} a_{\alpha}^{(2)^2}(r\mathrm{T}) + \frac{2m}{\hbar^2}(\gamma_{\alpha}^{21}(r) a_{\alpha}^{(2)^2}(r\mathrm{T}) + \gamma_{\alpha}^{22}(r) c_{\alpha}^{(2)^2}(r\mathrm{T}) + \gamma_{\alpha}^{23}(r) d_{\alpha}^{(2)^2}(r\mathrm{T})) f_{\alpha}^{(2)^2}]$$

$$+[f_{\alpha}^{(3)'^2} a_{\alpha}^{(3)^2}(r\mathrm{T}) + \frac{2m}{\hbar^2}(\gamma_{\alpha}^{31}(r) a_{\alpha}^{(3)^2}(r\mathrm{T}) + \gamma_{\alpha}^{22}(r) c_{\alpha}^{(3)^2}(r\mathrm{T}) + \gamma_{\alpha}^{23}(r) d_{\alpha}^{(3)^2}(r\mathrm{T})) f_{\alpha}^{(3)^2}]$$

$$+\frac{2m}{\hbar^2}\gamma_{\alpha}^{32}(r) b_{\alpha}^2(r\mathrm{T}) f_{\alpha}^{(3)} f_{\alpha}^{(2)} + \frac{1}{r^2}(f_{\alpha}^{(2)} - f_{\alpha}^{(3)})^2 b_{\alpha}^2(r\mathrm{T})\}$$

The normalization function:

$$\langle \Psi | \Psi \rangle = 1 + \frac{1}{2!}\chi_2 + \frac{1}{3!}\chi_3 + \ldots$$

employed constraint whichintroduces a Lagrange multiplier λ

$$\chi_2 = \frac{1}{A}\sum_{ij} \langle ij \, | h_{T_z}^2(12) - f^{\,2}(12) | \, ij - ji \rangle = 0$$

modified Pauli function

$$h_{T_z}(r) = \begin{cases} [1 - \frac{9}{2}(\frac{J_1(k_i^F r)}{k_i^F r})^2]^{-\frac{1}{2}} & T_z = \pm 1 \\ 1 & T_z = 0 \end{cases}$$

minimizing the two-body cluster energy → one uncoupled and two coupled Euler–Lagrange differential equations

$$g_{\alpha}^{\prime\prime(1)}-[\frac{a_{\alpha}^{\prime\prime(1)}}{a_{\alpha}^{(1)}}+\frac{m}{\hbar^2}(\gamma_{\alpha}^{11}(r)+\lambda)+\frac{m}{\hbar^2}\gamma_{\alpha}^{12}(r)\frac{c_{\alpha}^{(1)2}}{a_{\alpha}^{(1)}}]g_{\alpha}^{(1)}=0$$

$$g_{\alpha}^{(k)}(r)=a_{\alpha}^{(k)}(r)f_{\alpha}^{(k)}(r)$$

$$g_{\alpha}^{\prime\prime(2)}-[\frac{a_{\alpha}^{\prime\prime(2)}}{a_{\alpha}^{(2)}}+\frac{m}{\hbar^2}(\gamma_{\alpha}^{21}(r)+\lambda)+\frac{m}{\hbar^2}\gamma_{\alpha}^{22}(r)\frac{c_{\alpha}^{(2)2}}{a_{\alpha}^{(2)2}}+\frac{m}{\hbar^2}\gamma_{\alpha}^{23}(r)\frac{d_{\alpha}^{(2)2}}{a_{\alpha}^{(2)2}}+\frac{b_{\alpha}^{2}}{r^2a_{\alpha}^{(2)2}}]g_{\alpha}^{(2)}$$
$$+[\frac{1}{r^2}-\frac{m}{2\hbar^2}\gamma_{\alpha}^{32}(r)]\frac{b_{\alpha}^{2}}{a_{\alpha}^{(2)}a_{\alpha}^{(3)}}g_{\alpha}^{(3)}=0$$

