

**Compact stars and general constraints  
on exotic phases**

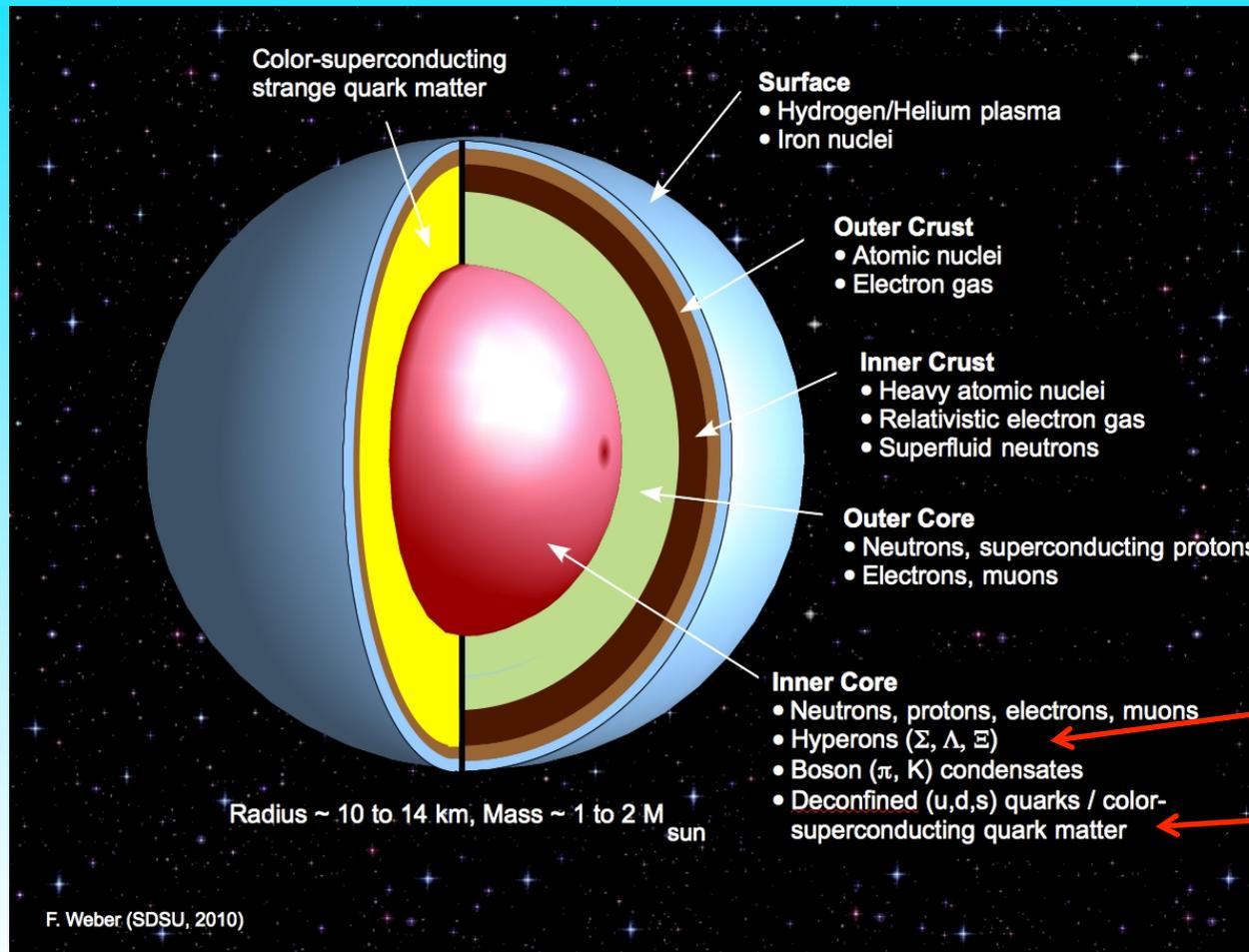
***OUTLINE***

- observational situation
- compact stars – nucleons + hyperons
- additional degrees of freedom
- adding quarks
- comparison to various lattice results
- magnetic fields

neutron stars are remnants of Type II supernovae

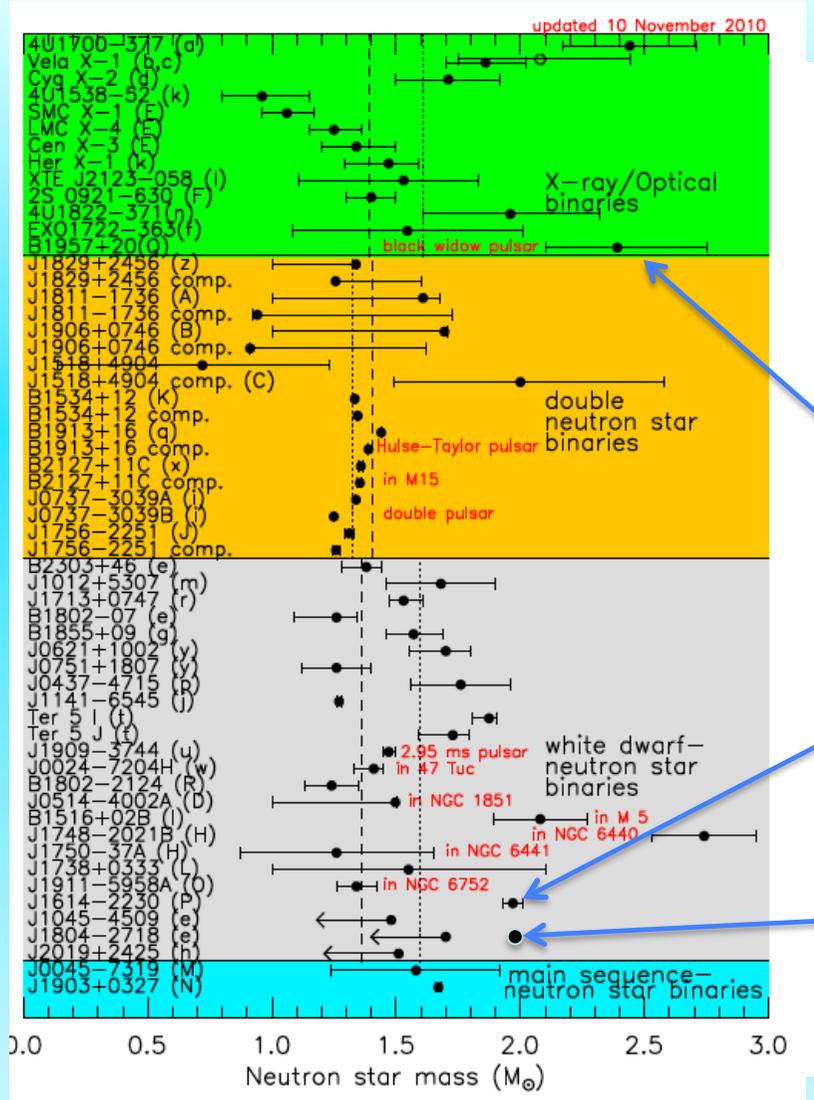
1 to 2 solar masses, radii around 10 - 15 km  
maximum central densities 4 to 10  $\rho_0$

about 2000 known neutron stars



hyper star

hybrid star



Lattimer, Prakash, astro-ph:1012.3208

## Masses of Neutron Stars

### Masses of radio pulsars

Kiziltan, Kottas, Thorsett, astro-ph:1011.4291

no signature for mass cut off

$M = (2.4 \pm 0.12) M_{\odot}$  ?  
van Kerkwijk et al., ApJ 728, 95 (2011)

*current benchmark for NS models*

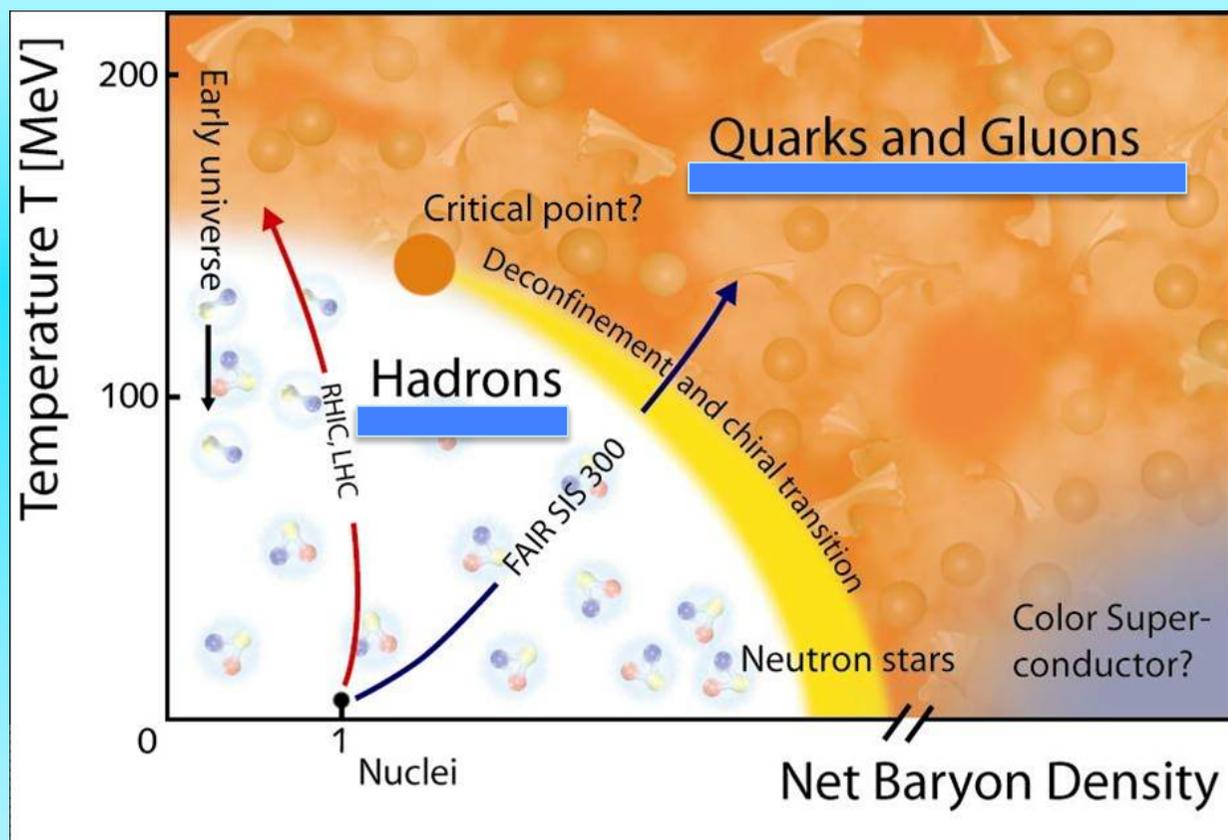
$M = (1.97 \pm .04) M_{\odot}$   
Demorest et al. Nature 467, 1081 (2010)

