Transport coefficients in superfluid neutron stars

the shear and bulk viscosities

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Outline

- ✧ r-mode instability window in rotating neutron stars
- ✧ EFT and superfluid phonon
- ✧ EoS for superfluid neutron star matter
- ✧ Shear viscosity due to superfluid phonons and the r-mode instability window
- ✧ Bulk viscosity due to superfluid phonons
- ✧ Summary

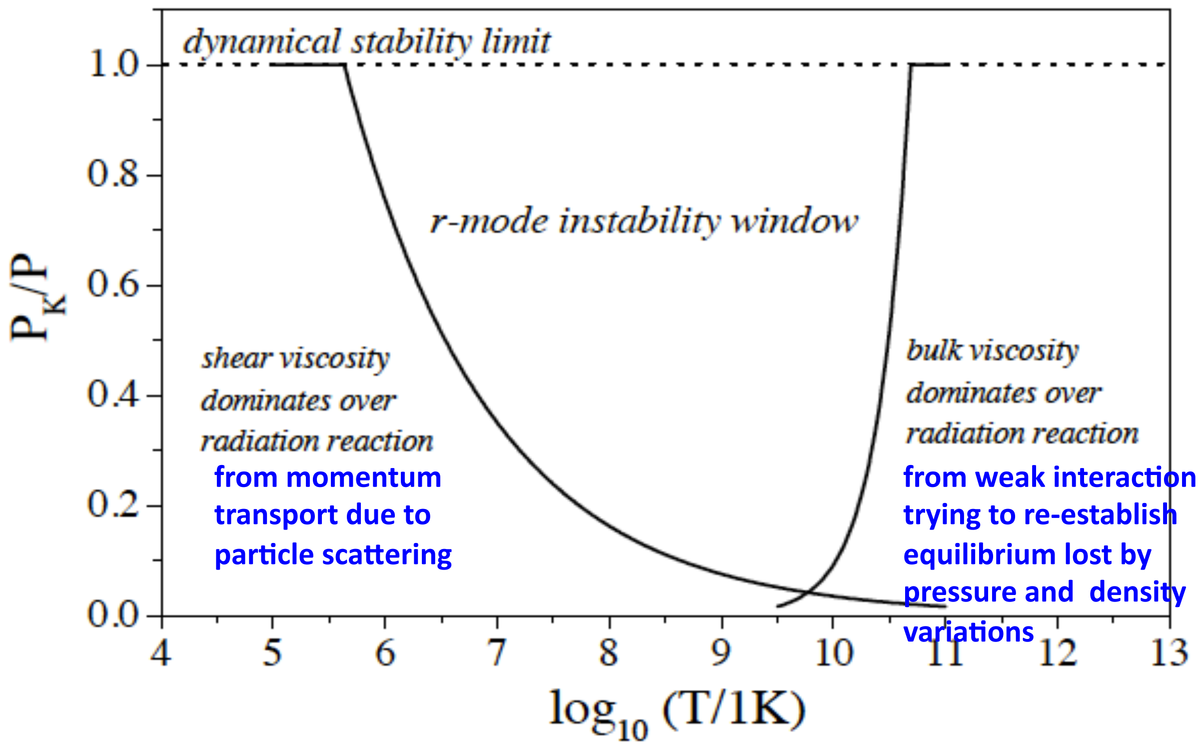
Manuel and Tolos, Physical Review D 84 (2011) 123007
Manuel and Tolos, Physical Review D 88 (2013) 043001
Manuel, Tarrus and Tolos, JCAP 07 (2013) 003

r-mode instability in rotating neutron stars

The low-frequency toroidal r-modes is one of the families of pulsation modes in rotating neutron stars.

r-modes are unstable via emission of gravitational waves. However, there are damping mechanisms (viscous processes) that may counteract the growth of an unstable r-mode.

r-modes can help to constrain the neutron star internal structure



if $\tau_{vp} \ll \tau_{gw}$, then r-modes are damped

Andersson and Kokkotas '00
Lindblom '01

EFT and superfluid phonon

Exploit the universal character of EFT at leading order by obtaining the effective Lagrangian associated to a superfluid phonon and implement the particular features of the system, associated to the coefficients of the Lagrangian, via the EoS

