g-modes in superfluid neutron stars

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Outline

- Introduction,
g-modes in non-superfluid NS matter

- thermal and composition g-modes
in superfluid NS matter
(original results)

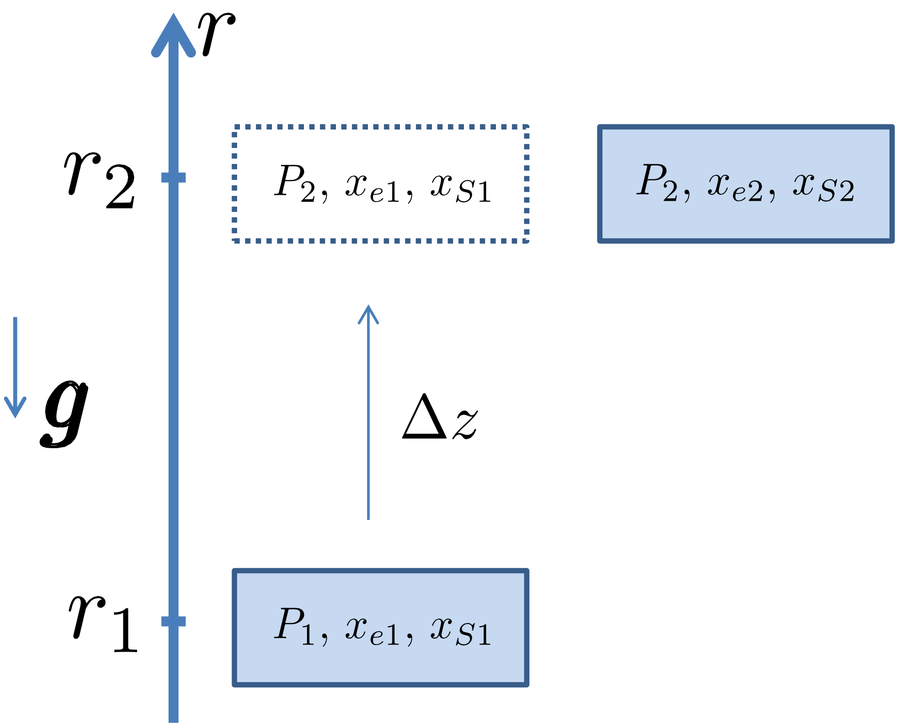
- Applications and Conclusions

g-modes in non-superfluid npe NS matter

relativistic inertial mass density $w \equiv \varepsilon + P = w(n_n,\, n_p,\, n_e, T)$

$$n_p = n_e \Rightarrow \qquad w = w(P,\, x_e,\, x_S)$$

$$x_e = \frac{n_e}{n_b} \qquad x_S = \frac{S}{n_b}$$



$$\nabla P + \frac{w}{c^2} \nabla \phi = 0$$

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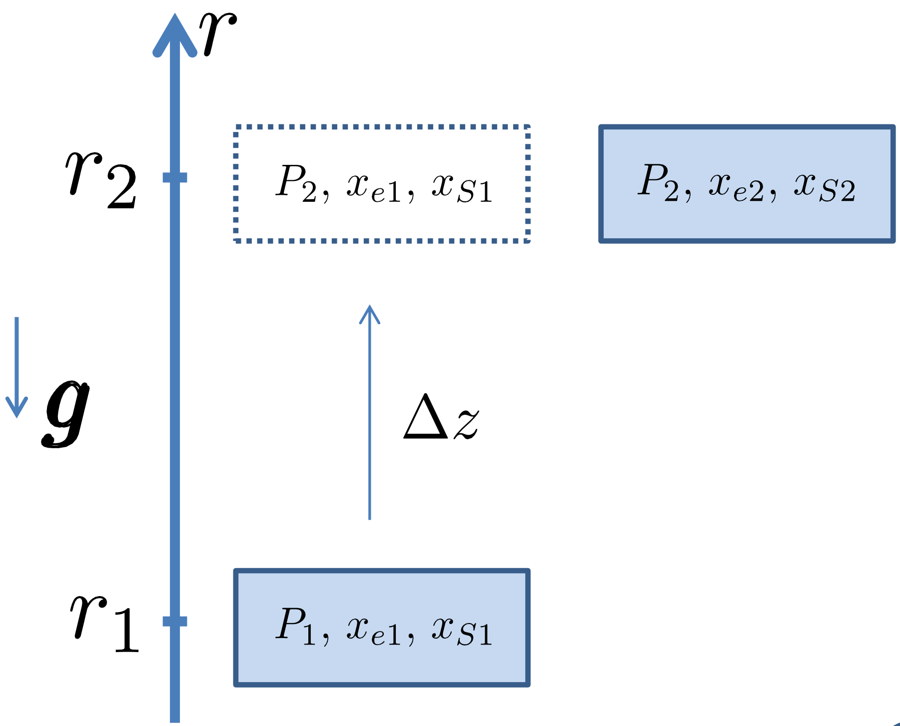

$$\nabla P + \frac{w}{c^2}\nabla\phi = 0$$

restoring force arises if:

$$w(P_2,\, x_{e1},\, x_{S1}) > w(P_2,\, x_{e2},\, x_{S2})$$

$$\boxed{\frac{\partial w(P,\, x_e,\, x_S)}{\partial x_e}\frac{dx_e}{dr} + \frac{\partial w(P,\, x_e,\, x_S)}{\partial x_S}\frac{dx_S}{dr} < 0}$$

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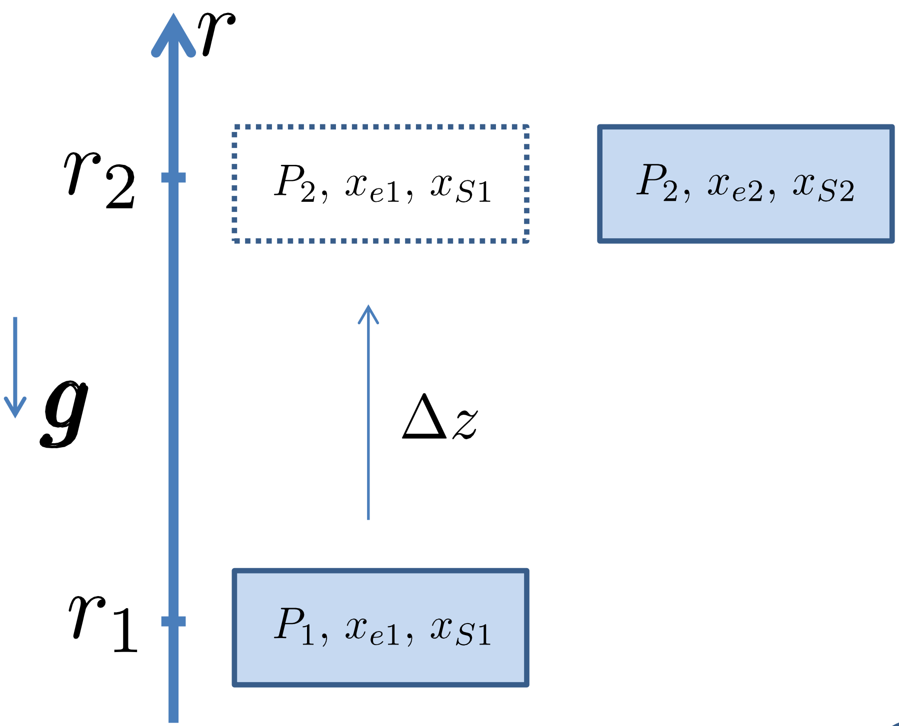

