Clusterized nuclear matter in PNS crust and the E_{sym}

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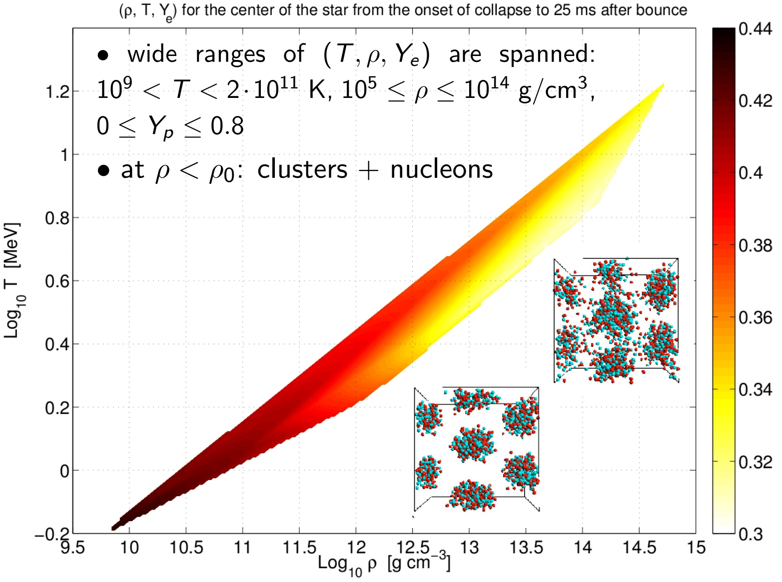
Overview

- The model
- Predictions along β-eq. path
- In medium effects / Cluster definition / Model improvements
- Symmetry energy
- Conclusions

Nuclear matter during CCSN

A. Fantina, PhD thesis;
Watanabe et al.. PRC69. 055805

- wide ranges of (T, ρ, Y_e) are spanned: $10^9 < T < 2 \cdot 10^{11}$ K, $10^5 \leq \rho \leq 10^{14}$ g/cm^3, $0 \leq Y_p \leq 0.8$
- at $\rho < \rho_0$: clusters + nucleons



The NSE model

- Phenomenological model: N(uclear) S(tatistical) E(quilibrium):

$$Z_{baryon} = Z_{cl} Z_{unbound}$$

Cluster gas (Fisher hypothesis):

$$Z_{cl} = \sum_{n_A} \prod_{A>1} \frac{\omega_{A,\mu_3}^{n_A}}{n_A!}$$

$$\omega_{A,\mu_3} = \frac{1}{2}\sqrt{2\pi\sigma_A^2} V_F \left(\frac{2\pi A m_0}{\beta h^2}\right)^{3/2} \exp -\beta \left(f_{A,\bar{I}} - \mu_3 \bar{I}\right)$$

Unbound nucleon gas: mean-field model

$$Z_{unbound} = Z_0^{(HM)} \exp \beta \left(\langle \hat{W} \rangle_0 - \langle \hat{H} \rangle_0\right)$$

Souza et al., Ap.J. (2009), Botvina & Mishustin, Nucl. Phys. A (2010); Hempel & Schaffner-Bielich, Nucl. Phys. A (2010);

Blinnikov et al., AA (2011), AR & Gulminelli, Phys. Rev. C (2010)

- Pros & cons: geometrical assumptions, coherence, in-medium-effects

The NSE model with coherent effective interactions

- Phenomenological model: N(uclear) S(tatistical) E(quilibrium):

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Cluster gas (Fisher hypothesis):

$$Z_{cl} = \sum_{n_A} \prod_{A>1} \frac{\omega_{A,\mu_3}^{n_A}}{n_A!}$$

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- Pros & cons: geometrical assumptions, coherence, in-medium-effects

- **Improved version** [AR, Gulminelli, Aymard, Eur.Phys.J (2014)]:

clusters and gas consistently described by the same effective interaction;

LDM parameters fitted on Skyrme [Danielewicz & Lee, Nucl. Phys. A (2009)]

- NM is well constrained at $(\rho_0, \delta = 0)$
- **Uncertainity:** iso-vector properties of the EOS

The EOS input & symmetry energy

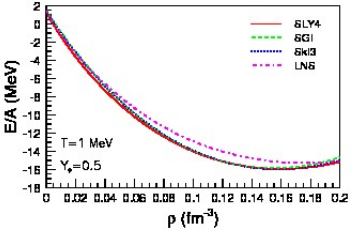
$$E(\rho,\delta) = E_0(\rho,\delta=0) + E_{sym}(\rho,\delta=0)\delta^2 + \mathcal{O}(\delta^4)$$

