Instability windows and evolution of rapidly rotating neutron stars

What limits the frequencies of rapidly rotating neutron stars?

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arXiv:1310.8103 (PRL, accepted); ***arXiv:1305.3825***

What limits the spin rate of a neutron star?

The obvious limiting factor is the Kepler (or mass-shedding) frequency, at which NS start to shed matter from its surface:

$$\nu_{\rm K} \approx 1233 \left(\frac{M}{1.4\,M_\odot}\right)^{1/2} \left(\frac{R}{10\,{\rm km}}\right)^{-3/2} {\rm Hz}.$$

Lattimer & Prakash (2007)

However, the most rapidly rotating neutron star observed so far is the millisecond pulsar PSR J1748-2446ad with $\nu = 716\,{\rm Hz}$.

Moreover, as follows from the statistical analysis, there should be an abrupt cut-off at $\sim 730\,{\rm Hz}$ in the spin frequency distribution of neutron stars (*Chakrabarty et al. Nature 2003*).

Why these frequencies are so different from $\nu_{\rm K}$?

This hints on the existence of a more efficient mechanism.

Possible mechanism limiting the spin rates of neutron stars is related to ***instability of r-modes***.

(Bildsten 1998; Andersson, Kokkotas & Stergioulas 1999)

Properties of r-modes:

- Oscillations of ***toroidal*** type; restoring force is the ***Coriolis*** force.

- Neglecting dissipation they are subject to gravitational-driven Chandrasekhar-Friedman-Schutz instability at ***arbitrary*** spin frequency of a neutron star; one has exponential growth of r-modes (*Andersson 1998; Friedman & Morsink 1998*).

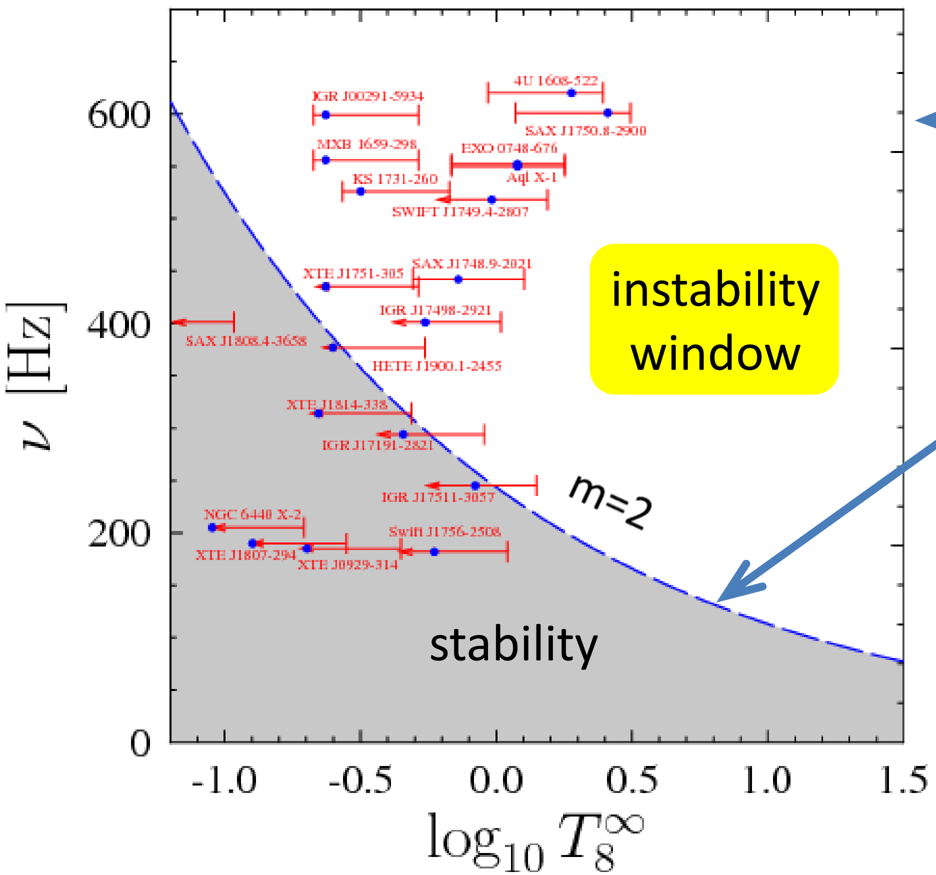
- Excited unstable r-mode radiates gravitational waves that carry off stellar angular momentum and ***decelerate*** a star.

- Account for dissipation stabilizes a star to some extent: it becomes unstable only at certain (high) spin frequencies ν and internal temperatures T^{∞} -- ***INSTABILITY WINDOWS!***

As it will clear just after a few slides,
this mechanism leads to a **problem**
if we consider
hot, rapidly rotating neutron stars, that is
neutron stars in low-mass X-ray binaries (LMXBs).

Standard instability window vs observation data

For some NSs in LMXBs we can measure spin frequency and estimate internal stellar temperature. These data are presented in this plot:



spin frequencies ν versus internal temperatures $T_8^\infty = T^\infty/10^8\mathrm{K}$ for NSs in LMXBs

Blue dashed curve shows the boundary of the instability window for the most unstable quadrupole (m=2) *r*-mode.

All sources in the instability window (white region) are unstable with respect to excitation of r-modes.

How probable is to find these sources in the instability region?

Equations governing NS evolution

Owen et al. (1998)

Amplitude of r-mode:

$$\frac{d\alpha}{dt} = -\alpha \left(\frac{1}{\tau_{\rm GR}} + \frac{1}{\tau_{\rm Diss}} \right)$$

amplitude grows due to emission of gravitational waves

damps due to viscous dissipation

Spin frequency:

$$\frac{d\nu}{dt} = -\frac{2\, Q\, \alpha^2\, \nu}{\tau_{\rm Diss}} + \dot{\nu}_{\rm acc}$$

spin frequency decreases due to dissipation of r-mode

increases due to accretion

Thermal balance:

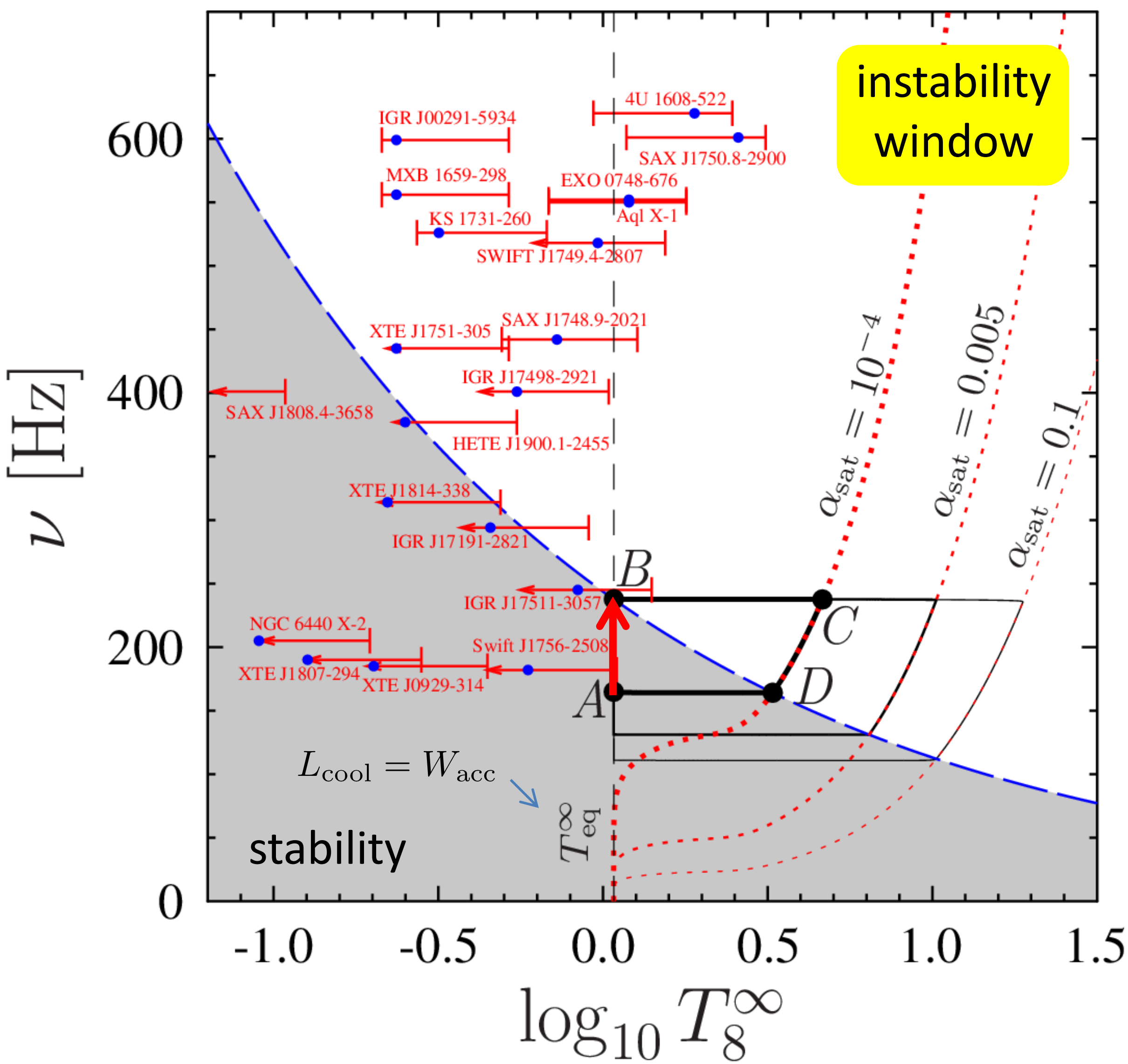
$$C_{\rm tot} \frac{dT^{\infty}}{dt} = W_{\rm Diss} + K_{\rm n} \dot{M} c^2 - L_{\rm cool}$$

an NS heats up due to viscous r-mode dissipation and accretion

cools down due to neutrino emission

Evolution of an NS in LMXB. Standard scenario.

Levin (1999)



A-B

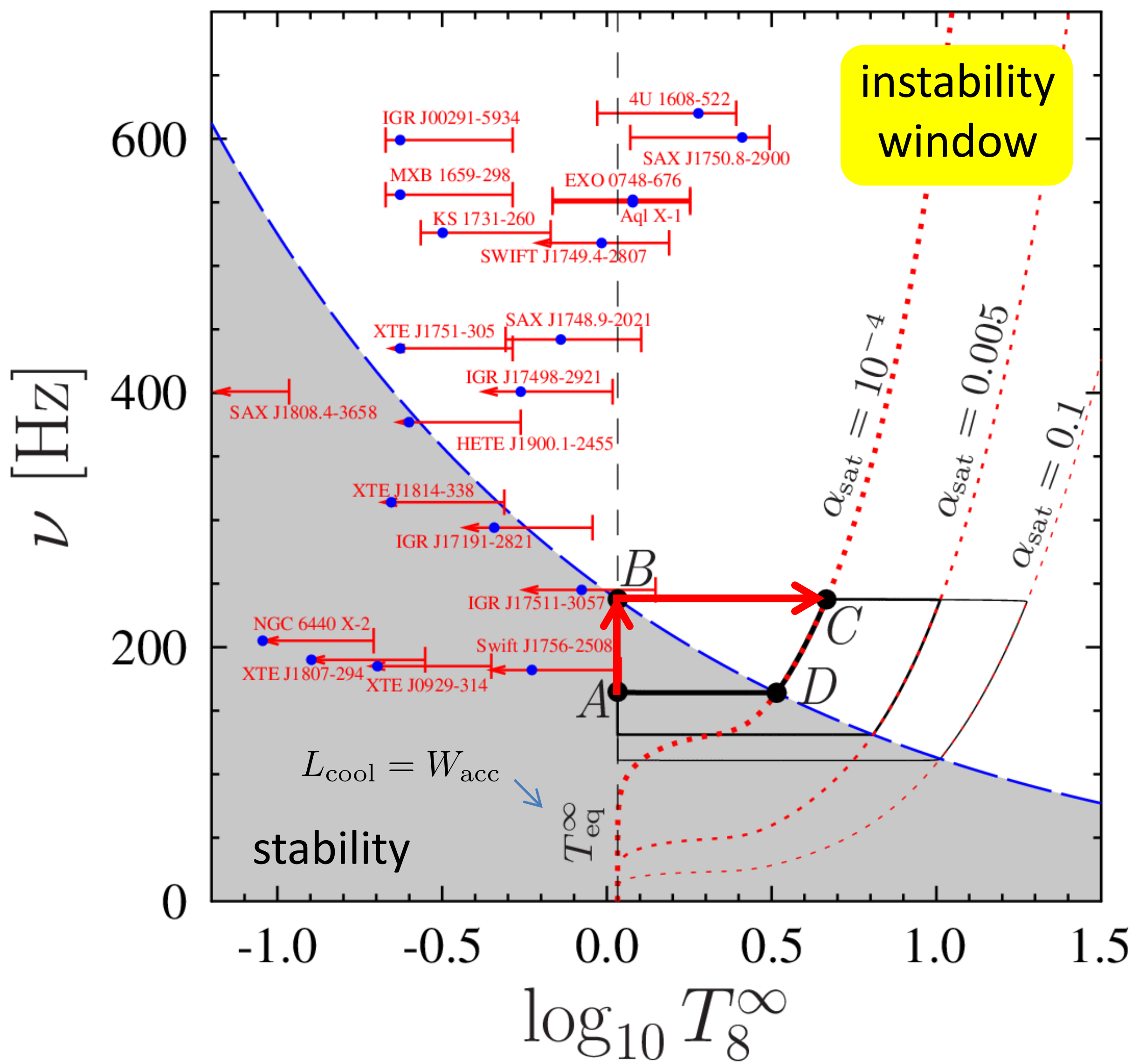
NS is spun up by accretion torque in the stability region.

