

# The neutron star equation of state

*Chiral forces in the Green's functions formalism*

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University of Surrey

with A. Carbone & A. Polls

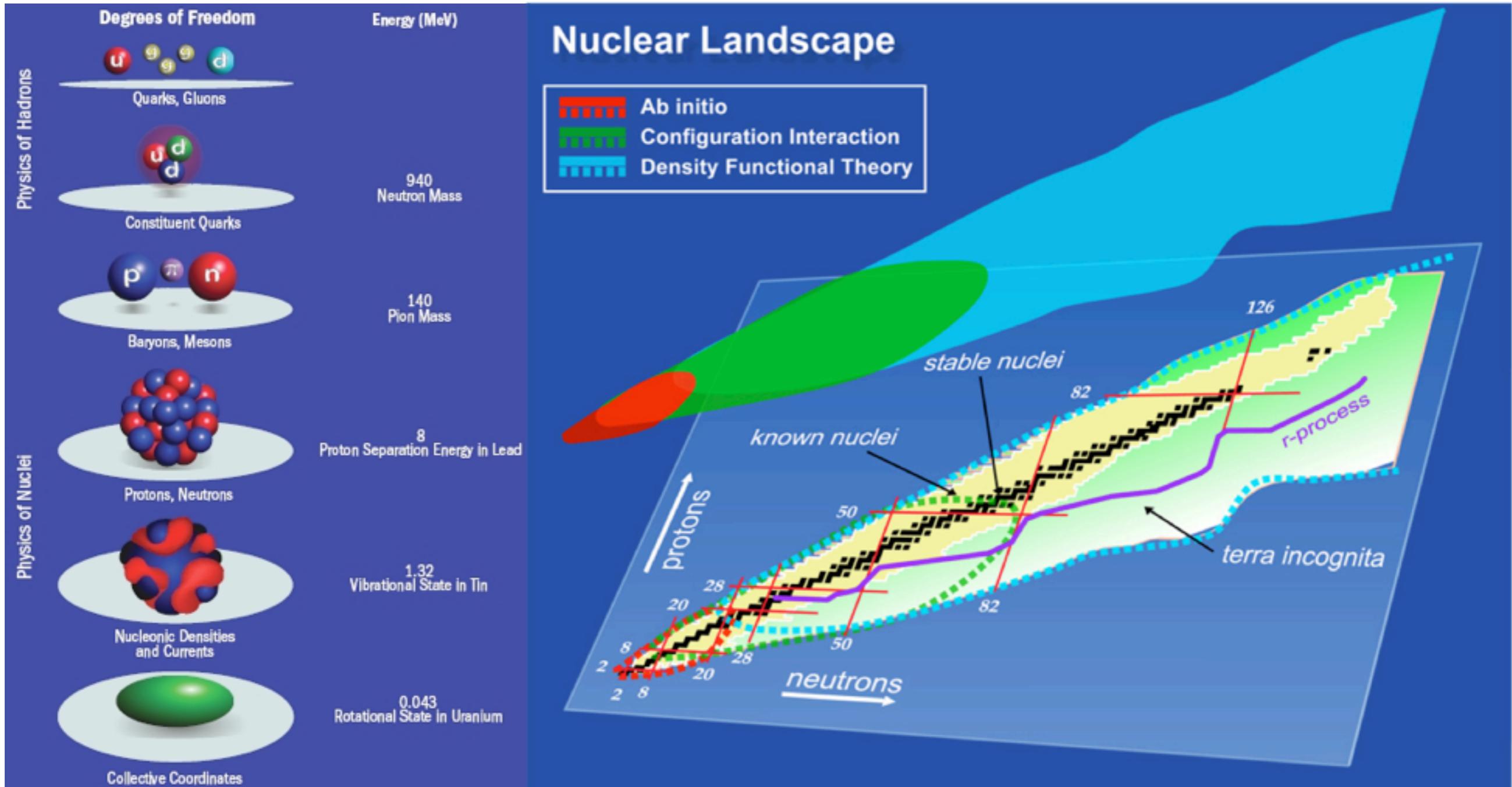


UNIVERSITAT DE BARCELONA



Compstar Conference, Florence, 25 March 2014

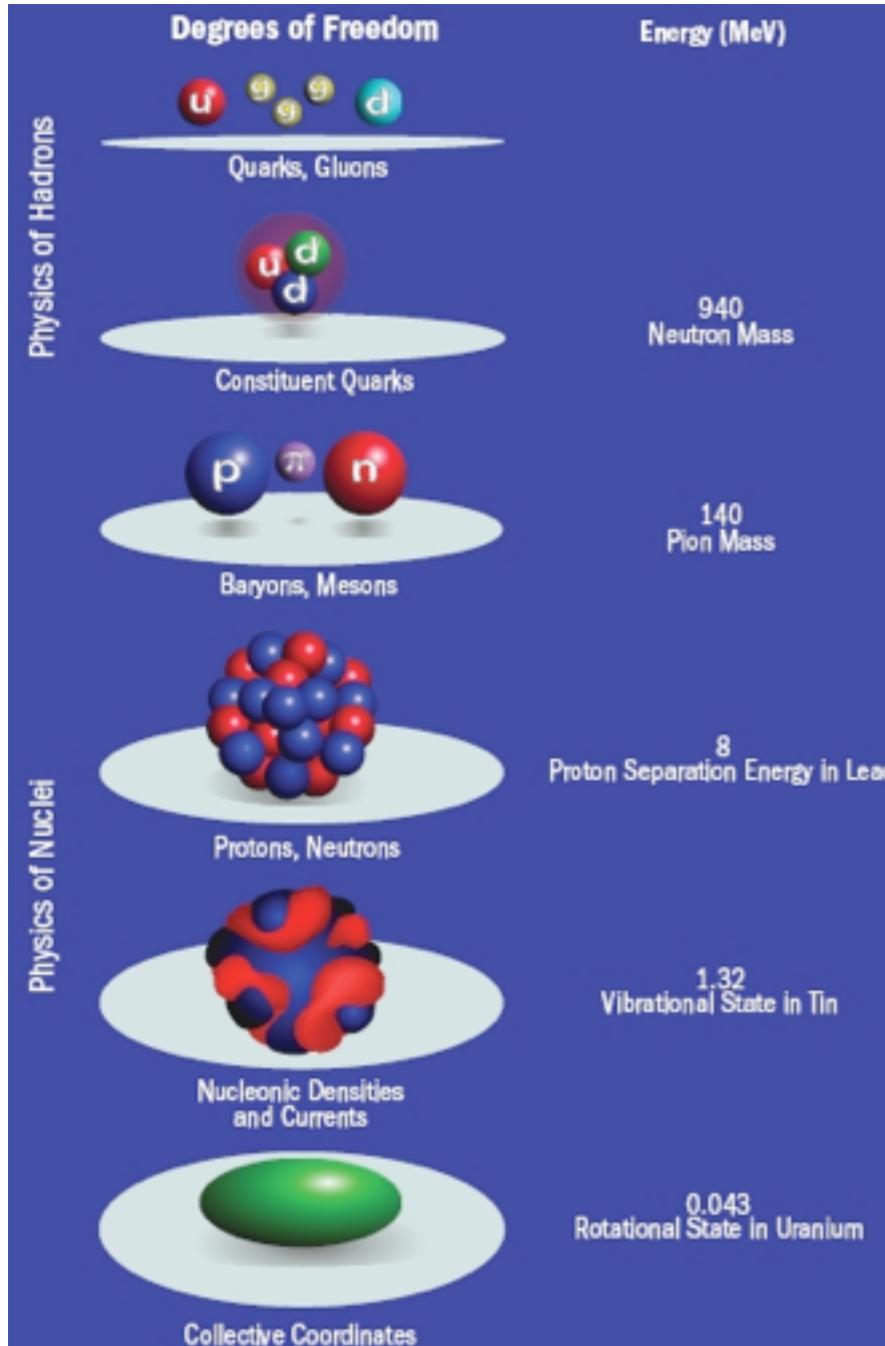
# Theoretical nuclear physics



DOE/NSF Nuclear Science Advisory Committee, The Frontiers of Nuclear Science: A Long-Range Plan, 2007.

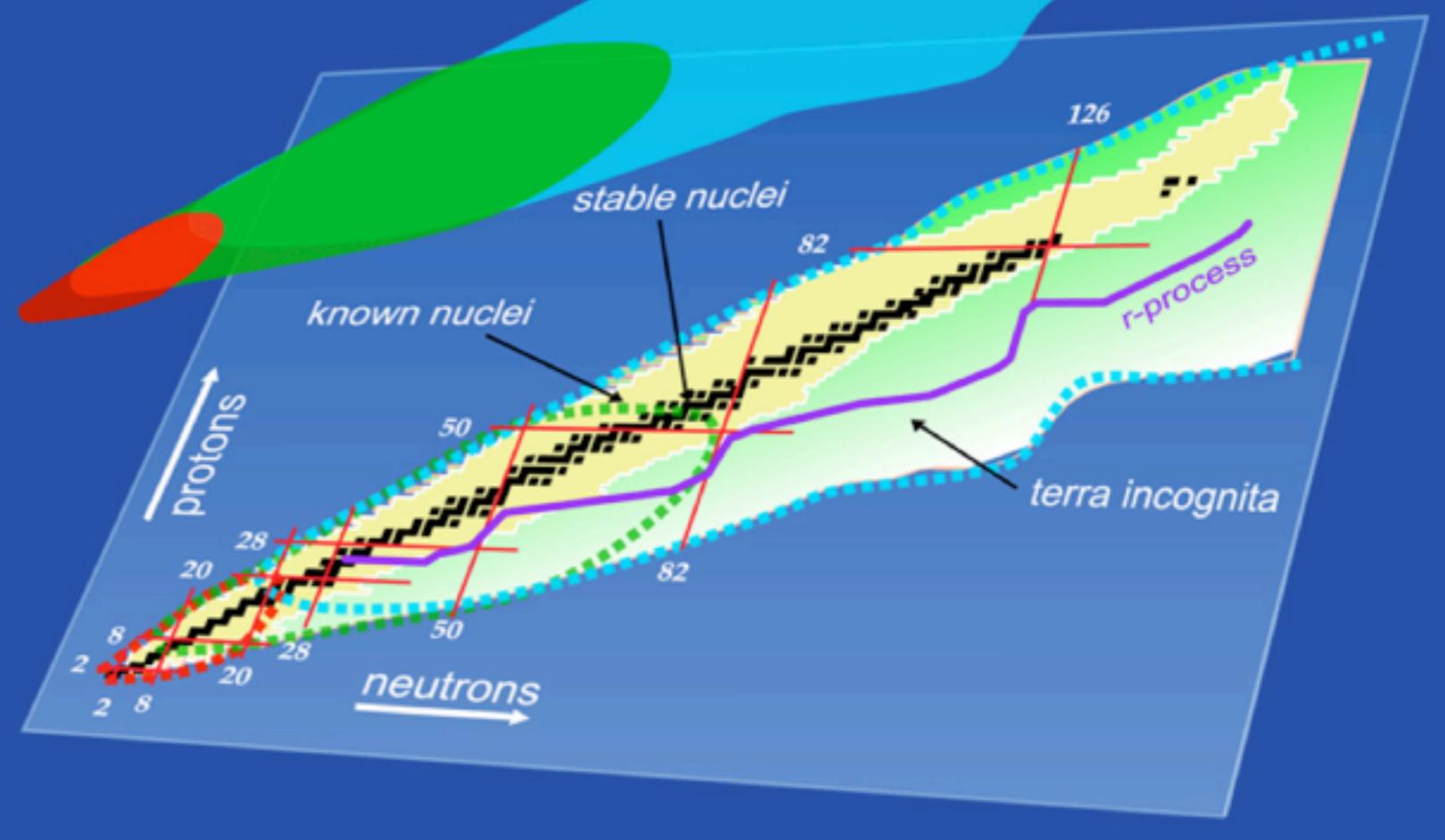
## Two philosophies

# Theoretical nuclear physics



## Nuclear Landscape

Ab initio  
Configuration Interaction  
Density Functional Theory



DOE/NSF Nuclear Science Advisory Committee, The Frontiers of Nuclear Science: A Long-Range Plan, 2007.

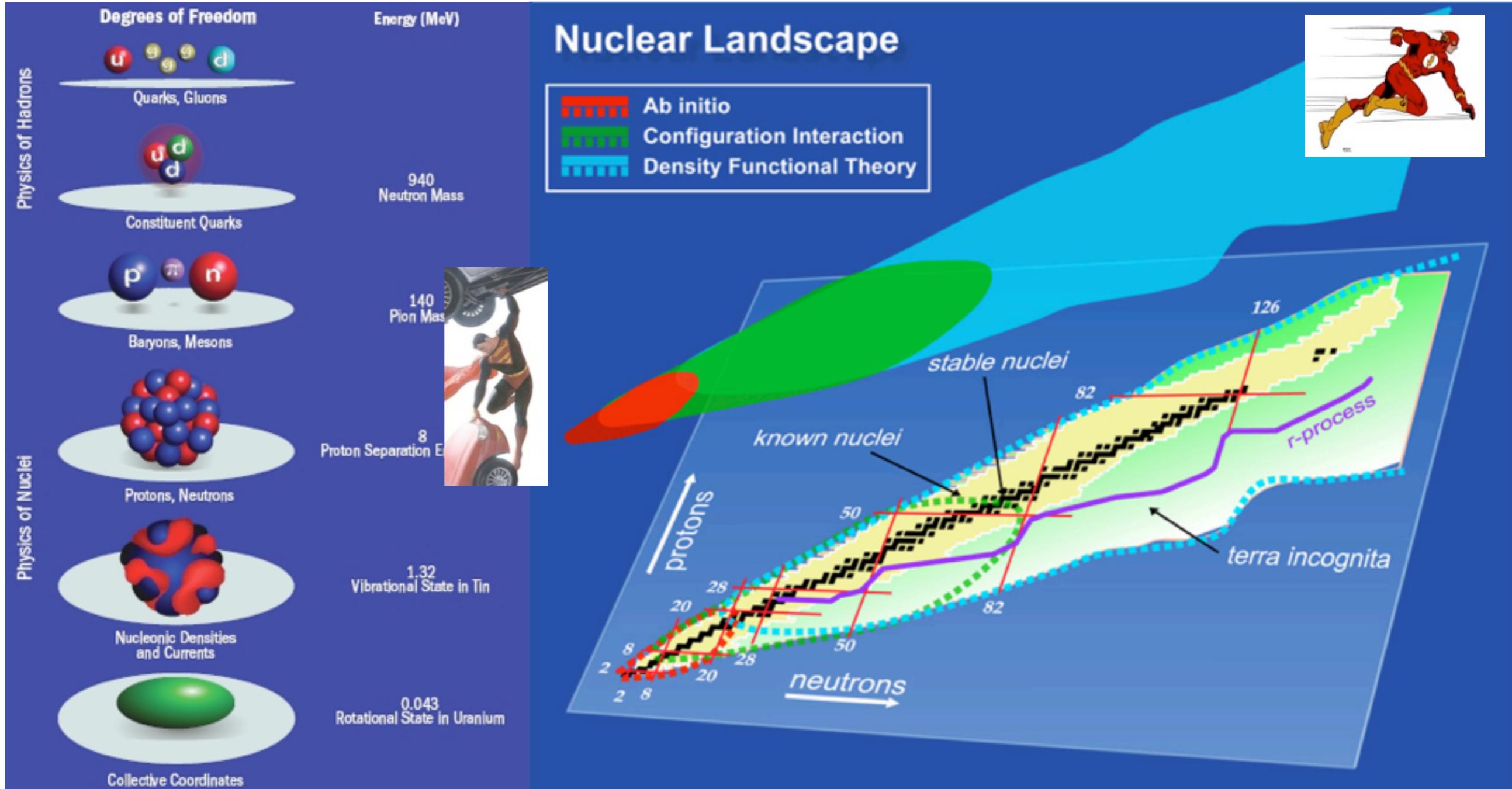
## Two philosophies

- Phenomenological: nuclear properties from energy density functional.