*new observation PSR J0348+0432*

$M = (2.01 \pm .04) M_{\odot}$   
Antoniadis et al. Science 340, 448 (2013)

well established - heavy neutron stars

*the usual phase diagram (sketch) of strong interactions*



connect both worlds  
in some reasonable way

Practical model useful for heavy-ion simulations and compact star physics

correct asymptotic degrees of freedom

reasonable description on a quantitative level for high T down to nuclei

possibility of studying first-order as well as cross-over transitions

## hadronic SU(3) approach

$$\begin{array}{ll}
 B = \{ p, n, \Lambda, \Sigma^{\pm/0}, X^{-/0} \} & \text{baryons} \\
 \text{diag}(V) = \{ (\omega + \rho) / \sqrt{2}, (\omega - \rho) / \sqrt{2}, \phi \} & \text{vector mesons} \\
 \text{diag}(X) = \{ (\sigma + \delta) / \sqrt{2}, (\sigma - \delta) / \sqrt{2}, \zeta \} & \text{scalar mesons}
 \end{array}$$

mean fields generate scalar attraction and vector repulsion

$$\text{scalar self interaction } L_0 = -\frac{1}{2} k_0 I_2 + k_1 (I_2)^2 + k_2 I_4 + 2 k_3 I_3 + L_{\text{ESB}}$$

$$\text{invariants} \quad I_1 = \text{Tr}(X) \quad I_2 = \text{Tr}(X)^2 \quad I_3 = \det(X)$$

$$V(M) \quad \langle \sigma \rangle = \sigma_0 \neq 0 \quad \langle \zeta \rangle = \zeta_0 \neq 0$$

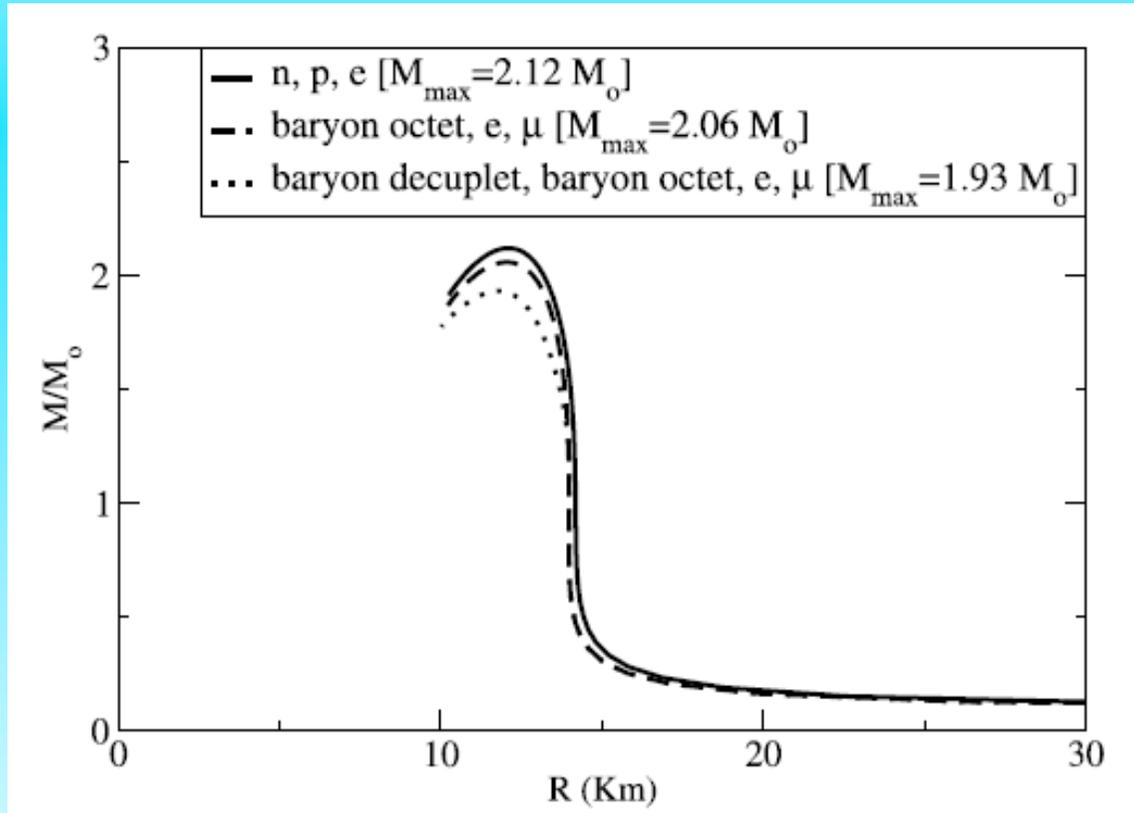
$$\sigma \sim \langle \bar{u} u + \bar{d} d \rangle \quad \zeta \sim \langle \bar{s} s \rangle \quad \delta^0 \sim \langle \bar{u} u - \bar{d} d \rangle$$

$$\text{explicit breaking} \sim \text{Tr}[c \sigma] \quad (\sim m_q \bar{q} q)$$

fix scalar parameters to baryon masses, decay constants, meson masses

*fit for large star masses T. Schürhoff*  $\varepsilon \sim 0.28$ ,  $\kappa \sim 275$  MeV,  $M \sim 2 M_\odot$

## Neutron star masses including different sets of particles

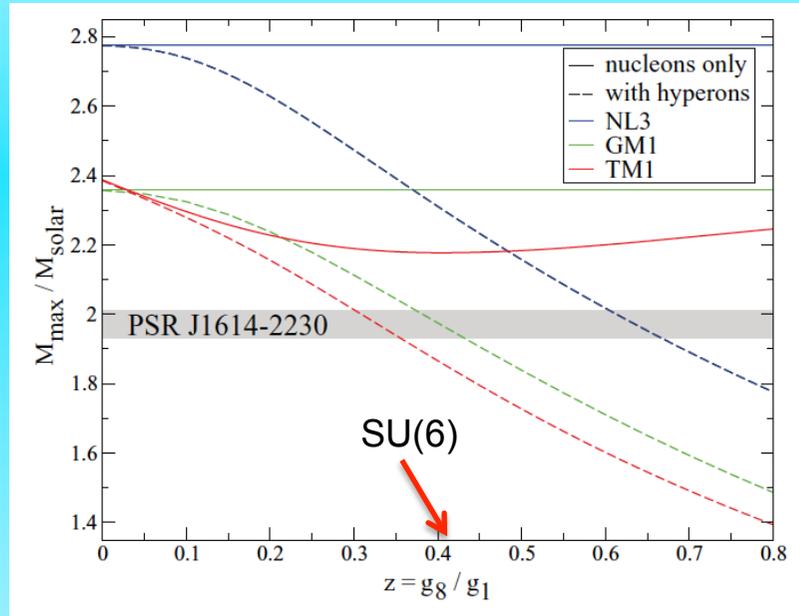


Tolman-Oppenheimer-Volkov equations, static spherical star

changing masses with degrees of freedom

large star masses even with spin 3/2 resonances

systematically “playing around” with vector couplings away from SU(6)



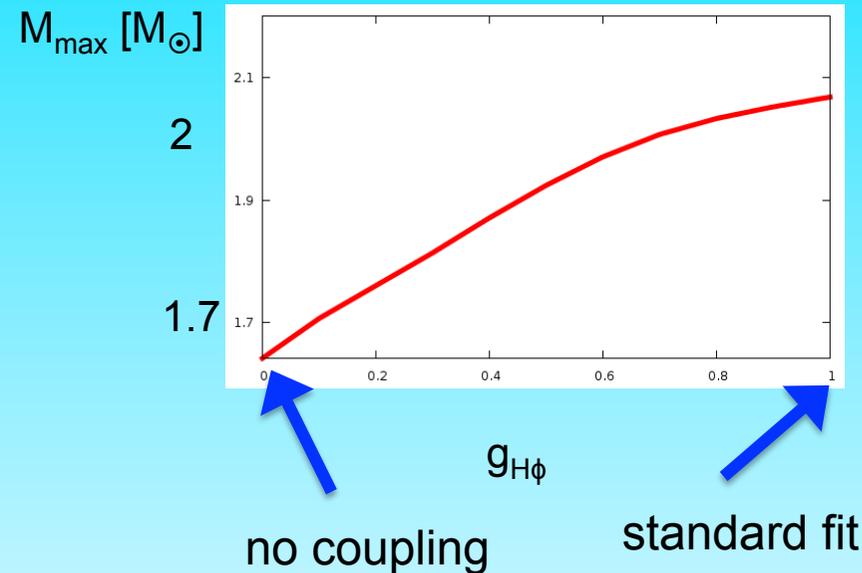
$$\frac{M_{\max}}{M_{\odot}} = \frac{M_{\max}(f_s = 0)}{M_{\odot}} - c \left( \frac{f_s}{0.1} \right)$$

with  $c \sim 0.6$

however, SU(6) limit:  $g_{N\phi} = 0$

$$\text{SU(3): } g_{N\phi} = \sqrt{2} \frac{1}{3} (-1 + 4 \alpha) g_8 - \frac{1}{\sqrt{3}} g_1$$