$$g_{\alpha}^{\prime\prime(3)}-[\frac{a_{\alpha}^{\prime\prime(3)}}{a_{\alpha}^{(3)}}+\frac{m}{\hbar^2}(\gamma_{\alpha}^{31}(r)+\lambda)+\frac{m}{\hbar^2}\gamma_{\alpha}^{22}(r)\frac{c_{\alpha}^{(3)2}}{a_{\alpha}^{(3)2}}+\frac{m}{\hbar^2}\gamma_{\alpha}^{23}(r)\frac{d_{\alpha}^{(3)2}}{a_{\alpha}^{(3)2}}+\frac{b_{\alpha}^{2}}{r^2a_{\alpha}^{(2)2}}]g_{\alpha}^{(3)}$$
$$+[\frac{1}{r^2}-\frac{m}{2\hbar^2}\gamma_{\alpha}^{32}(r)]\frac{b_{\alpha}^{2}}{a_{\alpha}^{(2)}a_{\alpha}^{(3)}}g_{\alpha}^{(2)}=0$$

THREE BODY FORCE

three-body forces are included in our nuclear Hamiltonian for the purpose of reproducing the empirical saturation properties of cold asymmetric nuclear matter

the Urbana interaction

$$V_{ijk} = V_{ijk}^{2\pi} + V_{ijk}^{R}$$

two-body contribution

$$V_{ijk}^{2\pi} = A\sum_{cyc}\left(\{X_{ij}, X_{jk}\}\{\tau_i.\tau_j, \tau_j.\tau_k\} + \frac{1}{4}[X_{ij}, X_{jk}][\tau_i.\tau_j, \tau_j.\tau_k]\right)$$

The repulsive part

$$V_{ijk}^{R} = U\sum_{cyc} T(r_{ij})^2 T(r_{jk})^2$$

Y(r) and T(r) are the Yukawa and tensor functions associated with the one-pion exchange interaction

one-pion exchange operator

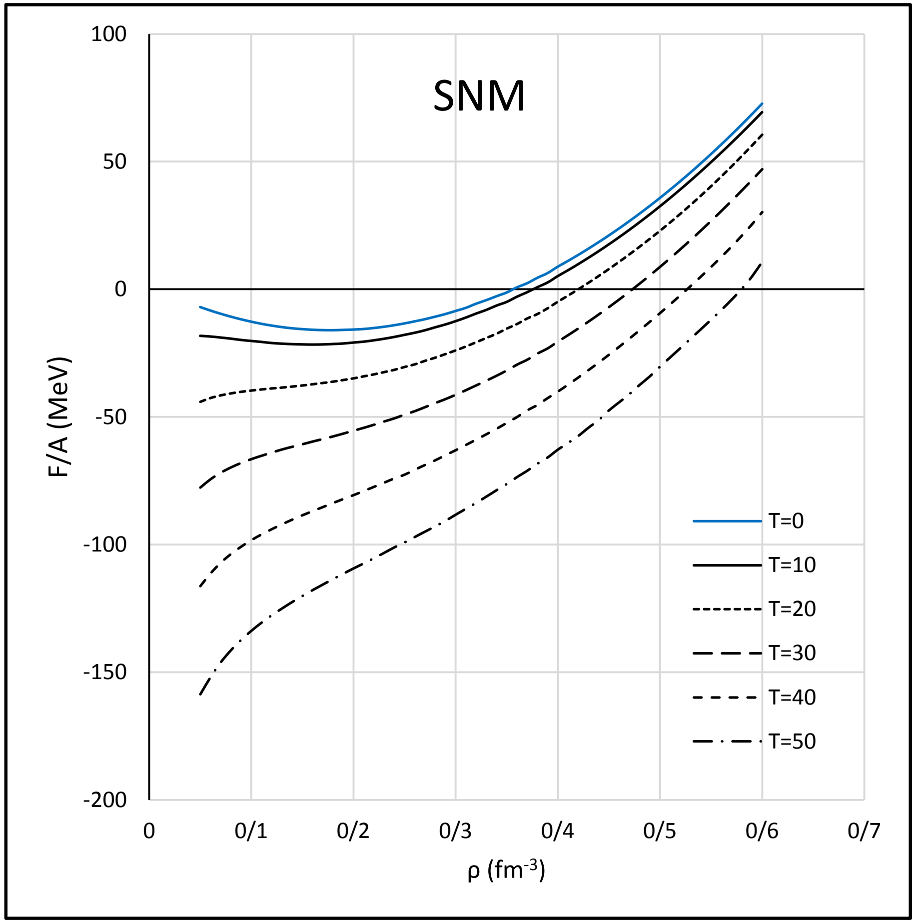
$$X_{ij} = Y(r_{ij})\sigma_i.\sigma_j + T(r_{ij})S_{ij}$$