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restoring force arises if:

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degenerate matter

$$\boxed{\frac{\partial w(P,\, x_e,\, x_S)}{\partial x_e}\frac{dx_e}{dr} + \cancel{\frac{\partial w(P,\, x_e,\, x_S)}{\partial x_S}\frac{dx_S}{dr}} < 0}$$

$$\boxed{\frac{\partial w(P,\, x_e)}{\partial x_e}\, \frac{dx_e}{dr} < 0}$$

This condition is always satisfied
in beta equilibrated ($\delta\mu \equiv \mu_n - \mu_p - \mu_e = 0$)
degenerate npe-matter.

Buoyancy is driven by matter composition gradient,
no dependence on temperature

$\Downarrow$

temperature independent composition g-modes!

Reisenegger & Goldreich, ApJ (1992)

g-modes in superfluid npe-matter

In contrast to normal matter, in superfluid matter two independent velocity fields can coexist:

$\boldsymbol{V}_{\mathrm{s}n}$ velocity of superfluid neutrons

$\boldsymbol{V}_{q}$ velocity of 'normal' particles

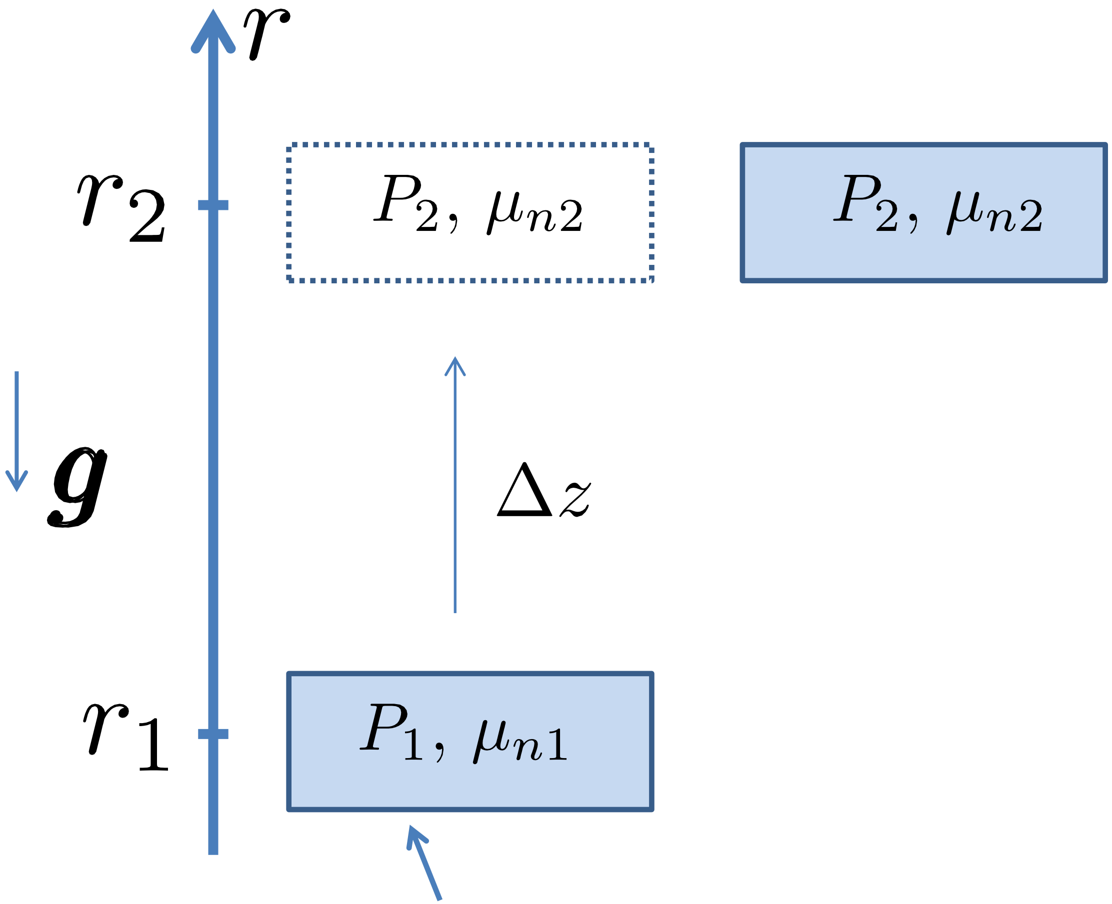
$\boldsymbol{\nabla}\mu_n, \quad \boldsymbol{\nabla}P$ generate $\dfrac{\partial \boldsymbol{V}_{\mathrm{s}n}}{\partial t}, \ \dfrac{\partial \boldsymbol{V}_{\mathrm{q}}}{\partial t}$

=> **<u>two</u>** conditions of hydrostatic equilibrium:

$$\nabla P + \frac{w}{c^2}\nabla\phi = 0$$ the same as in non-superfluid matter

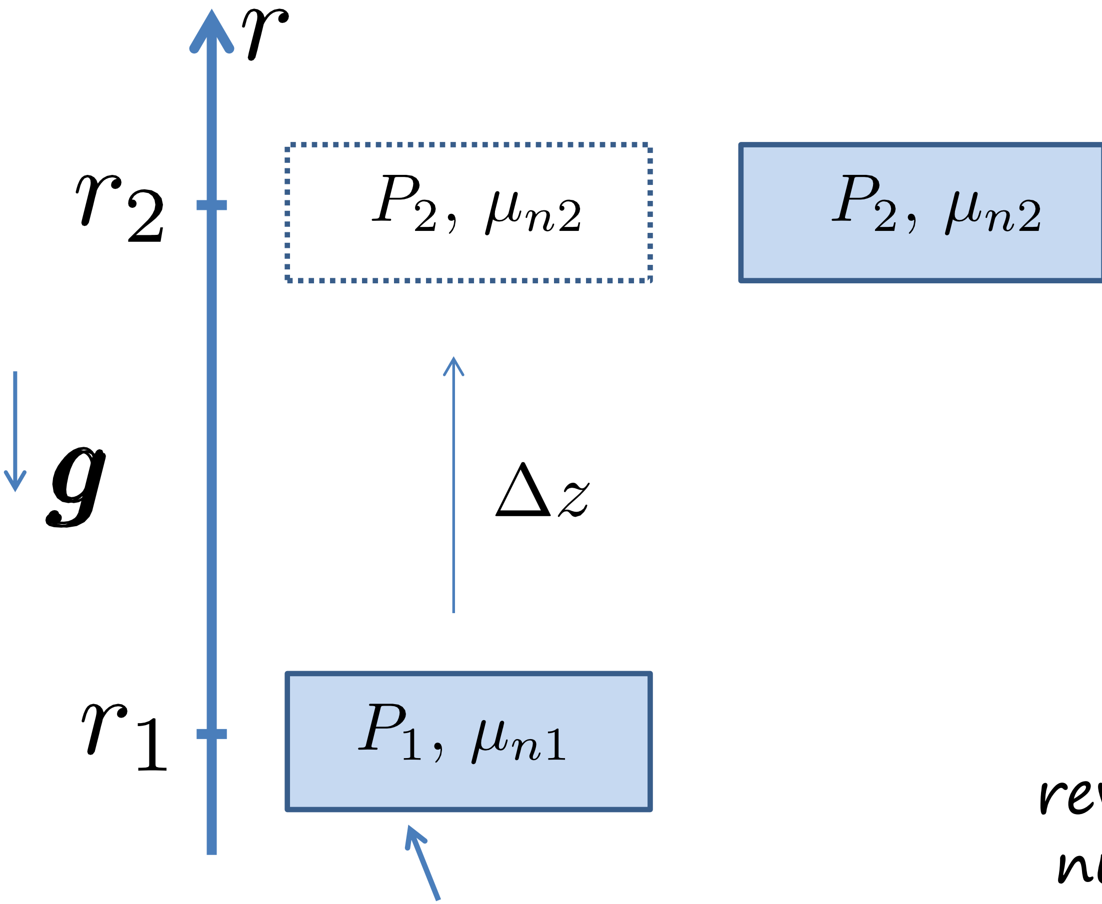
$$\nabla\mu_n + \frac{\mu_n}{c^2}\nabla\phi = 0$$ additional condition, valid in superfluid matter

$T = 0$ => $w = w(P,\ \mu_n)$



the imaginary volume element
is “sticked” to normal matter

$T = 0$ => $w = w(P,\, \mu_n)$



the imaginary volume element
is "sticked" to normal matter

no restoring force!

⇓

no g-modes!

revealed in a number of
numerical calculations:

U. Lee A&A (1995)
Andersson, Comer MNRAS (2001)
Andersson, Comer, Langlois PRD (2002)
Prix, Rieutord, A&A (2002)

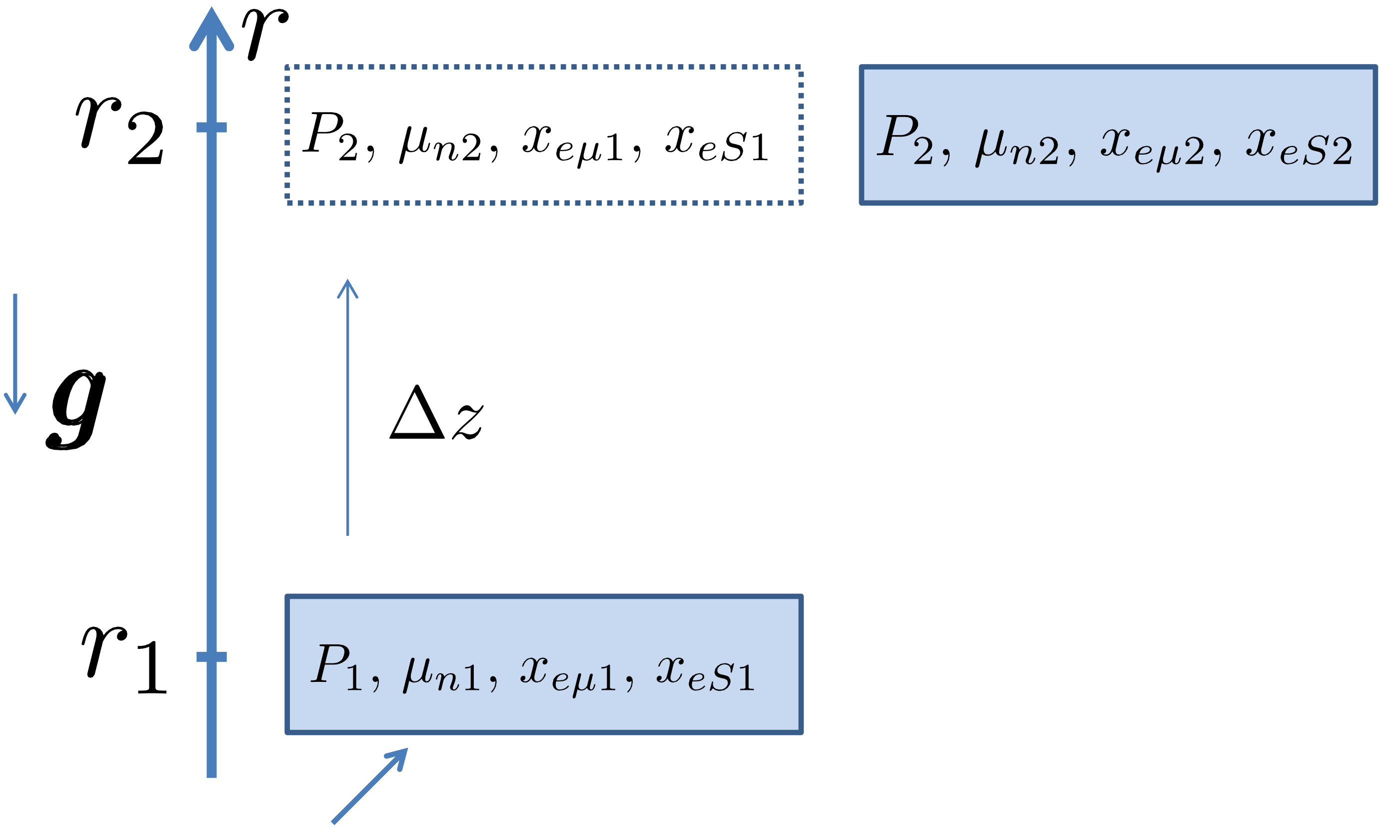
But account for temperature

or/and

admixture of additional particles (particularly muons)

leads to appearance of g-modes.

$$w = w(P,\, \mu_n,\, x_{e\mu},\, x_{eS}) \qquad x_{eS} = \frac{S}{n_e} \qquad x_{e\mu} = \frac{n_\mu}{n_e}$$