$f_s(\text{core})$  varies between 0.1 and 1



→ not a whole lot of strangeness

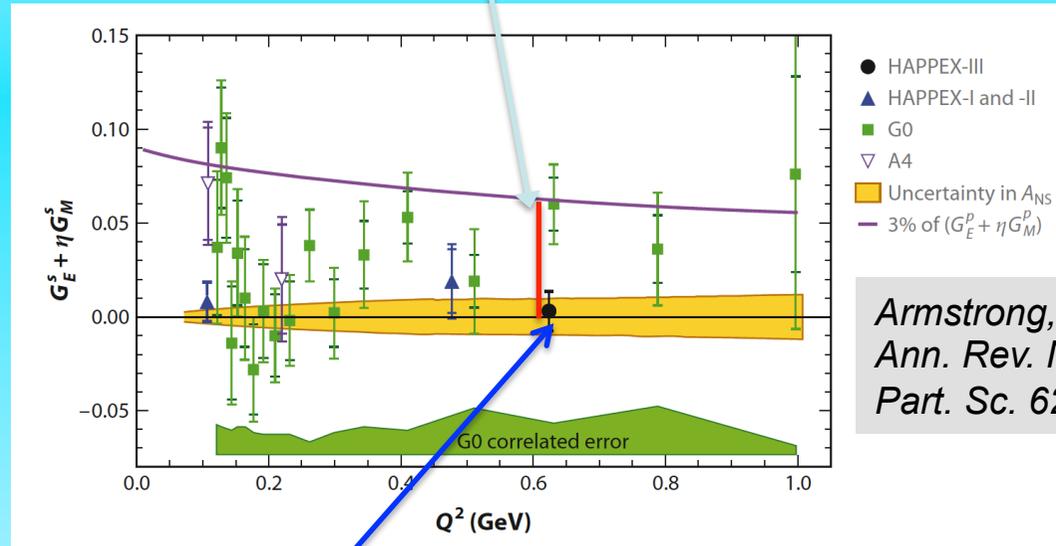
Strong coupling of N to  $\Phi$ ? - strange vector form factor of nucleon



band of possible values from calculation

SWS, MPLA 10 1201 (1995)

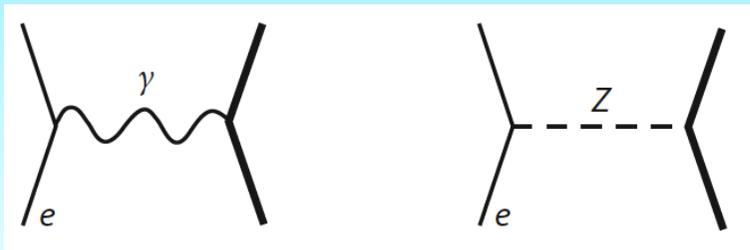
Summary of PV polarized eA scattering experiments



Armstrong, McKeown, *Ann. Rev. Nucl. & Part. Sc.* 62, 337 (2012)

most recent experiment - strangeness contribution consistent with 0

$M \sim$



interference term  $\sigma_{PV} \sim M_Y M_Z^*$

asymmetry  $A \sim G_F Q^2 / \alpha \sim 10^{-5}$

$\chi$ QCD collaboration small values for  $\mu_s$  and  $r_s$ ,

PRD80 094503 (2009)

playing around with the  $\Delta$  baryon

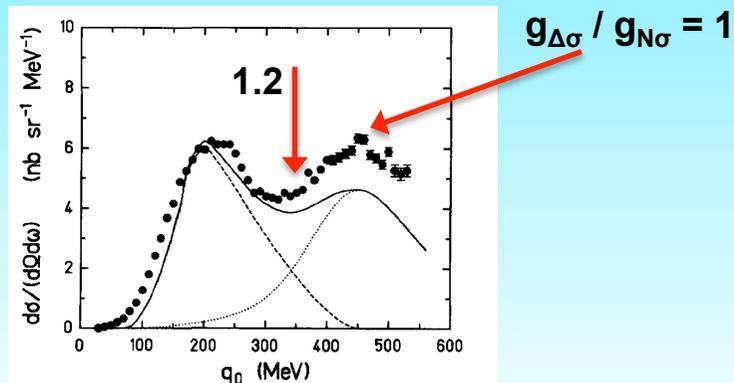
$\Delta$  resonances  
 scalar couplings  $\rightarrow$  vacuum masses

vector couplings unclear  
 moderate changes  $r_V = g_{\Delta\omega} / g_{N\omega}$

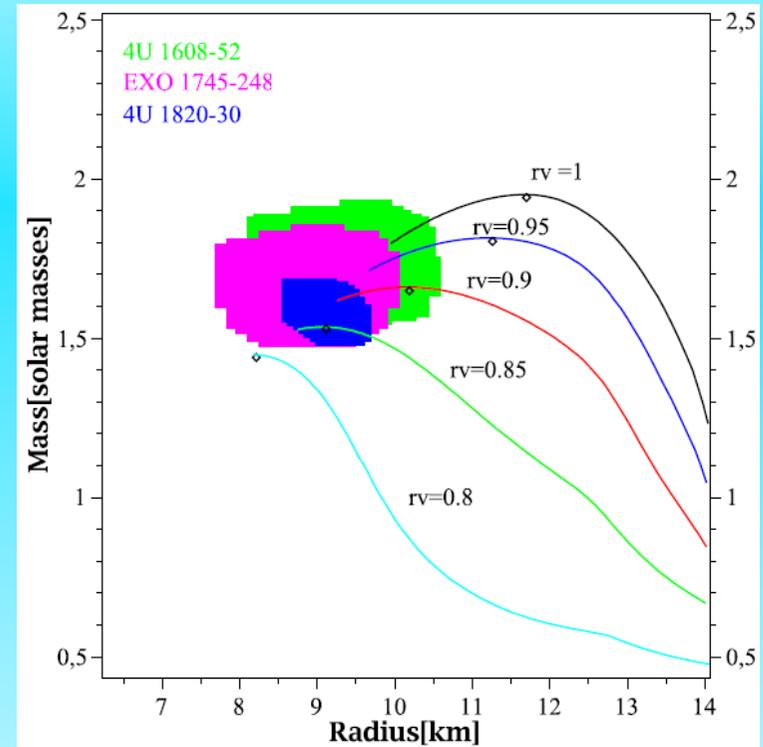
not far from SU(6)

same from quasielastic eA scattering

$E = 695 \text{ MeV}$   $^{40}\text{Ca}$



Wehrberger, *Phys. Rep.* 225, 273 (1993)

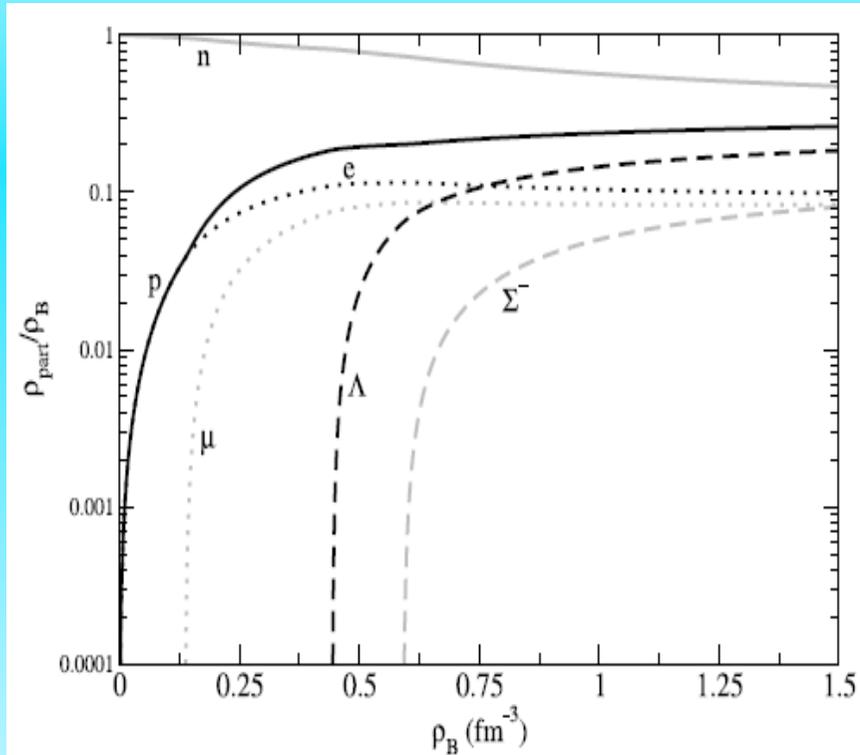


Schürhoff, SWS, Dexheimer, *APJL* 724 (2010) L74

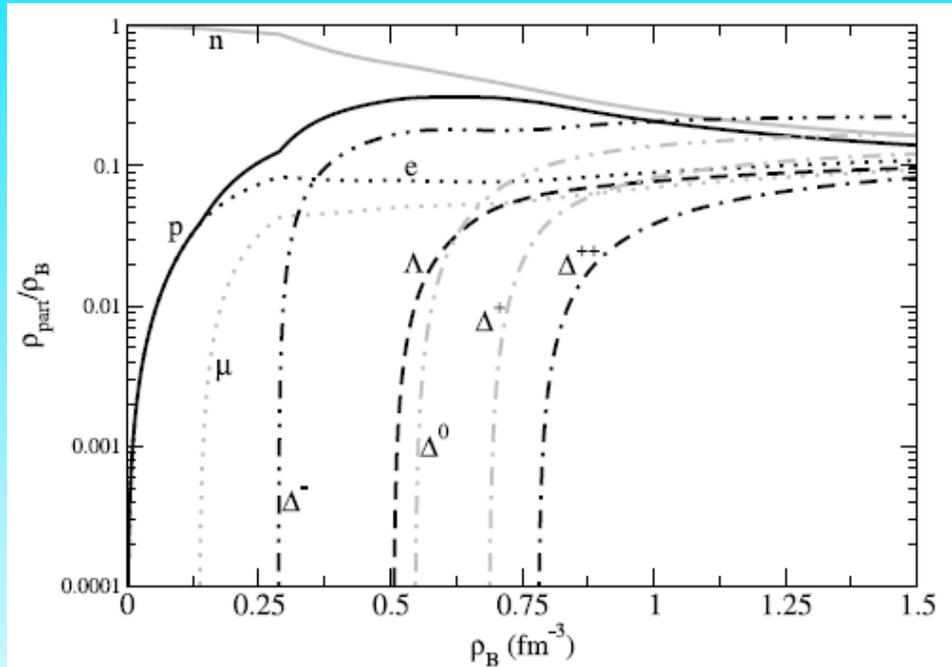
Data from Özel et al, *astro-ph:1002.3153*