# Theoretical nuclear physics



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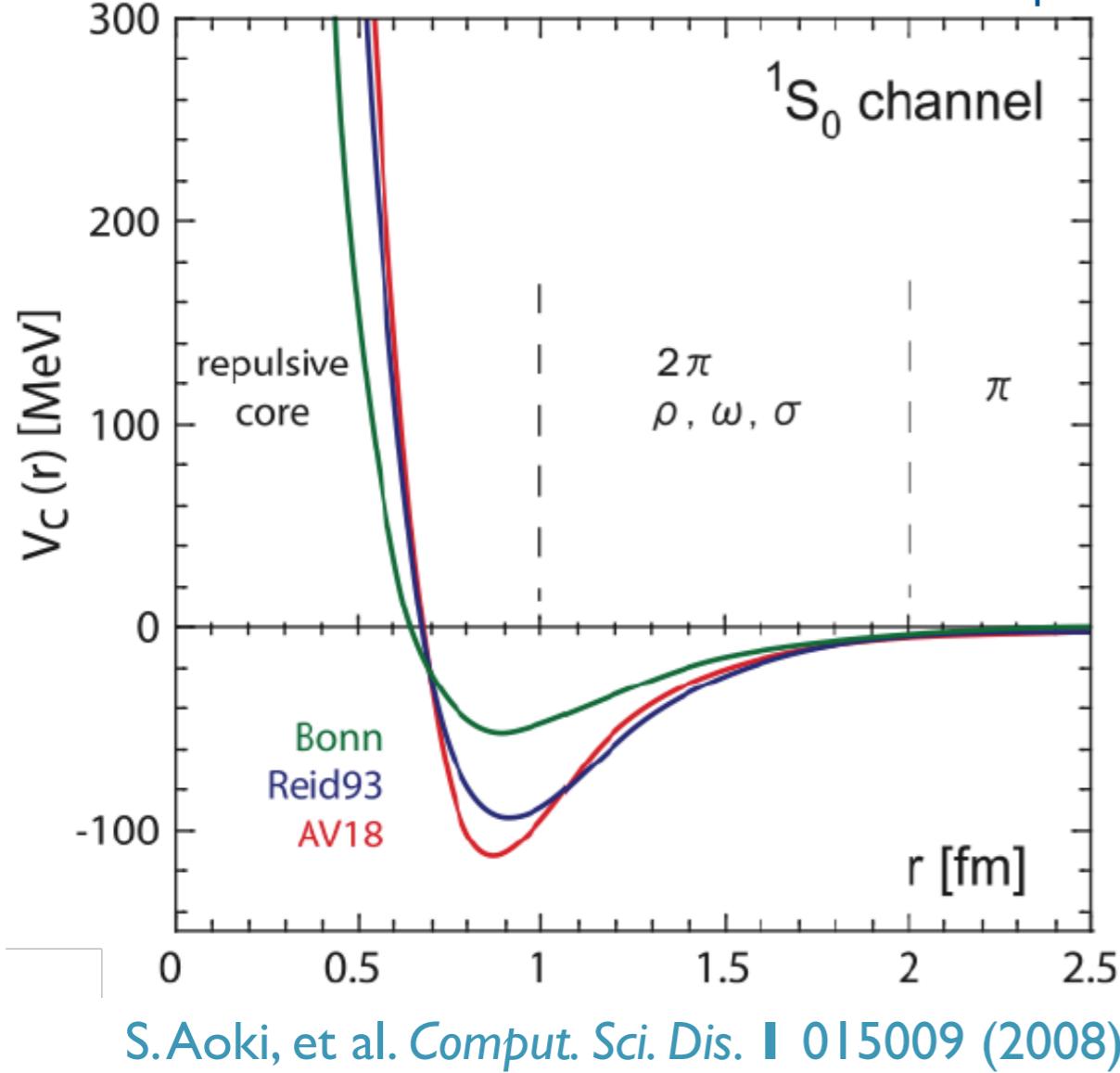


## Two philosophies

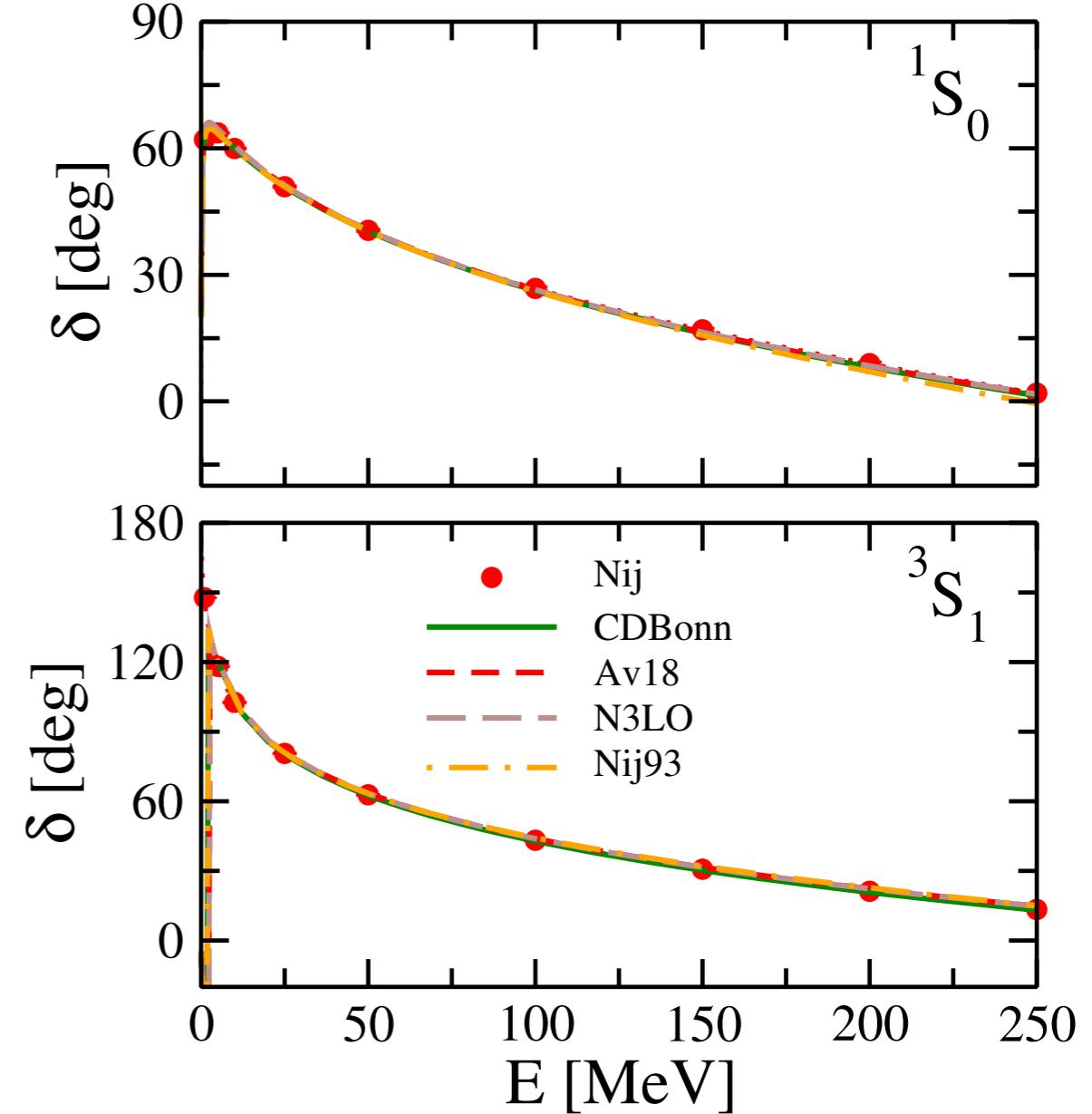
- Phenomenological: nuclear properties from energy density functional.
- Ab initio: from QCD nucleon-nucleon force to nuclei?

# Complications

NN interaction is not unique

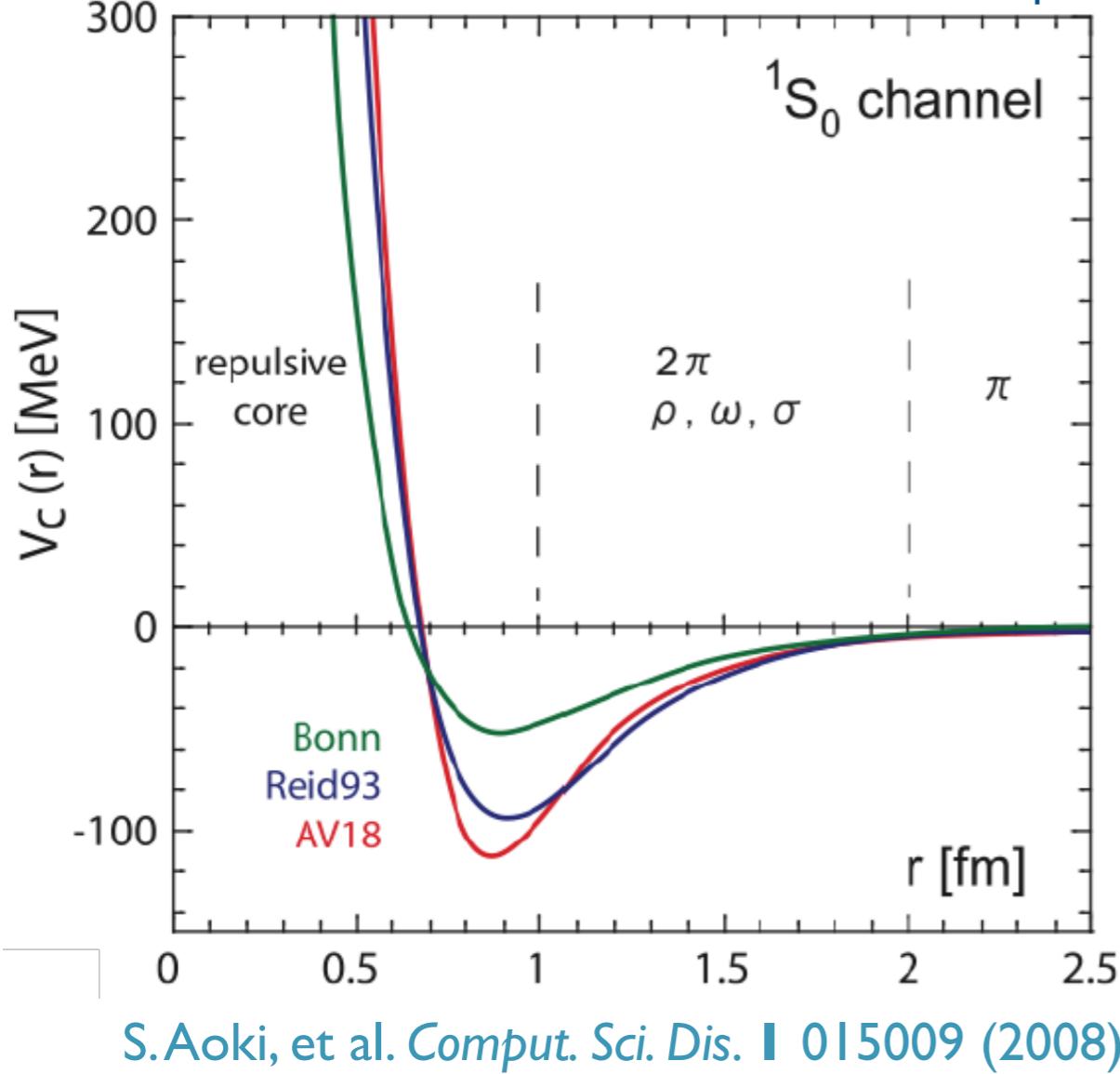


...but phase-shift equivalent!

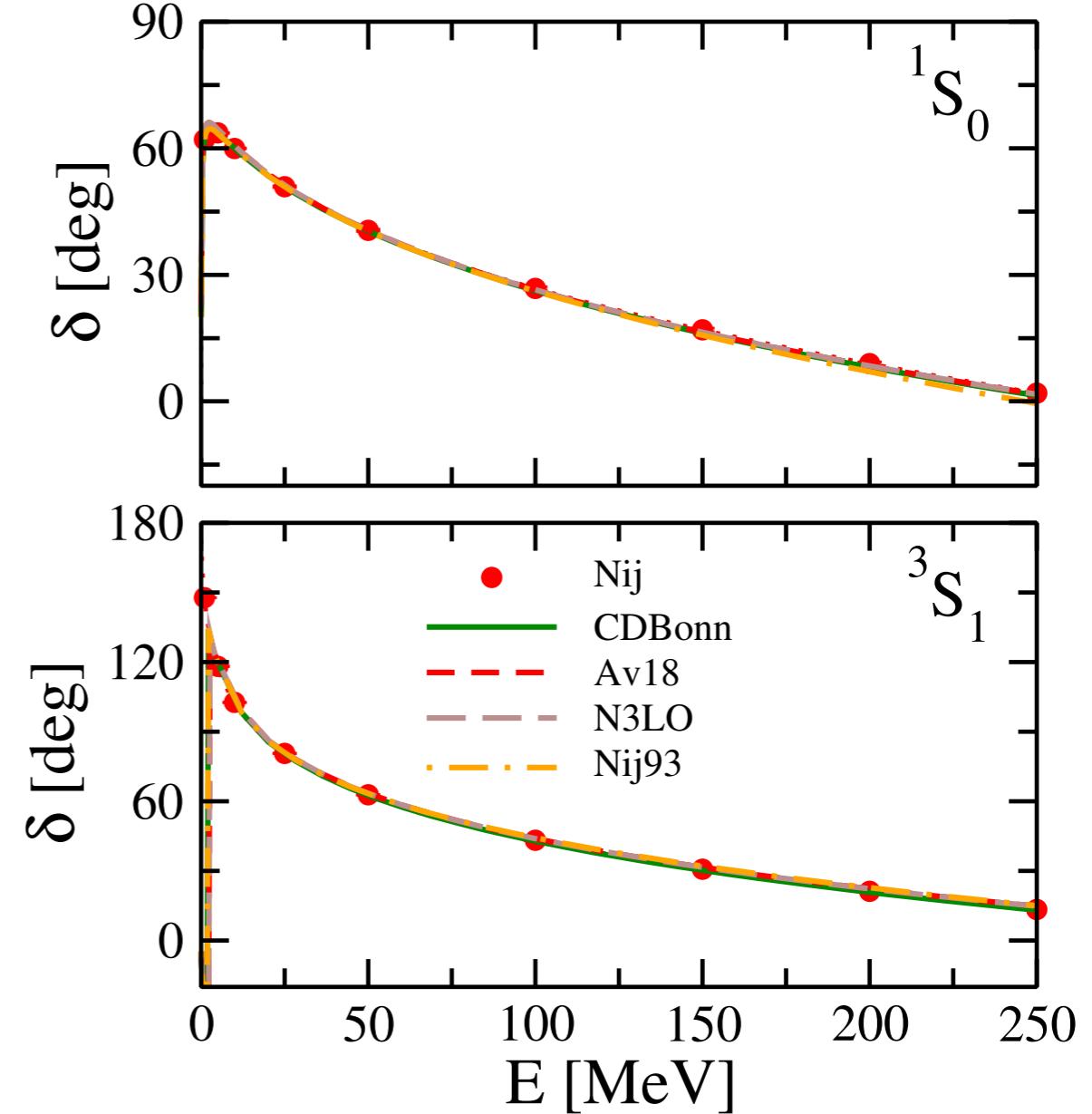


# Complications

NN interaction is not unique



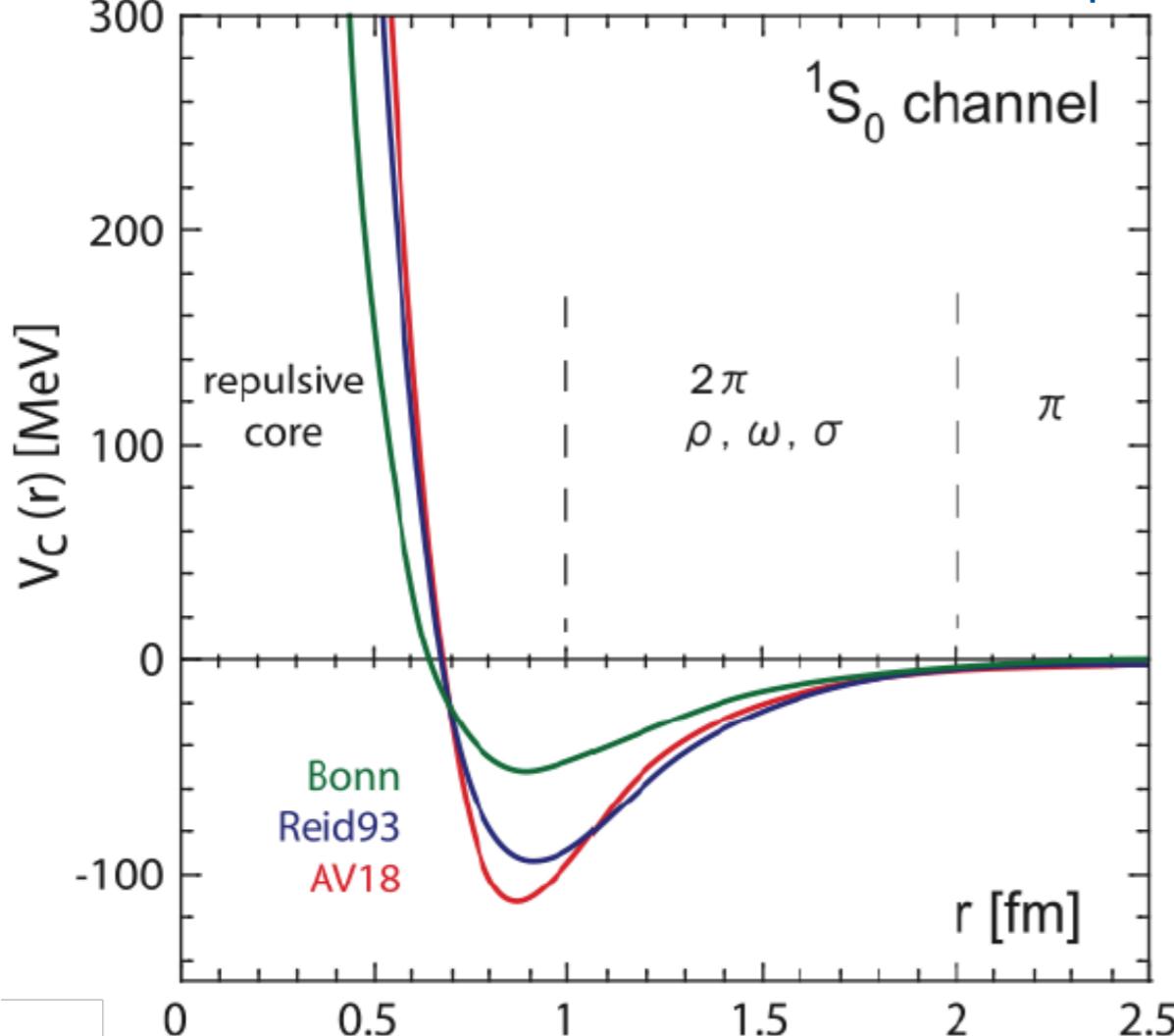
...but phase-shift equivalent!