Son '02
Son and Wingate '06

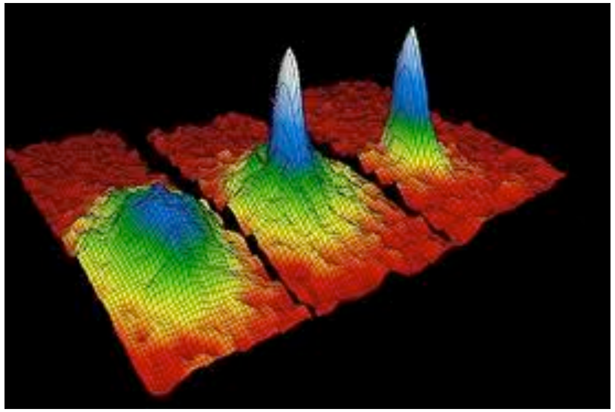
non-relativistic case

$$\mathcal{L}_{\rm LO} = P(X)$$

$$X = \mu - \partial_t \varphi - \frac{(\nabla \varphi)^2}{2m}$$

$P(\mu)$	pressure
μ	chemical potential
φ	phonon field
m	mass condense particles

Applicable in superfluid systems such as cold Fermi gas at unitary, ^{4}He or neutron stars



EoS for superfluid neutron star matter

In order to obtain the speed of sound at T=0 and the different phonon self-couplings one has to determine the EoS for neutron matter in neutron stars.

A common benchmark for nucleonic EoS is APR98
Akmal, Pandharipande and Ravenhall '98

which was later parametrized in a causal form Heiselberg and Hjorth-Jensen '00

$$E/A = \mathcal{E}_0 y \frac{y-2-\delta}{1+\delta y} + S_0 y^\beta (1-2x_p)^2$$

$$y = n/n_0 \qquad x_p = n/n_0$$

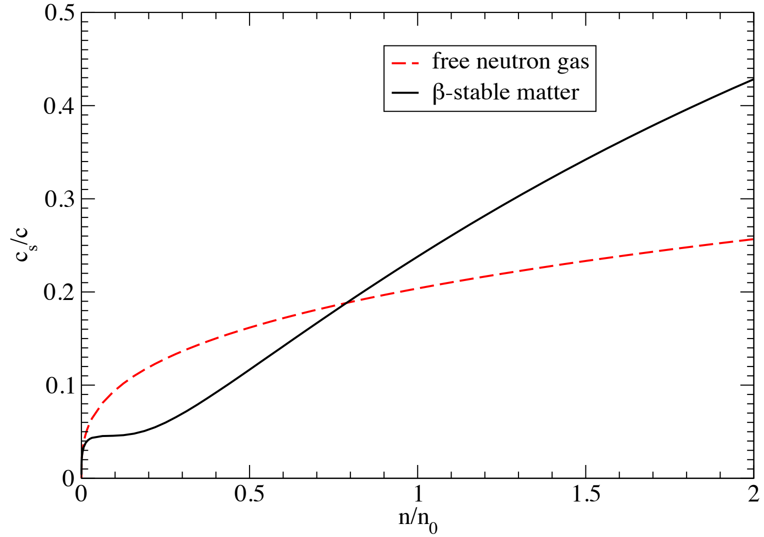
$$n_0 = 0.16\,\mathrm{fm}^{-3}$$

$$\mathcal{E}_0 = 15.8\,\mathrm{MeV} \qquad \delta = 0.2$$

$$S_0 = 32\,\mathrm{MeV} \qquad \beta = 0.6$$

For β-stable matter made up of neutrons, protons and electrons, the speed of sound at T=0 is

$$\sqrt{\frac{\partial P}{\partial \rho}} \equiv c_s$$



Effective Lagrangian for superfluid phonon at LO

$$\mathcal{L}_{\rm LO} = \frac{1}{2}((\partial_t\phi)^2 - v_{\rm ph}^2(\nabla\phi)^2) - g((\partial_t\phi)^3 - 3\eta_g\partial_t\phi(\nabla\phi)^2) + \lambda((\partial_t\phi)^4 - \eta_{\lambda,1}(\partial_t\phi)^2(\nabla\phi)^2 + \eta_{\lambda,2}(\nabla\phi)^4) + \cdots .$$

with Φ the rescaled phonon field, and where the different phonon self-couplings can be expressed in terms of the speed of sound at T=0

$$v_{ph} = \sqrt{\frac{\frac{\partial P}{\partial\mu}}{m\frac{\partial^2 P}{\partial\mu^2}}} = \sqrt{\frac{\partial P}{\partial\rho}} \equiv c_s$$

and derivatives with respect to mass density:

Escobedo and Manuel '10

$$u = \frac{\rho}{c_s}\frac{\partial c_s}{\partial\rho}, \quad w = \frac{\rho}{c_s}\frac{\partial^2 c_s}{\partial\rho^2},$$

$$g = \frac{1-2u}{6c_s\sqrt{\rho}}, \qquad \eta_g = \frac{c_s^2}{1-2u}, \qquad \lambda = \frac{1-2u(4-5u)-2w\rho}{24c_s^2\rho},$$

$$\eta_{\lambda,1} = \frac{6c_s^2(1-2u)}{1-2u(4-5u)-2w\rho}, \qquad \eta_{\lambda,2} = \frac{3c_s^4}{1-2u(4-5u)-2w\rho}$$

Results valid for neutrons pairing in $^1S_0$ channel and also valid for $^3P_2$ neutron pairing if corrections $\bar{\Delta}(^3P_2)^2/\mu_n^2$ are ignored Bedaque, Rupak and Savage '03

Including NLO corrections in the phonon dispersion law

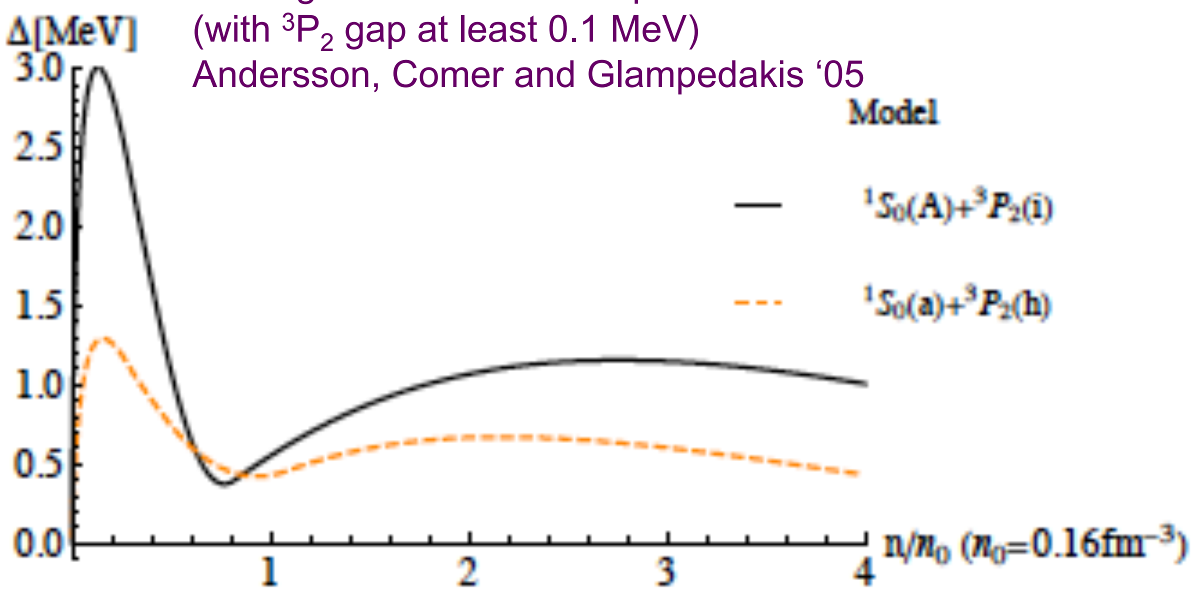
$$E_P = c_s p(1 + \gamma p^2)$$

$$\gamma = -\frac{v_F^2}{45\Delta^2}$$

v_F: Fermi velocity
Δ: gap function

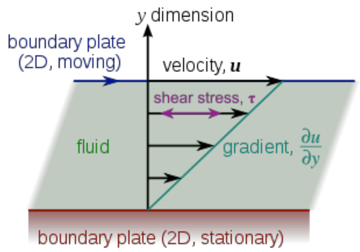
ϒ < 0: first allowed phonon scattering are binary collisions

liimiting values taken from parameterizations of different models
(with $^3P_2$ gap at least 0.1 MeV)
Andersson, Comer and Glampedakis '05