see however, Steiner et al, *astro-ph:1005.0811*

particle densities inside of the star



particle abundancies – no decuplet

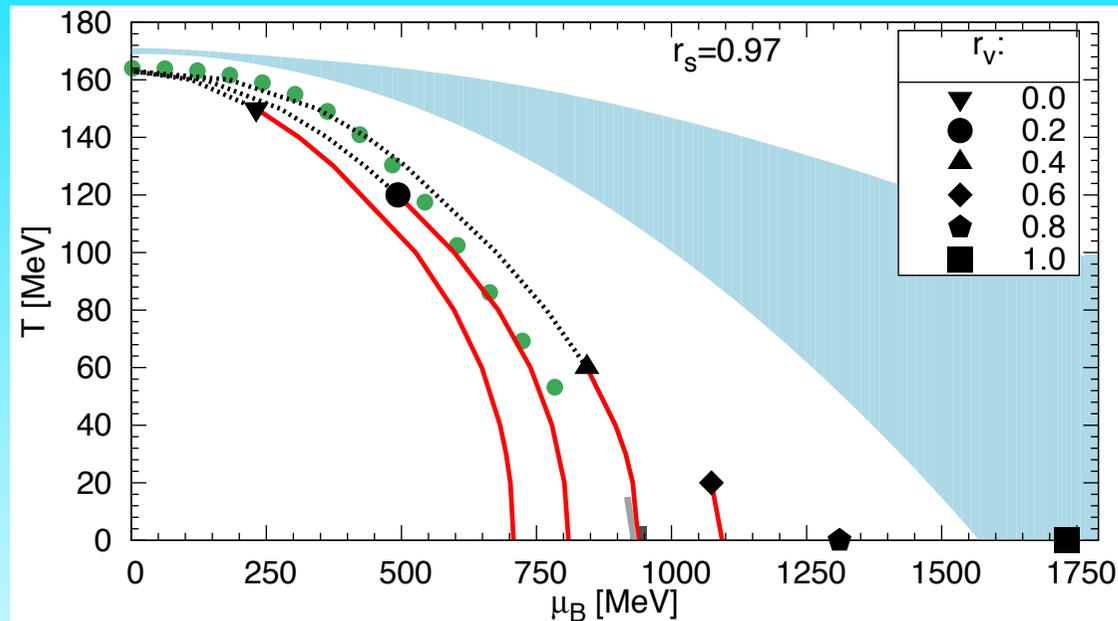


particle numbers as function of density  
uncertainties from  $g_{3/2}$  coupling

## Adding resonances up to 2.6 GeV

In principle - many (!! ) parameters

rescale lowest-multiplet coupling strengths



Advantage: EOS has same degrees of freedom as hybrid code  
Usually mismatch - d.o.f. hydro and cascade parts

## hadrons, quarks, Polyakov loop and excluded volume

Include modified distribution functions for quarks/antiquarks

$$\Omega_q = -T \sum_{j \in Q} \frac{\gamma_j}{(2\pi)^3} \int d^3k \ln \left( 1 + \Phi \exp \frac{E_j^* - \mu_j}{T} \right)^* \quad \Phi \quad \text{confinement order parameter}^*$$

Following the parametrization used in PNJL calculations

$$U = -\frac{1}{2} a(T) \Phi \Phi^* + b(T) \ln[1 - 6 \Phi \Phi^* + 4 (\Phi \Phi^*)^3 - 3 (\Phi \Phi^*)^2]$$

$$a(T) = a_0 T^4 + a_1 T_0 T^3 + a_2 T_0^2 T^2 \quad , \quad b(T) = b_3 T_0^3 T$$

The switch between the degrees of freedom is triggered by excluded volume corrections

thermodynamically consistent -

*no reconfinement!*

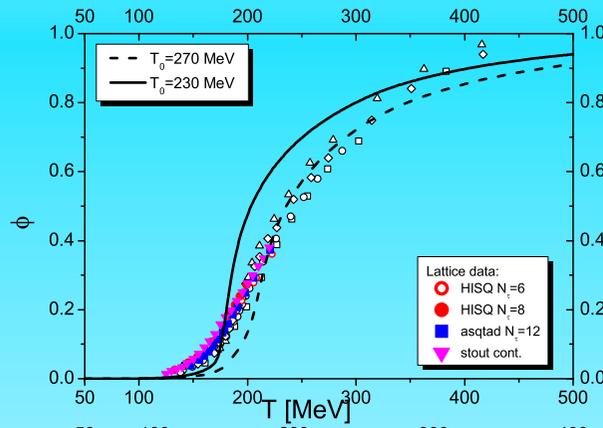
$$\begin{aligned} V_q &= 0 \\ V_h &= v \\ V_m &= v / 8 \end{aligned} \quad \tilde{\mu}_i = \mu_i - v_i P \quad e = \tilde{e} / (1 + \sum v_i \tilde{\rho}_i)$$

equation of state stays causal!

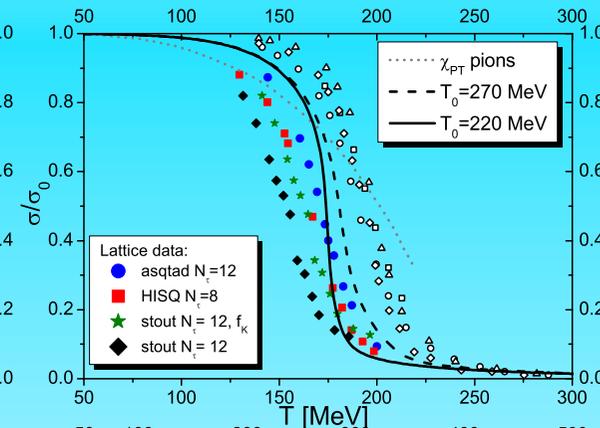
results for hot matter at vanishing chemical potential

points are various lattice results

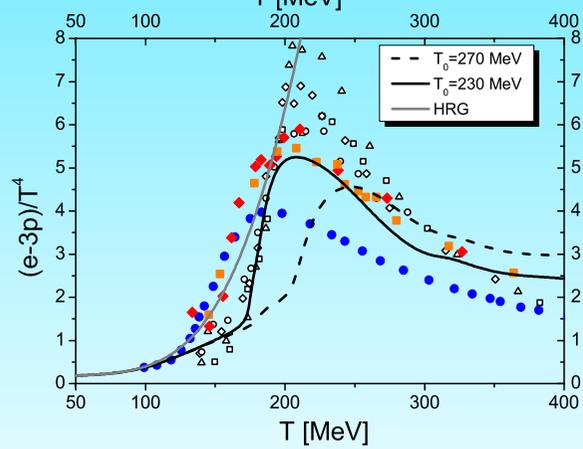
Polyakov loop



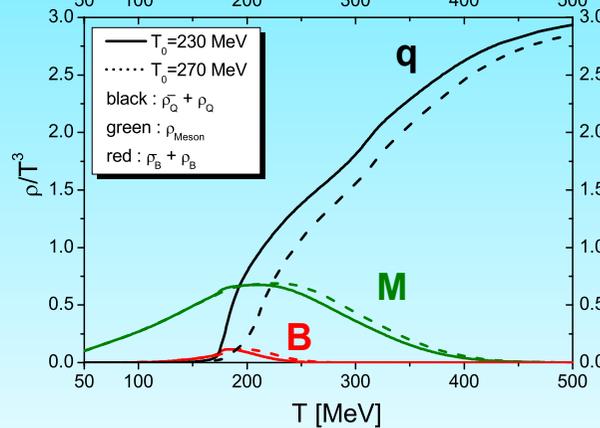
scalar condensate



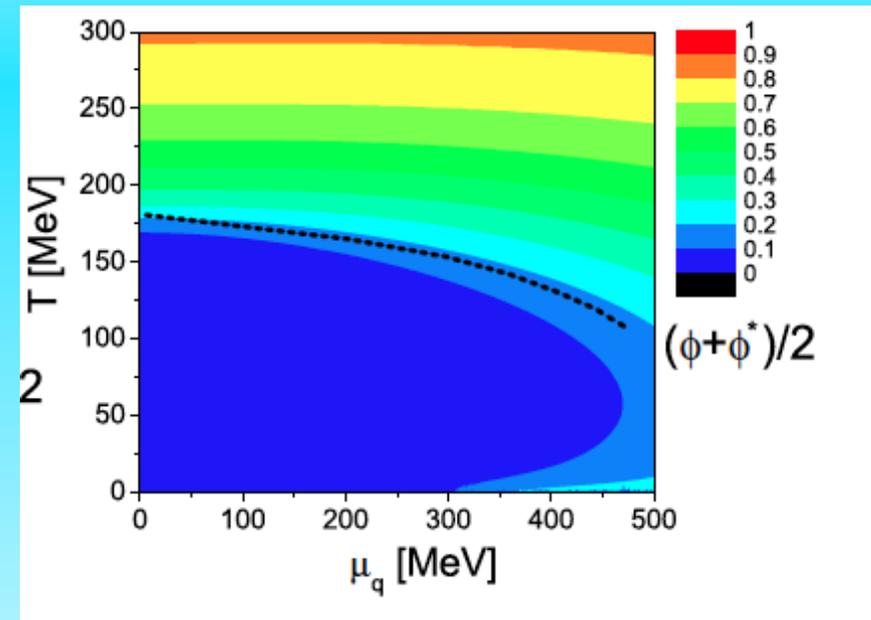
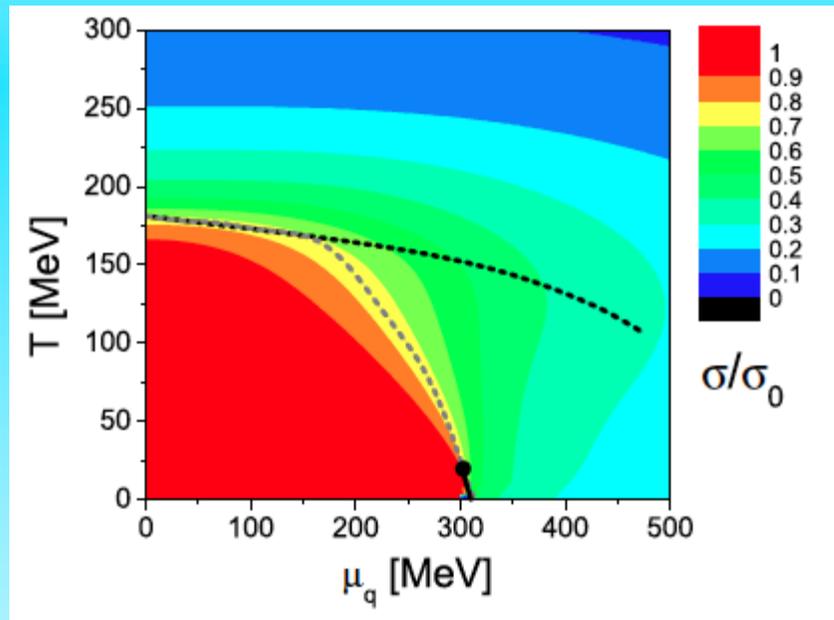
Interaction measure