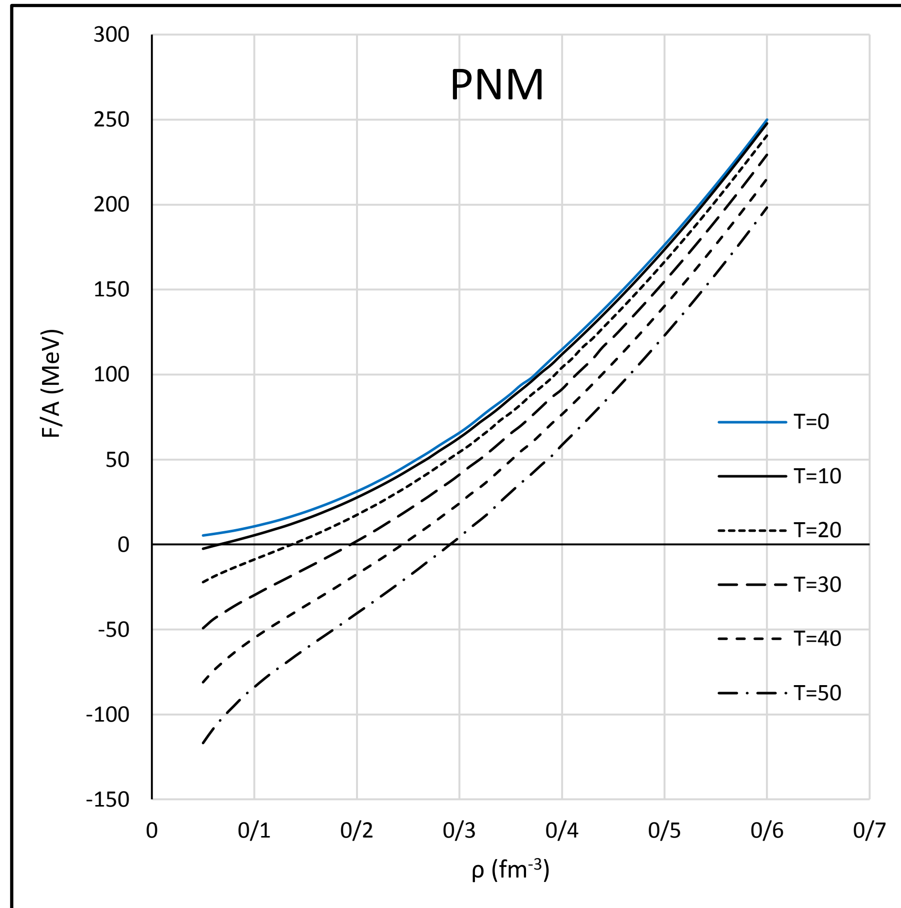
The strengths A and U are fitted in order to reproduce the empirical nuclear matter properties → A=1.21 and U=1.91