R-mode is not excited.

The temperature of the star keeps constant because heating by accretion is balanced by the neutrino cooling.

Evolution of an NS in LMXB. Standard scenario.

(*Levin 1999*)



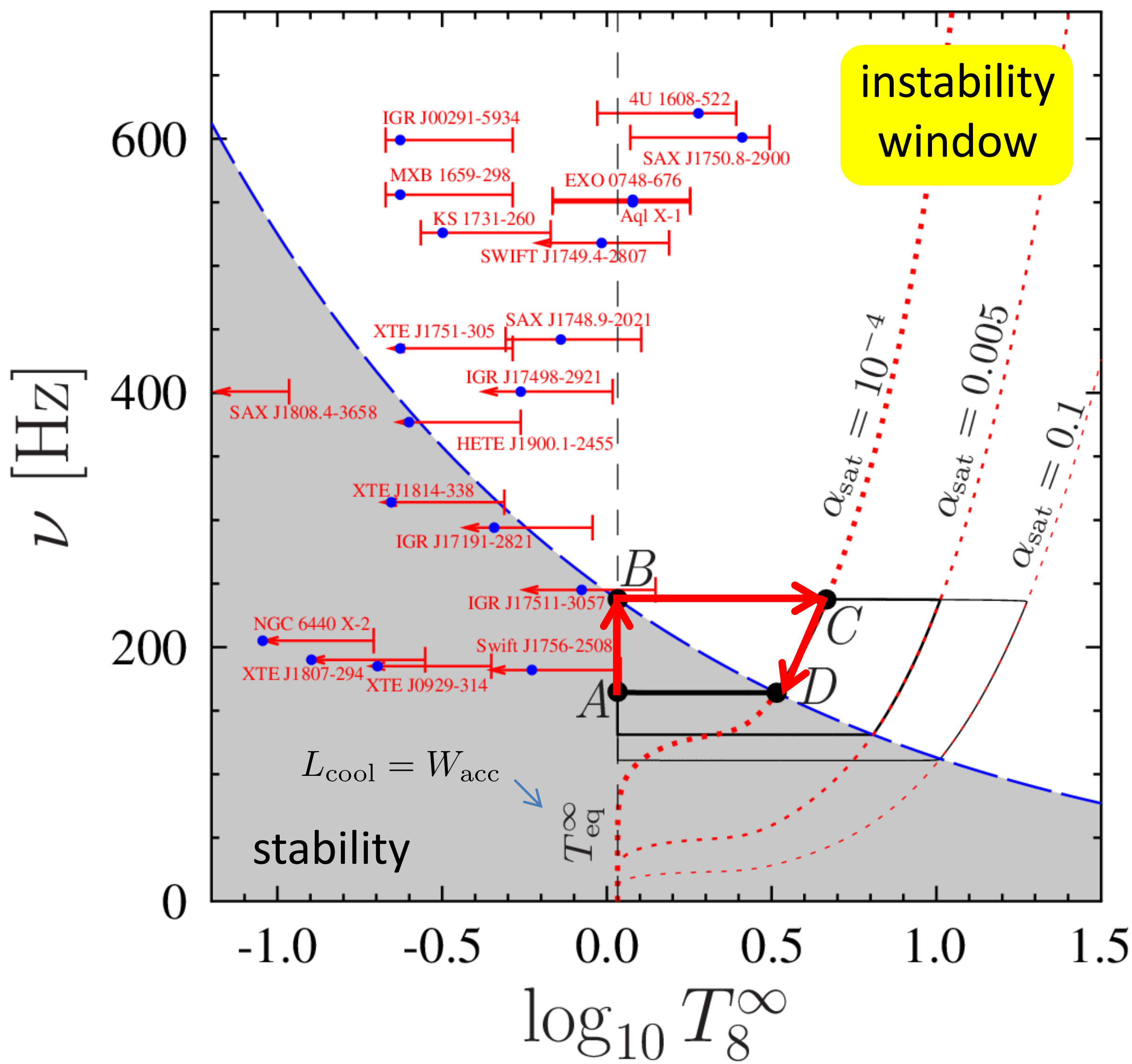
B-C

When the star crosses the instability curve at point B, it becomes unstable with respect to excitation of r-mode.

Then the star rapidly heats up because of viscous dissipation of excited r-mode and eventually reaches the point C.

Evolution of an NS in LMXB. Standard scenario.

(*Levin 1999*)



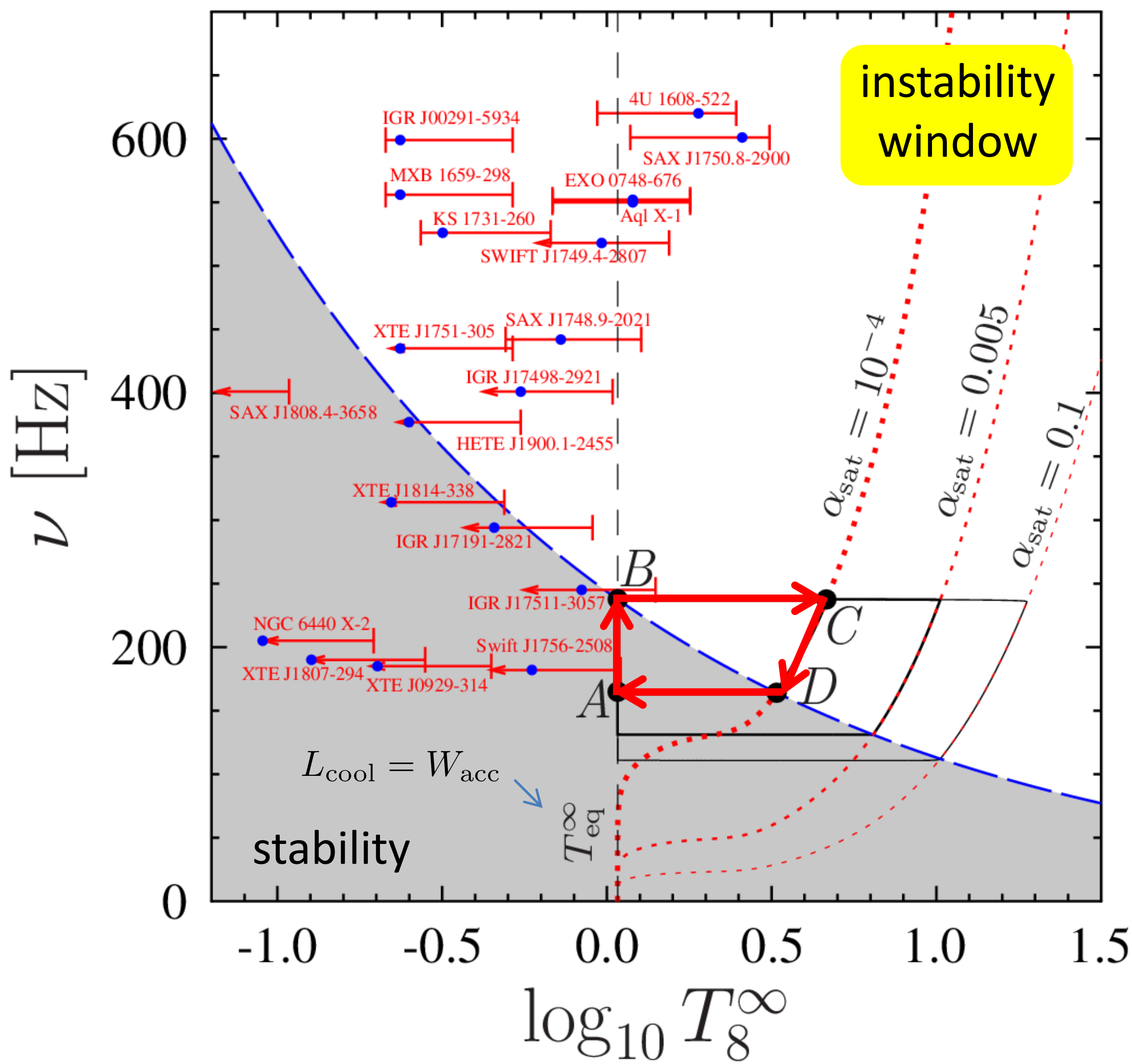
C-D

During this stage neutrino cooling exactly compensates viscous heating.

NS spins down due to radiation of gravitational waves by saturated r-mode until it reaches the point D.

Evolution of an NS in LMXB. Standard scenario.

(*Levin 1999*)



D-A

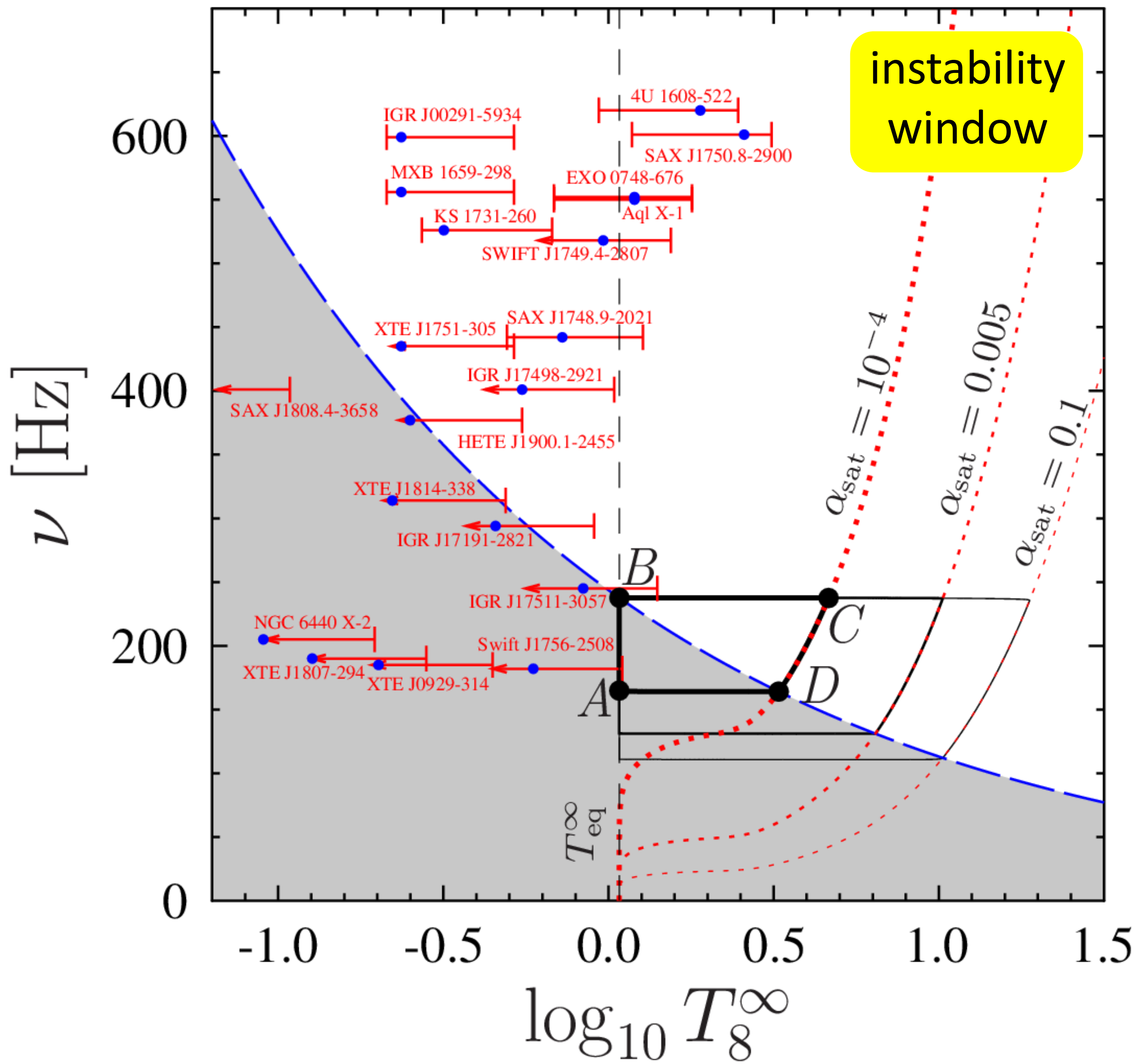
At point D a star enters the stability region where r-mode rapidly decays and the star cools down to its equilibrium temperature T_{eq}^{∞}.