densities



Order parameters for chiral symmetry and confinement in  $\mu$  and  $T$



except for liquid-gas no first-order transition

## dense matter and stars in a parity doublet model

- treat  $B, B^*$  as positive/negative parity doublets

transformation:

$$\begin{aligned}\psi_{1R} &\longrightarrow R\psi_{1R}, & \psi_{1L} &\longrightarrow L\psi_{1L}, \\ \psi_{2R} &\longrightarrow L\psi_{2R}, & \psi_{2L} &\longrightarrow R\psi_{2L}.\end{aligned}$$

chirally invariant mass term

$$\begin{aligned}m_0(\bar{\psi}_2\gamma_5\psi_1 - \bar{\psi}_1\gamma_5\psi_2) \\ = m_0(\bar{\psi}_{2L}\psi_{1R} - \bar{\psi}_{2R}\psi_{1L} - \bar{\psi}_{1L}\psi_{2R} + \bar{\psi}_{1R}\psi_{2L})\end{aligned}$$

Candidates –  $N(1535), \Lambda(1670), \Sigma(1750), \Xi(?)$  overall unclear

*single particle energies*  $E_{\pm} = \sqrt{(g_1\sigma + g_2\zeta)^2 + m_0^2} \pm (g'_1\sigma + g'_2\zeta)$

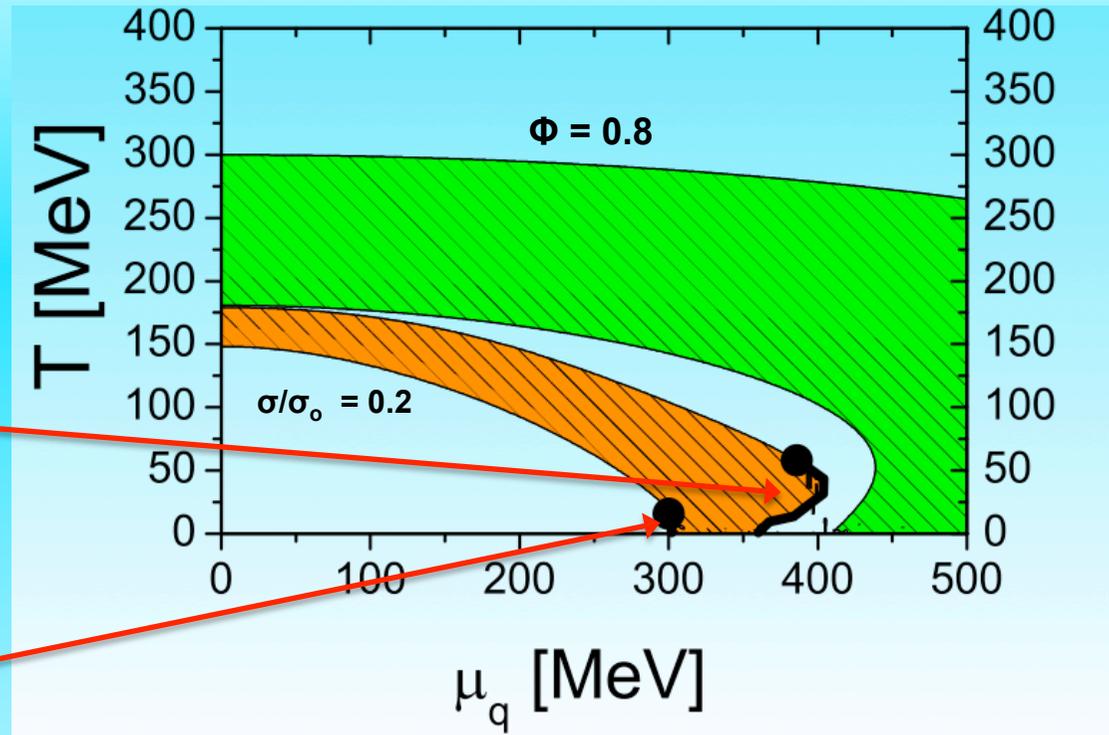
degenerate in the  
chiral limit

Steinheimer, SWS, Stöcker, PRC 84, 045208  
Dexheimer, Steinheimer, Negreiros, SWS, PRC 87, 015804

Excited quark-hadron matter in the parity-doublet approach

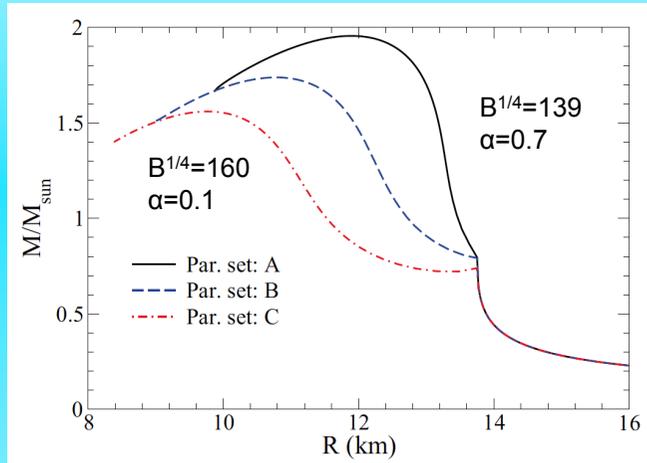
Chiral transition

Liquid-gas phase transition



First-order phase transition with low-T critical end point due to chiral parity partners

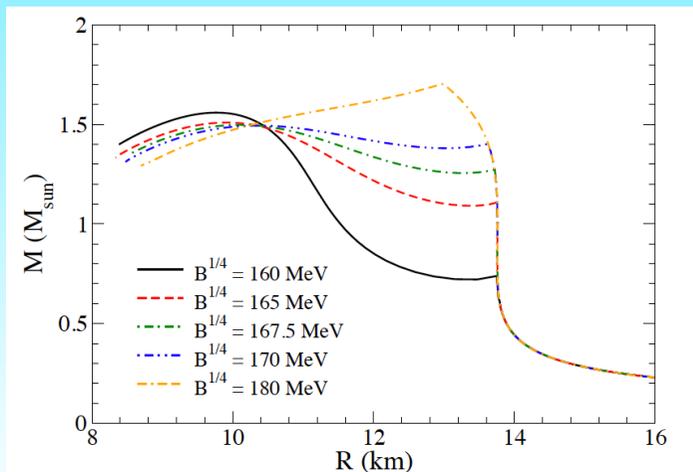
# Hybrid Stars, Quark Interactions



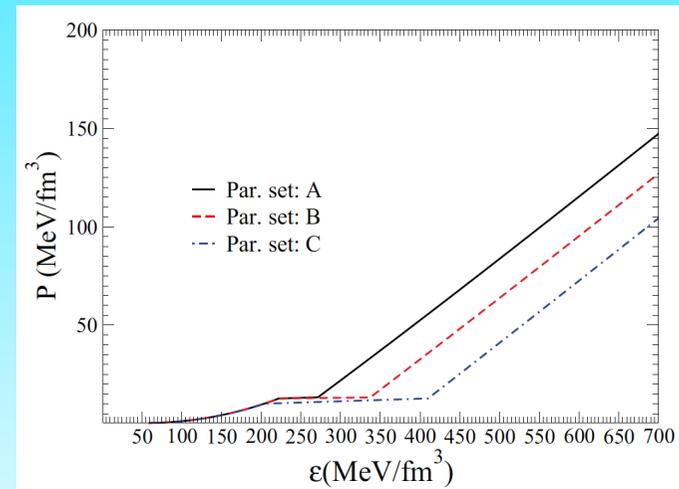
baryons alone  $M_{\text{max}} \sim 1.8 M_{\text{solar}}$

ingredients –  
Standard baryonic EOS (G300)  
plus MIT bag model +  $\alpha_s$  corrections