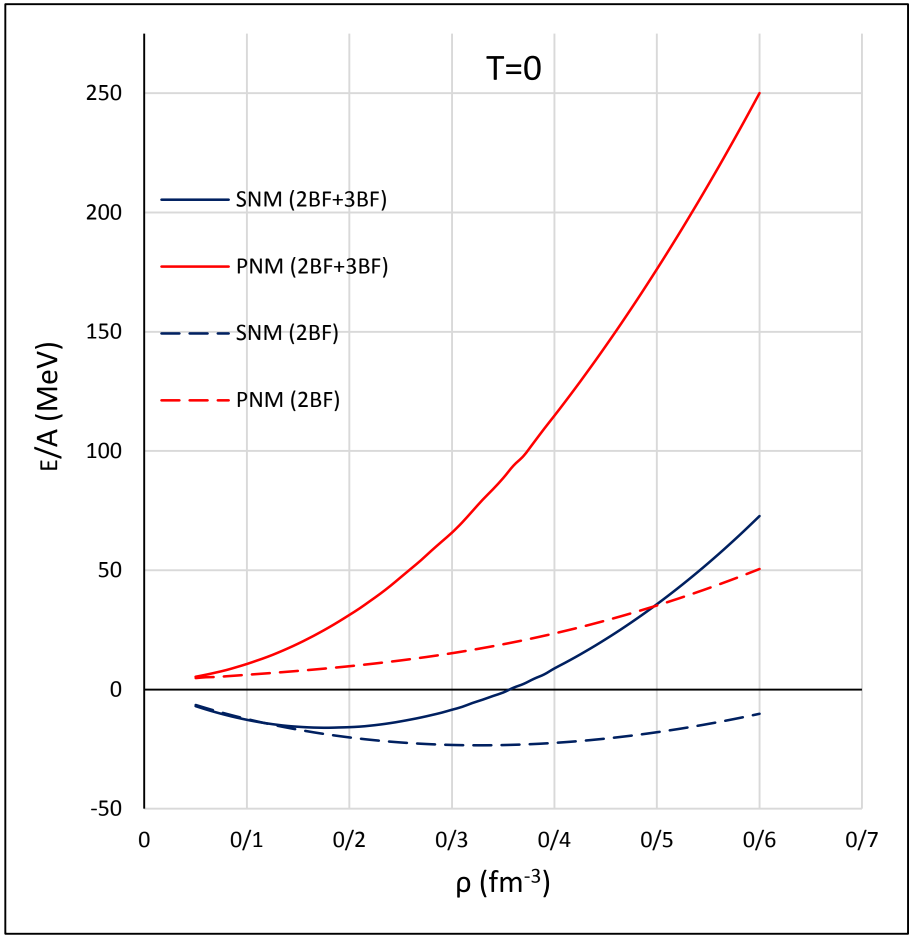
It may be more convenient to reduce the three-body interaction to an effective two-body one, by an averaging over the third particle coordinates . The final form for the attractive part can be written as

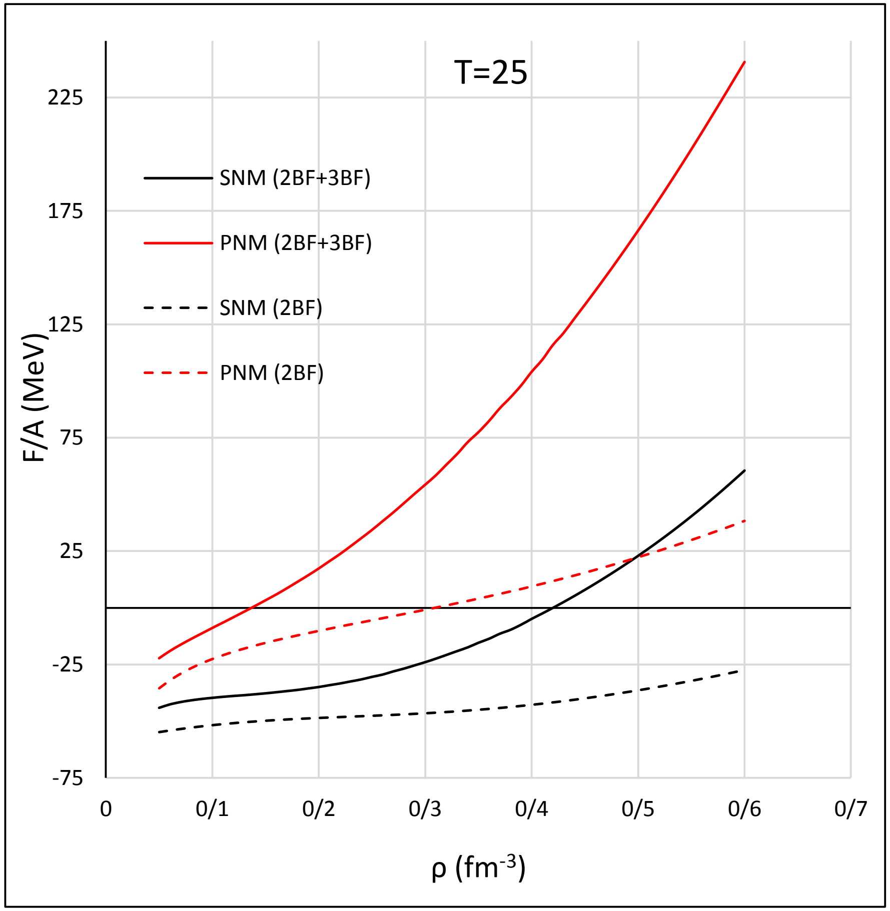
$$\upsilon_3^{eff} = \upsilon_S(r_{ij})(\sigma_i.\sigma_j) + \upsilon_T(r_{ij})S_{ij}$$

This effective force is added to the nuclear Hamiltonian and the calculation is proceed along the same ordinary LOCV method with only two-body forces.