Then the cycle repeats.

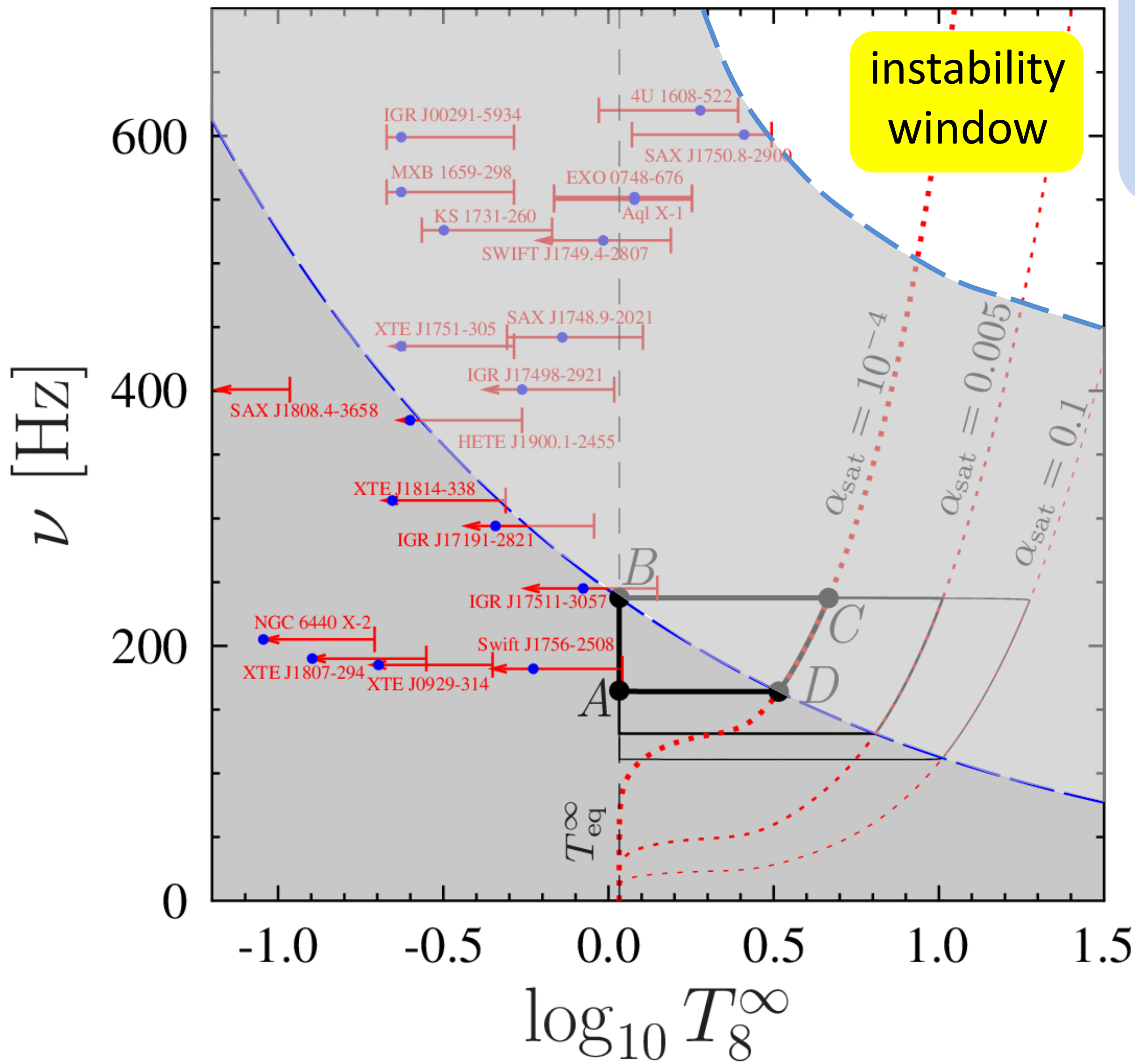
It is important that NS spends just a small part of the cycle period in the instability region

Thus the standard scenario cannot explain observations of the most hot and rapidly rotating NSs in LMXBs.

Problem: Many neutron stars fall well outside the stability region and cannot be explained within this scenario



A possible way around this problem is to assume that the observed sources are ***all*** contained in the stability region.



To stabilize these sources one has to increase dissipation by a factor of 1000. *This is unrealistic if we consider n-p-e-mu-matter.*

Other possibilities include:

- Stabilization by a high bulk viscosity in exotic (quark, hyperon) matter
- Low-saturation-amplitude scenarios, which require (unrealistically) low saturation amplitude of r-mode $\alpha \sim 10^{-9} - 10^{-6}$.

Here we present a new scenario explaining observations of hot and rapidly rotating NSs in LMXBs.

This scenario:

- does not need an exotic matter composition;

- works for a wide range of possible saturation amplitudes, including the amplitude $\alpha_{\rm sat} \sim 10^{-3} - 10^{-4}$, obtained by *Bondarescu, Teukolsky & Wasserman (2007, 2009);*

- employs the same ***minimal cooling model*** as that used to explain observation data on all cooling isolated neutron stars, including Cas A;

- ***is related to nucleon superfluidity.***

At $T \lesssim 10^8 \div 10^{10}\,\mathrm{K}$ nucleons in NS cores become superfluid.

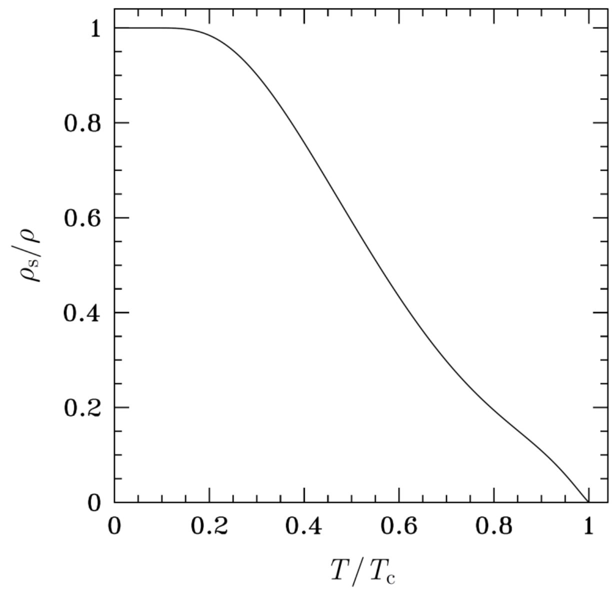
Superfluidity leads to appearance of two distinct classes of oscillation modes

"normal" modes,
which are almost the same as in nonsuperfluid matter
(*Lindblom & Mendell 1994*)

superfluid modes , which have very specific properties
(*Epstein 1988*)

Properties of superfluid modes:

• Their frequencies are very temperature-dependent (because neutron superfluid density is a strong function of *T*, *Gusakov & Andersson 2006*)

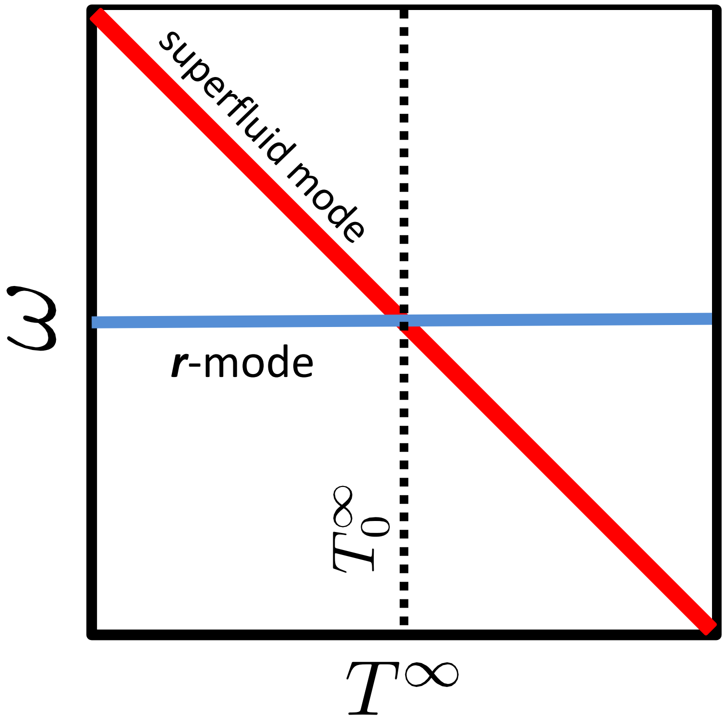


• They very weakly interact with ordinary r-modes (two weakly coupled oscillators). Interaction is strong only when their frequencies become ***close*** to one another (*Lindblom & Mendell 2000*).

Then we have ***avoided crossings*** of modes.

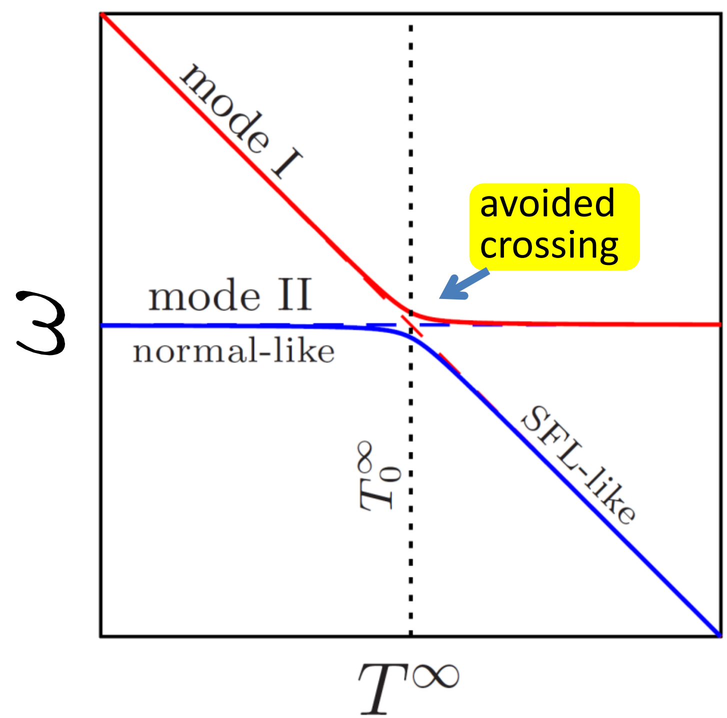
A schematic plot of frequency spectrum versus temperature near a crossing/avoided crossing of superfluid mode and normal r-mode

No interaction between the modes



Superfluid mode and normal r-mode do not “feel” each other

Allowing for the interaction



As the temperature varies, normal mode turns into superfluid mode and vice versa.