Fast cooling in the quark core  
need gaps in the quark phase

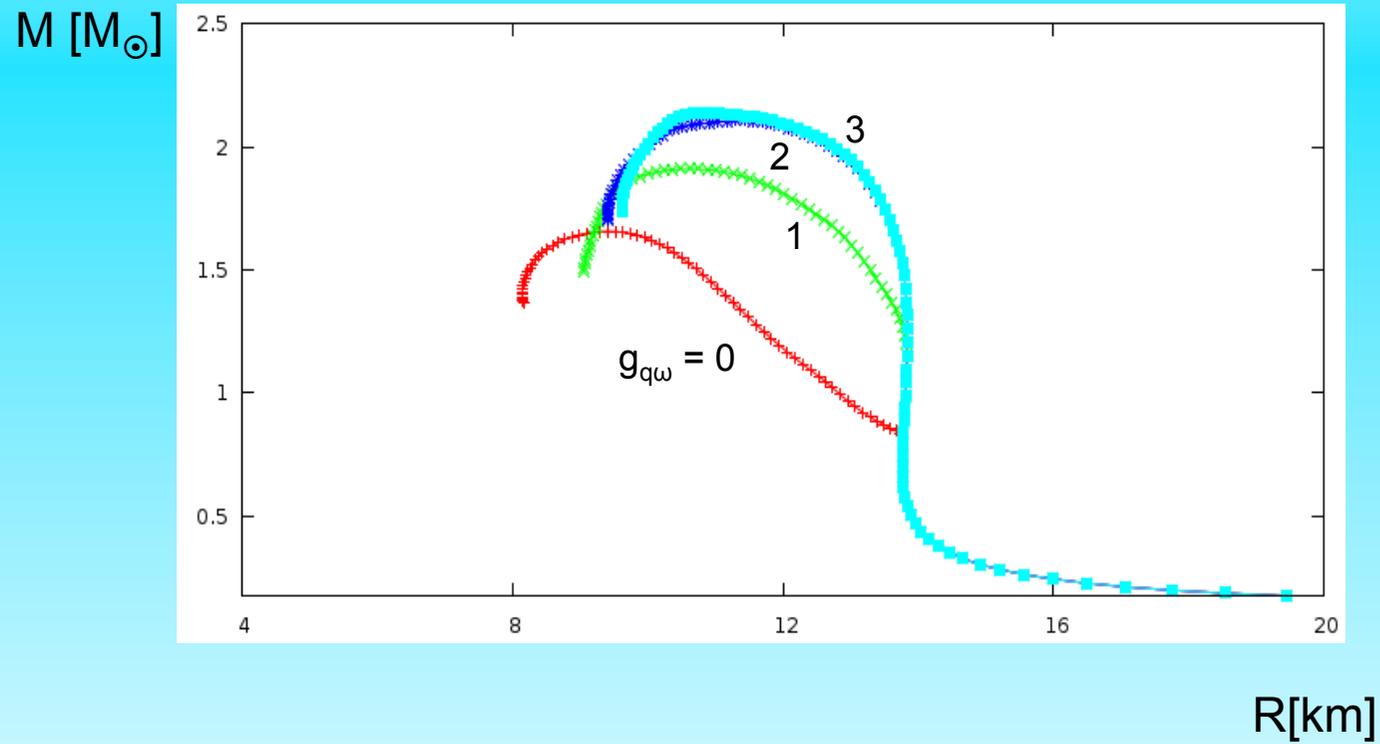


no  $\alpha_s$



Negreiros, Dexheimer, SWS, PRC 035805 (2012)

Including vector interaction for quarks



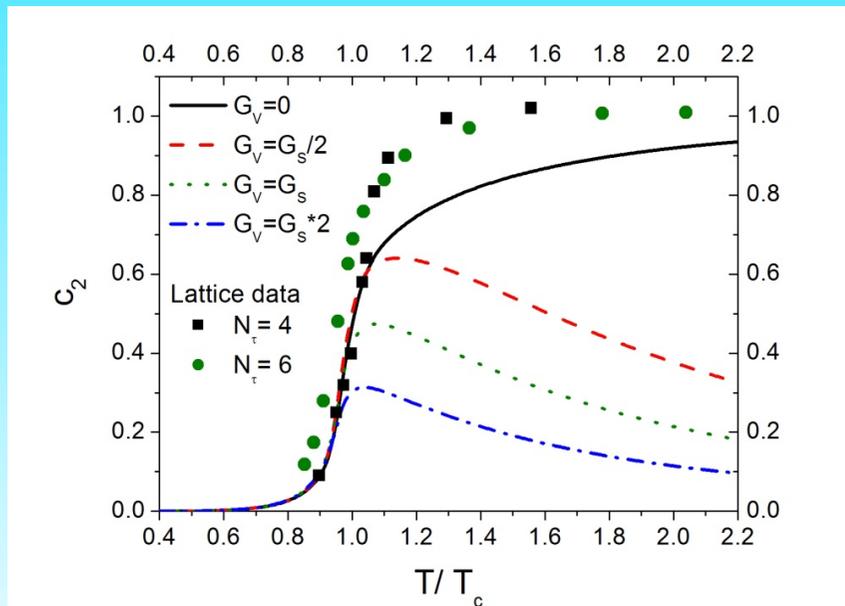
increase  $M / R$ , potential problems at  $\mu = 0$

# Susceptibility $c_2$ in PNJL and QH model for different quark vector interactions

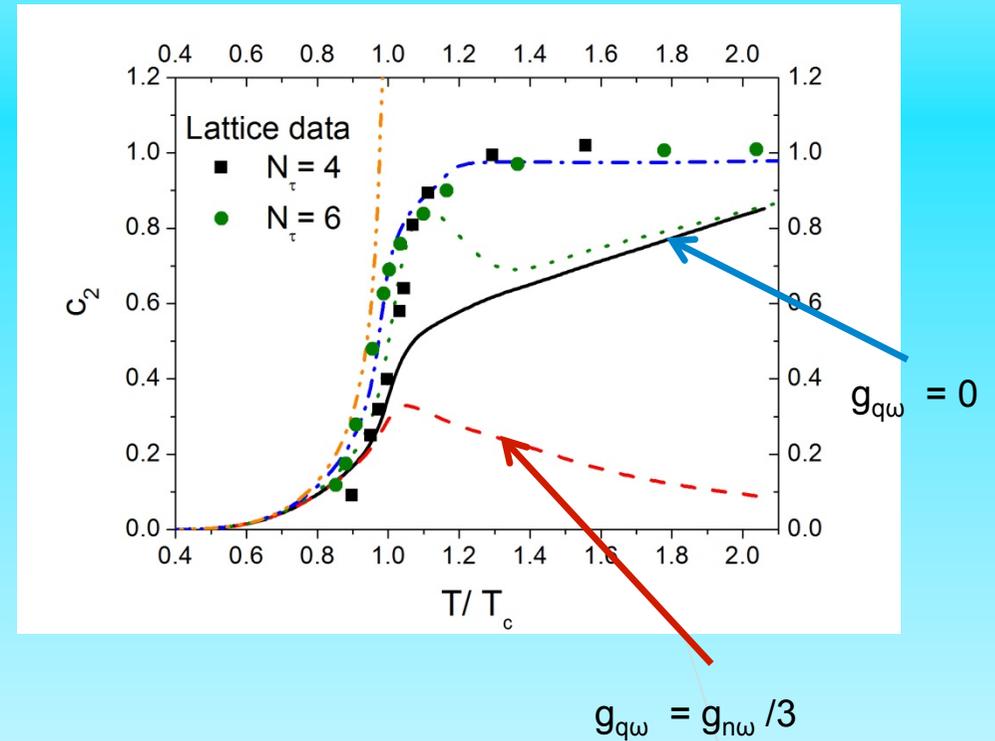
$$P(T, \mu) = P(T) + c_2(T) \mu^2 T^2 + \dots$$

small quark vector repulsion !!

PNJL

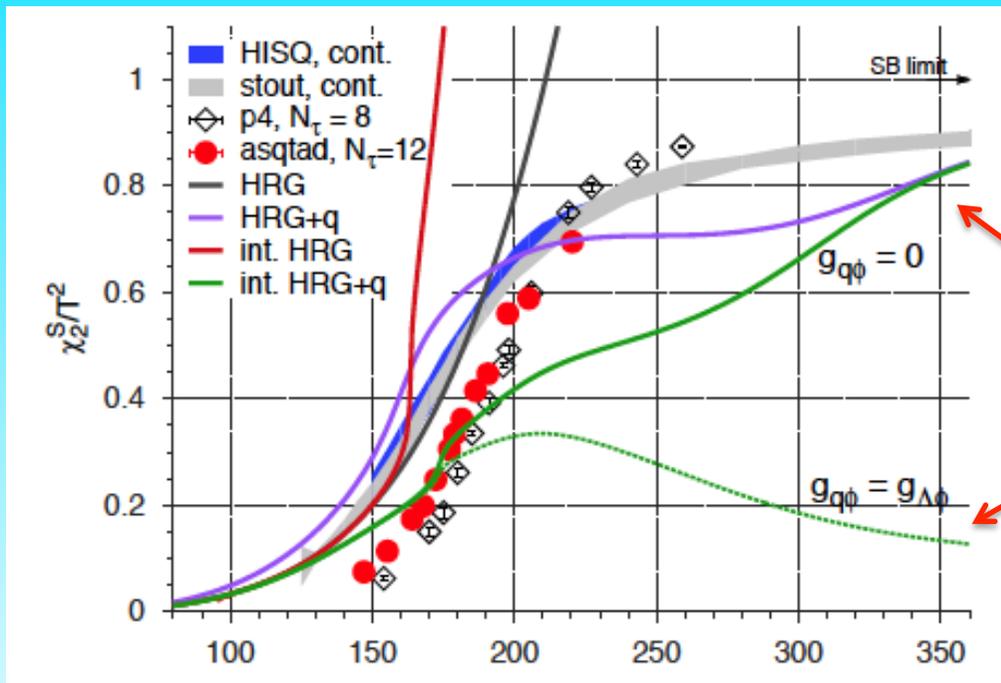


QH



analogous behaviour of strange susceptibility

$$\chi_s = T^2 d^2( P/T^4 ) / (d \mu_s)^2 |_{\mu_B, \mu_S = 0}$$

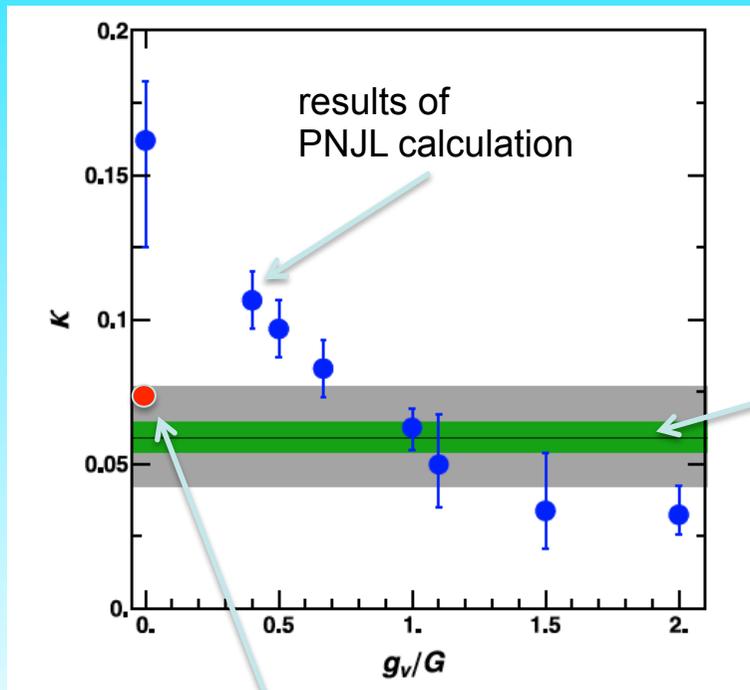


off  
strange quark repulsion

on

signs of vector repulsion in  $T_c(\mu)$  behavior

curvature of transition line  $\kappa = -T_c \left. \frac{dT_c(\mu)}{d\mu^2} \right|_{\mu=0}$