Shear viscosity due to superfluid phonons

Shear viscosity

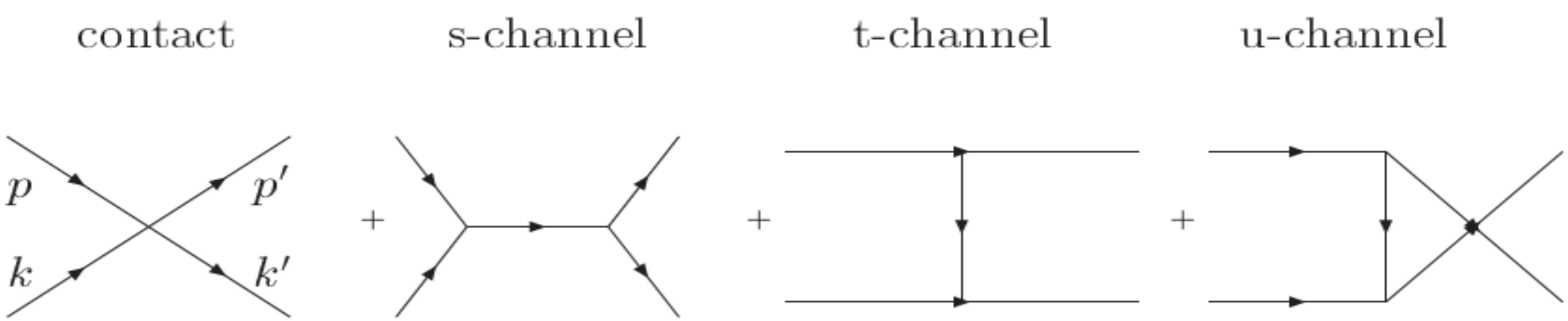


wikipedia

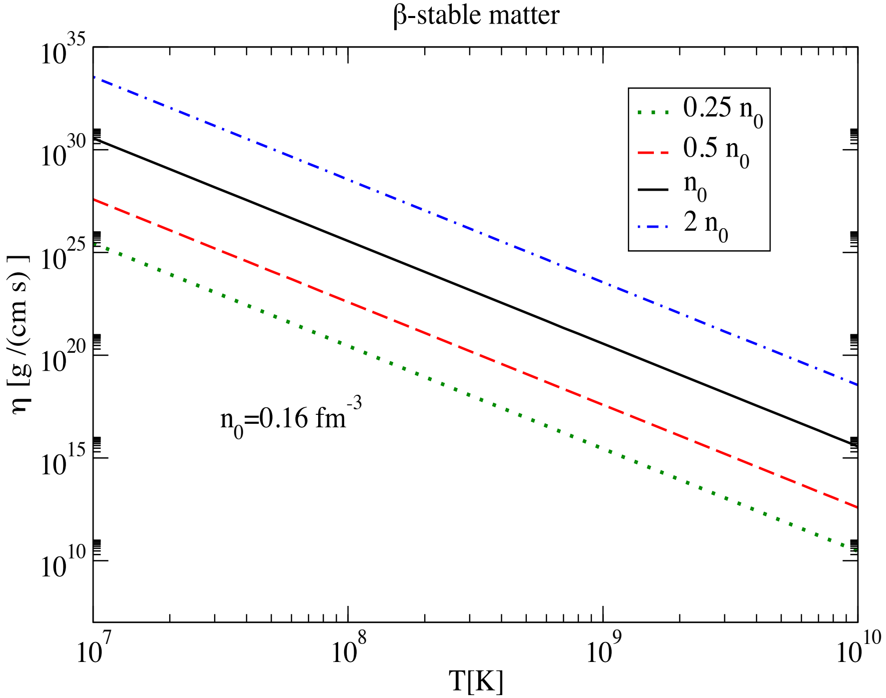
The shear viscosity is calculated using variational methods for solving the transport equation as

$$\eta = \left(\frac{2\pi}{15}\right)^4 \frac{T^8}{c_s^8} \frac{1}{M}$$

where M represents a multidimensional integral that contains the thermally weighted scattering matrix for phonons.



Shear viscosity due to binary collisions of phonons scales as $\eta \propto 1/T^5$ (also for ^{4}He and cold Fermi gas at unitary) while the coefficient depends on EoS.