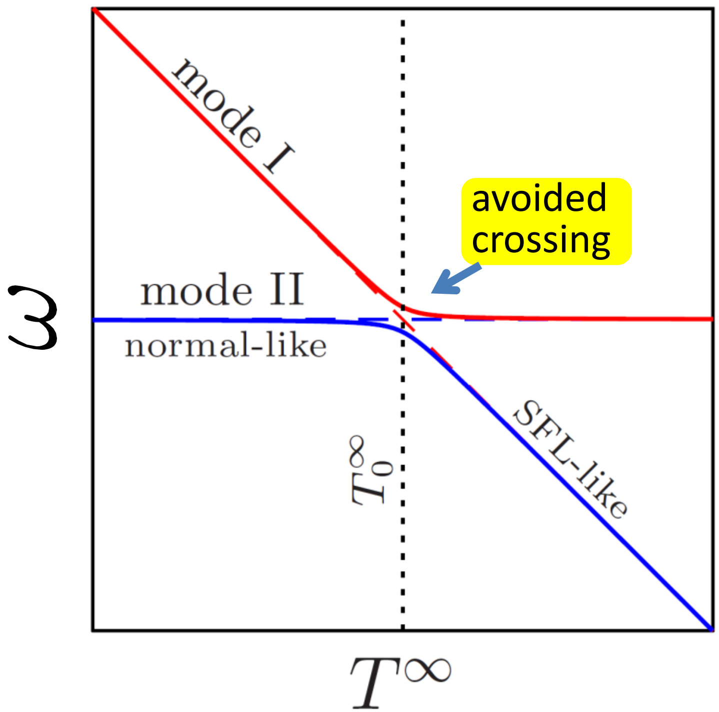
Properties of superfluid modes:

• Superfluid modes damp out much faster than ordinary r-modes due to the very effective mutual friction mechanism (*Lindblom & Mendell 2000; Lee & Yoshida 2002*).

$$\boxed{\tau_{\rm Diss}^{\rm (SFL)} \ll \tau_{\rm Diss}^{(r)}}$$

dissipation timescale for a superfluid mode

dissipation timescale for the normal m=2 r-mode



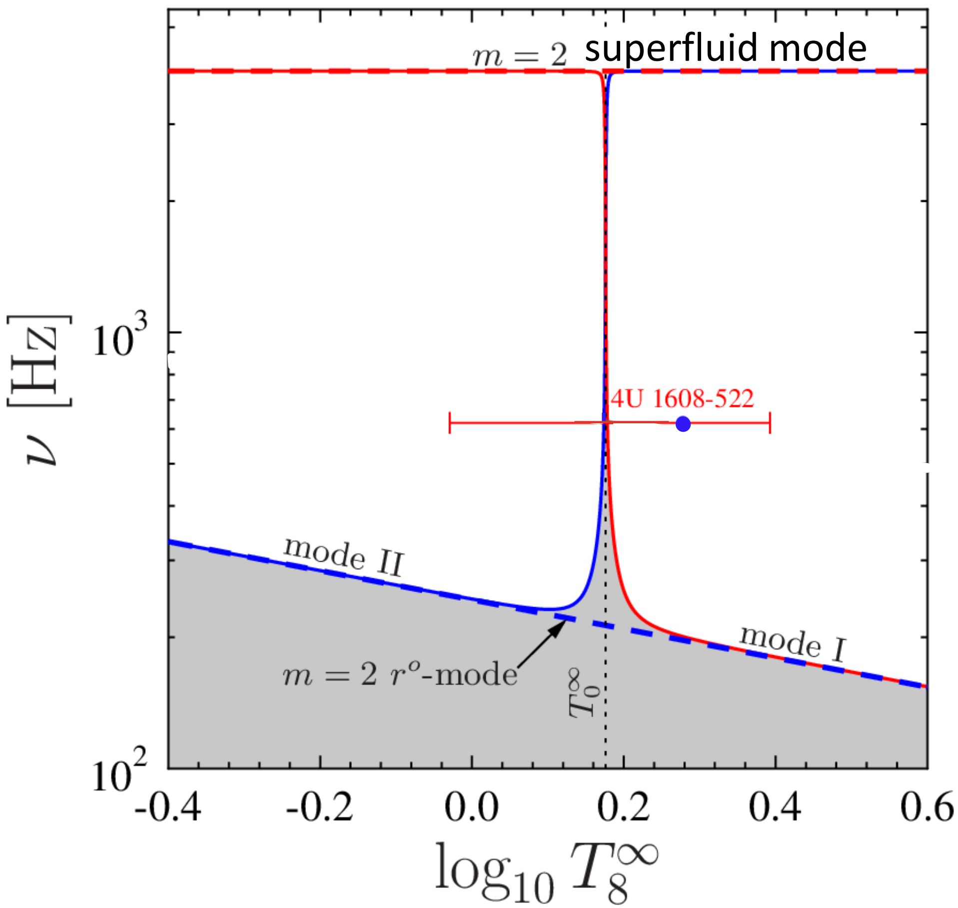
This results in an ***enhanced damping*** of the normal r-mode near avoided crossing!

How will this property affect the instability windows?

New instability windows

Assume that at some temperature T_0^∞ the frequencies of r-mode and a superfluid mode become close to one another so that avoided crossing of modes is formed.

Near avoided crossing r-mode start to transform into superfluid mode which leads to strong enhancement of its dissipation rate and, as a result, to formation of a *left edge of the stability peak* in the $\nu - T^\infty$ plane. Similarly, superfluid mode that turns into a normal r-mode, forms a *right edge of the stability peak*.

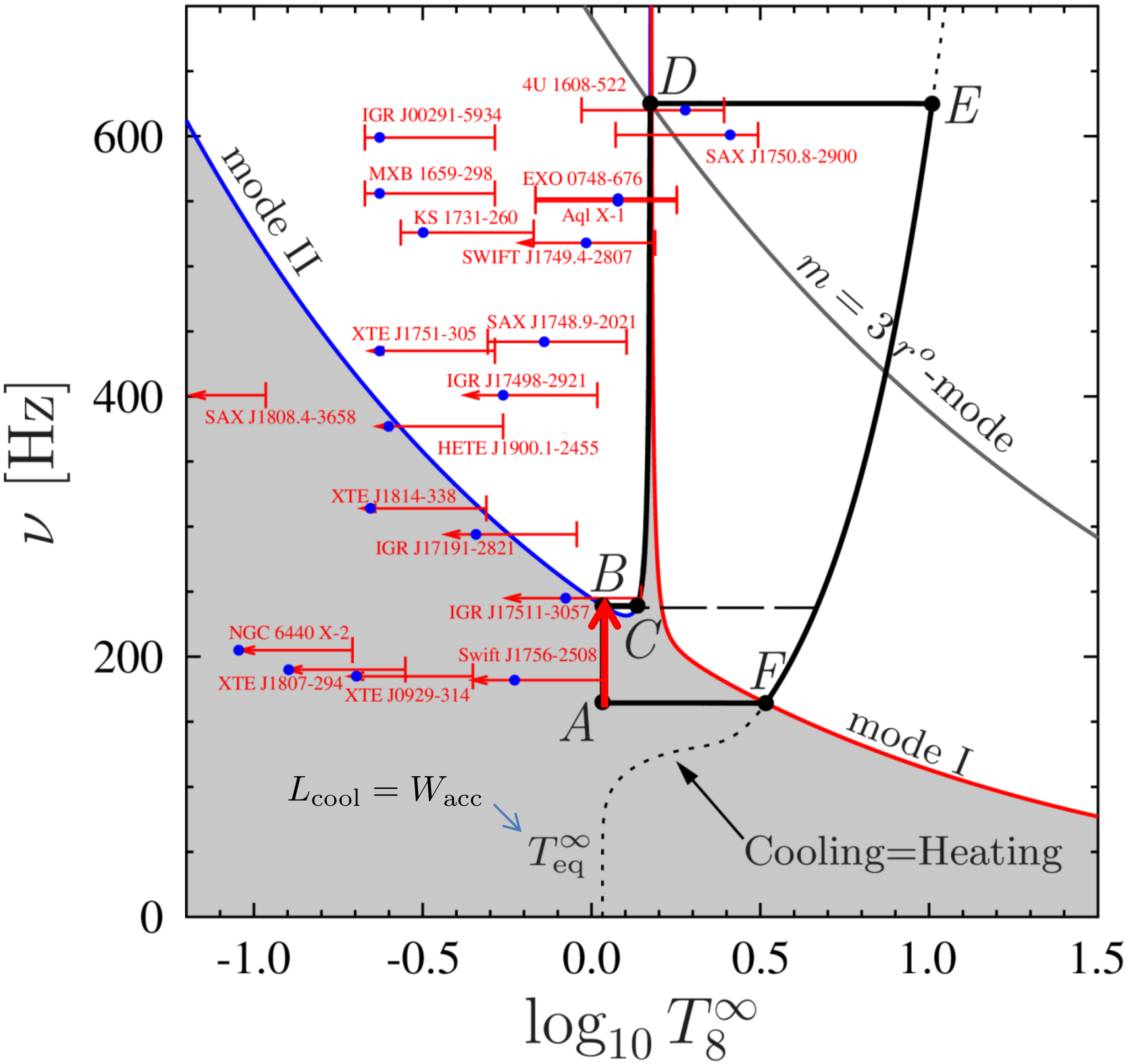


Blue dashed line: Instability curve for the normal r-mode (***the same as in previous slides***; obtained neglecting interaction between superfluid and normal modes);

Red dashed line: Instability curve for a superfluid mode (***this mode is much more stable(!)***; obtained neglecting interaction between superfluid and normal modes);

Shaded region is stable with respect to excitation of any oscillation mode.

Evolution of NSs in LMXBs: scenario with the stability peak



A-B

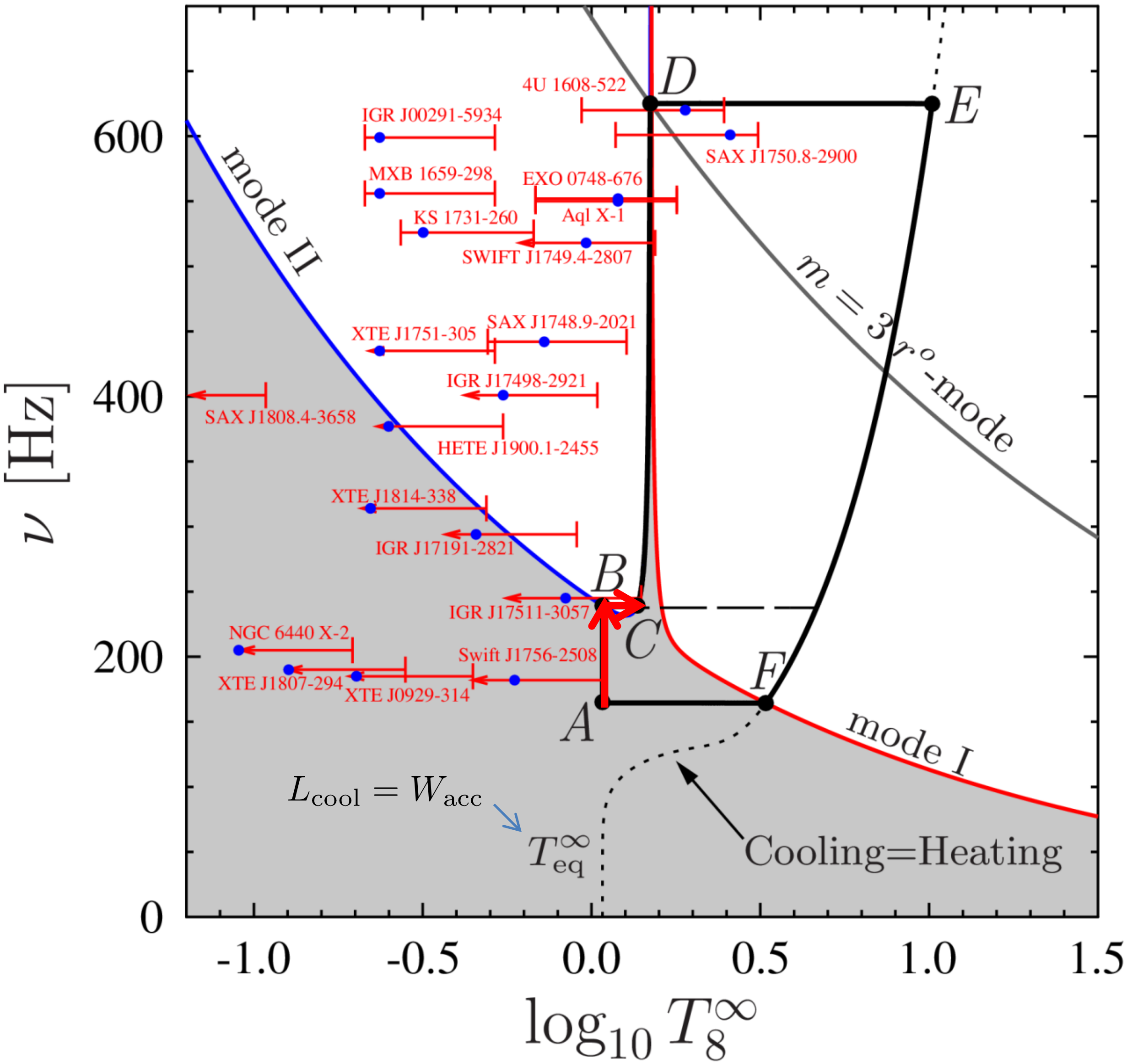
The same as in the standard scenario.