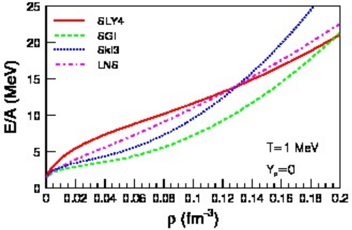
$$E_{sym}(\rho,0) = \frac{1}{2!}\frac{\partial^2 E(\rho,\delta)}{\partial\delta^2}\Big|_{\delta=0}$$
$$= E_{sym}(\rho_0,\delta=0) + L\chi + \frac{K_{sym}}{2!}\chi^2 + \mathcal{O}(\chi^3)$$

NN-potential	ρ_0 (fm^{-3})	K (MeV)	L (MeV)	K_{sym} (MeV)
SLY4	0.1595	230.0	46.0	-119.8
SGI	0.1544	261.8	63.9	-52.0
SkI3	0.1577	258.2	100.5	73.0
LNS	0.1746	210.8	61.5	-127.4

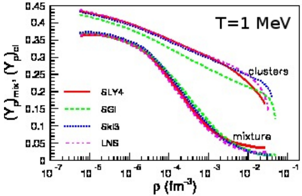
- similar iso-scalar properties
- different iso-vector properties

asym. matter is sensitive to iso-vector EoS

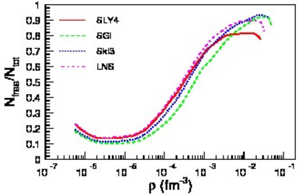




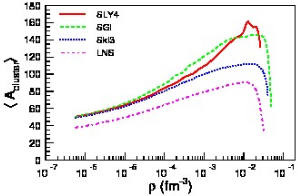
Predictions at $\beta-$equilibrium: composition



$(Y_p)_{cl}$, $(Y_p)_{total}$ decrease monotonically
$(Y_p)_{total}$ is universal



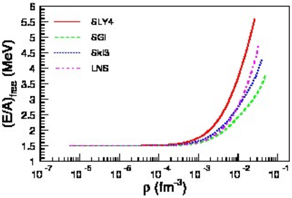
$\lim_{\rho\to 0} N_{free}/N_{tot} = 0$
$\lim_{\rho\to \rho_0} N_{free}/N_{tot} = 1$
crust-core transition
EoS-dependence



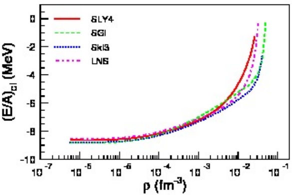
$\rho \ll \rho_0$: $\langle A_{cl} \rangle$ increases with ρ
$\rho \to \rho_0$: $\langle A_{cl} \rangle \to 0$
strong EoS-dependence

AR, Gulminelli, Aymard, Eur.Phys.J (2014)

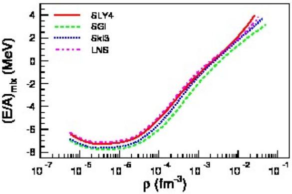
Predictions at $\beta-$equilibrium: energetics



$\lim_{\rho\to 0}(E/A)_{free} \to 0$
$\lim_{\rho\to \rho_0}(E/A)_{free}$ EOS-dependent



$\lim_{\rho\to 0}(E/A)_{cl} \to -8.5$
$\lim_{\rho\to \rho_0}(E/A)_{cl}$ EOS-dependent



the effective dens. $\neq$ average dens.
moderate EoS sensitivity
uncertainities due to nm EOS are washed out

AR, Gulminelli, Aymard, Eur.Phys.J (2014)

In-medium corrections

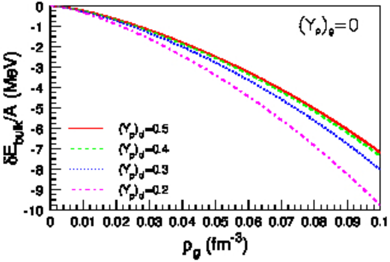
- clusters are treated as hard sphere

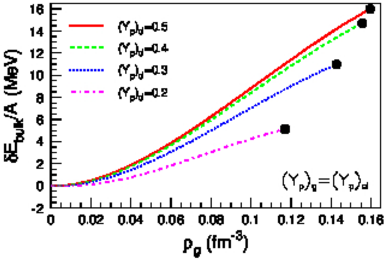
$$E_{WS} = E_{A,I}(\rho_e) + \epsilon(\rho_g, \delta_g)\left(V_{WS} - \frac{A}{\rho_0(A,I)}\right)$$