SNM
100
50
0
-50
-100
-150
-200
F/A (MeV)
0
0/1
0/2
0/3
0/4
0/5
0/6
0/7
ρ (fm-3)
T=0
T=10
T=20
T=30
T=40
T=50


PNM
300
250
200
150
100
50
0
-50
-100
-150
F/A (MeV)
0
0/1
0/2
0/3
0/4
0/5
0/6
0/7
ρ (fm-3)
T=0
T=10
T=20
T=30
T=40
T=50

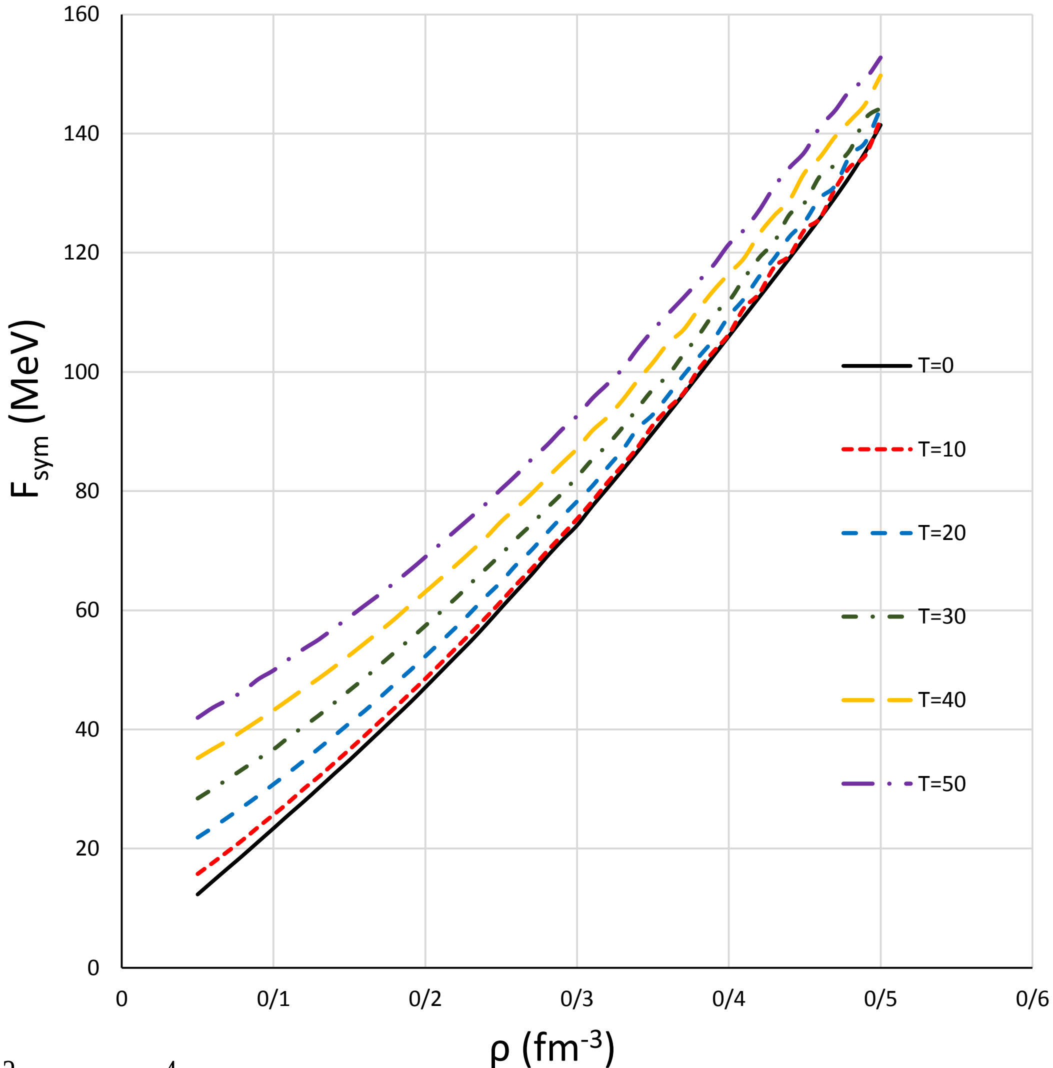

T=0
SNM (2BF+3BF)
PNM (2BF+3BF)
SNM (2BF)
PNM (2BF)
E/A (MeV)
250
200
150
100
50
0
-50
0
0/1
0/2
0/3
0/4
0/5
0/6
0/7
ρ (fm-3)


T=25
SNM (2BF+3BF)
PNM (2BF+3BF)
SNM (2BF)
PNM (2BF)
F/A (MeV)
225
175
125
75
25
-25
-75
0
0/1
0/2
0/3
0/4
0/5
0/6
0/7
ρ (fm-3)