- Non-uniqueness of nucleon forces ✗

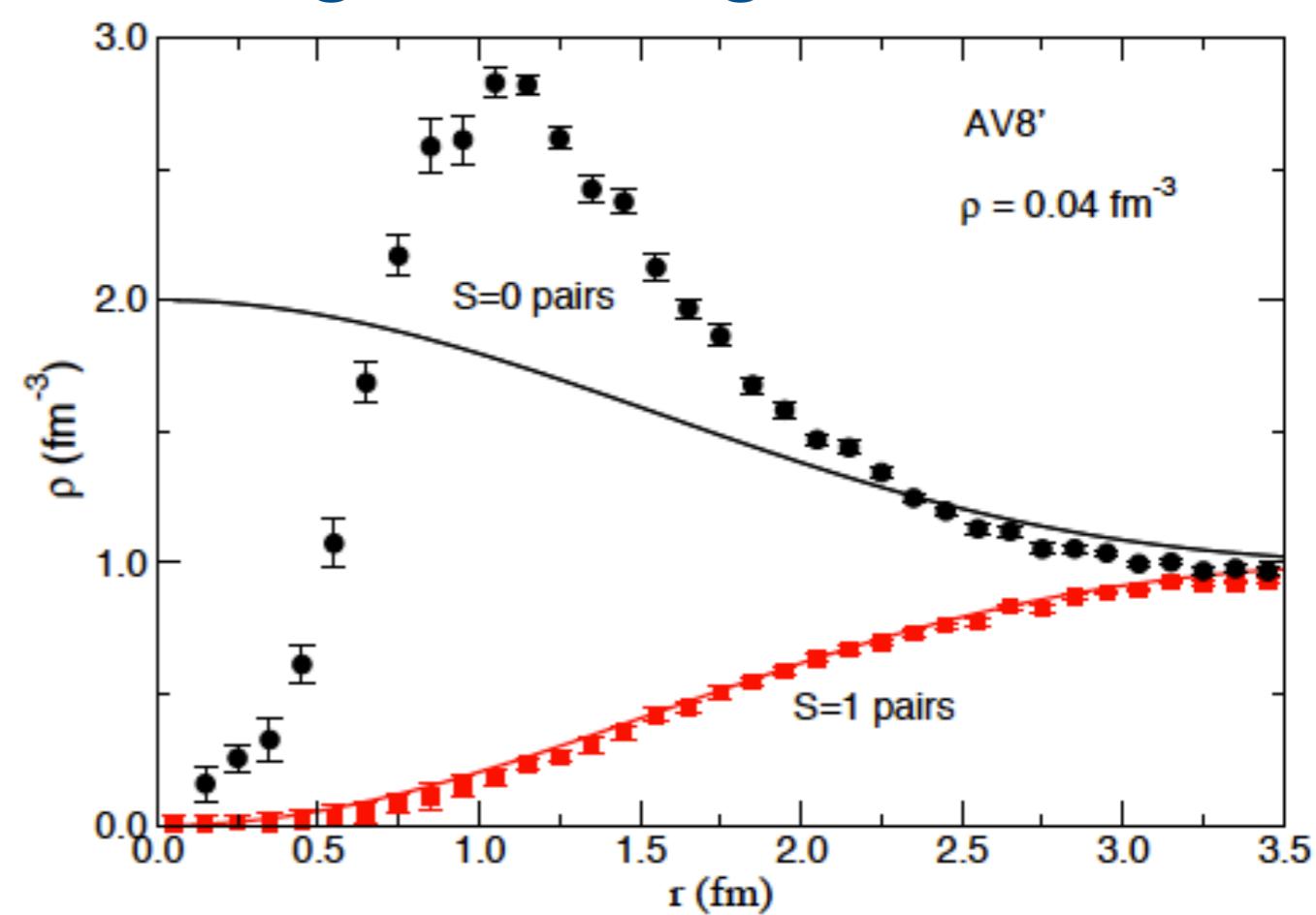
# Complications

NN interaction is not unique



S.Aoki, et al. *Comput. Sci. Dis.* **I** 015009 (2008)

Strong short-range correlations

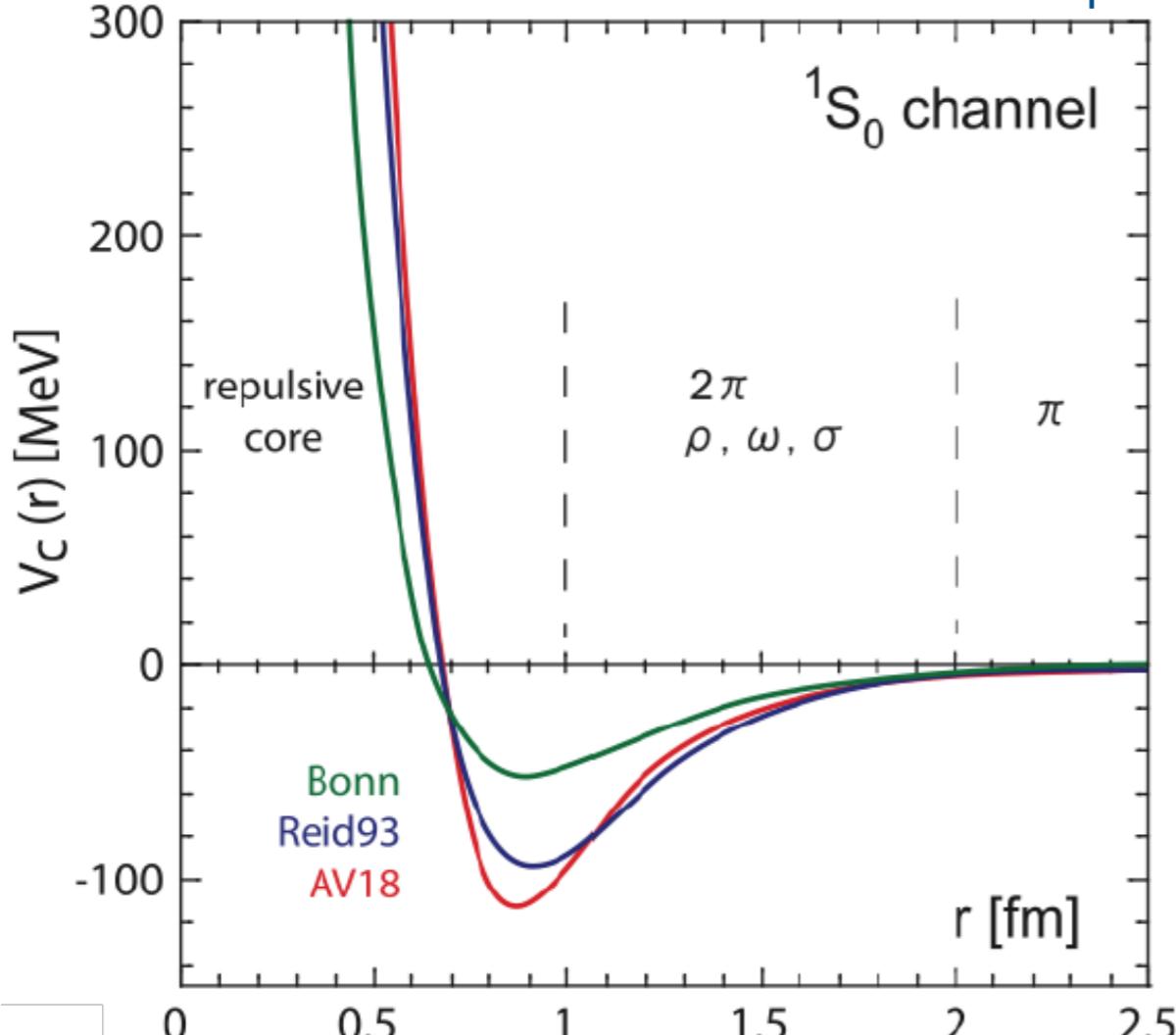


Carlson et al., *Phys. Rev. C* **68** 025802 (2003)

- Non-uniqueness of nucleon forces ✗
- Short-range core needs many-body treatment ✗

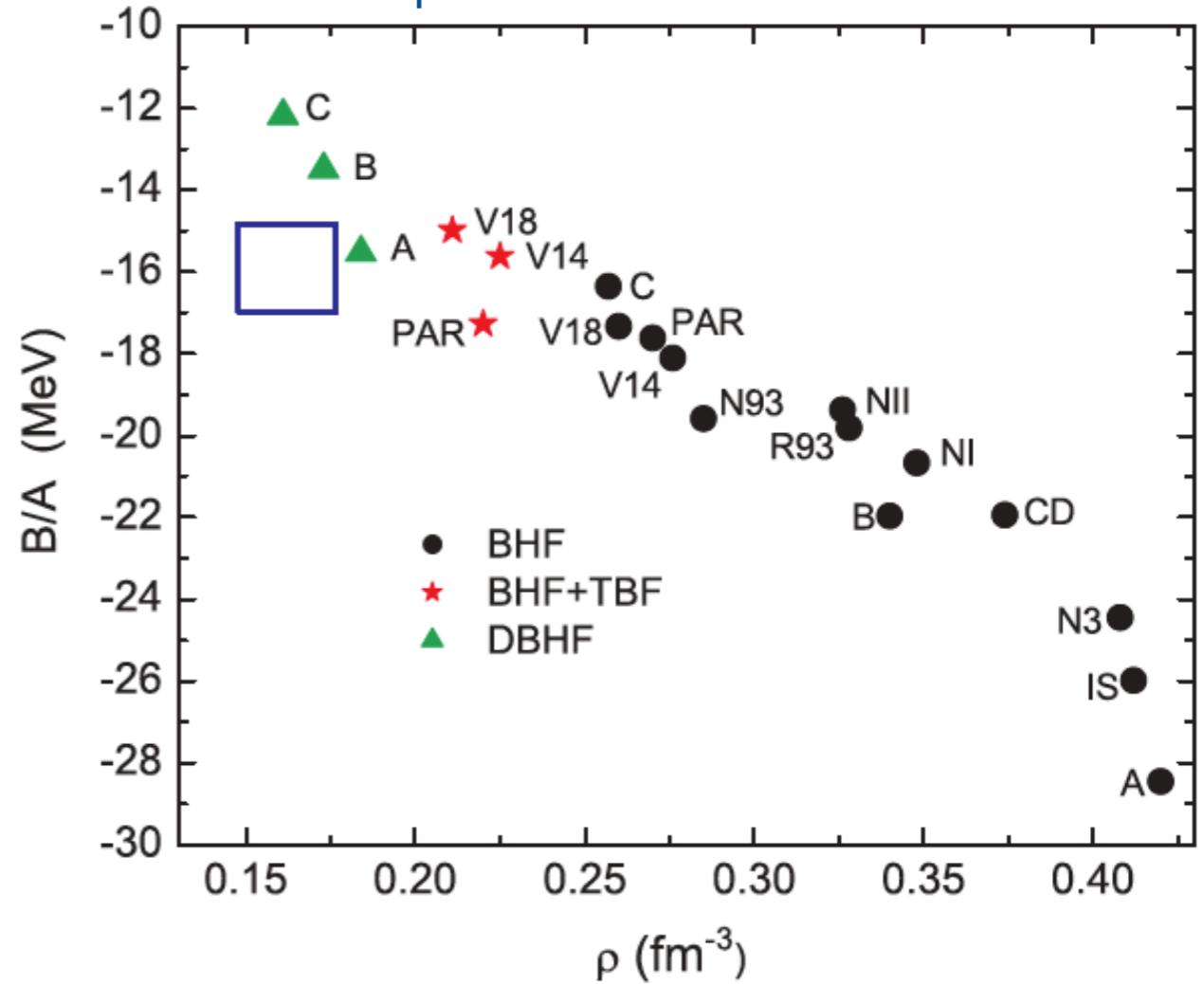
# Complications

NN interaction is not unique



S.Aoki, et al. *Comput. Sci. Dis.* I 015009 (2008)

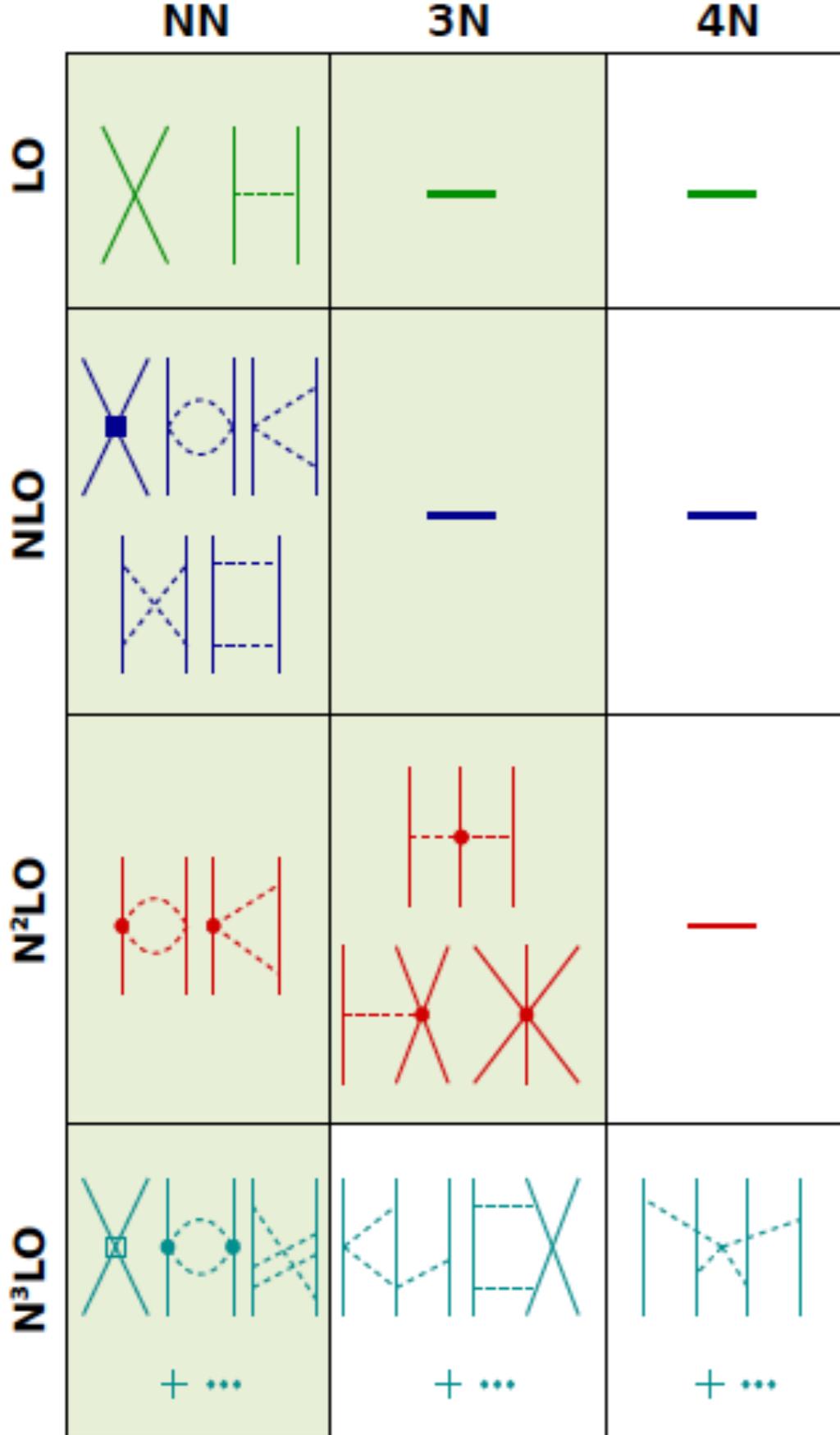
Saturation point of nuclear matter



Li, Lombardo, Schulze et al. *PRC* 74 047304 (2006)

- Non-uniqueness of nucleon forces ✗
- Short-range core needs many-body treatment ✗
- Three-body forces needed for saturation ✗

# NN forces from EFTs of QCD

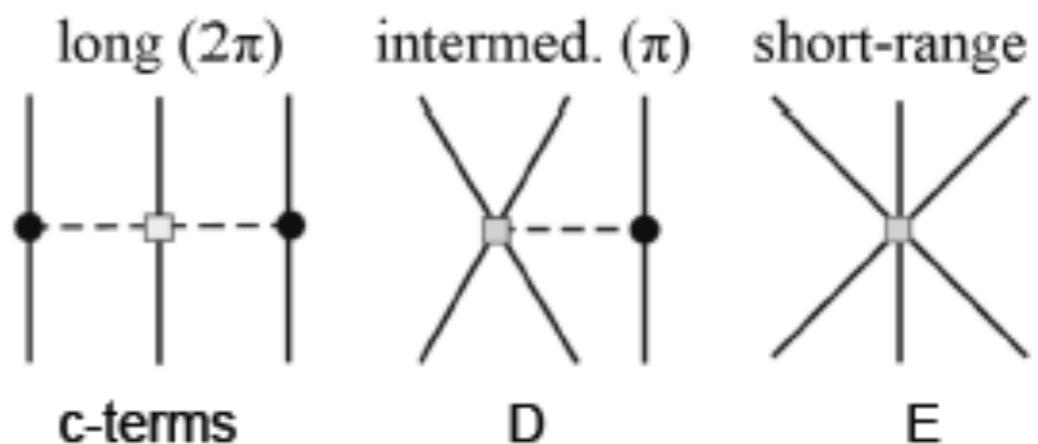


$$\mathcal{O}\left(\frac{Q}{\Lambda}\right)$$

$$\Lambda \sim 1 \text{ GeV}$$

## Chiral perturbation theory

- $\pi$  and N as dof
- Systematic expansion
- 2N at  $N^3LO$  - LECs from  $\pi N$ , NN
- 3N at  $N^2LO$  - 2 more LECs
- (Often further renormalized)



Weinberg, Phys. Lett. B **251** 288 (1990), Nucl. Phys. B **363** 3 (1991)