NS is spun up by accretion torque in the stability region.

R-mode is not excited.

The temperature of the star keeps constant because heating by accretion is balanced by the neutrino cooling.

Evolution of NSs in LMXBs: scenario with the stability peak



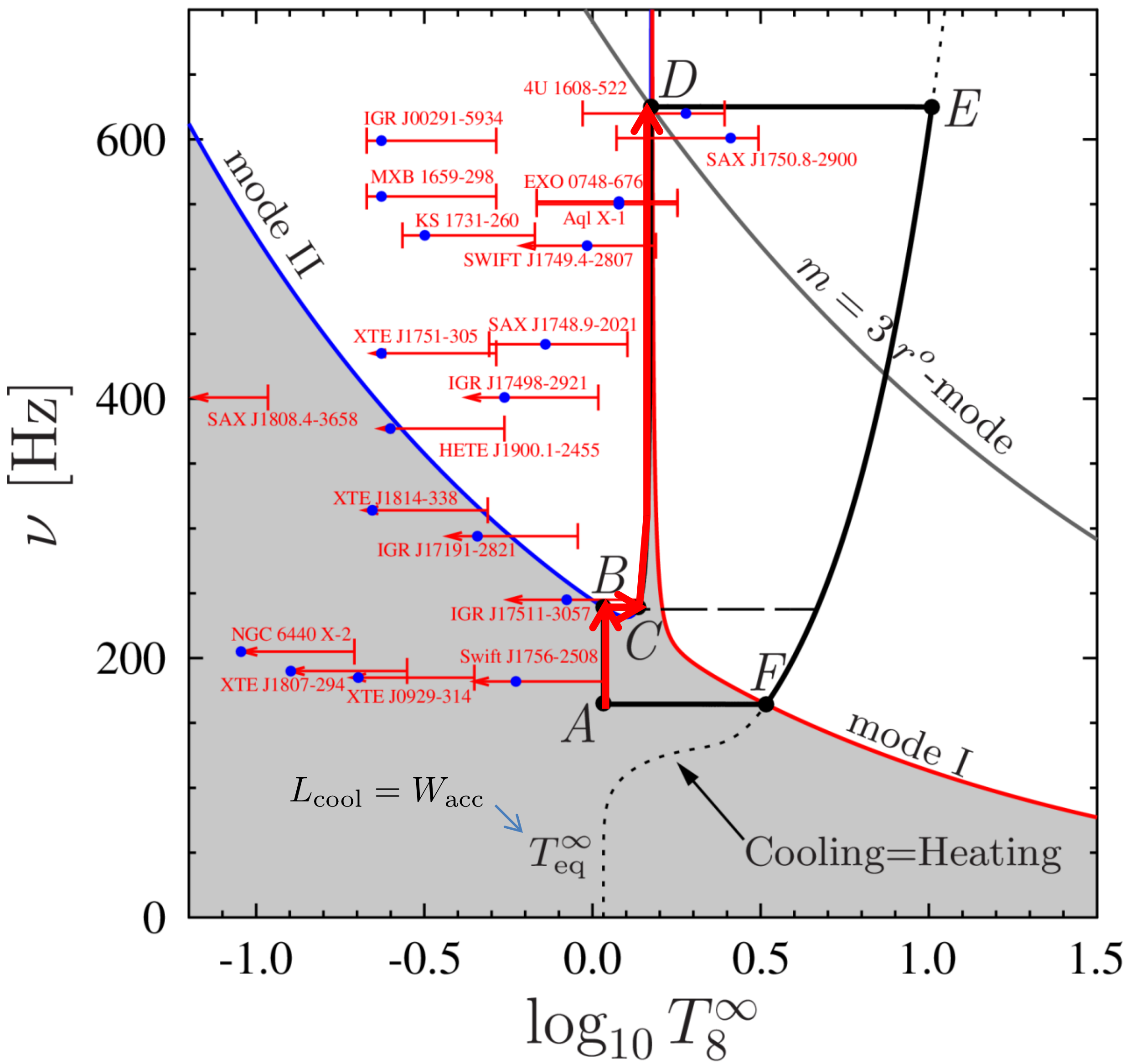
B-C

The same as in the standard scenario.

When the star crosses the instability curve at point B, it becomes unstable with respect to excitation of r-mode.

Then the star rapidly heats up because of viscous dissipation of excited r-mode and eventually reaches the foot of the stability peak at point C.

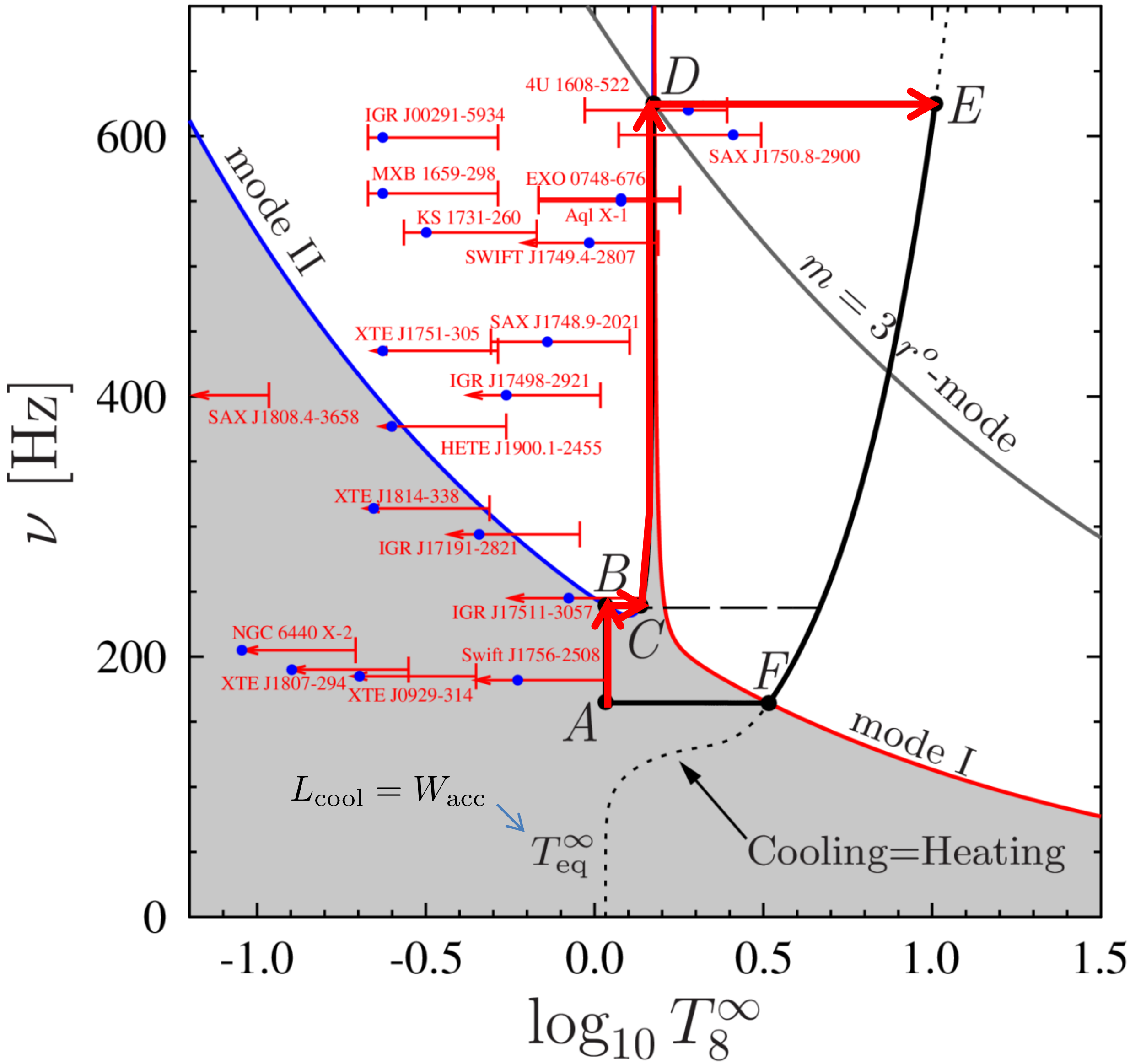
Evolution of NSs in LMXBs: scenario with the stability peak



C-D

During the stage C-D an NS climbs up the stability peak; this stage is **absent in the standard scenario** and will be discussed after a few slides.

Evolution of NSs in LMXBs: scenario with the stability peak

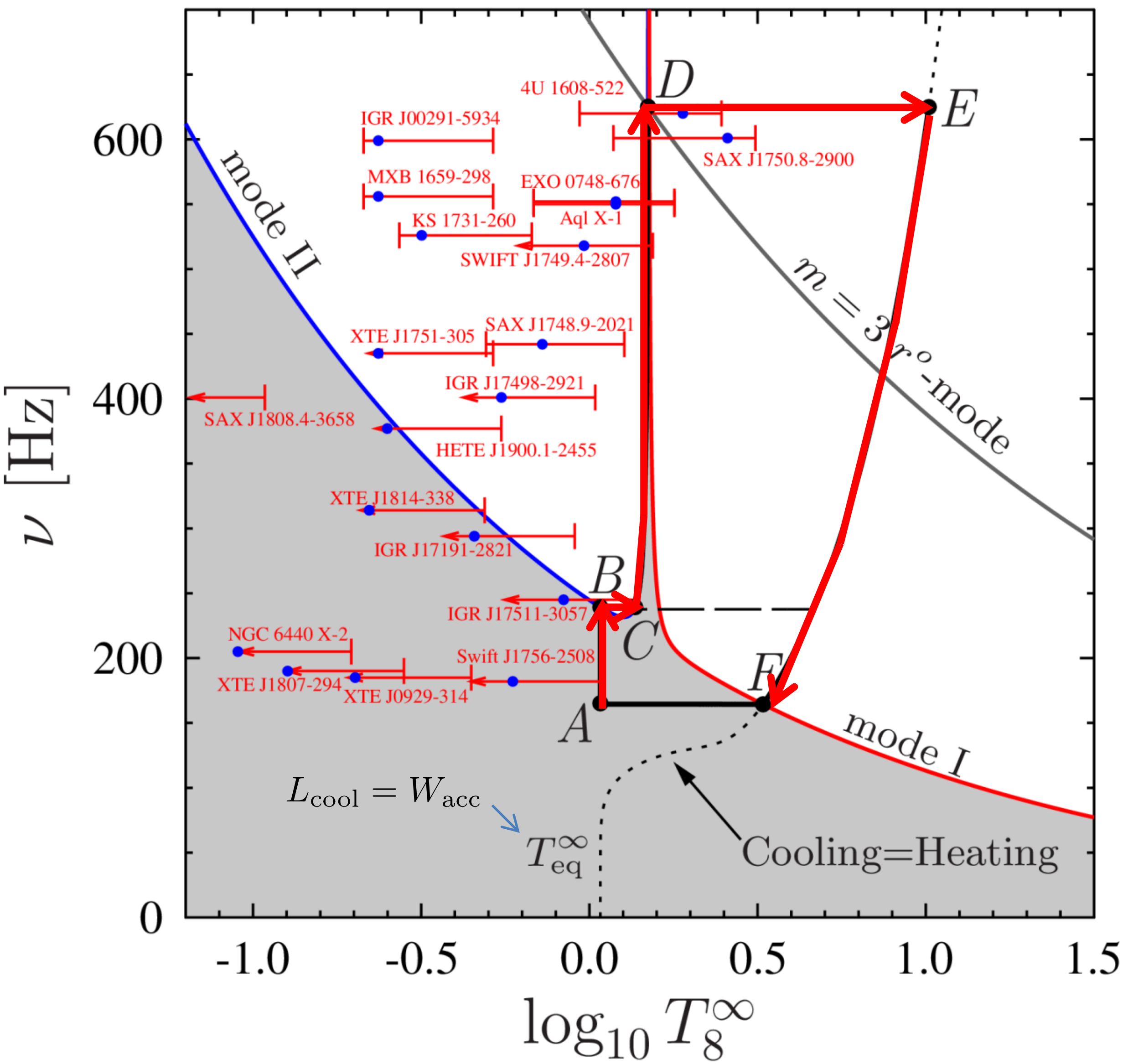


D-E

The stage C-D finishes at point D, when the star crosses the instability curve for the octupole r-mode with multipolarity m=3 (this mode is the most unstable after m=2 r-mode).