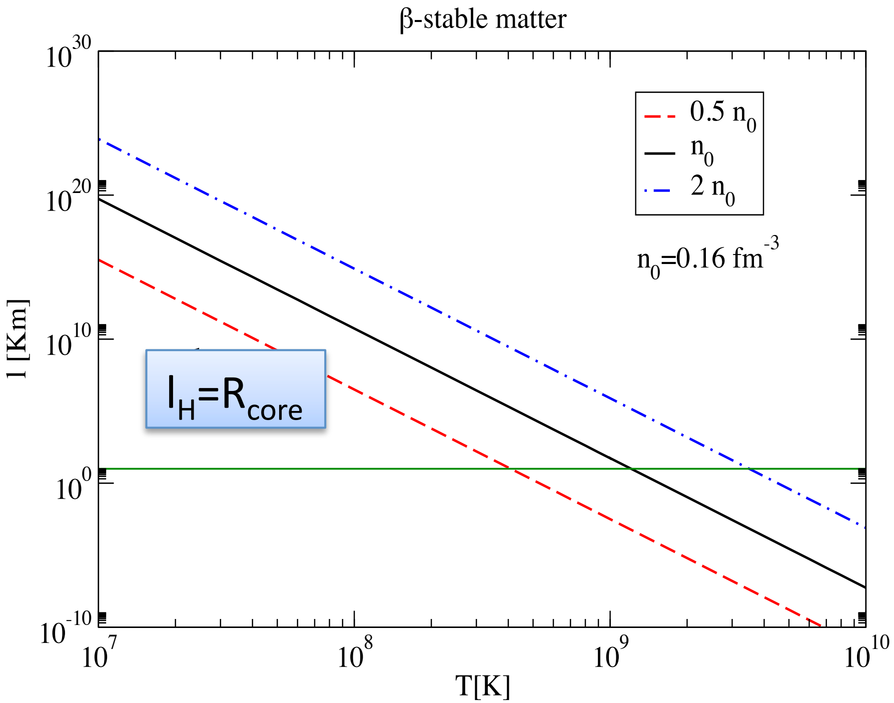


Mean free path of phonons: establish when phonons become hydrodynamic

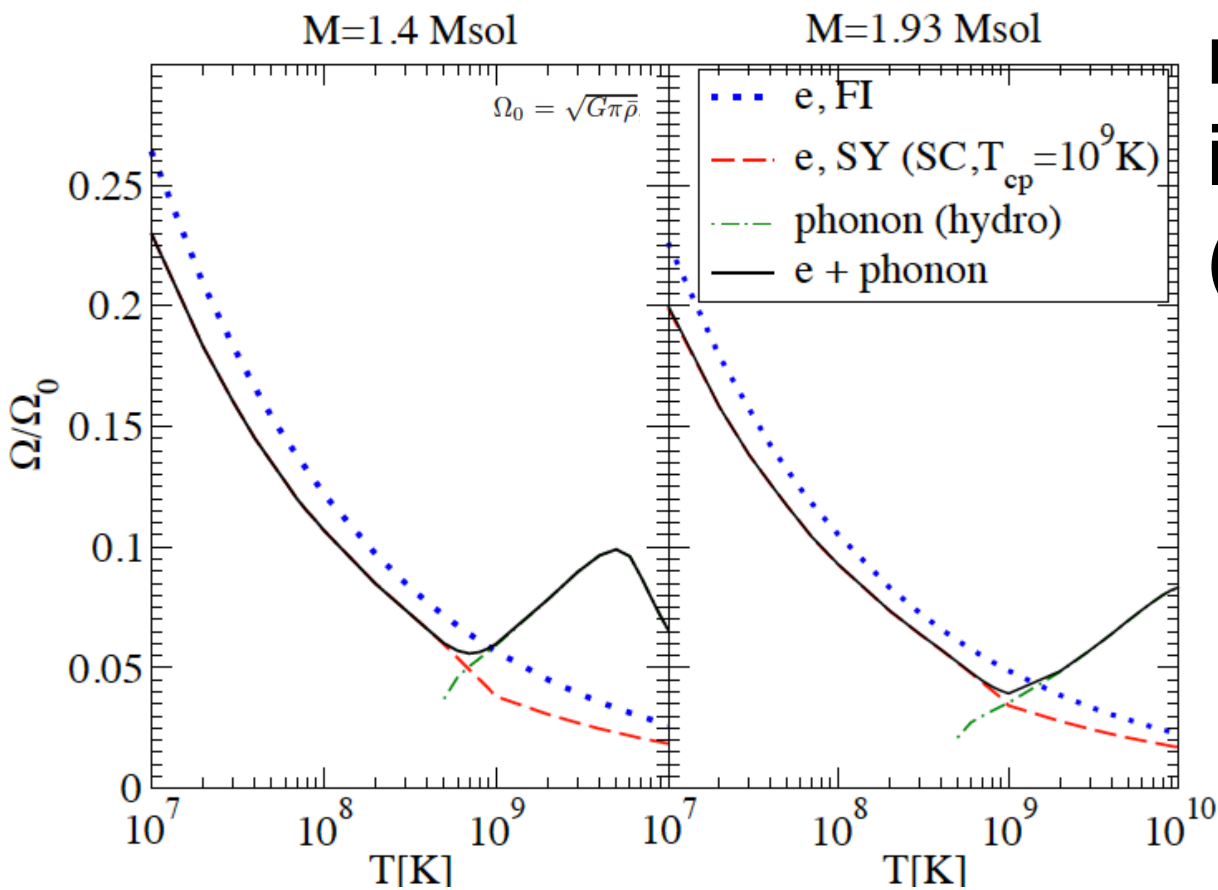
$$l = \frac{\eta}{n < p>}$$

$< p >$: thermal average

n : phonon density



Alford, Braby, and Mahmoodifar '10



r-mode instability window (only shear)

$$-\frac{1}{|\tau_{\mathrm{GR}}(\Omega)|} + \frac{1}{\tau_{\eta_{\text{hydro}}}(T)} = 0$$