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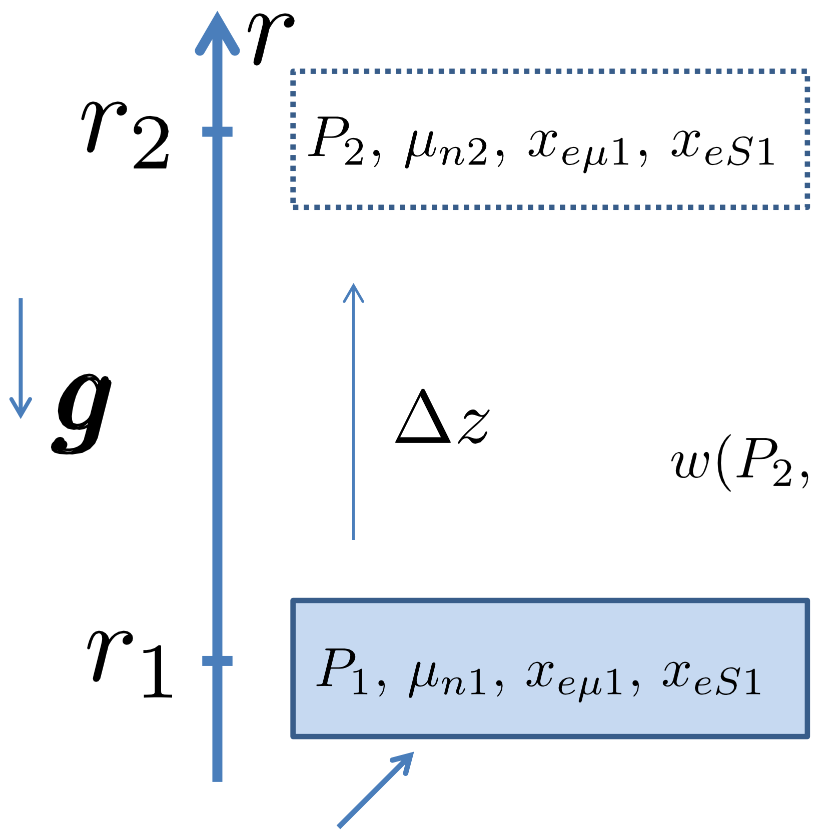

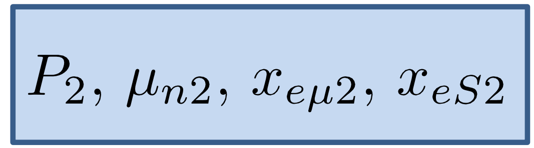

the imaginary volume element is "sticked" to normal matter

Restoring force arises if:

$$w(P_2,\, \mu_{n2},\, x_{e\mu1},\, x_{eS1}) > w(P_2,\, \mu_{n2},\, x_{e\mu2},\, x_{eS2})$$

$$\boxed{\frac{\partial w(P,\, \mu_n,\, x_{e\mu},\, x_{eS})}{\partial x_{e\mu}} \frac{dx_{e\mu}}{dr} + \frac{\partial w(P,\, \mu_n,\, x_{e\mu},\, x_{eS})}{\partial x_{eS}} \frac{dx_{eS}}{dr} < 0}$$

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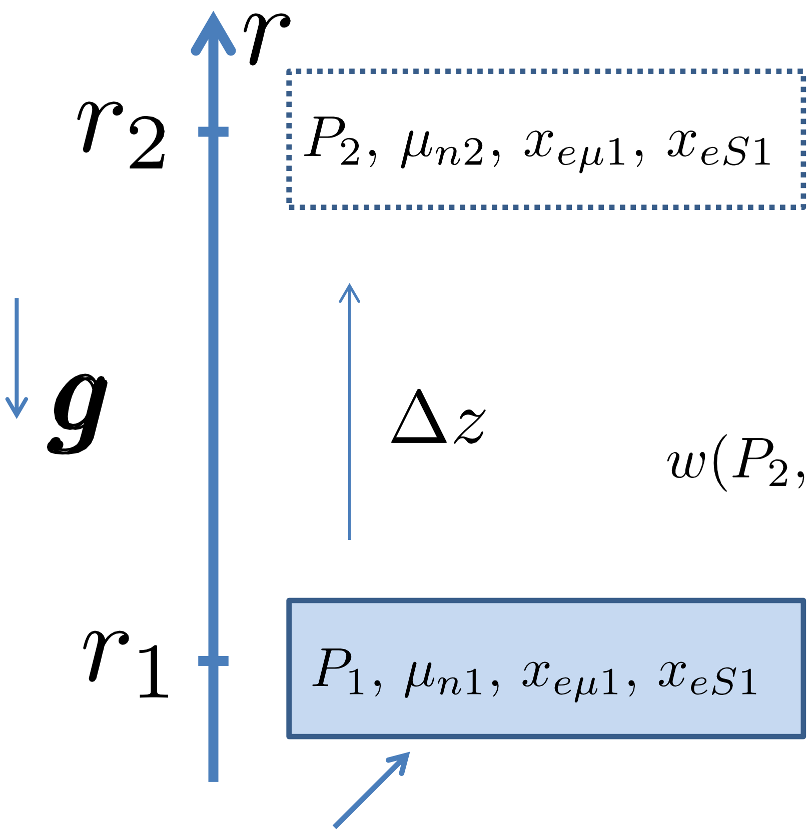


$P_2,\, \mu_{n2},\, x_{e\mu 2},\, x_{eS2}$

Restoring force arises if:

$$w(P_2,\, \mu_{n2},\, x_{e\mu 1},\, x_{eS1}) > w(P_2,\, \mu_{n2},\, x_{e\mu 2},\, x_{eS2})$$

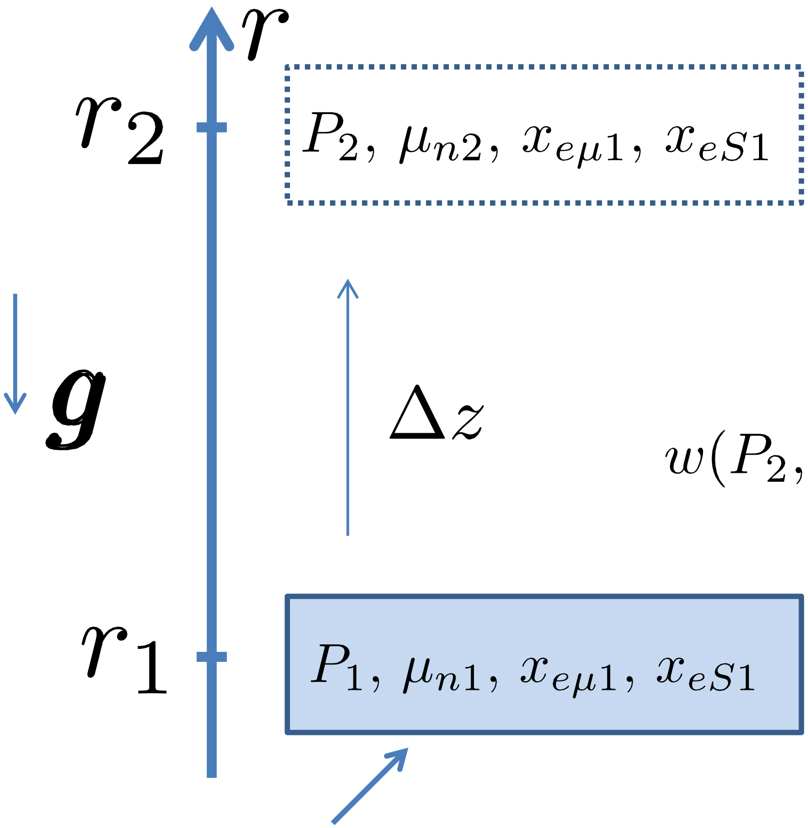
no muons

$$\cancel{\frac{\partial w(P,\, \mu_n,\, x_{e\mu},\, x_{eS})}{\partial x_{e\mu}} \frac{dx_{e\mu}}{dr}} + \\ + \frac{\partial w(P,\, \mu_n,\, x_{e\mu},\, x_{eS})}{\partial x_{eS}} \frac{dx_{eS}}{dr} < 0$$