- clusters can be defined by the en. dens. functional

$$E_{WS} = \int_{V_{WS}} \epsilon\left[\{\rho_i(r), \tau_i(r)\}\right] d^3r$$
$$= E_{A,I}(\rho_e) + \delta E_{bulk} + \delta E_{surf} + \epsilon(\rho_{gn}, \rho_{gp}) V_{WS}$$

$$\delta E_{bulk} = -\epsilon(\rho_g, \delta_g)\frac{A}{\rho_0(A,I)}$$





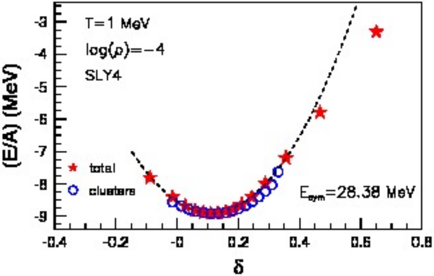
- δE_{surf} *see F. Aymard talk on Wednesday*

E_{sym} in unhomogeneous matter

Test of the parabolic approximation

$$(E/A)_{tot} = e_{cl}(<A_{cl}>, <\delta_{cl}>, \rho_g, \delta_g)x_{cl} + e_g(\rho_g, \delta_g)(1 - x_{cl})$$

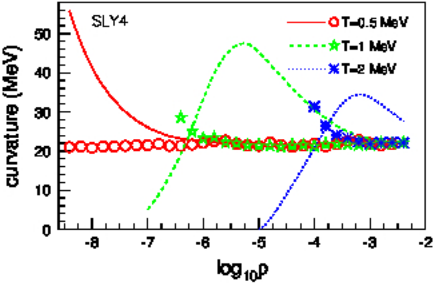
$$\delta = x_{cl} < \delta_{cl} > +(1 - x_{cl})\delta_g$$



- charge inv. is broken (Coulomb effect) $e_{sym}^{(1)} = \frac{1}{2}\frac{\partial^2 e}{\partial \delta^2}|_{\delta_0(\rho)}$
- $e_{sym}^{(1)} \neq e(\rho, \delta = 1) - e(\rho, \delta = 0)$
- higher order correlations

$E_{sym}(T, \rho)$

$$e_{sym}^{(1)} = \frac{1}{2} \frac{\partial^2 e}{\partial \delta^2}\Big|_{\delta=\delta_0(\rho)}$$



- $\lim_{x_{cl} \to 0} e_{sym}^{(1)} \approx e_{sym}^{g} \approx 0$ (e.g. $T = 1$ MeV, $\rho < 10^{-6.5}$ fm^{-3})
- $\lim_{x_{cl} \to 1} e_{sym}^{(1)} \approx e_{sym}^{cl}$ (e.g. $T = 1$ MeV, $\rho > 10^{-4}$ fm^{-3})
- $x_{cl} \approx x_g$, $e_{sym}^{(1)} >> e_{sym}(\rho_0)$ (e.g. $T = 1$ MeV, $10^{-6.5} < \rho < 10^{-4}$ fm^{-3})

AR, Gulminelli, Aymard, Eur.Phys.J (2014)

Conclusions

• an improved NSE where clusterized and unbound components are described by the same energy functional is proposed

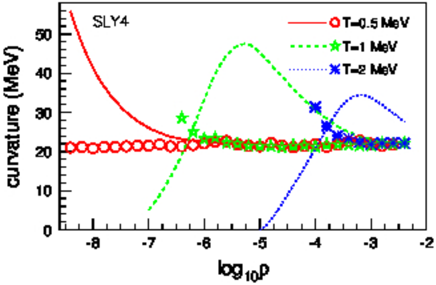
• PNS composition and energetics is investigated along β-eq. path for different effective interactions

• because clusterization, the explored effective density differs from the average density

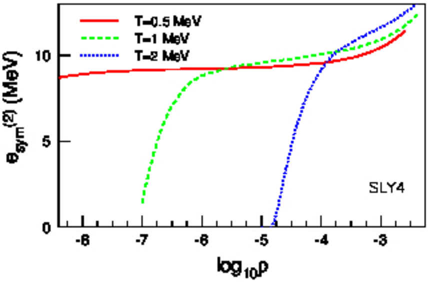
• the isovector properties dependence of stellar matter is reduced with respect to the one of homogeneous matter

E_{sym} meaning

$$e_{sym}^{(1)} = \frac{1}{2}\frac{\partial^2 e}{\partial \delta^2}\Big|_{\delta=\delta_0(\rho)}$$



$$e_{sym}^{(2)} = e(\rho,\delta=1) - e(\rho,\delta=0)$$



$e_{sym}^{(1)}$ - dependence on the iso-vector properties of NM