Dissipation due to superfluid phonons start to be relevant at T≈7 x 10^8 K for 1.4 $M_\odot$ and T≈10^9 K for 1.93 $M_\odot$

$$\frac{1}{|\tau_{\mathrm{GR}}(\Omega)|} = \frac{32\,\pi\,G\,\Omega^{2l+2}}{c^{2l+3}}\,\frac{(l-1)^{2l}}{((2l+1)!!)^2}\left(\frac{l+2}{l+1}\right)^{2l+2}\int_0^R \rho r^{2l+2}dr$$

$$\frac{1}{\tau_{\eta_{\text{hydro}}}(T)} = (l-1)(2l+1)\int_{R_c}^R \eta_{\text{hydro}} r^{2l}dr\left(\int_0^R \rho r^{2l+2}dr\right)^{-1}$$

l=2 (dominant)

Bulk viscosities due to superfluid phonons

The bulk viscosity coefficients are calculated from the dynamical evolution of the phonon number density[1] or, equivalently, by using the Boltzmann equation for phonons in the relaxation time approximation

$$\zeta_i(\omega) = \frac{1}{1+\left(\omega I_1^2 \frac{\partial\rho}{\partial n}\frac{\partial\rho}{\partial\mu}\frac{T}{\Gamma_{ph}}\right)^2}\frac{T}{\Gamma_{ph}} C_i \ , \qquad i = 1,2,3,4$$

$$C_1 = C_4 = -I_1 I_2 \ , \qquad C_2 = I_2^2 \ , \qquad C_3 = I_1^2 \ ,$$

$$I_1 = \frac{60 T^5}{7 c_s^7 \pi^2}\left(\pi^2\zeta(3) - 7\zeta(5)\right)\left(c_s \frac{\partial B}{\partial\rho} - B\frac{\partial c_s}{\partial\rho}\right) \ , \qquad B = c_s\gamma$$

NLO corrections in phonon dispersion law

$$I_2 = -\frac{20 T^5}{7 c_s^7 \pi^2}\left(\pi^2\zeta(3) - 7\zeta(5)\right)\left(2Bc_s + 3\rho\left(c_s\frac{\partial B}{\partial\rho} - B\frac{\partial c_s}{\partial\rho}\right)\right) \ ,$$

Three independent coefficients: $\zeta_1 = \zeta_4 \quad \zeta_1^2 \leq \zeta_2\zeta_3 \quad \zeta_2, \zeta_3 \geq 0$