the imaginary volume element is "sticked" to normal matter

thermal g-modes (Gusakov & Kantor, PRD, 2013)

$$w = w(P,\, \mu_n,\, x_{e\mu},\, x_{eS}) \qquad x_{eS} = \frac{S}{n_e} \qquad x_{e\mu} = \frac{n_\mu}{n_e}$$



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Restoring force arises if:

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$$\frac{\partial w(P,\, \mu_n,\, x_{e\mu},\, x_{eS})}{\partial x_{e\mu}} \frac{dx_{e\mu}}{dr} + \cancel{+ \frac{\partial w(P,\, \mu_n,\, x_{e\mu},\, x_{eS})}{\partial x_{eS}} \frac{dx_{eS}}{dr}} < 0$$

degenerate
matter

composition g-modes (submitted to MNRAS Letters)

We will consider non-rotating star with background metric:

$$-\mathrm{d}s^2 \equiv g_{\alpha\beta}\mathrm{d}x^\alpha \mathrm{d}x^\beta = -\mathrm{e}^\nu \mathrm{d}t^2 + \mathrm{e}^\lambda \mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2)$$

in Cowling approximation (no perturbations of $g_{\alpha\beta}$)

Consider small non-radial polar perturbations
of oscillation equations

$$\propto \exp(\mathrm{i}\omega t)\,\mathrm{Y}_{lm}(\theta,\,\varphi)$$

Local analysis of the equations describing linear oscillations of superfluid $npe\mu$ matter (short wave perturbations)

$$\boxed{\omega^2 = \mathcal{N}^2 \, \frac{l(l+1)\mathrm{e}^{\lambda}}{l(l+1)\mathrm{e}^{\lambda} + k^2 r^2}}$$

Brunt-Vaisala frequency squared

$$\boxed{\mathcal{N}^2 = -\frac{\nu'}{2} \frac{1}{\mu_\mathrm{n} n_\mathrm{b}} \mathrm{e}^{\nu-\lambda} \, Y^2 \, \frac{\partial w(P, \mu_\mathrm{n}, x_{\mathrm{e}\mu})}{\partial x_{\mathrm{e}\mu}} \; \nabla x_{\mathrm{e}\mu}}$$

Y is a function of superfluid density which depends on temperature

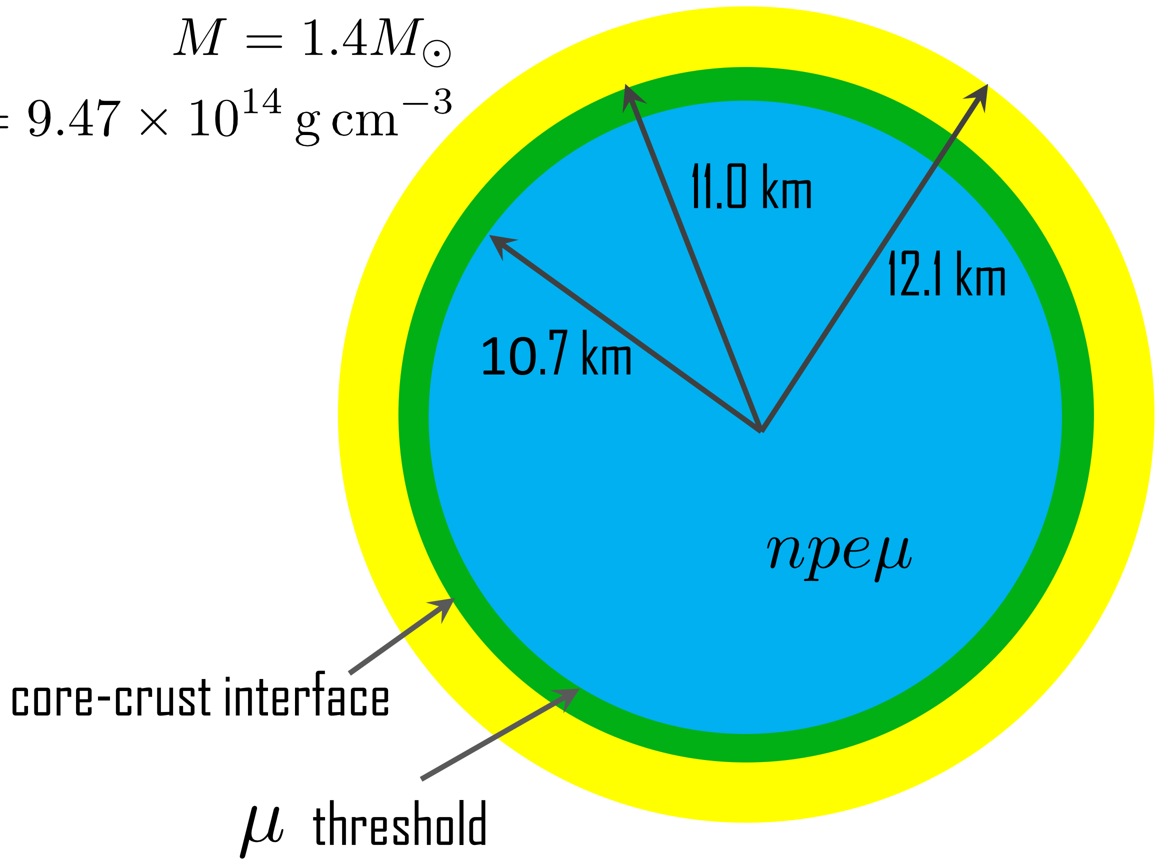
temperature dependent composition g-modes
do not vanish at T=0