Then the octupole r-mode will be excited and the star rapidly heats up to point E.

Evolution of NSs in LMXBs: scenario with the stability peak

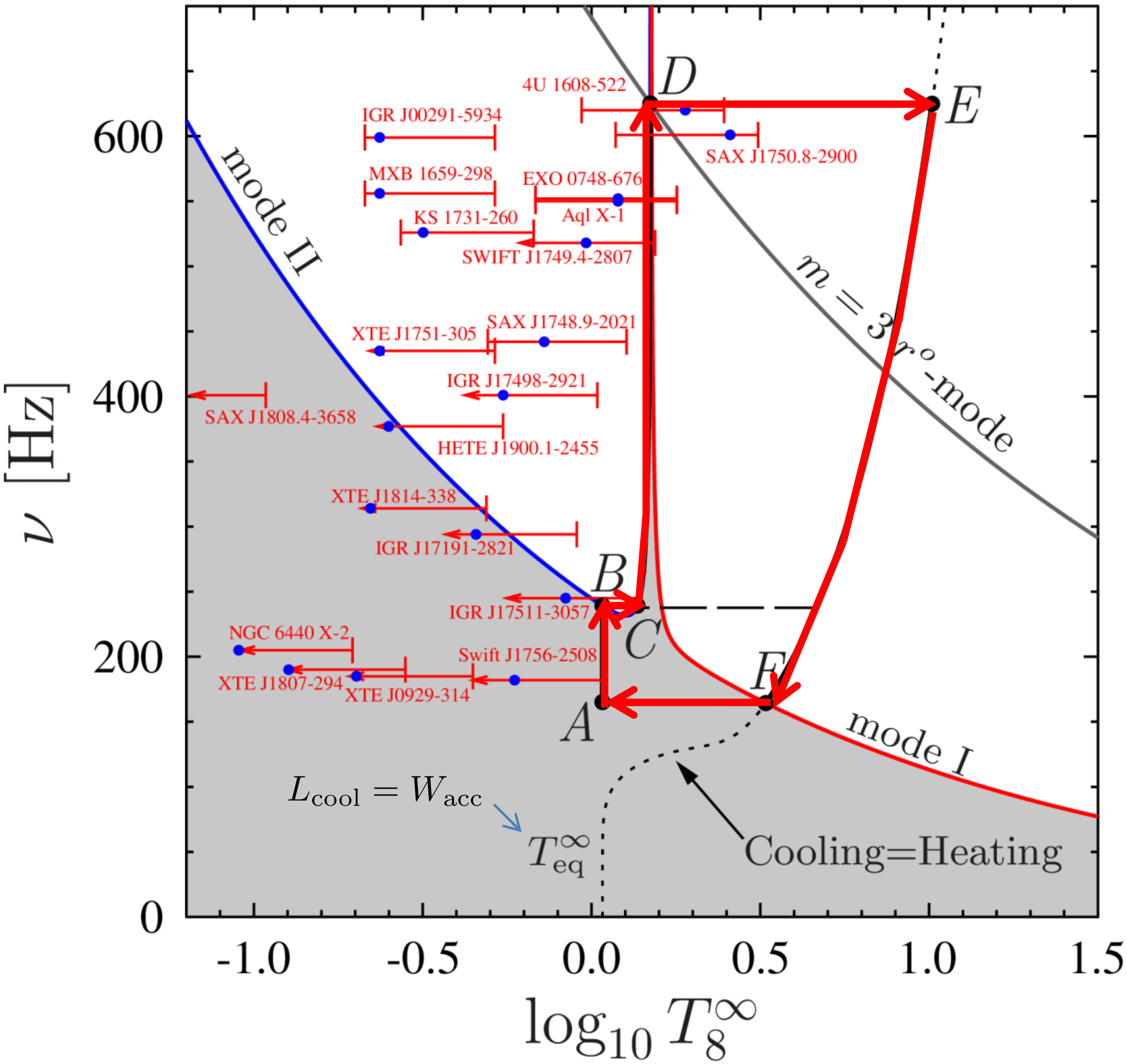


E-F

At point E NS reaches the curve E-F where viscous heating by r-mode balances the neutrino cooling.

Then the star spins down due to radiation of gravitational waves by saturated r-mode.

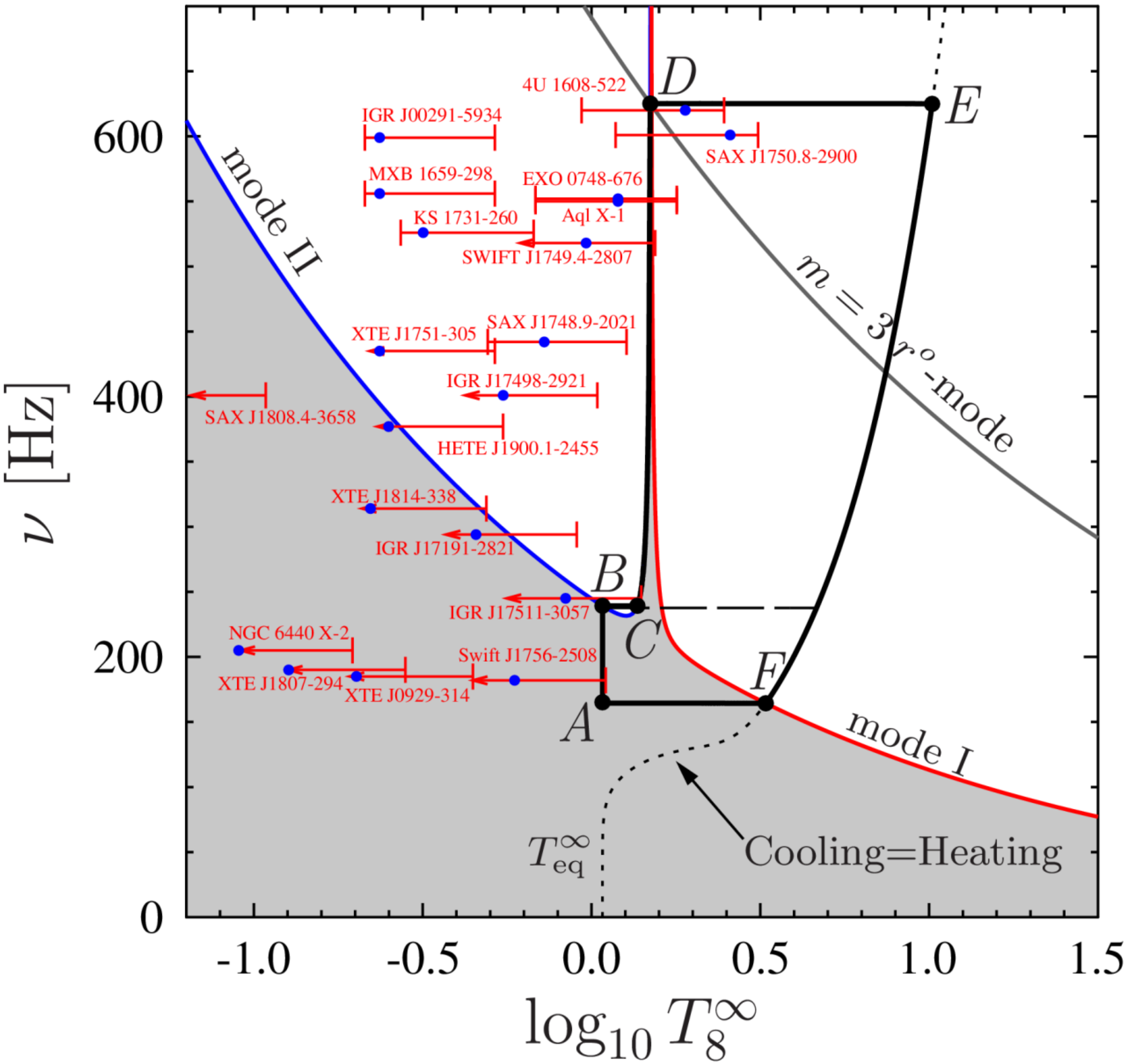
Evolution of NSs in LMXBs: scenario with the stability peak



F-A

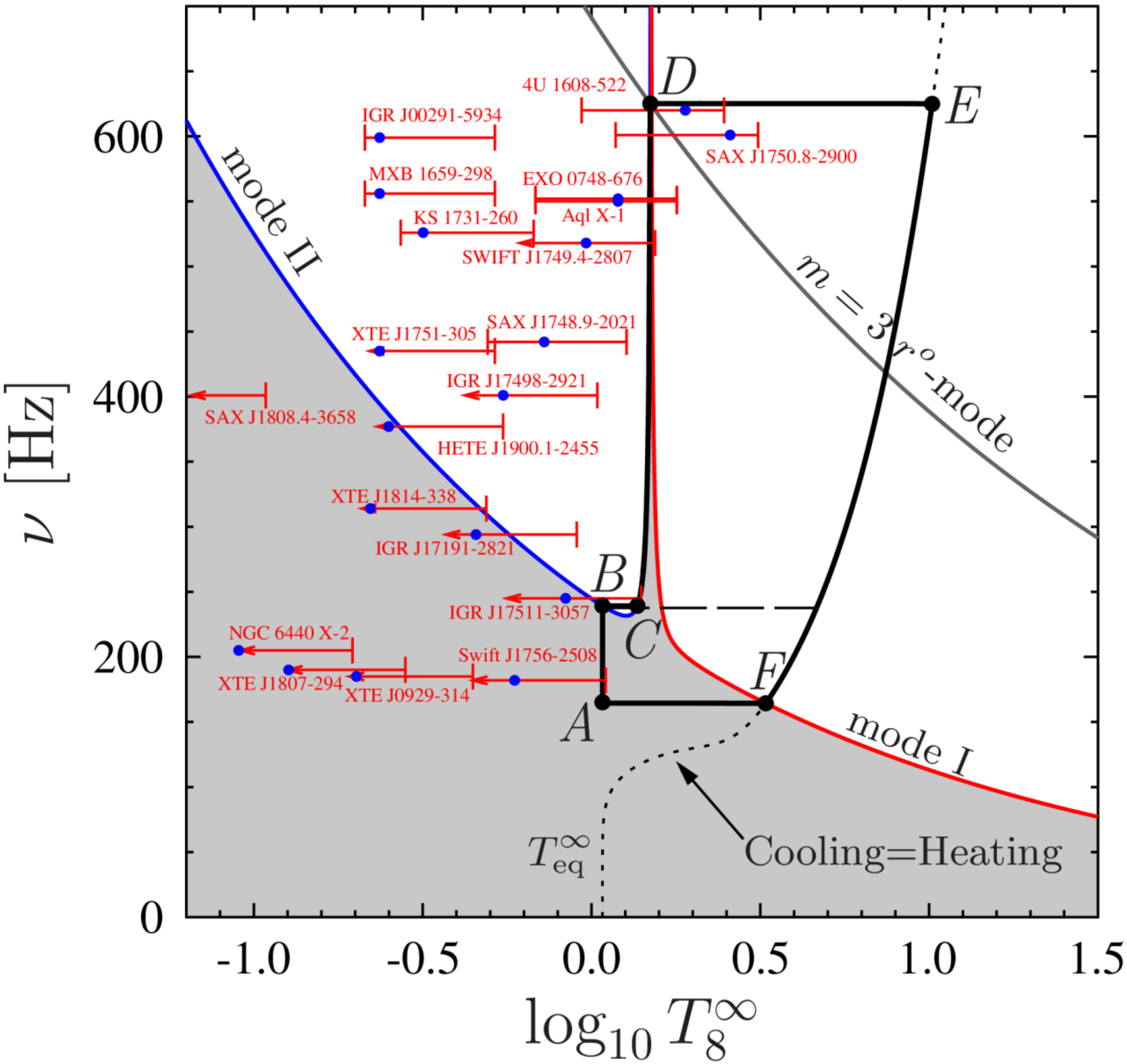
Finally, at point F a star enters the stability region where r-mode rapidly decays and the star cools down to its equilibrium temperature $T_{\rm eq}^\infty$. Then the cycle repeats.