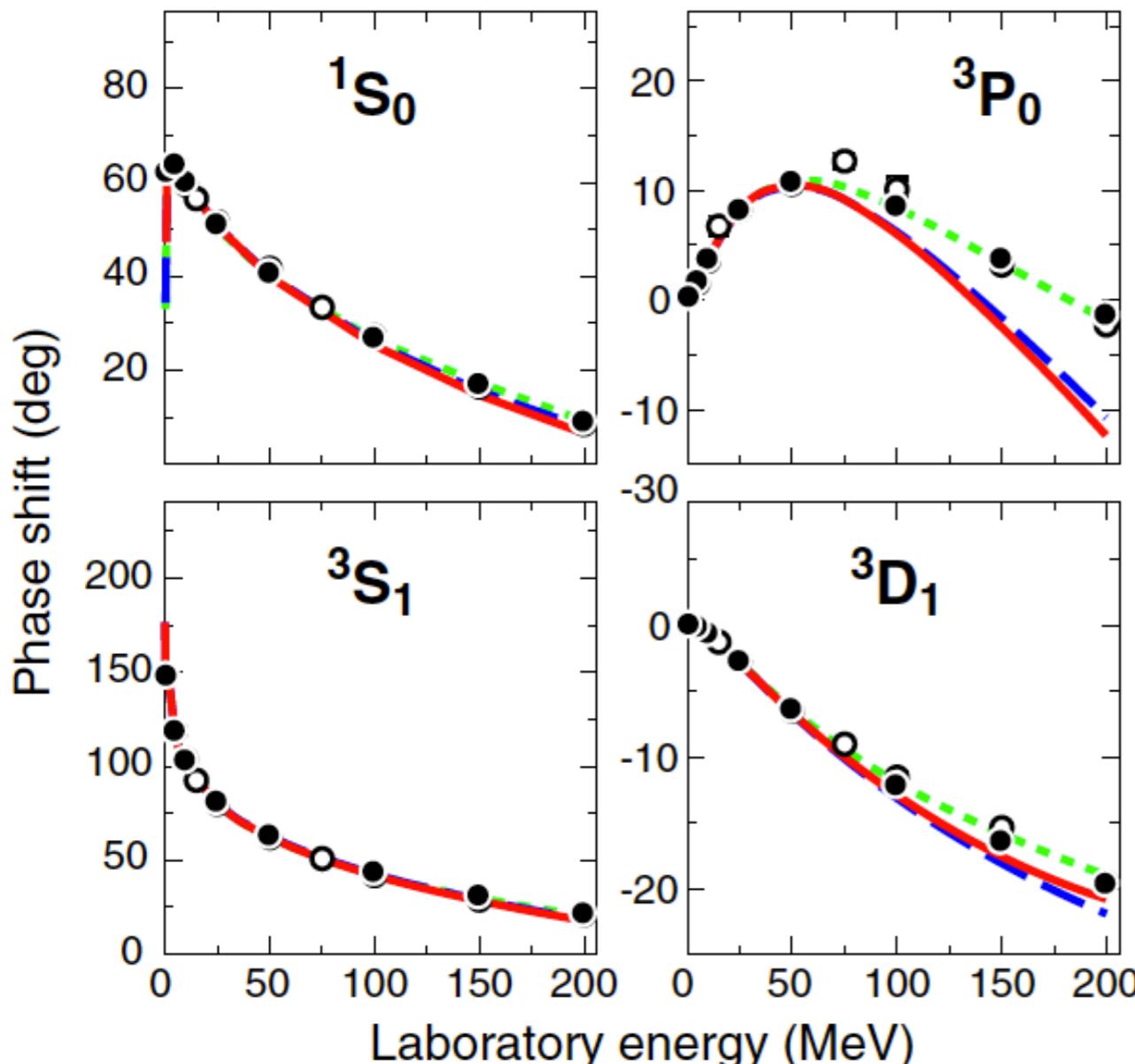
Entem & Machleidt, Phys. Rev. C **68**, 041001(R) (2003)

Tews, Schwenk et al., Phys. Rev. Lett. **110**, 032504 (2013) 4

# LECs fitting

2NF constants:

deuteron + phase-shifts

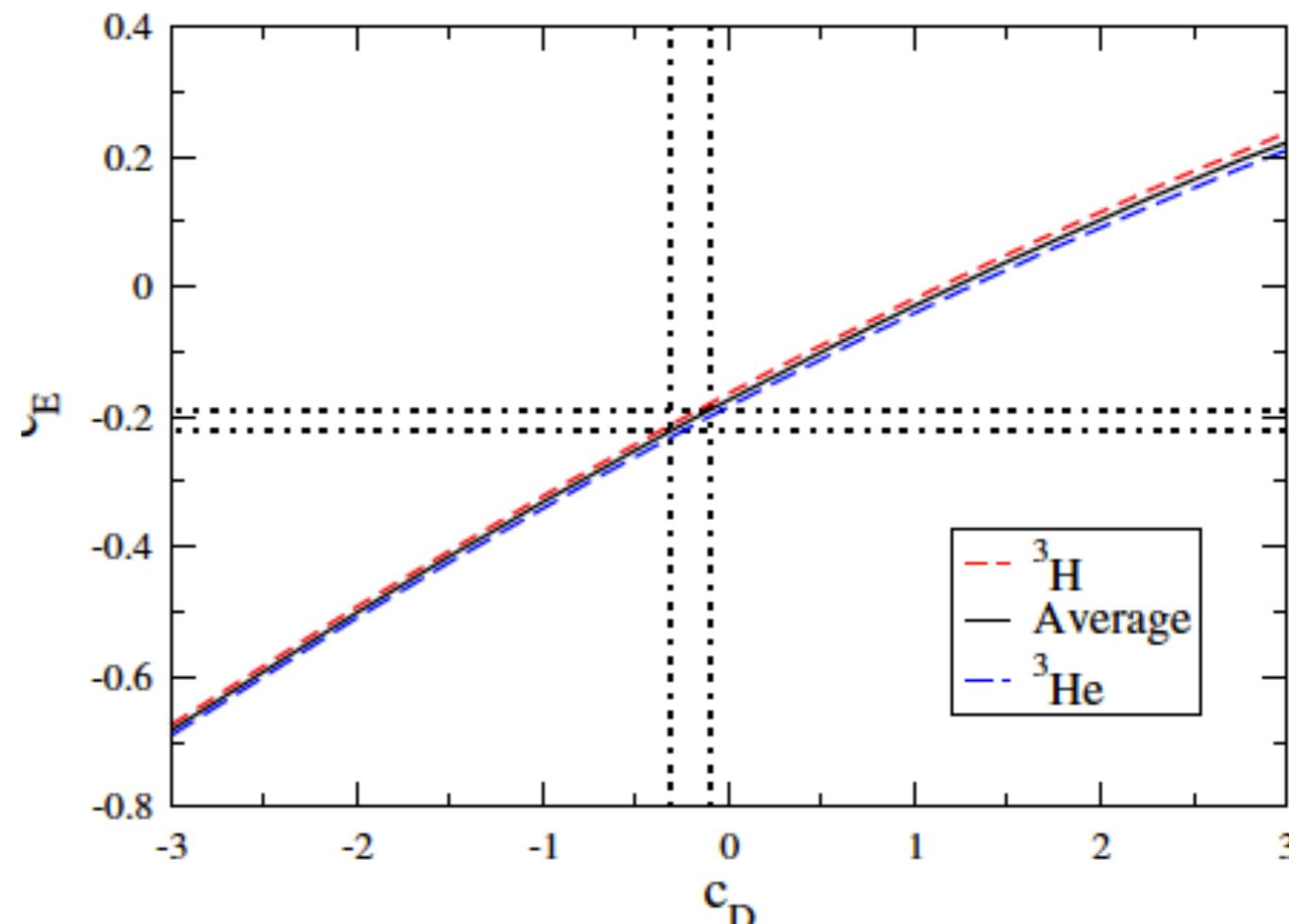


$N^3LO$ : Entem & Machleidt, Phys. Rev. C **68** 041001 (2003)

$N^2LO$ : Ekstrom et al., PRL **110** 192502 (2013)

3NF constants:

$A=3 + \beta$  decay



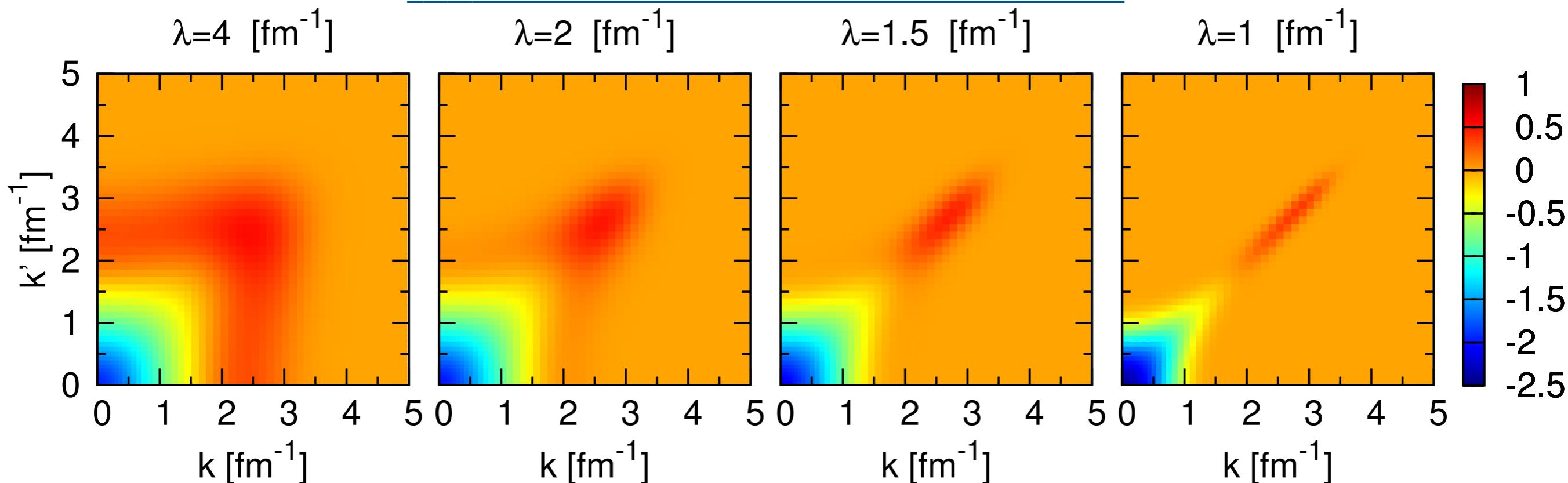
Nogga et al., Phys. Rev. C **73** 064002 (2006)

Gazit et al., PRL **103** 102502 (2009)

- Non-uniqueness now is quantifiable!

# Further renormalizations

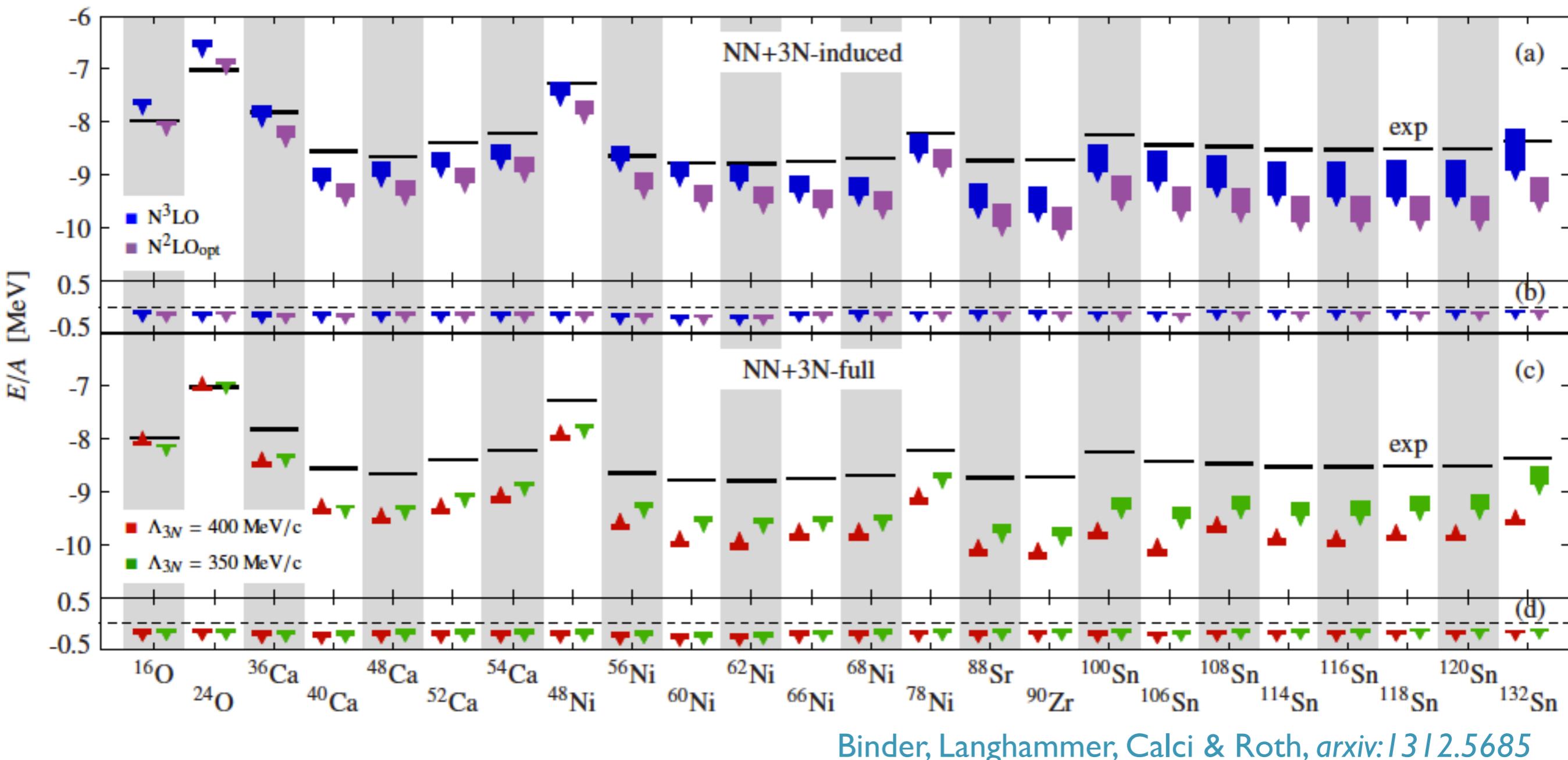
$^1S_0$  NN matrix elements from N3LO



$$\frac{dH_s}{ds} = [[T_{rel}, H_s], H_s] \Leftrightarrow \lambda = s^{-1/4}$$

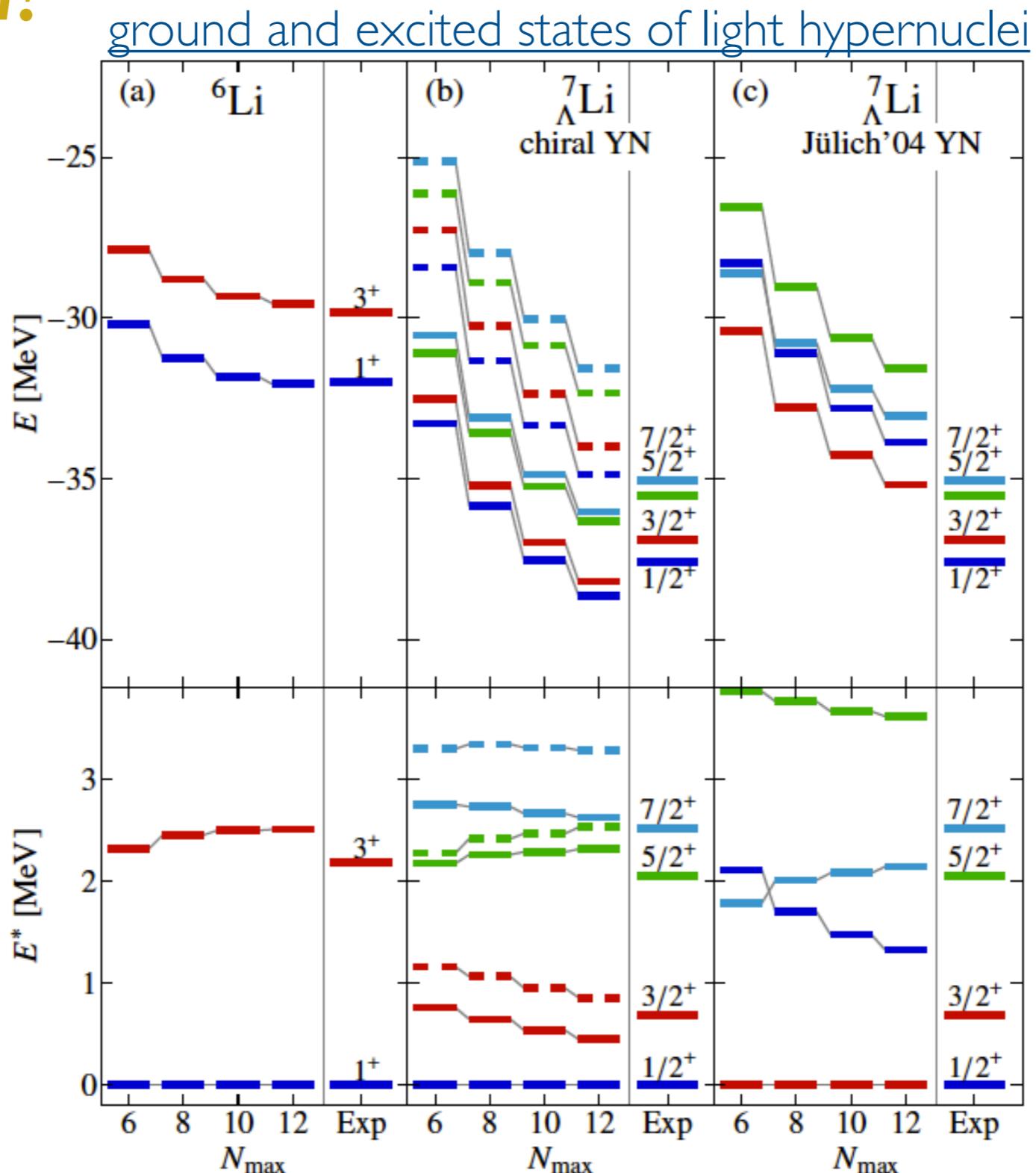
- Series of **unitary** transformation
- Observables **unaltered**, but force becomes **perturbative**
- Induces 3-, 4- and up to A-body forces...
- If these can be treated perturbatively, calculation is **easier**