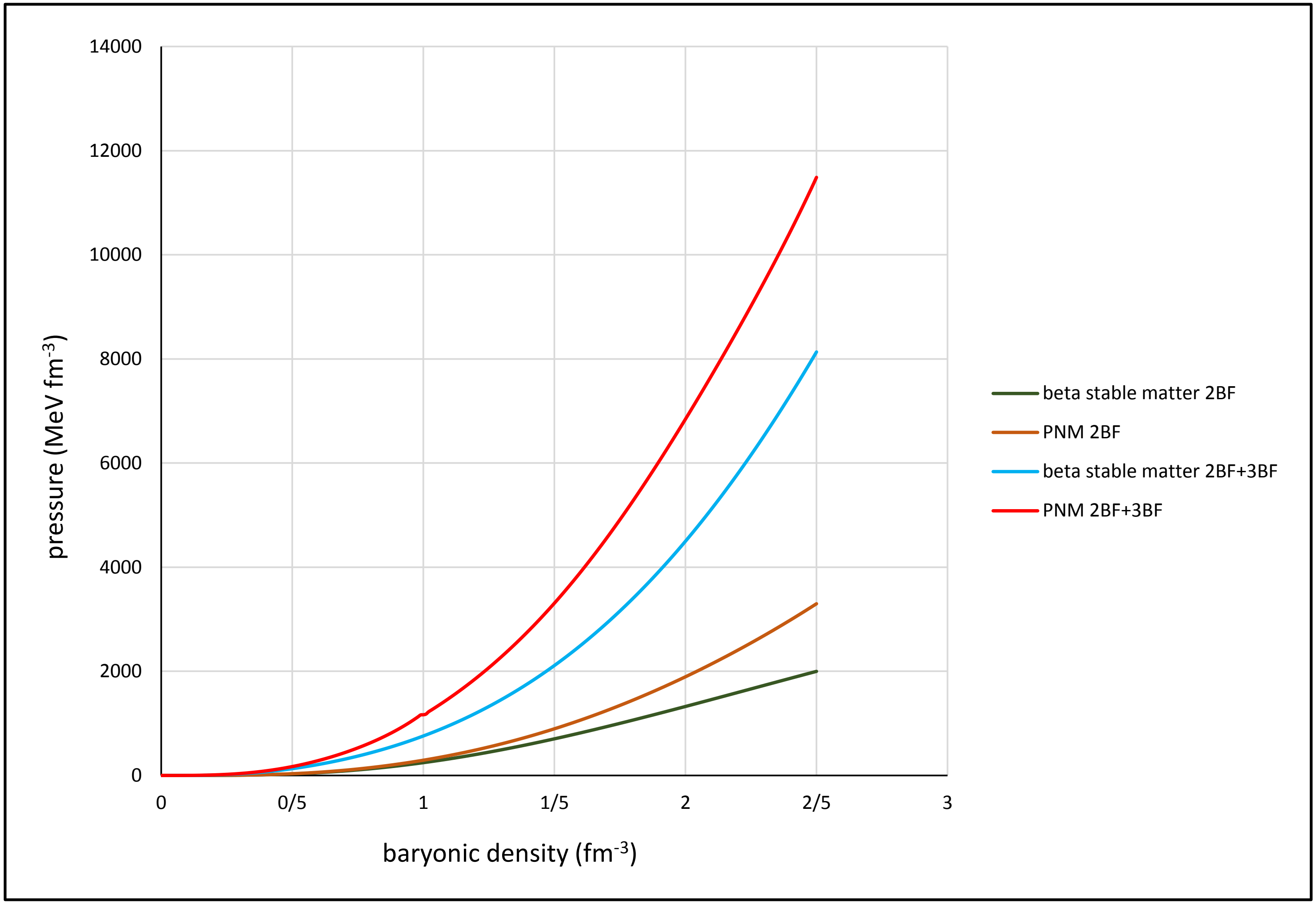
THE NUCLEAR SYMMETRY ENERGY

The Helmholtz free energy per nucleon of asymmetric nuclear matter at given density and temperature