Evolution of NSs in LMXBs: stage C-D



- NS climbs up along the left edge of the stability peak.
- It can't move to the right because then r-mode dies out and a star cools down to the edge of the peak.
- It can't move to the left because excited r-mode prevents it from cooling.

Evolution of NSs in LMXBs: stage C-D



- During the phase C-D stellar temperature stays almost constant and is higher than the equilibrium temperature $T_{\rm eq}^{\infty}$.

- Additional heating is provided by excited r-mode with the amplitude: $\alpha_{CD} \sim 10^{-6}$

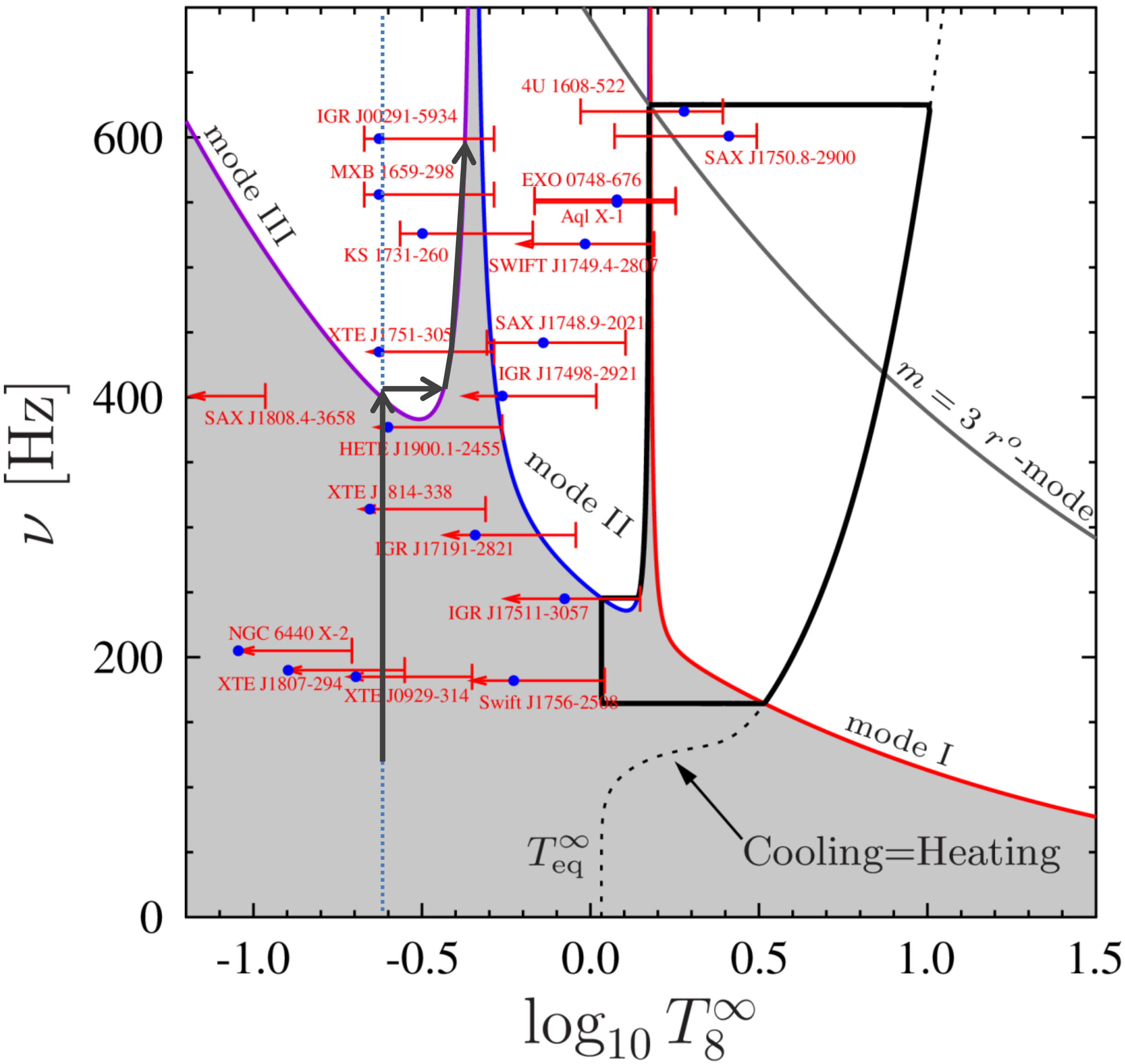
Then: $W_{\rm Diss} + W_{\rm acc} \approx L_{\rm cool}$

- The amplitude α_{CD} is sufficient to keep an NS at the peak, but is not sufficient to spin it down. Hence, NS spins up due to accretion.

$$\Delta t_{CD} = \frac{\nu_D - \nu_C}{\dot{\nu}_{\rm acc}} \approx 2 \times 10^8 \text{ yrs}$$

$82\% t_{\rm cycle}$

Colder NSs: low-temperature stability peak



In reality normal r-mode can experience more than one avoided crossing with superfluid modes (*Lee & Yoshida 2002; Gualtieri et al. 2014, in prep., …*). In that case several stability peaks should appear, and some of them may have lower temperatures (see figure).

If $T_{\rm eq}^{\infty}$ is sufficiently small, an NS will eventually find itself at the *low temperature peak*.

Thus, we interpret the colder sources as climbing up the low-temperature stability peak.

Summary

(see also ***arXiv:1310.8103*** (PRL, accepted); ***arXiv:1305.3825***)

• We propose a new scenario that can explain observations of hot rapidly rotating NSs in LMXBs. This scenario uses the same microphysics input as `minimal cooling' scenarios (*Page et al. 2004; Gusakov et al. 2004*) developed to explain observations of isolated NSs (including Cas A).

• Resonance interaction of a normal r-mode and superfluid inertial modes plays a key role in the evolution of neutron stars in LMXBs.

• Resonance interaction leads to enhanced damping of oscillations and to formation of a "stability peak" limited by the instability of octupole ($m=3$) r-mode.

• NSs spend significant amount of time climbing up the stability peak.

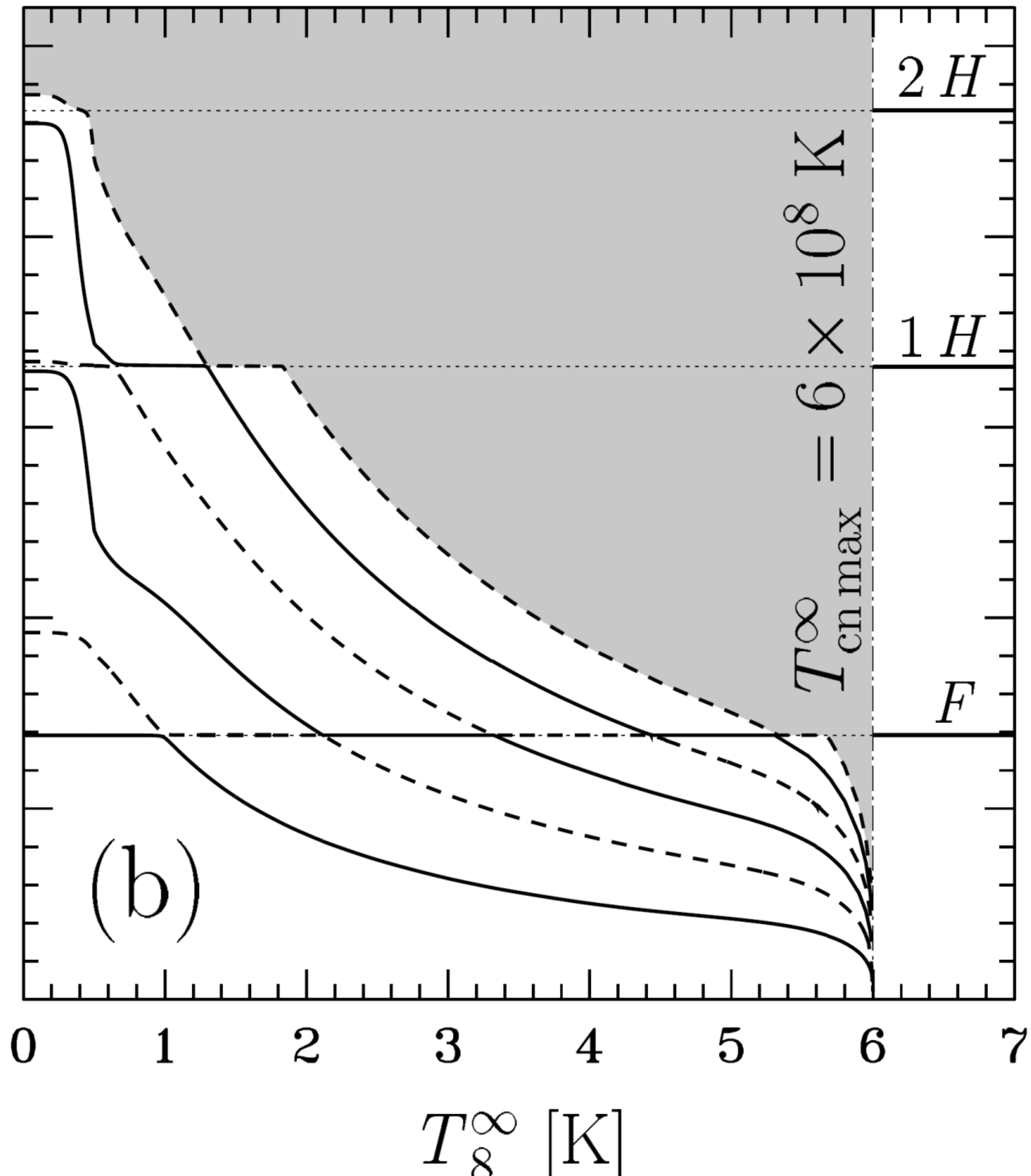
• According to our model, temperatures of the observed neutron stars coincide with the temperatures T_0^∞, at which one has avoided crossings of superfluid modes and the normal quadrupole (m=2) r-mode.