# The reach of *ab initio*: 2014



- Consistent calculations up to  $A=132$
- Many-body (CC) errors under control
- Overbinding even when 3NF accounted for
- Radii are too small as well

# Hypernuclei!



- IT-NCSM with chiral NN & different YN forces
- Hypernuclear structure hints at nonperturbative YNN

# SCGF: diagrammatic expansion

*N-body Green's function*

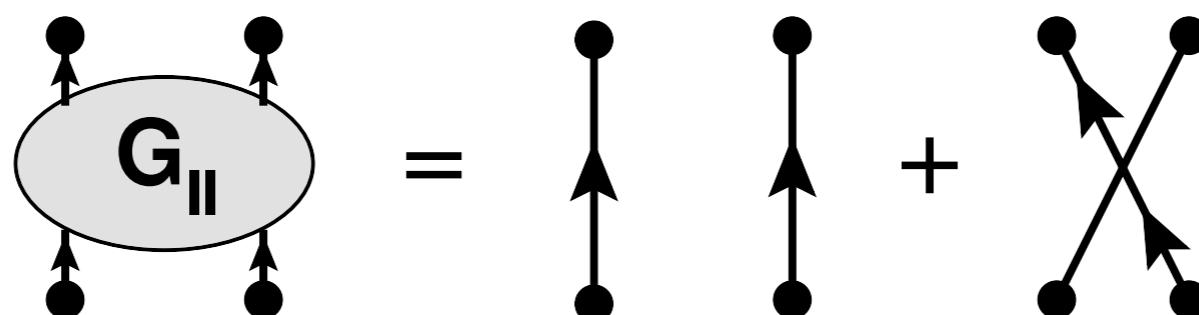
$$i^N G(\mathbf{1}, \dots, \mathbf{N}; \mathbf{1}', \dots, \mathbf{N}') = \left\langle \mathcal{T} \{ a(\mathbf{1}) \cdots a(\mathbf{N}) a^\dagger(\mathbf{N}') \cdots a^\dagger(\mathbf{1}') \} \right\rangle$$

$\langle \cdot \rangle \rightarrow$  average over states

$\mathcal{T} \rightarrow$  some sort of time ordering (real, imaginary, on a contour...)

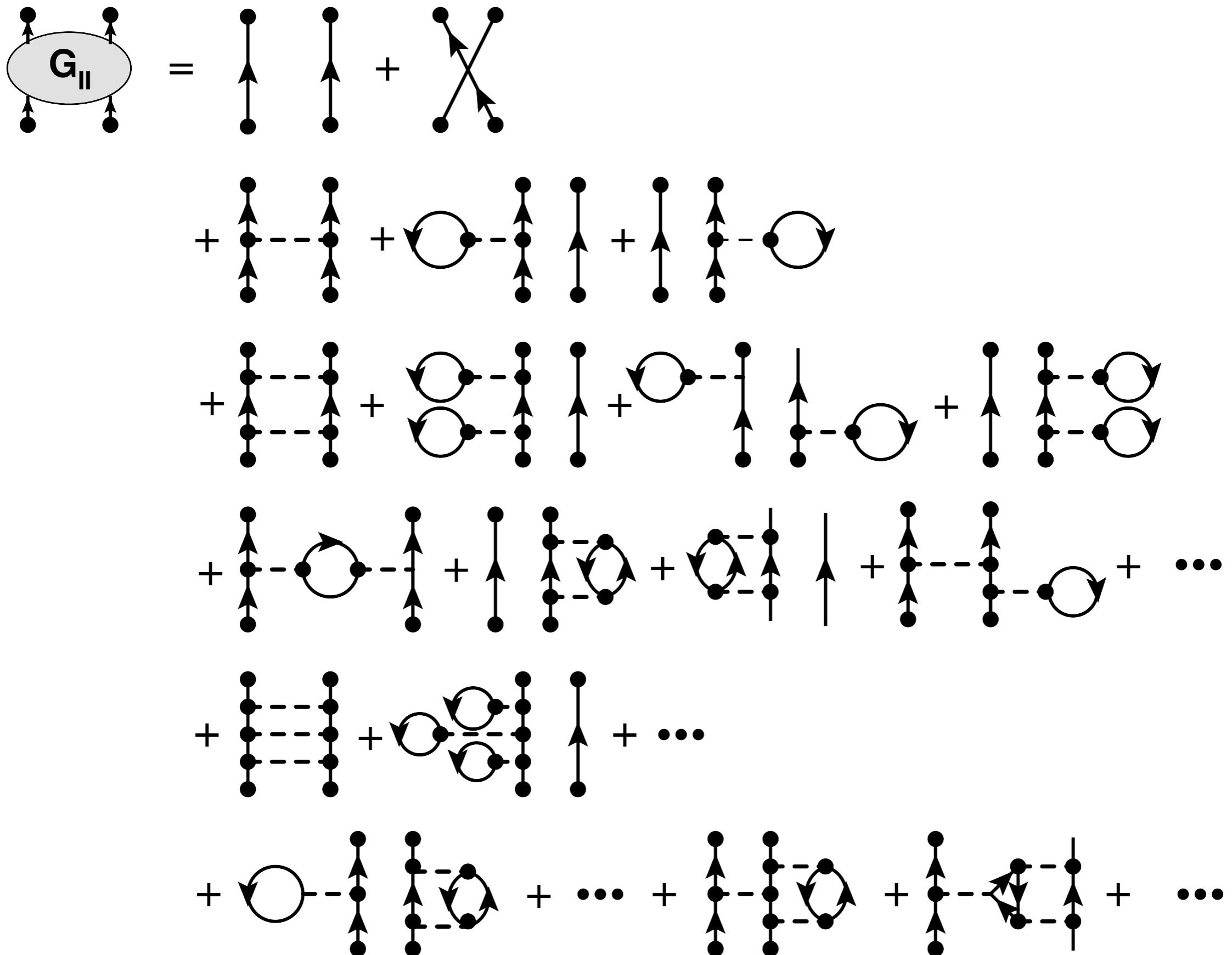
$\mathbf{1} \rightarrow \mathbf{r}_1, t_1, \sigma_1, \tau_1$

*Two-body GF, lowest order diagrams*

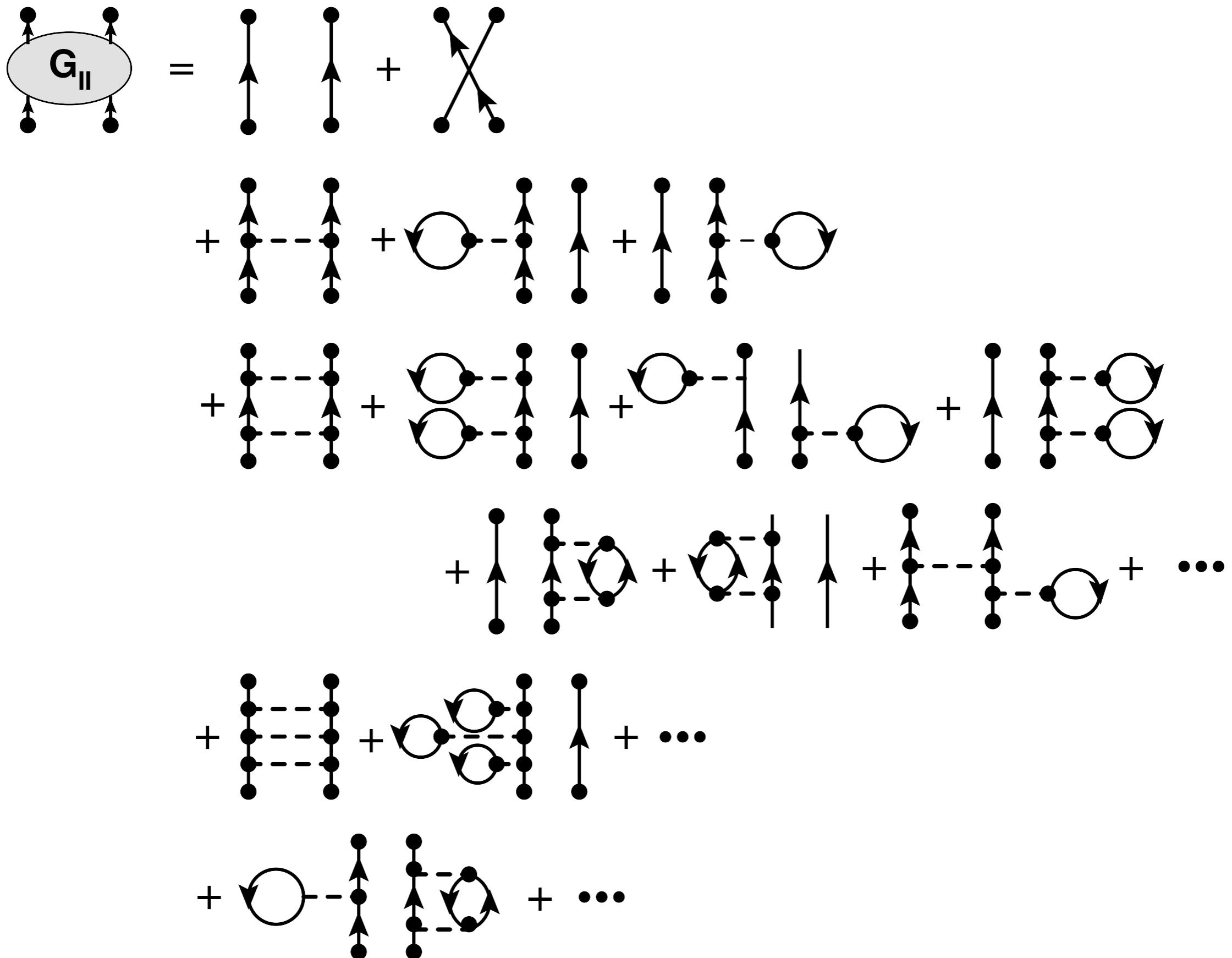


$$G_{II}(1, 2; 1', 2') = G(1, 1')G(2, 2') - G(1, 2')G(1', 2)$$

# SCGF: diagrammatic expansion



# SCGF: diagrammatic expansion

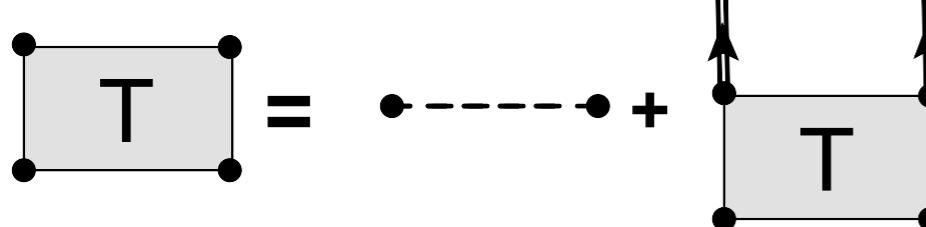


# *Ladder approximation with 3BF*

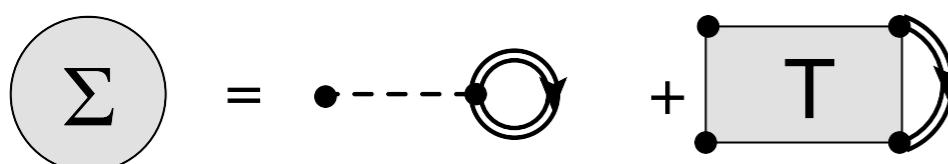
## Two-body interaction



# In-medium **T**-matrix



## Self-energy



## Effective interactions

## *Effective one-body force*

$$\bullet = \bullet - \bullet + \frac{1}{4} \bullet$$

## *Effective two-body force*

$$\text{Diagram A} = \text{Diagram B} + \text{Diagram C}$$

# In-medium **T**-matrix

$$T = \text{spring} + T_{\text{ext}}$$

## Self-energy

$$\Sigma = \bullet - \text{---} \times + \text{---} T + \frac{1}{12} \text{---} \text{---} \text{---} + \dots$$

The diagram illustrates the decomposition of the genus-0 surface  $\Sigma$  into simpler components. It shows a large circle labeled  $\Sigma$ , followed by a subtraction sign ( $-$ ) with a red dot and a red horizontal line crossed out. This is followed by a plus sign ( $+$ ) and a blue square labeled  $T$  with a clockwise arrow around its perimeter. Below this, another plus sign ( $+$ ) is followed by a term involving a vertical line with two loops attached, each with a clockwise arrow, and a horizontal line connecting them. The fraction  $\frac{1}{12}$  is placed next to this term, and three horizontal dashed lines extend from the left and right ends of the term to the right, indicating an ellipsis.

Dyson equation

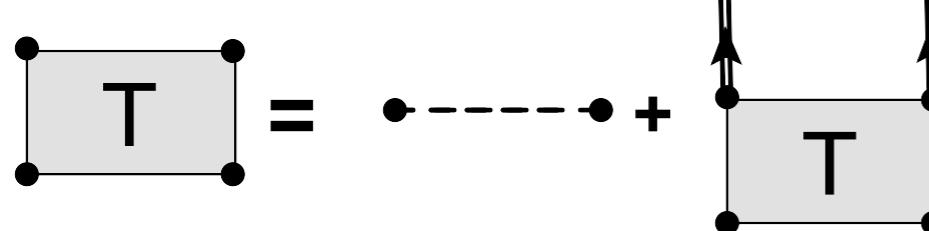
The diagram illustrates the Dyson equation as a decomposition of a double vertical line with an upward arrow into a single vertical line with an upward arrow plus a circular loop with a summation symbol ( $\Sigma$ ) attached below it.

# *Ladder approximation with 3BF*

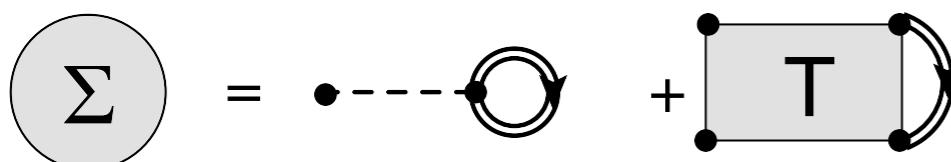
## Two-body interaction



# In-medium T-matrix



## Self-energy



## Effective interactions

Effective one-body force

$$\bullet = \bullet - \bullet + \frac{1}{4} \bullet$$

The diagram illustrates the decomposition of a single loop diagram (represented by a red dot connected by a dashed line to a black dot) into three components. The first component is a tree-level diagram (red dot connected by a solid line to a black dot). The second component is a loop diagram (black dot connected to a dashed line forming a circle with an arrow). The third component is one-fourth of a loop diagram with a shaded central region labeled  $G_{II}$ , where the loop is closed by a dashed line and has two arrows indicating direction.

Effective two-body force

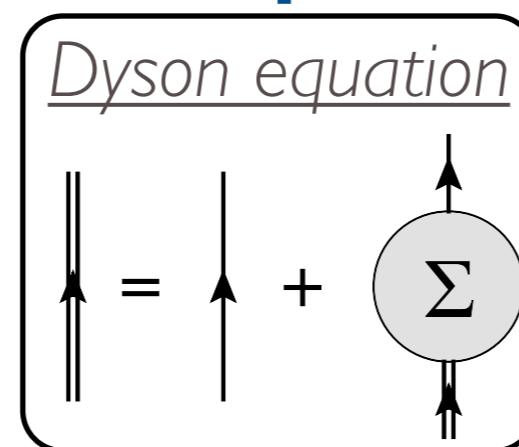
$$\text{Diagram A} = \text{Diagram B} + \text{Diagram C}$$

## In-medium **T**-matrix

$$T = \text{spring} + T$$

The diagram illustrates the decomposition of a total system  $T$  into a spring and a subsystem  $T$ . On the left, a gray rectangle representing the system  $T$  is shown with blue dots at its corners. An equals sign follows it. To the right of the equals sign is a blue spring icon consisting of two blue dots connected by a wavy line. A plus sign follows the spring icon. To the right of the plus sign is another gray rectangle representing the subsystem  $T$ , identical to the one on the far left.