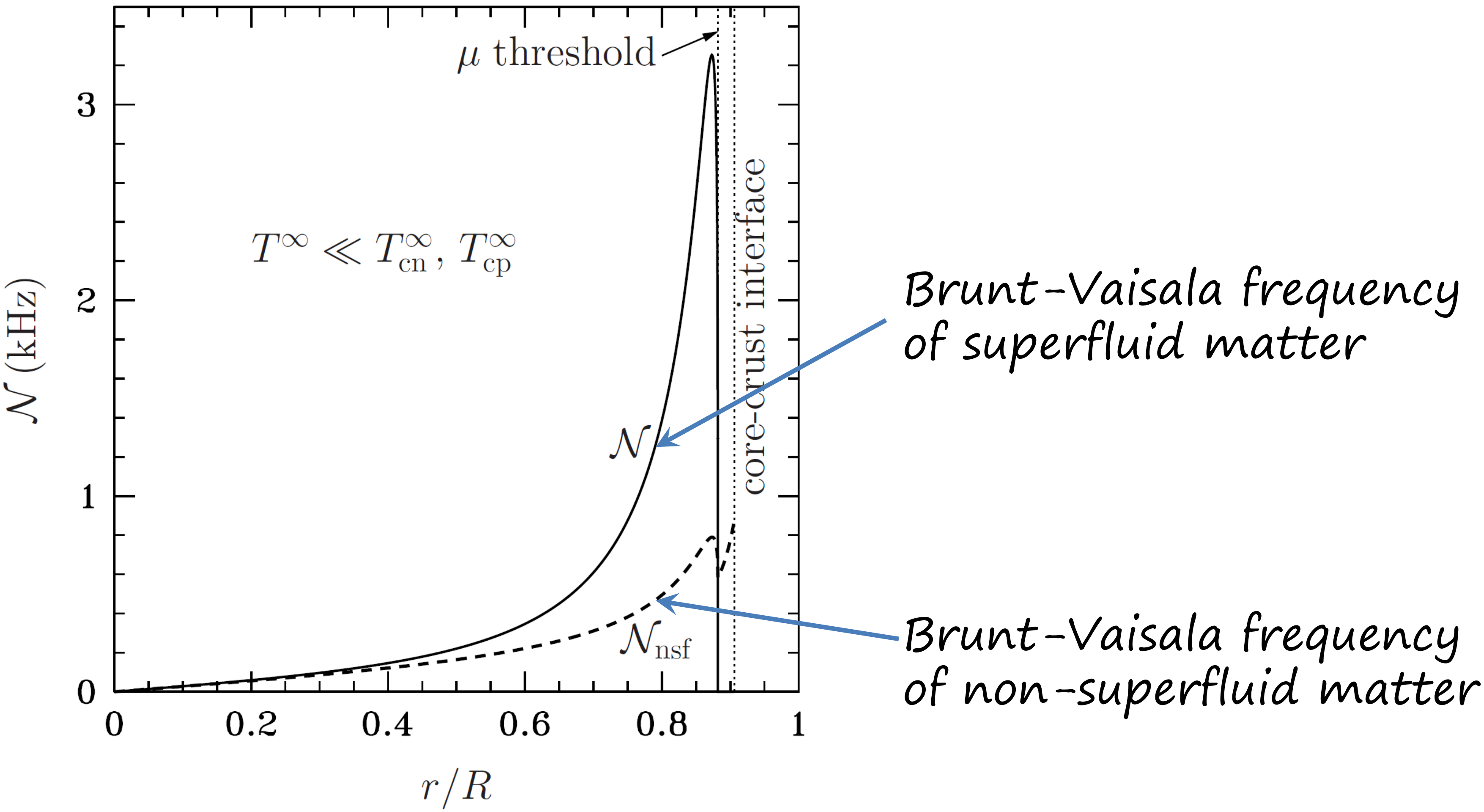
Numerical results

Heiselberg & Hjorth-Jensen (1999) parametrization of APR (Akmal, Pandharipande, and Ravenhall, 1998) equation of state (EOS) in the core.

$M = 1.4 M_\odot$

$\rho_\mathrm{c} = 9.47 \times 10^{14}\,\mathrm{g\,cm^{-3}}$

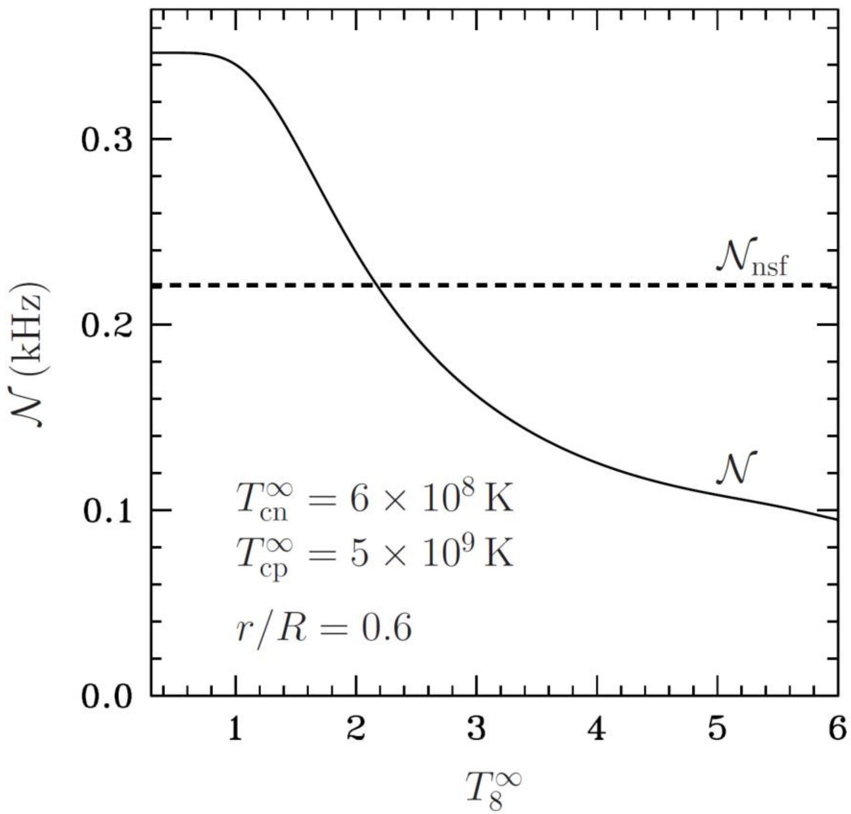


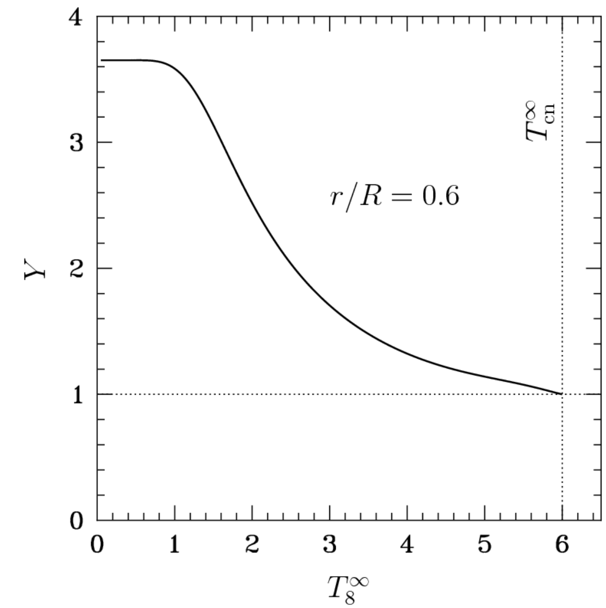


Low-temperature limit of Brunt-Väisälä frequency of SF npeμ-matter, $\mathcal{N}$ (solid line), and Brunt-Väisälä frequency of non-SF npeμ-matter, $\mathcal{N}_{\rm nsf}$ (dashed line), versus r/R. Dotted lines indicate the threshold for muon appearance and core-crust interface.