• Comparison of the observed temperatures with the results of theoretical calculations may impose tight constraints on the properties of superdense matter and parameters of superfluidity. Already now we can estimate the maximum critical temperatures of neutrons in the NS core:

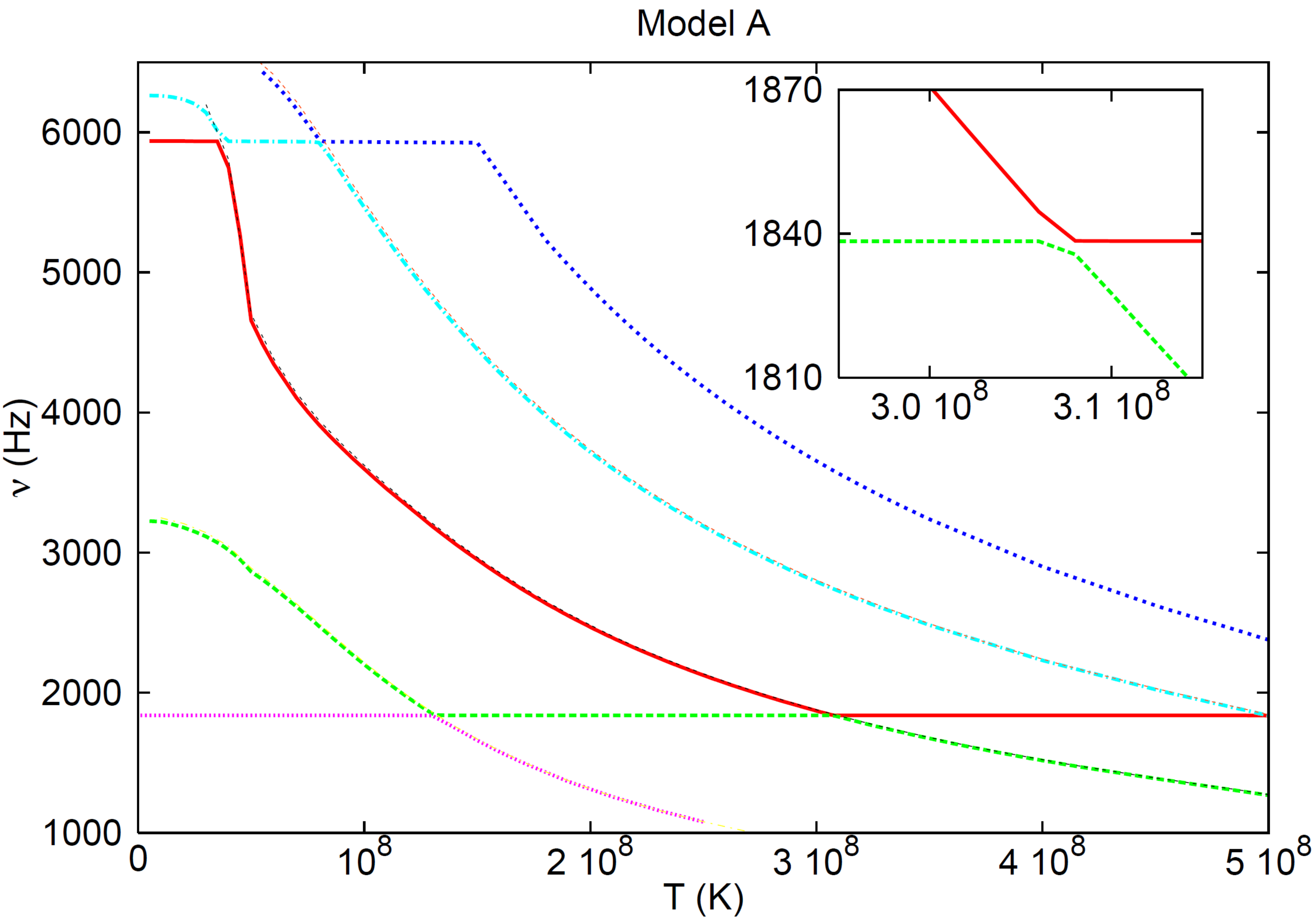
$$2 \times 10^8\, K < T_{cn} < 8 \times 10^8\, K$$

At smaller *Tcn* there is no high-temperature peak (no superfluid modes); at larger *Tcn* there will be no low-temperature peak because the eigenfrequencies of the stars climbing up the low-temperature peak will not depend on *T*.

• Other interesting implications of our results will be discussed in the next talk by Andrey Chugunov!

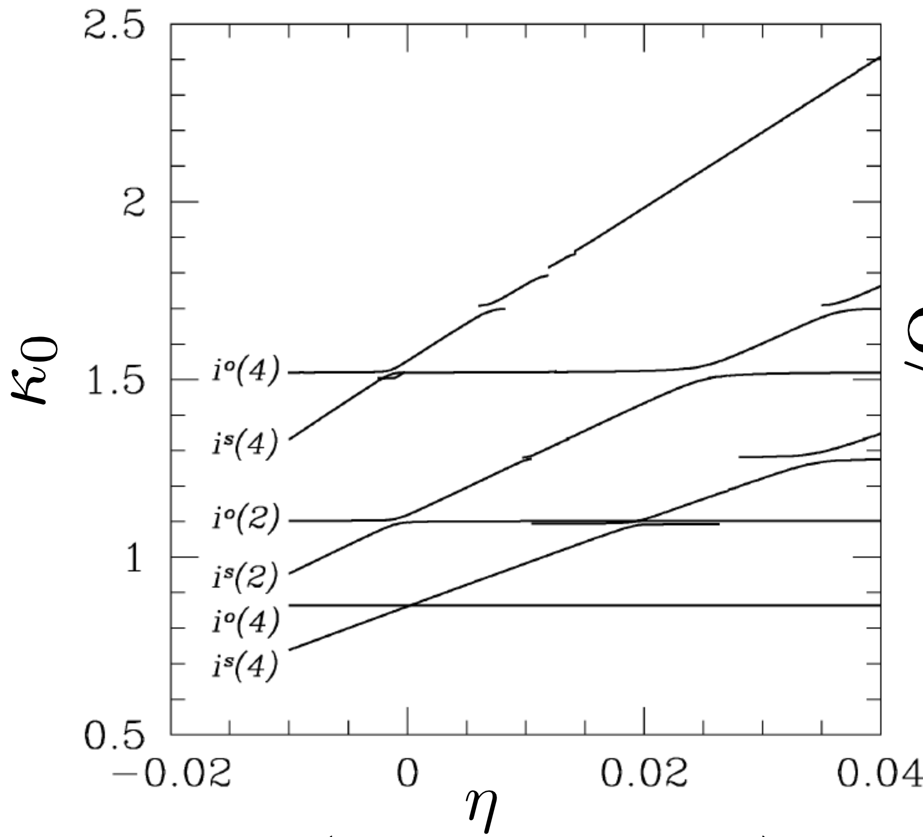


Gusakov et al. (2013)

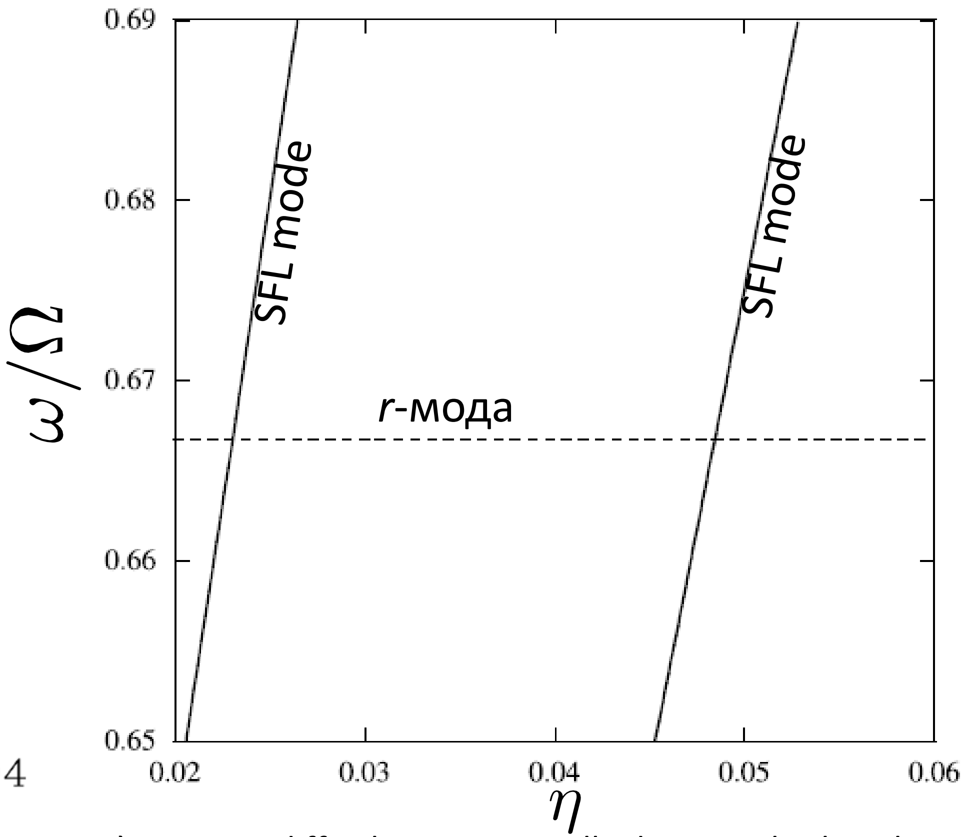


Gualtieri et al, in prep. (2014)

Inertial modes at *T=0*



$$\omega = \Omega\ \left(\kappa_0 + \kappa_2 \Omega^2 + \ldots\right)$$



\`it is quite difficult to numerically discern whether the mode crossings result in avoided crossings or degeneracy of the mode frequencies at the crossing point'

Yoshida & Lee, MNRAS **344** (2003), 207

Lee & Yoshida, ApJ **586** (2003), 403

Neutron Stars in LMXB: observations

Source	ν [Hz]	$\frac{T^{\infty}_{\rm eff}}{10^6\,{\rm K}}$	Ref.	$\frac{T^{\infty}_{\rm acc}}{10^8\,{\rm K}}$	$\frac{T^{\infty}_{\rm fid}}{10^8\,{\rm K}}$	$\frac{T^{\infty}_{\rm Fe}}{10^8\,{\rm K}}$	$\frac{\dot M}{M_\odot}$ [yr^{-1}]	Ref.
4U 1608-522	620	1.51	[2]	0.93	1.90	2.47	3.6×10^{-10}	[3]
SAX J1750.8-2900	601	1.72	[4]	1.18	2.57	3.11	2×10^{-10}	[4]
IGR J00291-5934	599	0.63^a	[5]	0.21	0.24	0.52	2.5×10^{-12}	[5]
MXB 1659-298	556	0.63	[6]	0.21	0.24	0.52	1.7×10^{-10}	[3]
EXO 0748-676 ^{b}	552	1.26	[7]	0.68	1.20	1.79		
Aql X-1	550	1.26	[8]	0.68	1.20	1.79	4×10^{-10}	[3]
KS 1731-260	526	0.73	[9]	0.27	0.32	0.67	$< 1.5 \times 10^{-9}$	[3]
SWIFT J1749.4-2807	518	< 1.16	[10]	0.59	0.96	1.54		
SAX J1748.9-2021	442	1.04	[11]	0.49	0.72	1.27	1.8×10^{-10}	[3]
XTE J1751-305	435	$< 0.63^a$	[5]	0.21	0.24	0.52	6×10^{-12}	[5]
SAX J1808.4-3658	401	$< 0.27^a$	[5]	0.05	0.05	0.11	9×10^{-12}	[5]
IGR J17498-2921	401	< 0.93	[12]	0.41	0.55	1.04		
HETE J1900.1-2455	377	< 0.65	[13]	0.22	0.25	0.55		
XTE J1814-338	314	$< 0.61^a$	[5]	0.20	0.22	0.49	3×10^{-12}	[5]
IGR J17191-2821	294	< 0.86	[13]	0.36	0.45	0.90		
IGR J17511-3057	245	< 1.1	[13]	0.54	0.84	1.40		
NGC 6440 X-2	205	< 0.37	[13]	0.09	0.09	0.20	1.3×10^{-12}	[14]
XTE J1807-294	190	$< 0.45^a$	[5]	0.12	0.13	0.28	$< 8 \times 10^{-12}$	[5]
XTE J0929-314	185	< 0.58	[15]	0.19	0.20	0.45	$< 2 \times 10^{-11}$	[5]
Swift J1756-2508	182	< 0.96	[13]	0.43	0.59	1.10		

[1] A. Patruno, ApJ **722** (2010), 909.
[2] R. E. Rutledge et al., ApJ **514** (1999), 945.
[3] C. O. Heinke et al., ApJ **660** (2007), 1424.
[4] A. W. Lowell et al., ApJ **749** (2012), 111.
[5] C. O. Heinke et al., ApJ **691** (2009), 1035.
[6] E. M. Cackett et al., ApJ Lett. **687** (2008), L87.
[7] N. Degenaar et al., MNRAS **412** (2011), 1409.
[8] E. M. Cackett et al., MNRAS **414** (2011), 3006
[9] E. M. Cackett, et al., ApJ Lett. **722** (2010), L137.
[10] N. Degenaar et al., ApJ **756** (2012), 148.
[11] E. M. Cackett et al., ApJ **620** (2005), 922
[12] P. G. Jonker et al., ATel #3559 (2011), 1.
[13] B. Haskell et al., MNRAS **424** (2012), 93.
[14] C. O. Heinke et al., ApJ **714** (2010), 894.
[15] R. Wijnands et al., ApJ **619** (2005), 492.

$$T^{\infty}_{\rm eff} \to T^{\infty} :$$

A.Y. Potekhin et al., A&A **323** (1997), 415

Three models:

- Fully accreted envelope
- Accreted up to $P/g = 10^9\ {\rm g\,cm^{-2}}$
- Fe

$$M = 1.4\,M_\odot,\ R = 10\ {\rm km}$$