## Self-energy



# Ladder approximation with 3BF

Two-body interaction



In-medium T-matrix

$$\boxed{T} = \text{---} + \text{---} \quad \text{---} \quad \boxed{T}$$

Self-energy

$$\boxed{\Sigma} = \text{---} \quad \text{---} \quad \boxed{T}$$

Dyson equation

$$\boxed{\Gamma} = \boxed{\Gamma} + \boxed{\Sigma}$$

Effective interactions

*Effective one-body force*

$$\text{---} = \text{---} + \frac{1}{2} \cdot \dots$$

*Effective two-body force*

$$\text{---} = \text{---} + \text{---} \quad \text{---} \quad \text{---} \quad \text{---}$$

In-medium T-matrix

$$\boxed{T} = \text{---} + \text{---} \quad \text{---} \quad \boxed{T}$$

Self-energy

$$\boxed{\Sigma} = \text{---} + \text{---} + \frac{1}{12} \cdot \dots$$

# Ladder approximation with 3BF

Two-body interaction



In-medium T-matrix

$$\boxed{T} = \text{---} + \text{---} \quad \text{---} \quad \boxed{T}$$

Self-energy

$$\boxed{\Sigma} = \text{---} \quad \text{---} \quad \boxed{T}$$

Effective interactions

*Effective one-body force*

$$\text{---} = \text{---} + \frac{1}{2} \text{---} \quad \text{---}$$

*Effective two-body force*

$$\text{---} = \text{---} + \text{---} \quad \text{---}$$

In-medium T-matrix

$$\boxed{T} = \text{---} + \text{---} \quad \text{---} \quad \boxed{T}$$

Self-energy

$$\boxed{\Sigma} = \text{---} + \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \quad \text{---}$$

~~$\frac{1}{12}$~~

Dyson equation

$$\text{---} = \text{---} + \boxed{\Sigma}$$

# Ladder approximation with 3BF

Two-body interaction



In-medium T-matrix

$$\boxed{T} = \text{---} + \text{---} \quad \text{---} \quad \boxed{T}$$

Self-energy

$$\boxed{\Sigma} = \text{---} \quad \text{---} \quad \boxed{T}$$

Dyson equation

$$\boxed{\Gamma} = \boxed{\Gamma} + \boxed{\Sigma}$$

Effective interactions

*Effective one-body force*

$$\text{---} = \text{---} + \cancel{\frac{1}{2} \dots}$$

*Effective two-body force*

$$\text{---} = \text{---} + \text{---}$$

In-medium T-matrix

$$\boxed{T} = \text{---} + \text{---} \quad \text{---} \quad \boxed{T}$$

Self-energy

$$\boxed{\Sigma} = \cancel{\text{---}} + \boxed{T}$$

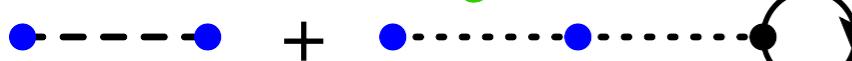
$$+ \frac{1}{12} \dots + \dots$$

# Density-dependent interaction

*Two-body N3LO*



*Uncorrelated average*<sup>1</sup>



*Correlated average*<sup>2</sup>

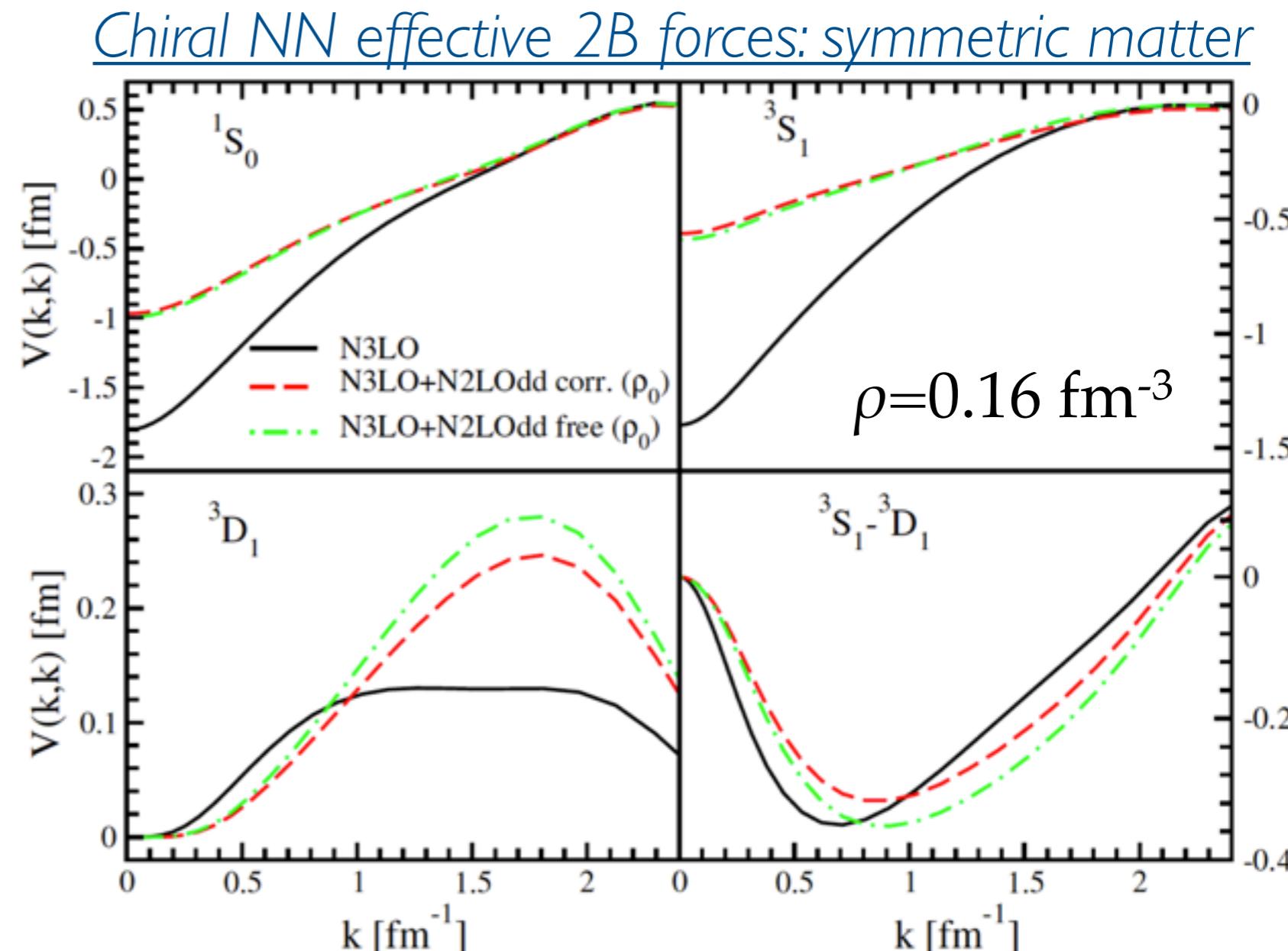


LECs

$$c_D = -1.11$$

$$c_E = -0.66$$

$$k \neq k' \Rightarrow \frac{1}{2}(k + k')$$



- 3NF bring repulsion: correlated & uncorrelated averages are similar
- Correlated average brings small corrections to 1/2 of terms
- Diagonal  $k=k'$  matrix elements computed
- Off-diagonal extrapolated & regulated

<sup>1</sup>Holt et al. Phys. Rev. C **81** 024002 (2010)

<sup>2</sup> Carbone, Polls & Rios, *in preparation*; A. Carbone, PhD thesis 11

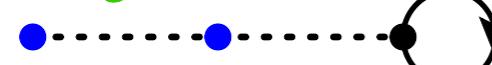
# Symmetric matter

Theoretical uncertainties: average procedure

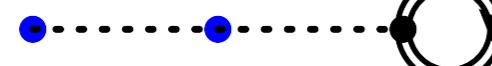
*Two-body N3LO*

• 

*Uncorrelated average*

• 

*Correlated average*

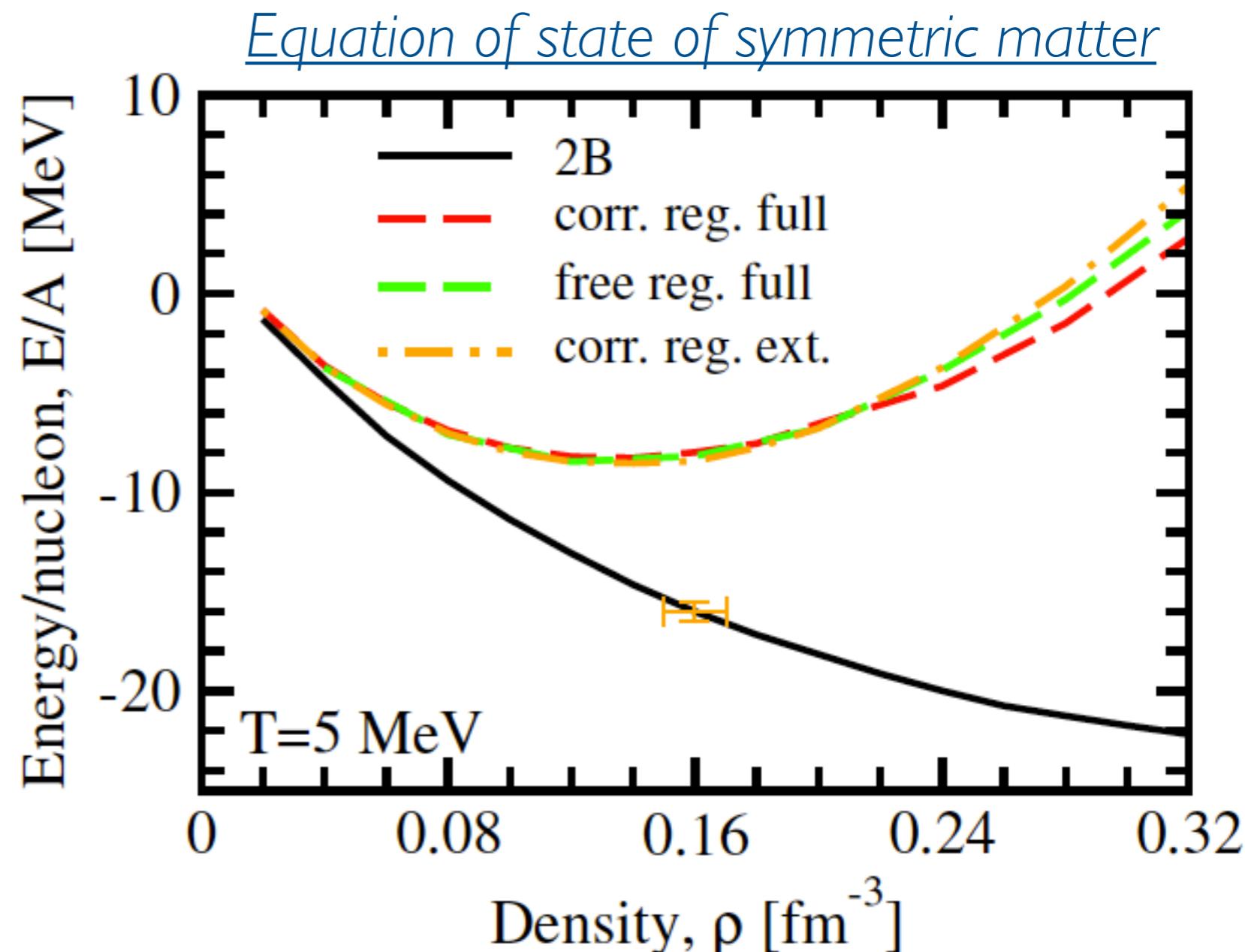
• 

LECs

$$c_D = -1.11$$

$$c_E = -0.66$$

Energy from GMK sum-rule  
... with 3N corrections<sup>1</sup> ...



- Cor. reg. full = best we can do now is underbound
- Previous work with uncorrelated averages is validated
- Regulation at high momentum is irrelevant

<sup>1</sup>Carbone, Polls & Rios, Phys. Rev. C **88** 044302 (2014)  
Carbone, Polls & Rios, *in preparation*; A. Carbone, PhD thesis 12

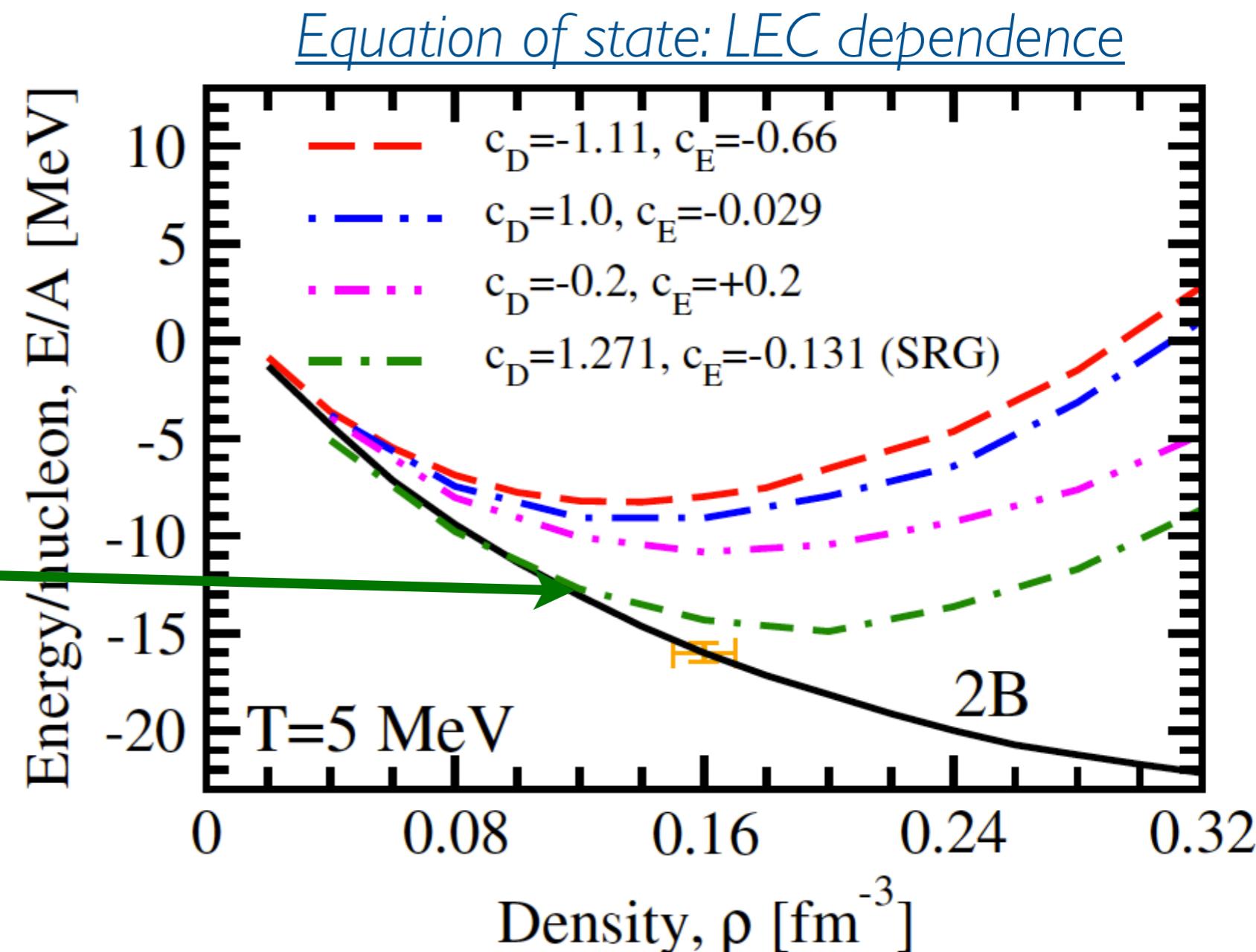
# Symmetric matter

Theoretical uncertainties: NN force

*Uncorrelated average*



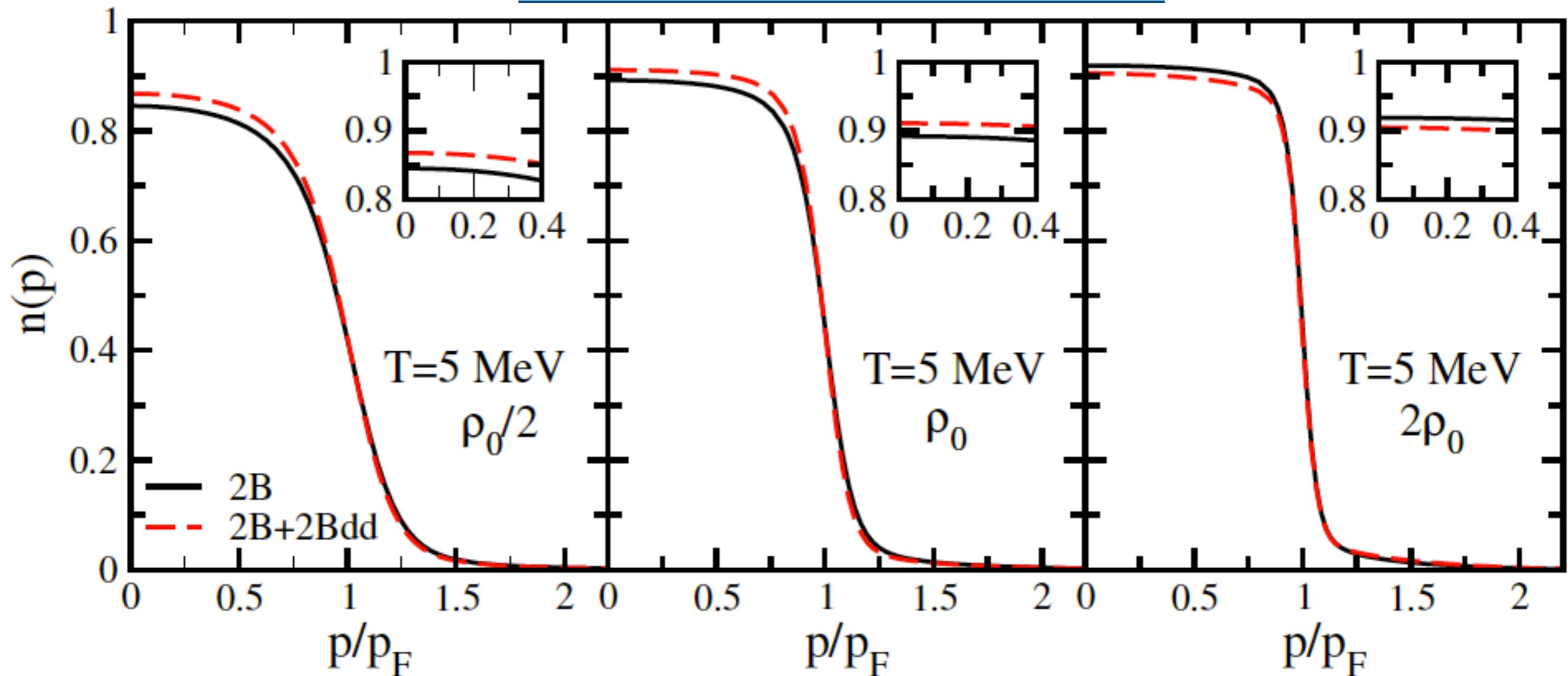
Further renormalized NN force  
Nuclear structure calculations