*this calculation*

Plot taken from  
Bratovic, Hatusda, Weise, PLB 719, 131 (2013)

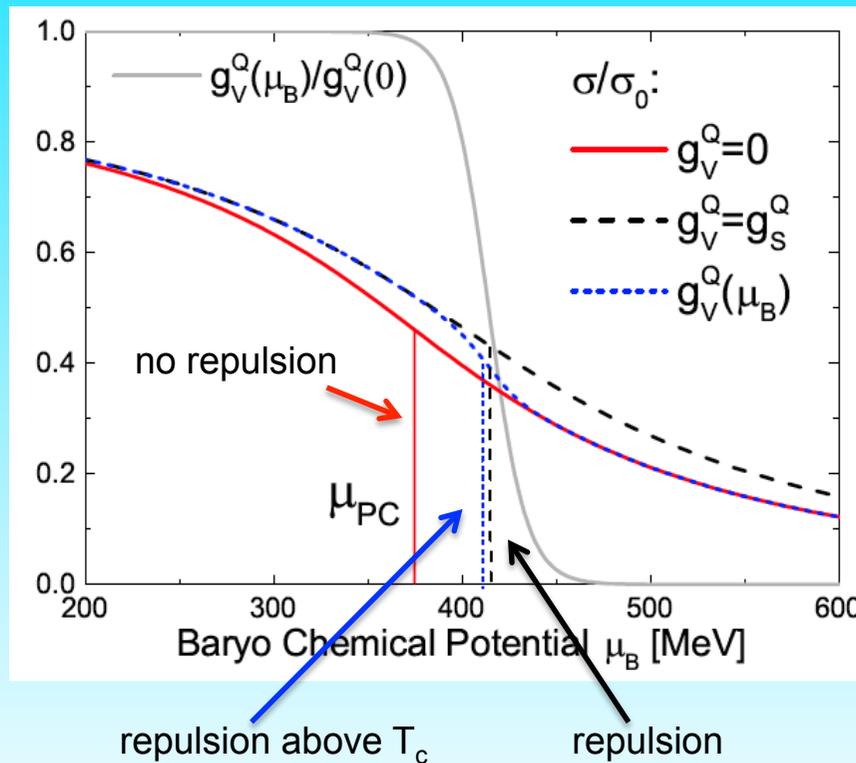
Lattice results Kacmarek et al  
PRD 83, 014504 (2011)

large quark vector repulsion?

What is the source for the shift in  $T_c$  ?

Simple exercise – switch off vector coupling above  $T_c$

scalar condensate as function of T



Conclusion: transition temperature governed by low-T behaviour

$T = 140$  MeV

## Include magnetic field effects

observed surface fields up to  $\sim 10^{15}$  G - magnetars

might be significantly larger in the interior of the star

Landau levels:

$$E_{i\nu s}^* = \sqrt{k_{z_i}^2 + \left( \sqrt{m_i^{*2} + 2\nu|q_i|B^*} - s_i\kappa_i B^* \right)^2}$$

anomalous magnetic moment

simple parameterization of the field

$$B^*(\mu_B) = B_{surf} + B_c \left[ 1 - e^{b \frac{(\mu_B - 938)^a}{938}} \right]$$

## Impact on M(R) diagram for neutron/hybrid star

$$T_{\mu\nu}^F = \text{diag}(B^2, B^2, B^2, -B^2) / (8\pi^2)$$

energy-momentum tensor not isotropic  
consistent modeling of star needed

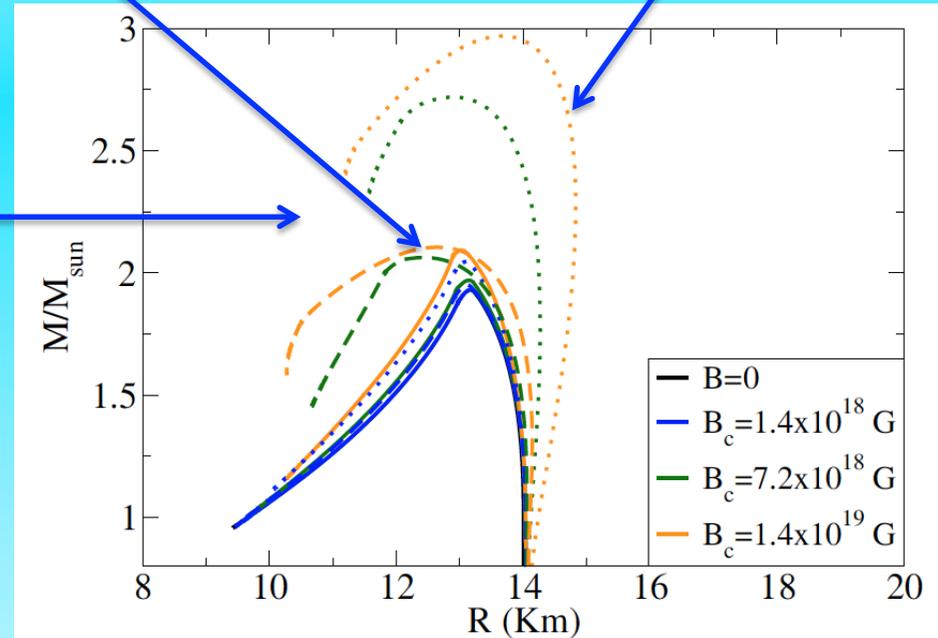
2d calculations:

Bocquet et al. A&A 301, 757 (1995)

Cardall et al. APJ 554, 322 (2001)

add average pressure

add isotropic pressure



for “interesting” field strengths field energy dominates changes in the EOS

include deformation in approximate fashion

include effect of magnetic field in GR

$$\begin{aligned}\epsilon &= \epsilon_m + \frac{B^2}{8\pi} \\ P_{\perp} &= P_m + \frac{B^2}{8\pi} \\ P_{\parallel} &= P_m - \frac{B^2}{8\pi}.\end{aligned}$$

assume a dipole field

$$P = P_m + \frac{B^2}{8\pi}(1 - 2\cos^2\theta)$$

$$P = P_m + [p_0 + p_2 P_2(\cos\theta)].$$

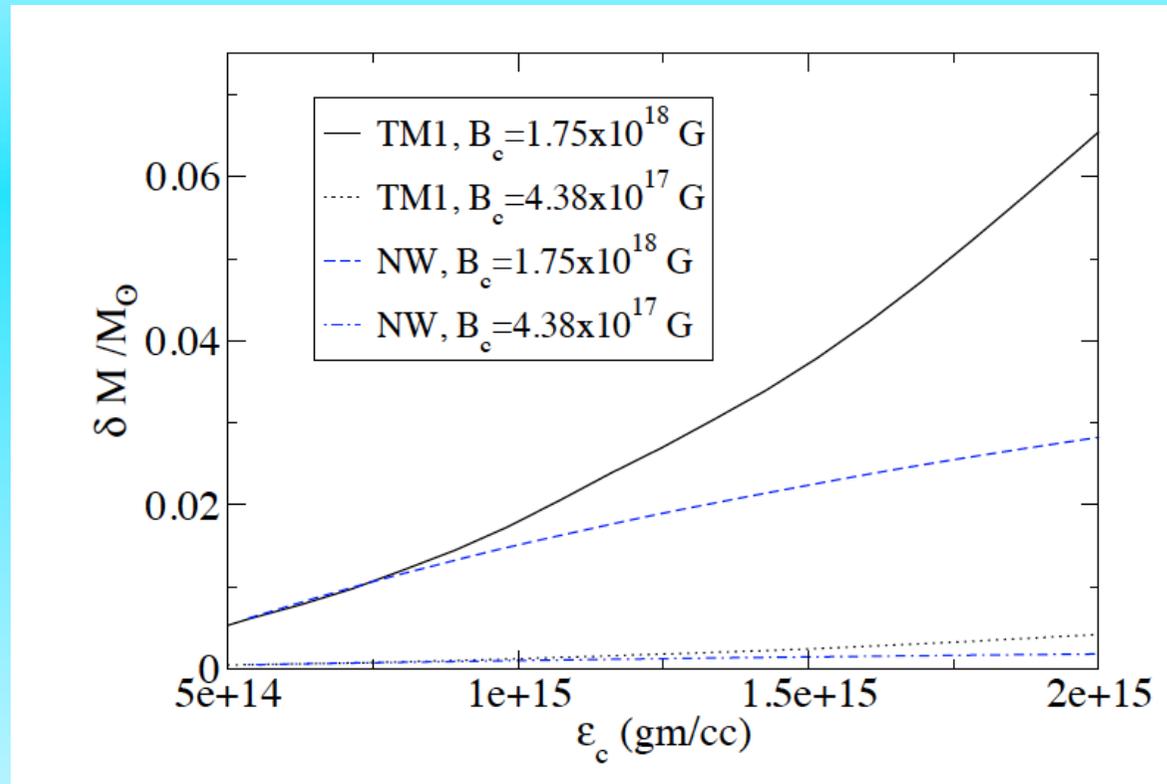
expand metric into multipoles

analogous to Hartle/rotation

$$\begin{aligned}ds^2 &= -e^{\nu(r)}[1 + 2(h_0(r) + h_2(r)P_2(\cos\theta))]dt^2 \\ &+ e^{\lambda(r)}\left[1 + \frac{e^{\lambda(r)}}{r}(m_0(r) + m_2(r)P_2(\cos\theta))\right]dr^2 \\ &+ r^2[1 + 2k_2(r)P_2(\cos\theta)](d\theta^2 + \sin^2\theta d\phi^2),\end{aligned}$$