Temperature dependence of Brunt-Vaisala frequency

$$\mathcal{N}^2 = -\frac{\nu'}{2}\frac{1}{\mu_\mathrm{n} n_\mathrm{b}} \mathrm{e}^{\nu-\lambda}\, Y^2\, \frac{\partial w(P, \mu_\mathrm{n}, x_{\mathrm{e}\mu})}{\partial x_{\mathrm{e}\mu}}\ \nabla x_{\mathrm{e}\mu}$$

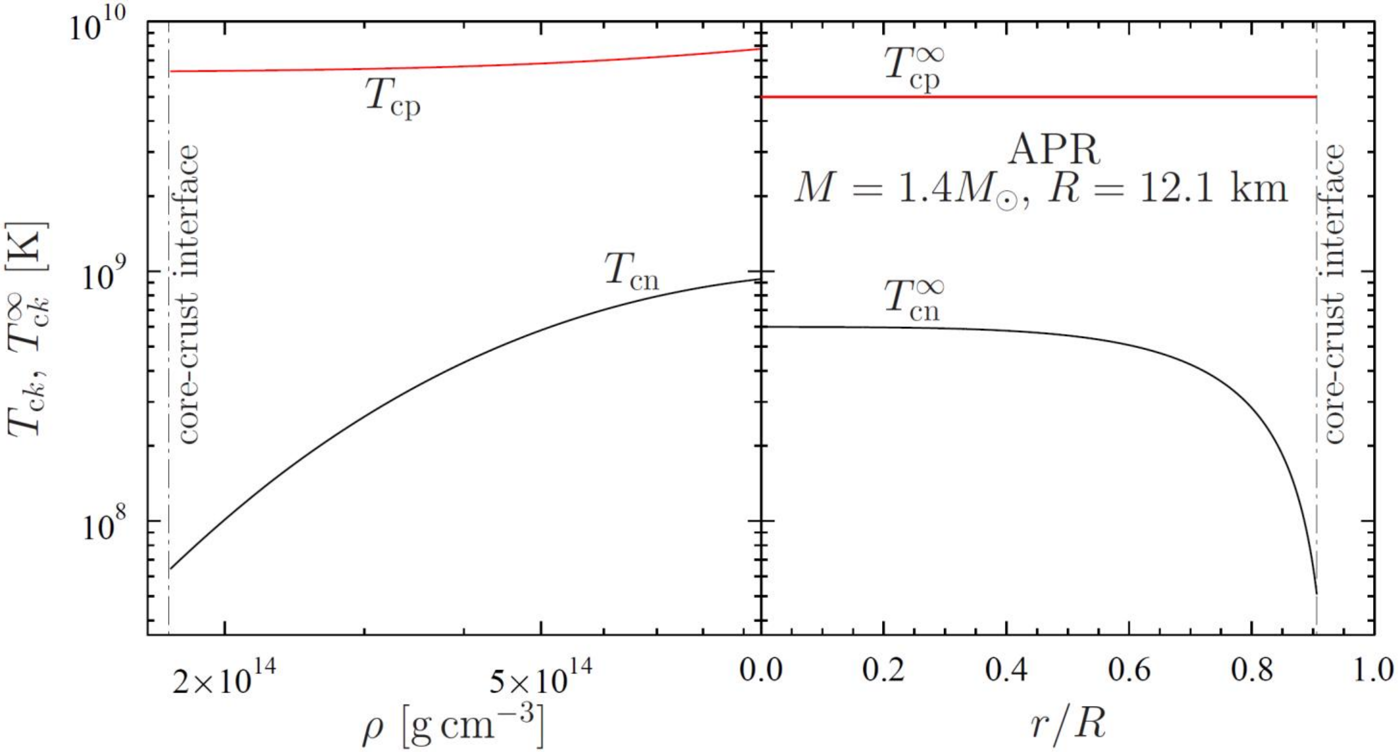




Brunt-Väisälä frequency of SF npeμ-matter, $\mathcal{N}$ (solid line), and non-SF npeμ-matter, $\mathcal{N}_\mathrm{nsf}$ (dashed line), at the distance $r/R = 0.6$ from stellar center versus $T_8^\infty \equiv T^\infty/(10^8\ \mathrm{K})$

Eigenfrequencies of stellar oscillations

Critical temperature profiles



Left panel: Nucleon critical temperatures T_{ck} ($k =$ n, p) versus density ρ. Right panel: Redshifted critical temperatures T_{ck}^{∞} versus r (in units of R). Dot-dashed lines indicate the crust-core interface.

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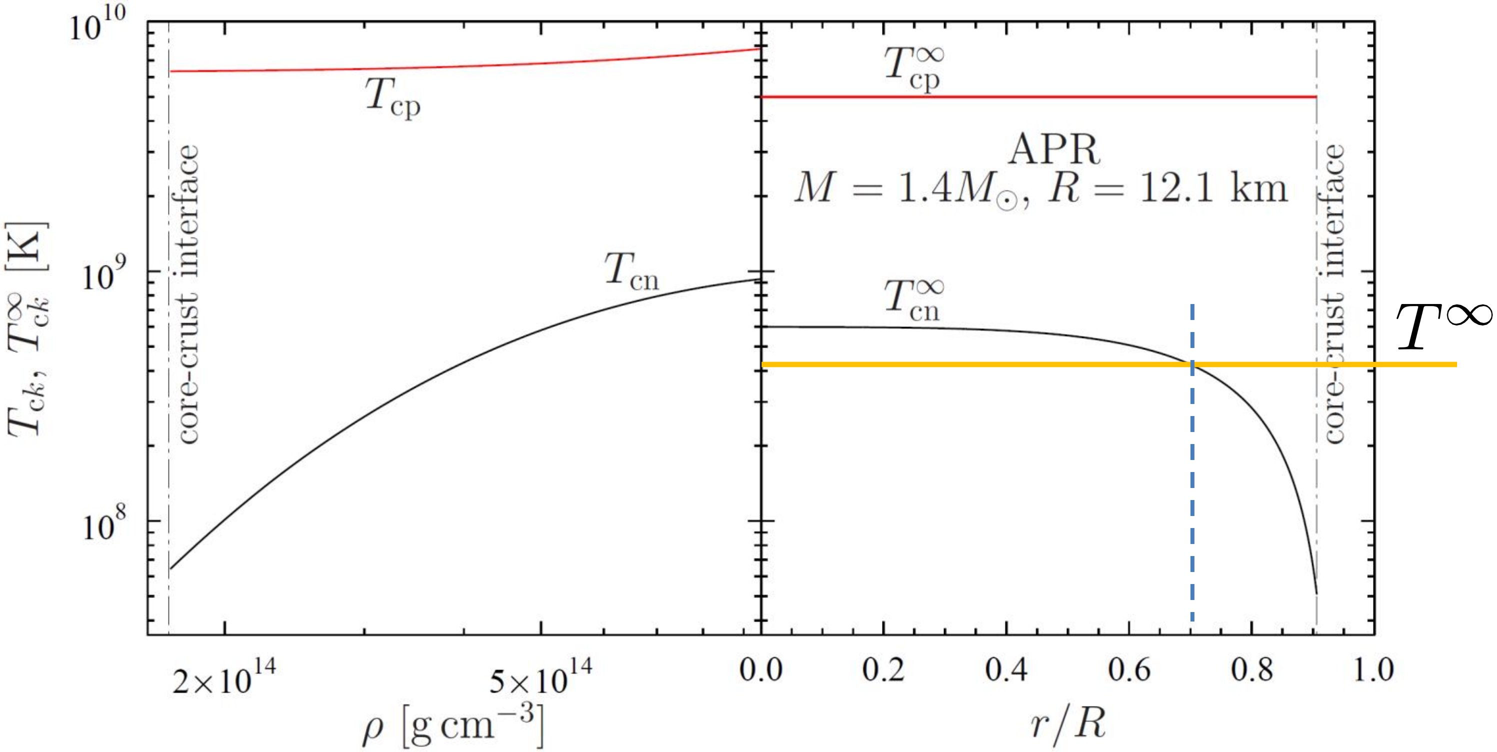