- LECs dependence is strong
- Renormalization via SRG: nuclear structure calculations?
- Small 3NF effects with larger saturation densities  $\Rightarrow$  smaller radii

# Microscopic properties

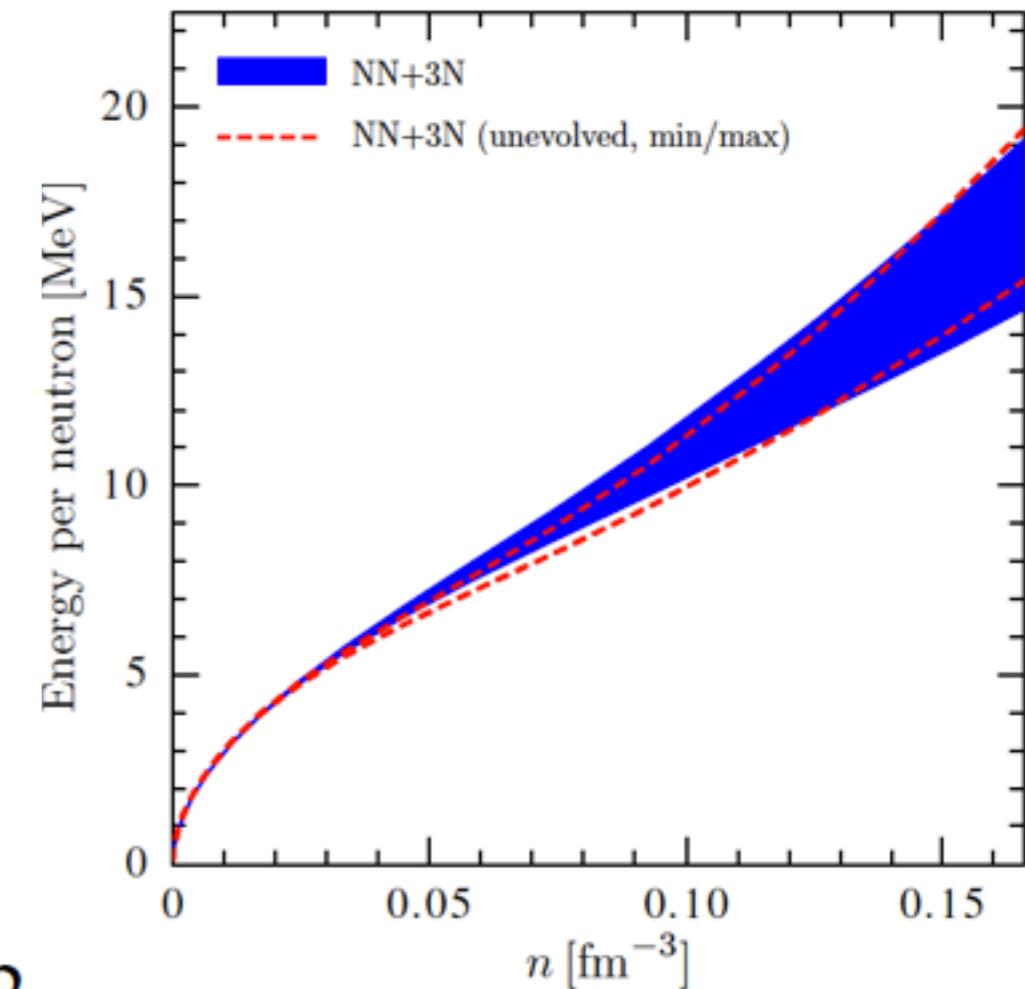
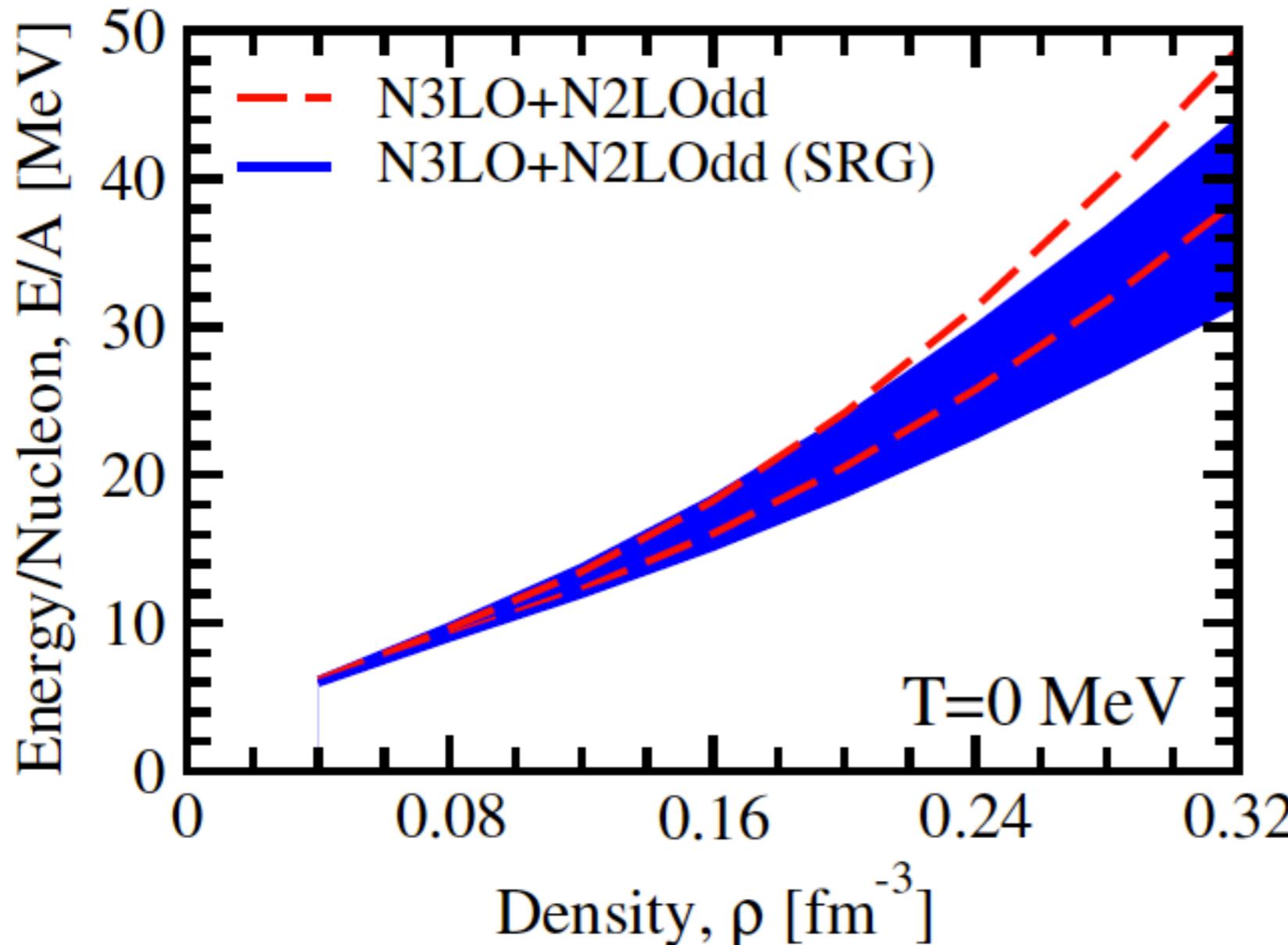
## Correlated momentum distribution



- 3NFs induce **different density dependence**
- Error band from unknown ChPT parameters
- Finite temperature & higher densities available

# Neutron matter

EoS for neutron matter: SRG

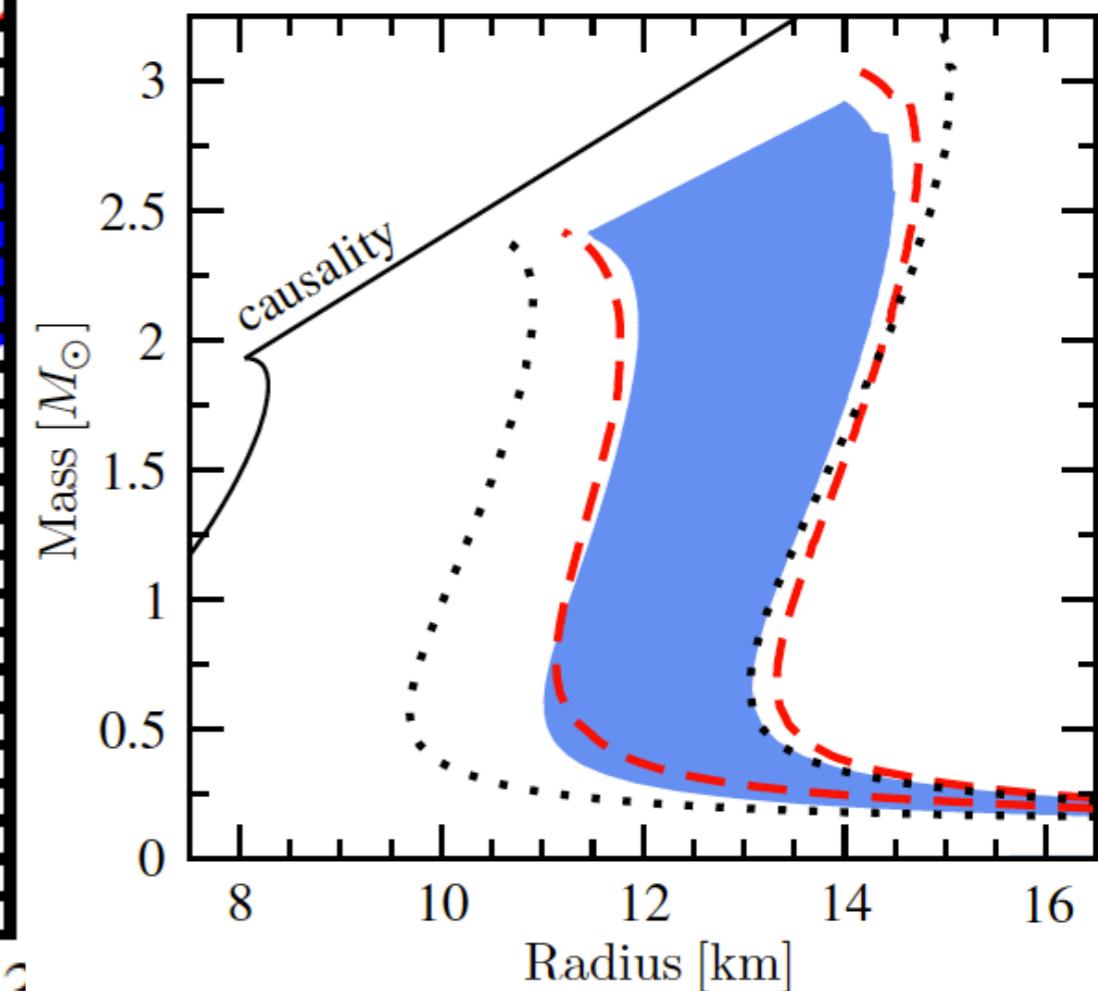
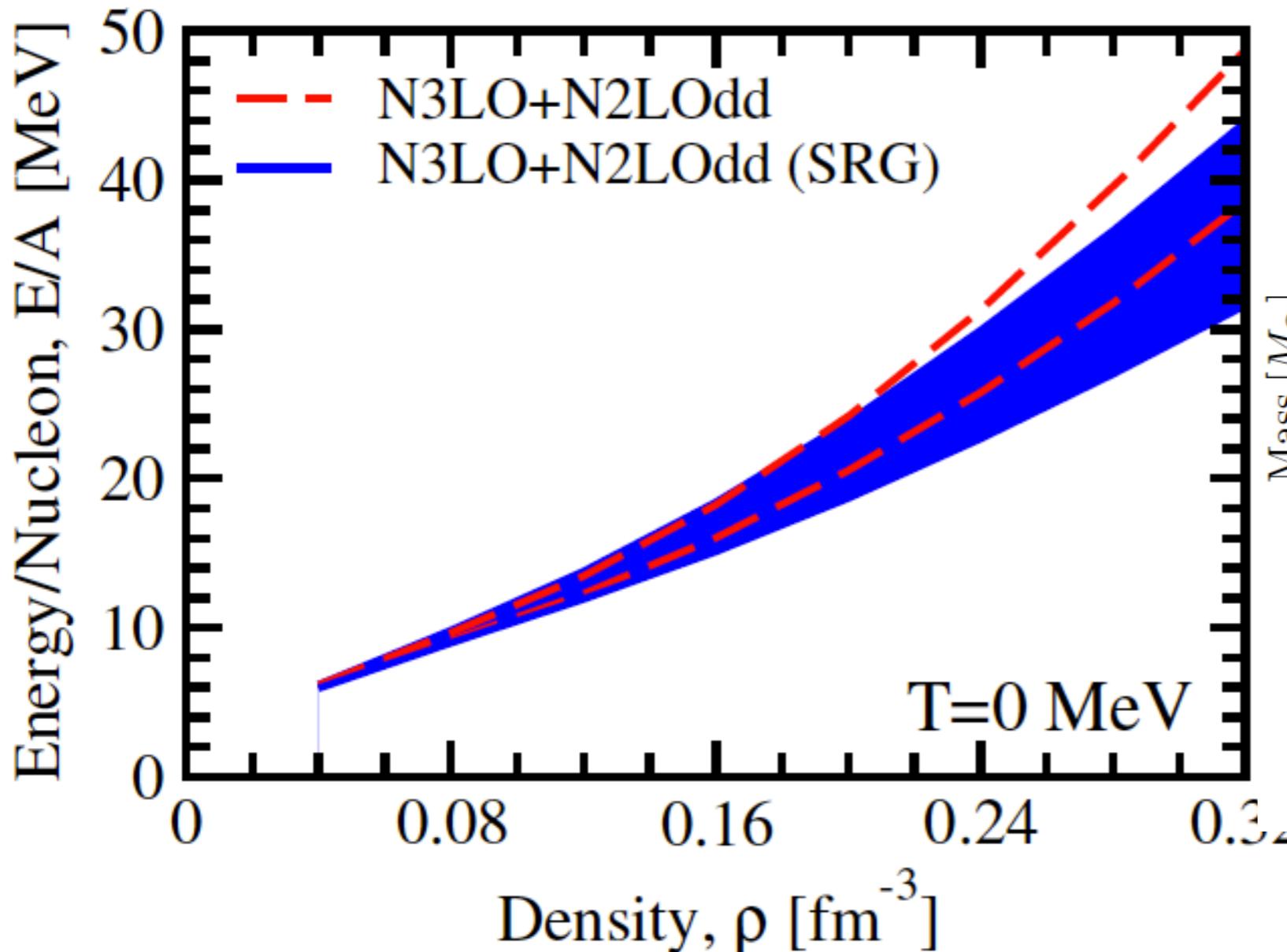


Hebeler, Lattimer, Pethick, Schwenk  
*ApJ* **773** 11 (2013)

- Error band from unknown ChPT  $c_1, c_3$  parameters
- Finite temperature & higher densities available