*Hartle, APJ, 1967*

## Mass shift for different values of central energy density and magnetic field

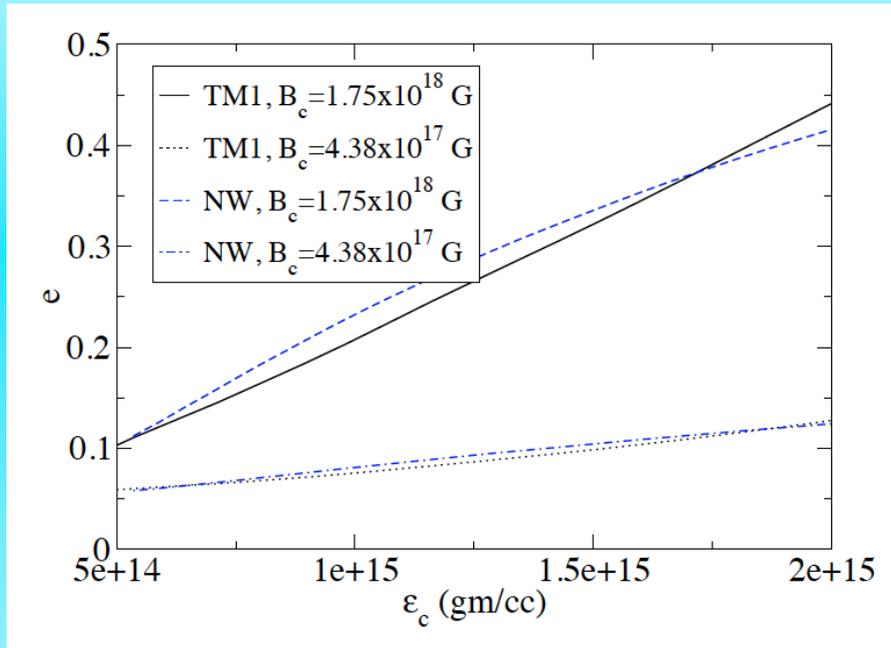


NW , TM1 hard / soft equation of state

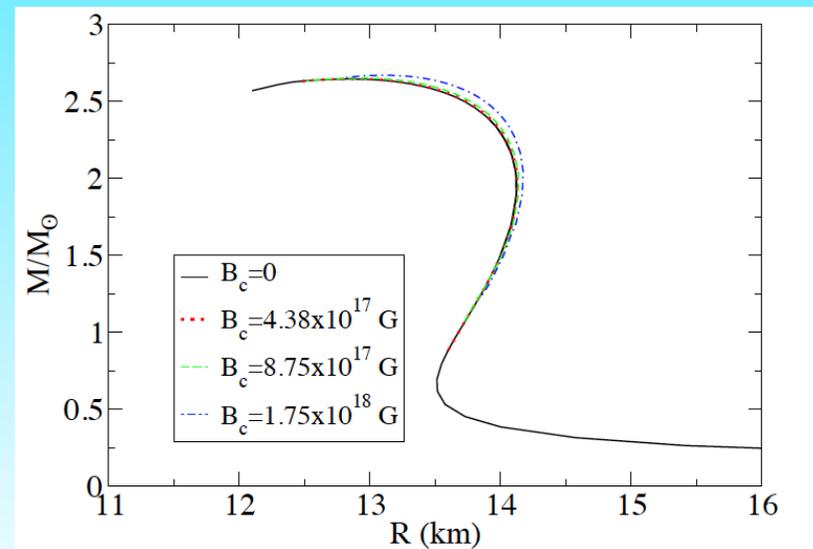
assume a profile of the magnetic field

$$B(n_b) = B_s + B_0 \left\{ 1 - e^{-\alpha \left( \frac{n_b}{n_0} \right)^\gamma} \right\}$$

$$\text{eccentricity} = (1 - R_p^2 / R_e^2)^{1/2}$$



M(R) change due to B field

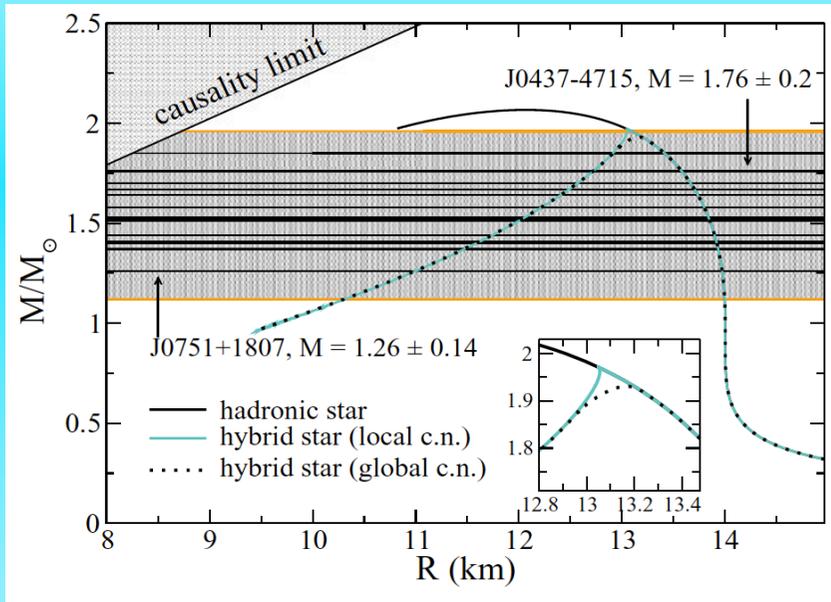


## Conclusions, Outlook

- observations lead to significant modeling constraints
- hadronic and quark-hadron approaches
- SU(3) schemes – no vector strangeness in nuclei
- limits on couplings of higher resonances
- phase structure with critical endpoint?
- constraints from lattice QCD results

***Many thanks to the organizers!***

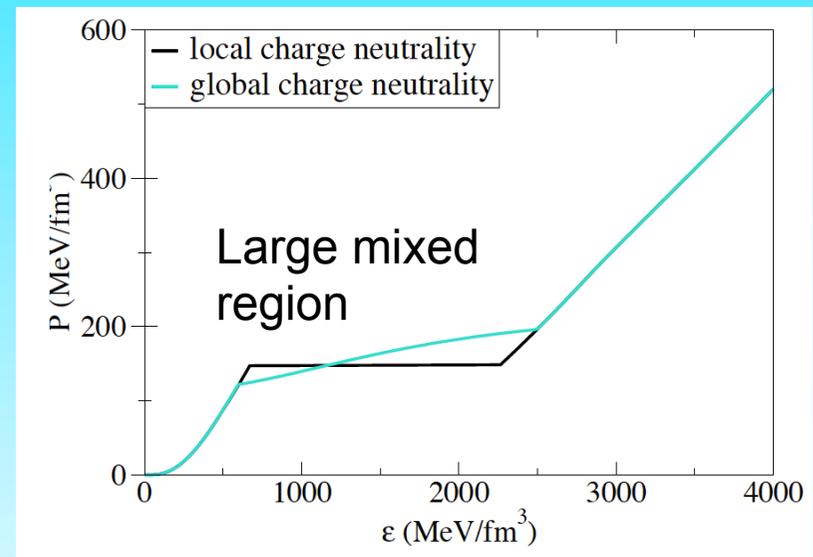
# Hybrid Stars



Maxwell / Gibbs construction for local / global charge neutrality

## M-R diagram in QH model

baryonic star with a 2km core of quarks

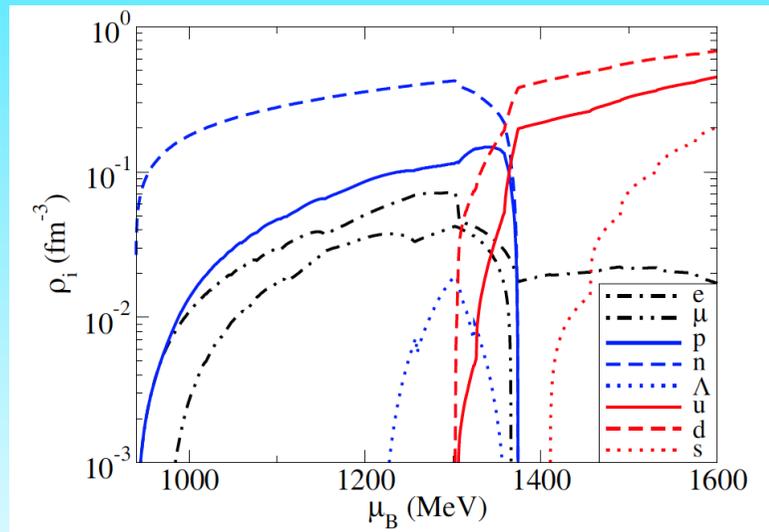


## QH model with PT

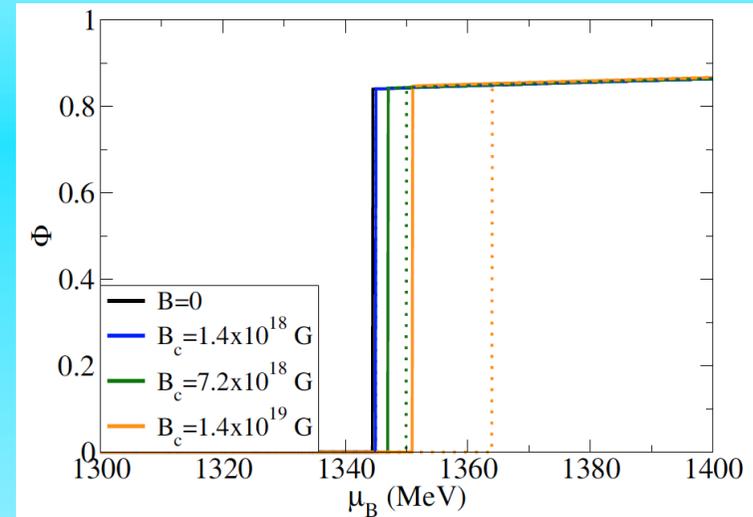
$$e B_{\text{cr}} = m_e^2, \quad B_{\text{cr}} = 4.4 \cdot 10^{13} \text{ G}$$

$$(1.5 \cdot 10^{20} \text{ G})$$

particle densities  $B_c = 7.2 \times 10^{18} \text{ G}$



Polyakov loop and 1<sup>st</sup> order transition



PT gets shifted somewhat to higher  $\mu$