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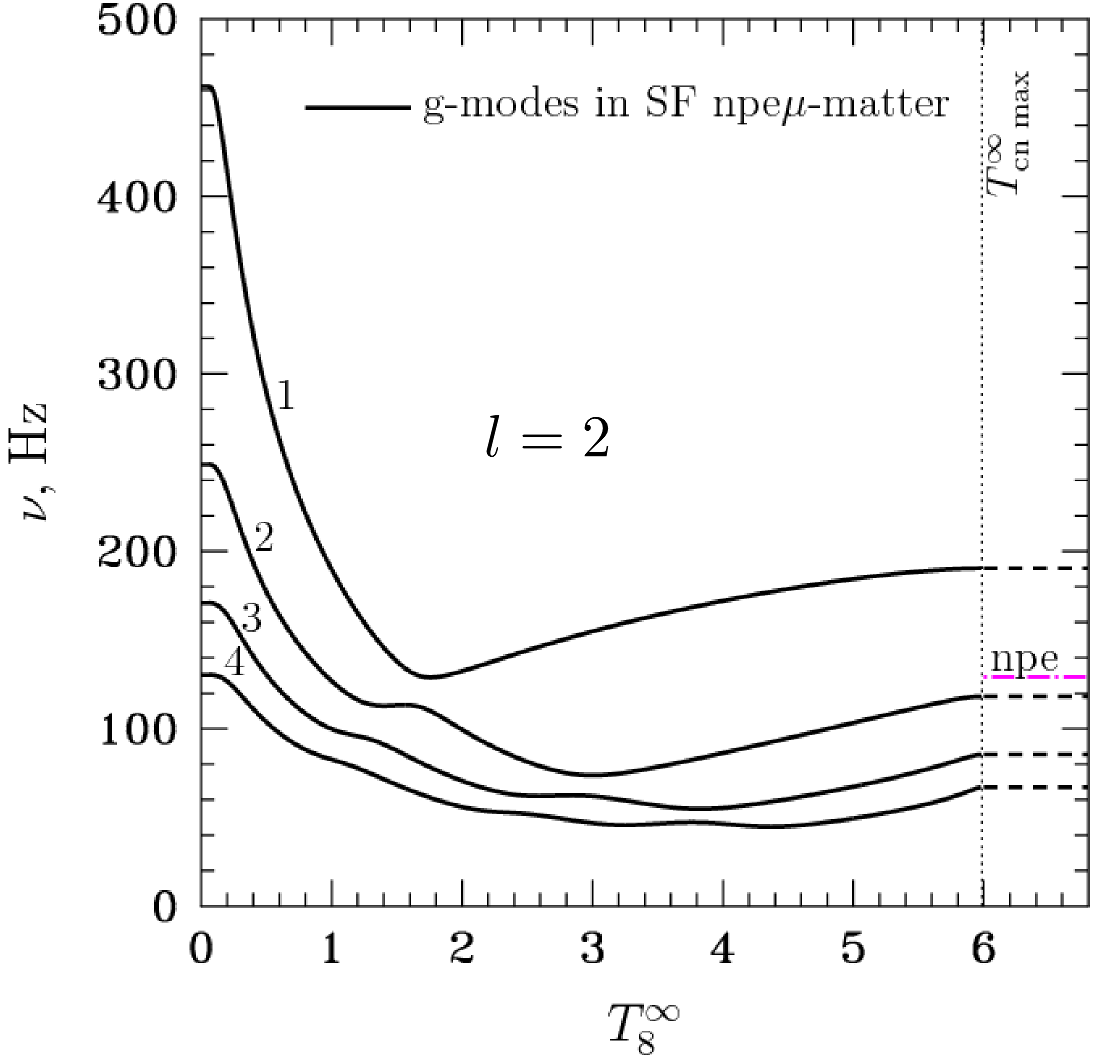
Two layers:

inner superfluid

and

outer non-superfluid



$T^\infty > T^\infty_{\mathrm{cn\,max}}$

no superfluid neutrons in the star and the star oscillates as a non-superfluid one => normal g-modes

$T^\infty \ll T^\infty_{\mathrm{cn}}$

low temperature asymptote for superfluid g-modes, no temperature dependence

Spectrum of quadrupolar ($l = 2$) g-modes versus $T_8^\infty \equiv T^\infty/(10^8\,\mathrm{K})$. Solid/dashed lines show eigenfrequencies ν (in Hz) for the first 4 g-modes in SF/non-SF NS with npeμ core composition; dot-dashed line shows ν for the fundamental $l = 2$ g-mode in non-SF NS with npe core composition. Dotted line indicates maximum T^∞_{cn} in the core.

Conclusions

• A peculiar class of temperature dependent composition g-modes is shown to exist in superfluid $npe\mu$ matter of NS cores.

• Their frequencies appear to be rather high, of the order of the spin frequencies of the most rapidly rotating neutron stars.

• This means that oscillation spectra of rotating neutron star will be significantly affected by these g-modes.

• Analogous composition superfluid g-modes should exist in laboratory superfluids (one superfluid, say He II, + two non-superfluid species).

•Probably are already observed as the coherent frequency 249 Hz identified in the light curves of a millisecond X-ray pulsar XTE J1751-305?