can be expanded to second order in isospin asymmetry parameter X as

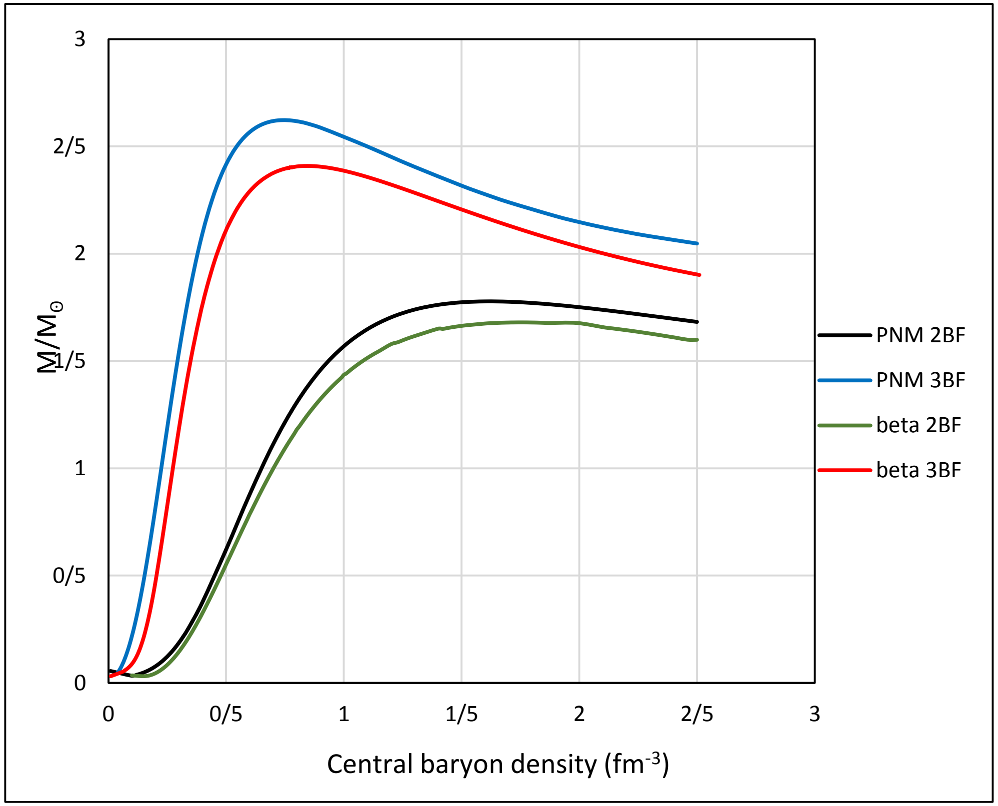
$$X = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$