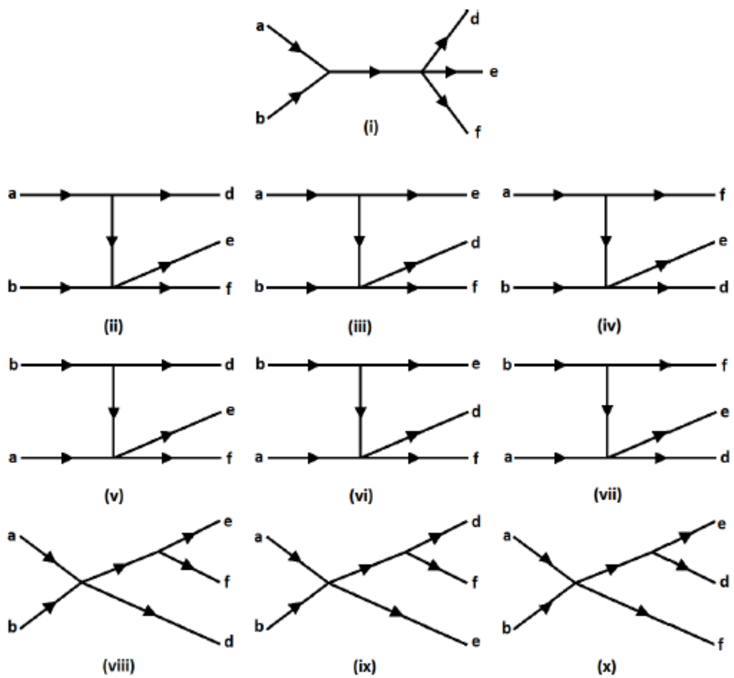
In the static limit

$$\zeta_i = \frac{T}{\Gamma_{ph}} C_i \ , \qquad i = 1,2,3,4 \ ,$$

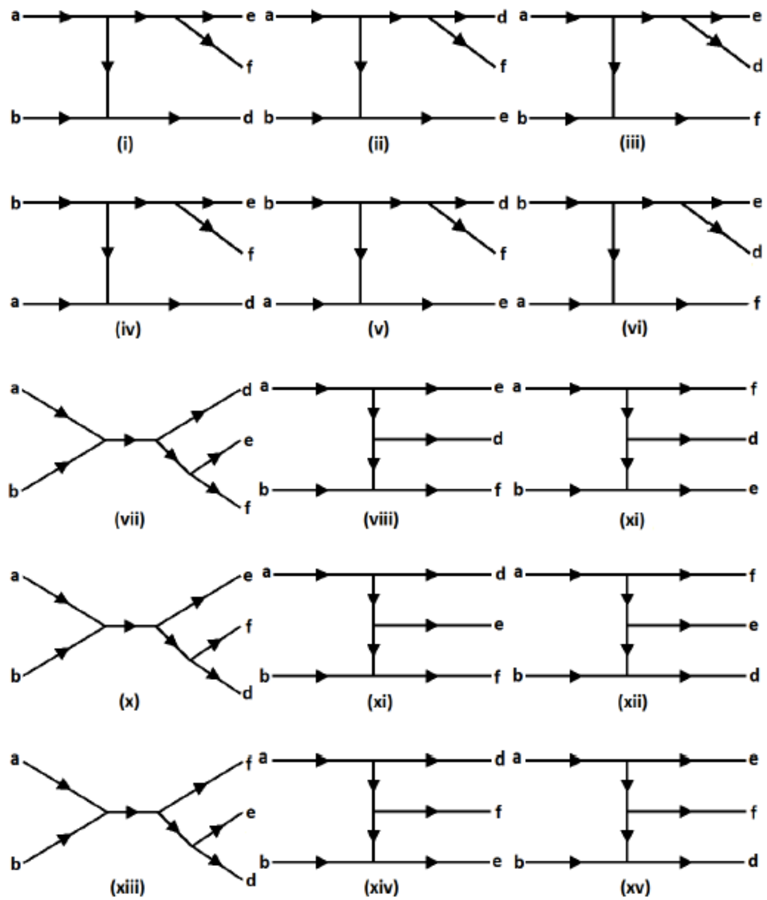
[1] Khalatnikov

Phonon decay rate for phonon number changing processes: 2 <-> 3

$$\Gamma_{ph} = \int d\Phi_5(p_a, p_b; p_d, p_e, p_f) \|\mathcal{A}\|^2 f(E_a) f(E_b) \left(1 + f(E_d)\right) \left(1 + f(E_e)\right) \left(1 + f(E_f)\right)$$

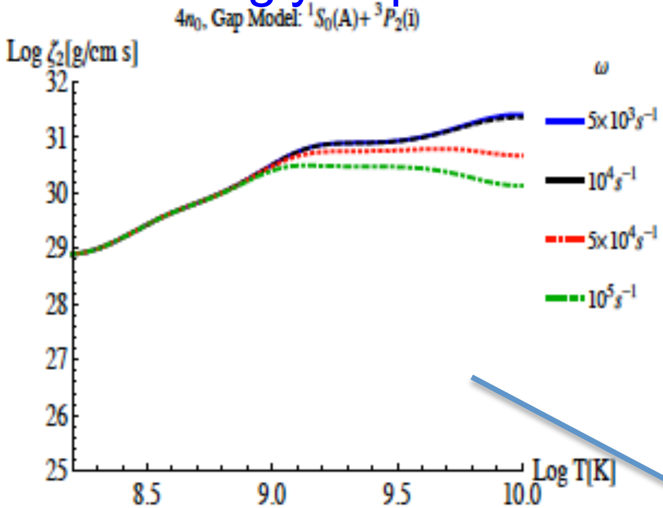


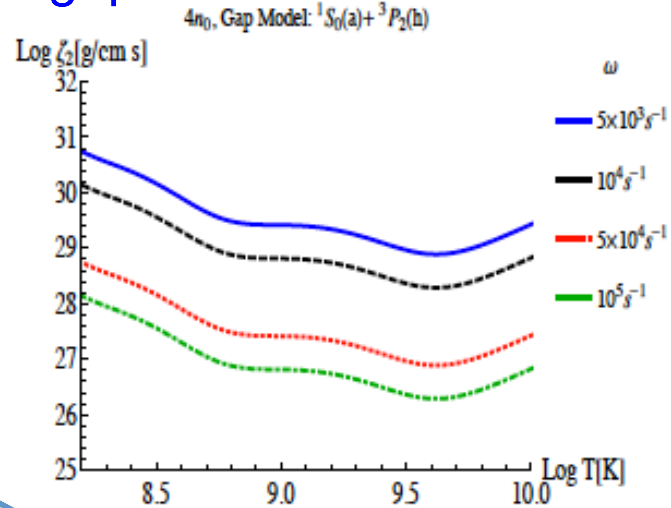
with one 4-phonon vertex
and one 3-phonon vertex



with only 3-phonon vertices

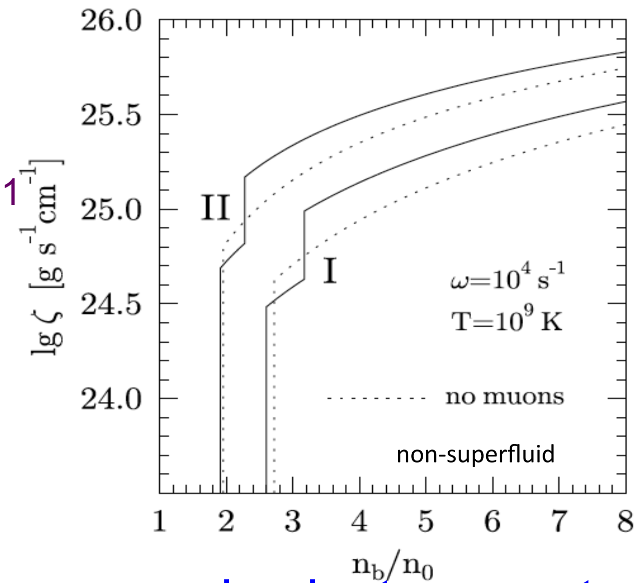
ξ_2 at $n \geq 4n_0$ is within 10% of the static value for $T \leq 10^9$ K and for the case of maximum values of the $^3P_2$ gap > 1 MeV, while, otherwise, the static solution is not a valid: strongly dependent on the gap!



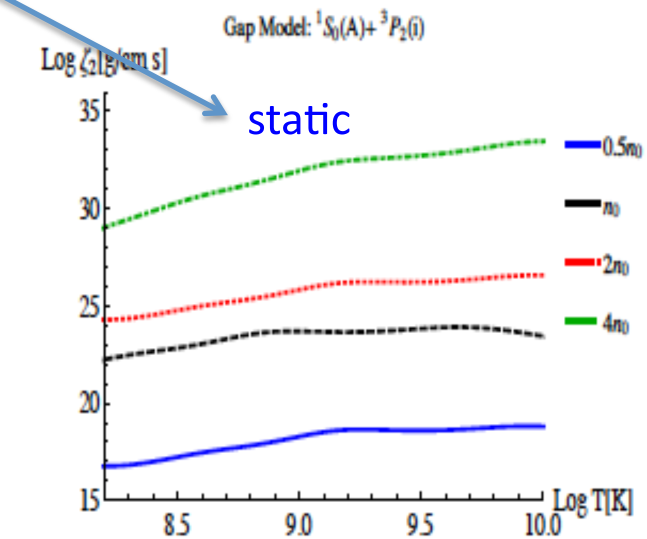


Compared to Urca (and modified Urca) processes....

Haensel, Levenfish and Yakovlev '00 '01



Urca



static

phonon processes dominate except $n \approx 2n_0$ when Urca opens

Summary

Starting from a general formulation for the collisions of superfluid phonons, we compute the shear and bulk viscosities in terms of the EoS of the system (and the gap function)

• Binary collisions of phonons produce a shear viscosity that scales with $1/T^5$ (universal feature seen for ^{4}He and cold Fermi gas at unitary) while the coefficient depends on EoS (microscopic theory).

r-mode window modified for $T \geq (10^8\text{-}10^9)$ K due to phonon shear viscosity

• For typical $\omega \approx 10^4\ s^{-1}$, bulk viscosity coefficients at $n \geq 4n_0$ are within 10% from its static value for $T \leq 10^9$ K and for the case of maximum values of the $^3P_2$ gap above 1 MeV, while, otherwise, the static solution is not a valid approximation to the bulk viscosity coefficients

• Phonon bulk viscosities dominate in the core except for $n \approx 2n_0$ when the opening of the Urca processes takes place

Future: r-mode instability window considering other dissipative processes, such as bulk viscosity or rubbing core-crust, …