$$F(\rho, X, T) = F_0(\rho, T) + F_{sym}(\rho, T) X^2 + O(X^4)$$

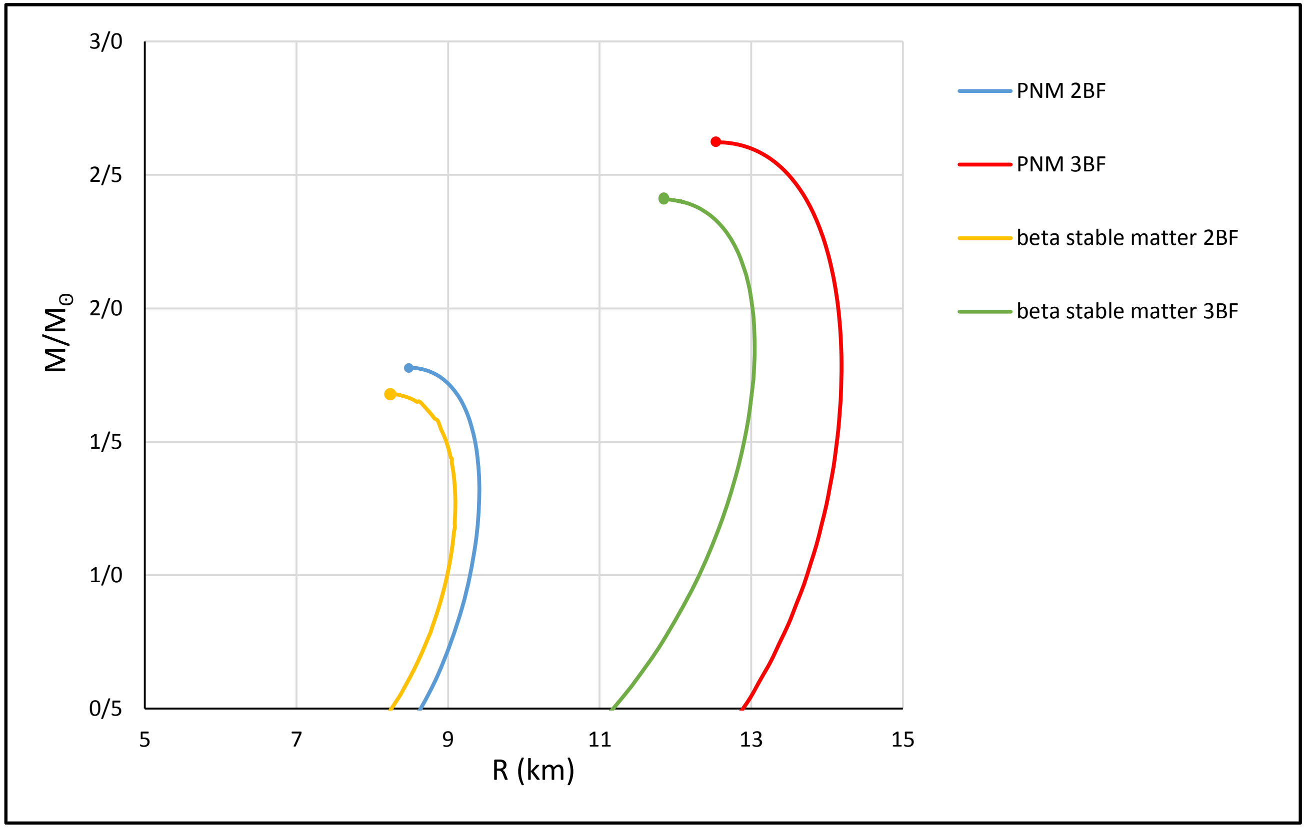

14000
12000
10000
8000
6000
4000
2000
0
pressure (MeV fm-3)
0
0/5
1
1/5
2
2/5
3
baryonic density (fm-3)
beta stable matter 2BF
PNM 2BF
beta stable matter 2BF+3BF
PNM 2BF+3BF

THE TOLMAN-OPPENHEIMER-VOLKOV EQUATION



$$\frac{dM(r)}{dr} = \frac{4\pi\varepsilon(r)r^2}{c^2}$$

$$\frac{dP}{dr} = -\frac{G}{c^2}\frac{M(r)\varepsilon(r)}{r^2}\left(1+\frac{P(r)}{\varepsilon(r)}\right)\left(1+\frac{4\pi r^3 P(r)}{M(r)c^2}\right)\left(1-\frac{2GM(r)}{rc^2}\right)^{-1}$$



	Mass (in M_sun)	Radius (km)
PNM 2BF	1.78	8.50
Beta stable matter 2BF	1.68	8.18
PNM 2BF+3BF	2.62	12.56
Beta stable matter 2BF+3BF	2.41	11.82

Conclusion

- EOS from a microscopic, self consistent method

-use the pure variational method

-Thermal properties , phase transition etc.

Maximum mass of NS more 2 solar mass so maybe including hyperons could reduce to 2 solar mass