# Neutron matter

EoS for neutron matter: SRG

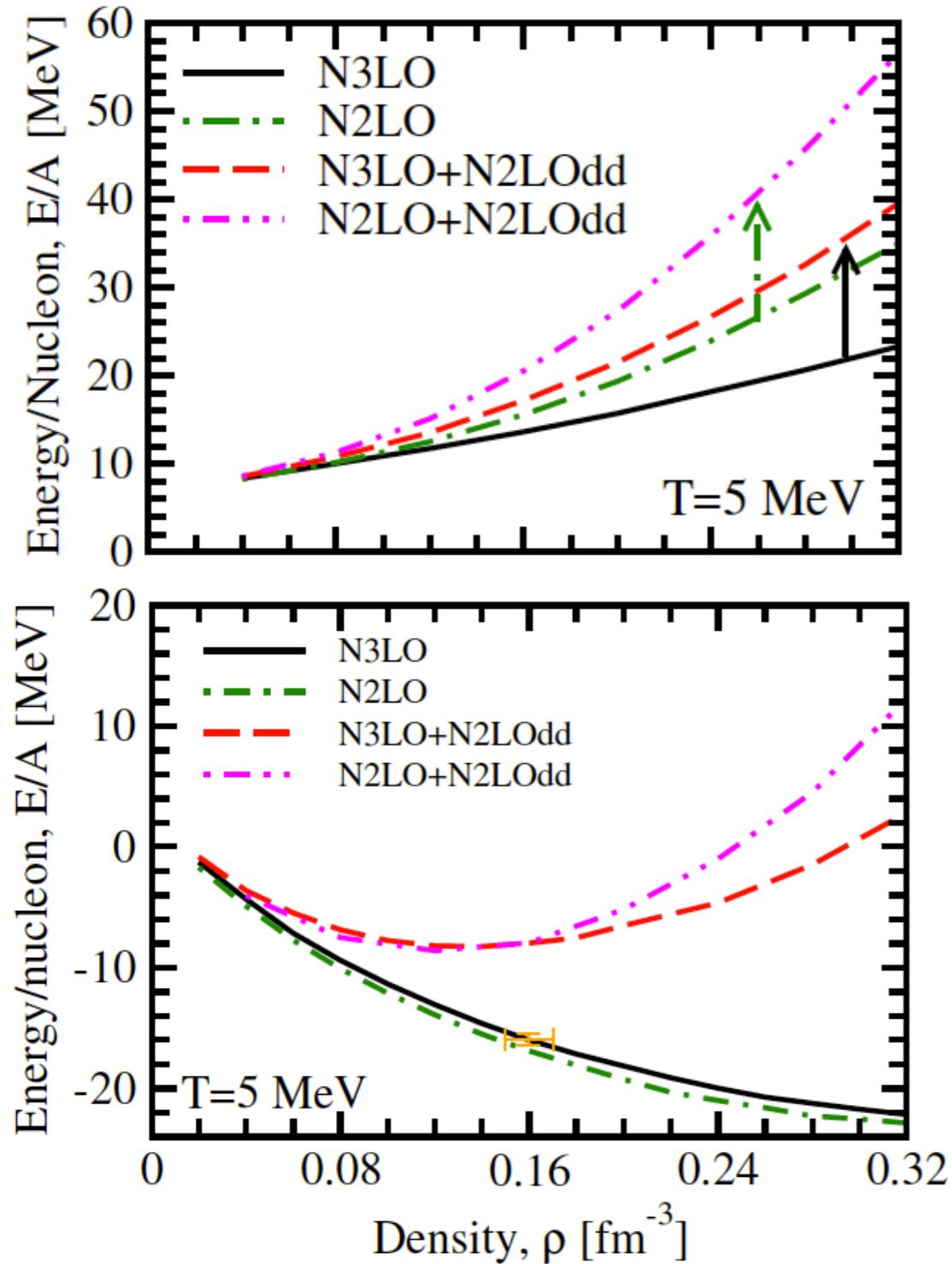


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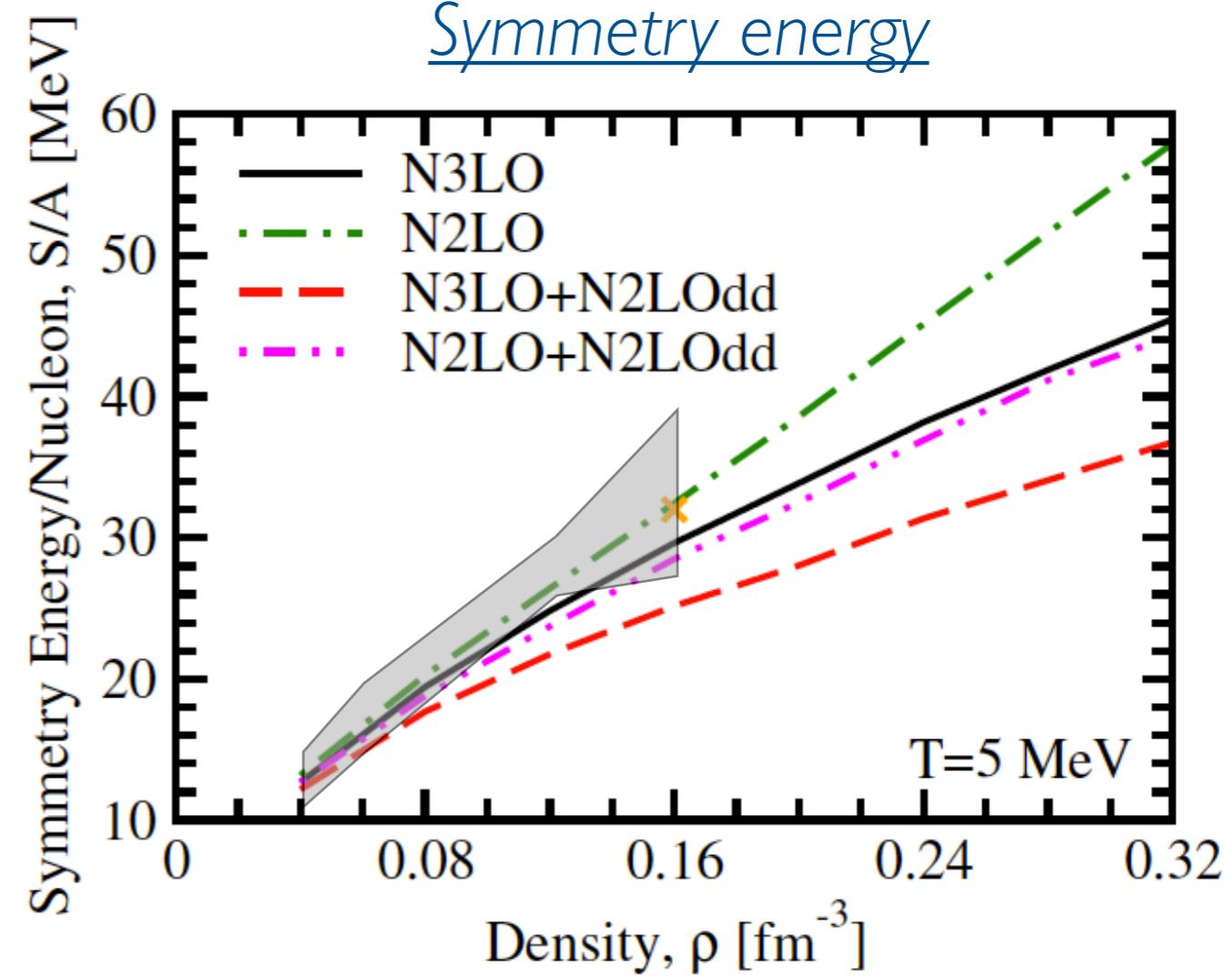
# Isovector properties

## EoS for SNM & PNM



$$S(\rho) = E_{PNM}(\rho) - E_{SNM}(\rho)$$

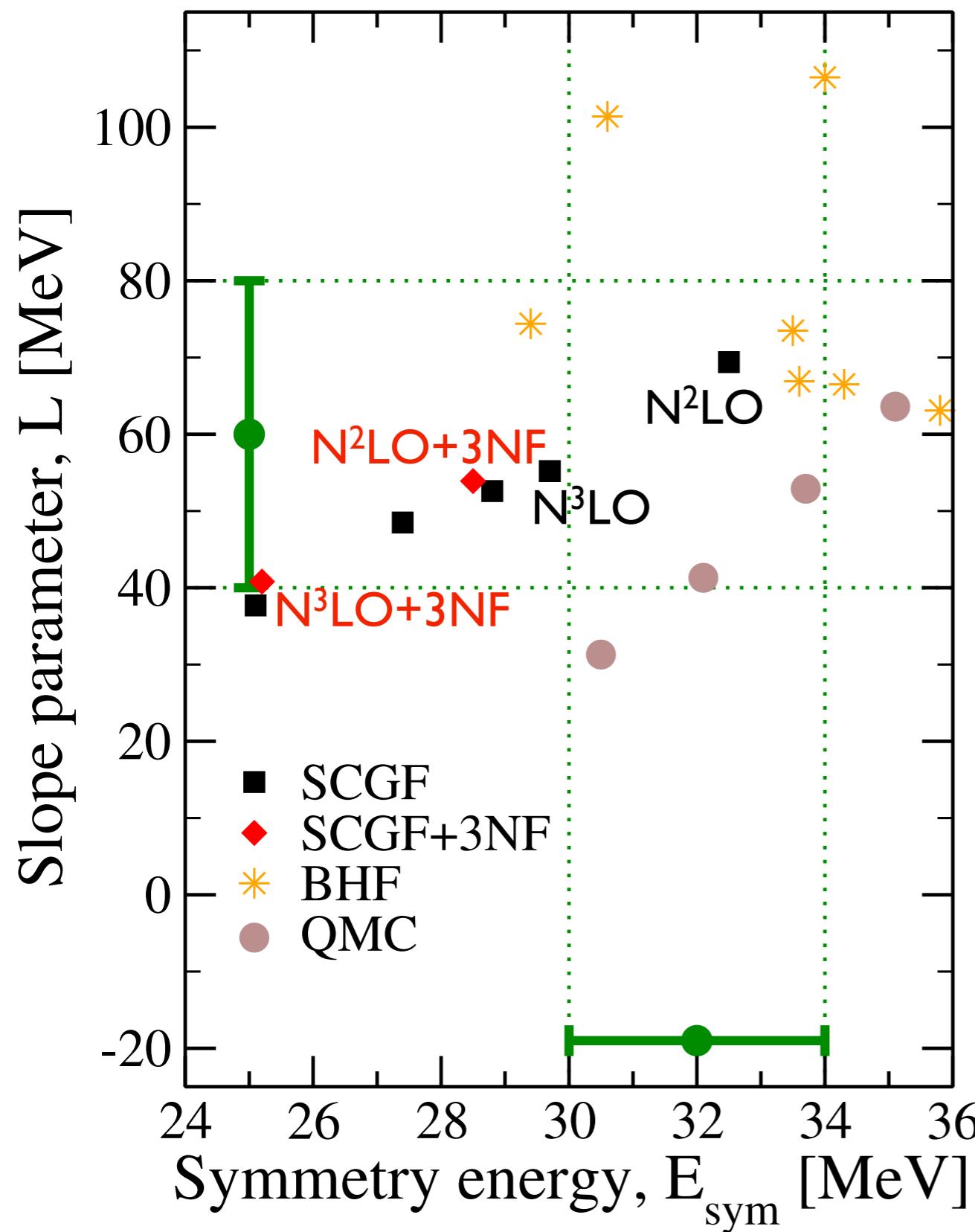
## Symmetry energy



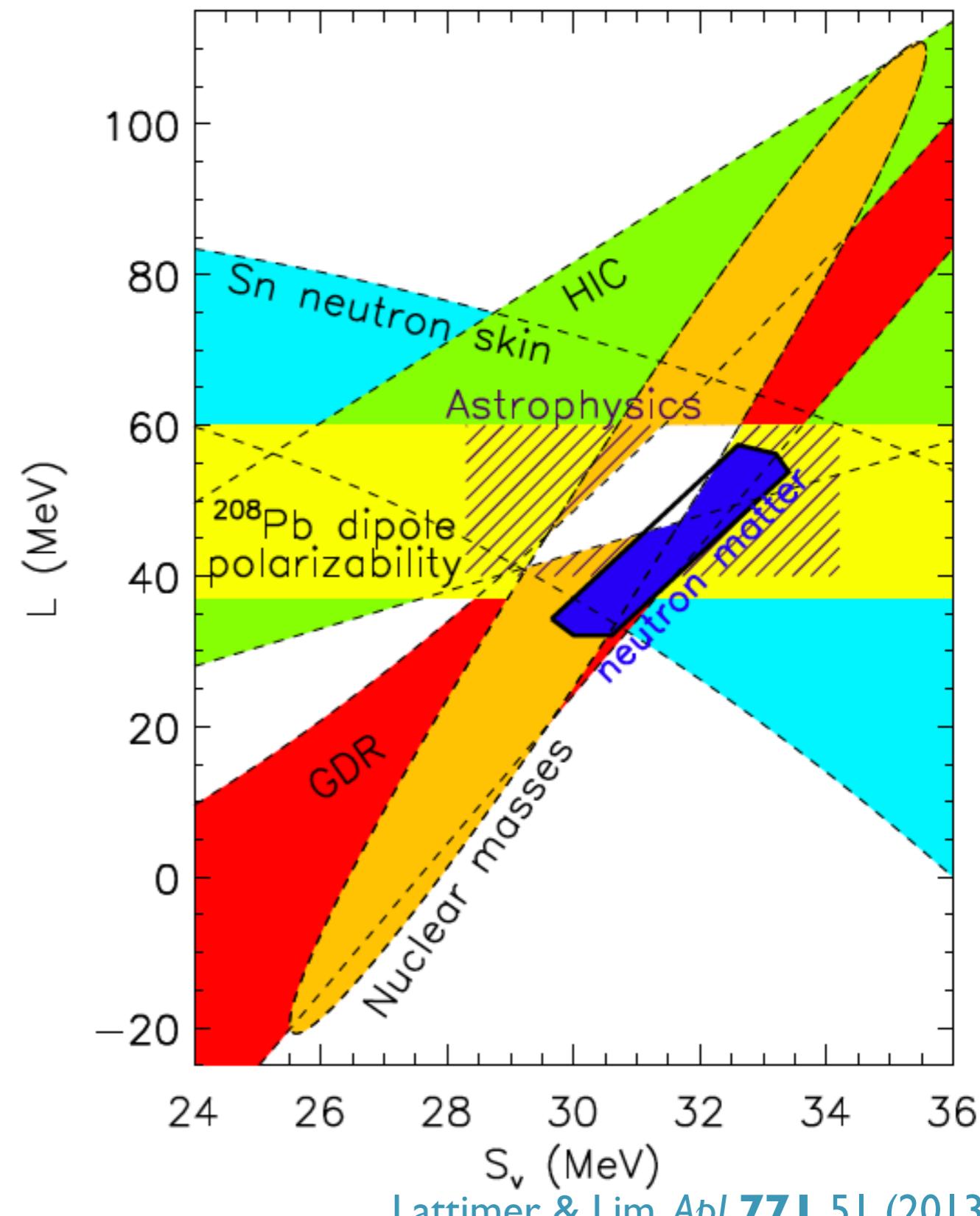
$\rho_0 = 0.16 \text{ fm}^{-3}$	$E_{PNM}/A$	$E_{SNM}/A$	$S/A$	$L$
N3LO	13.6	-16.0	29.6	52.9
N2LOopt	15.6	-16.9	32.5	69.4
N3LO+N2LOOdd	17.2	-7.99	25.2	40.8
N2LOopt+N2LOOdd	20.5	-7.96	28.5	53.9

# Comparison to phenomenology

Correlations from nuclear theory

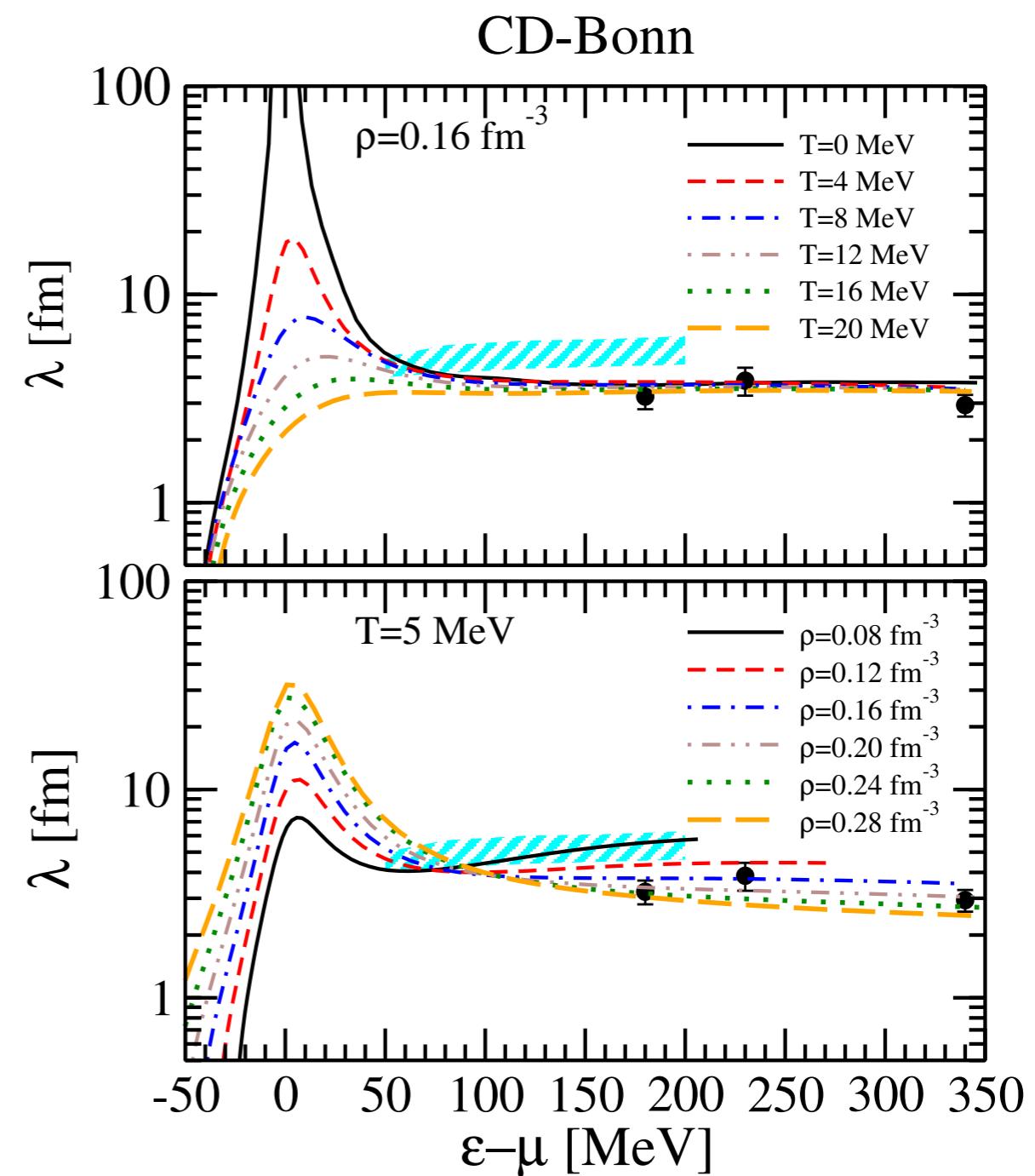
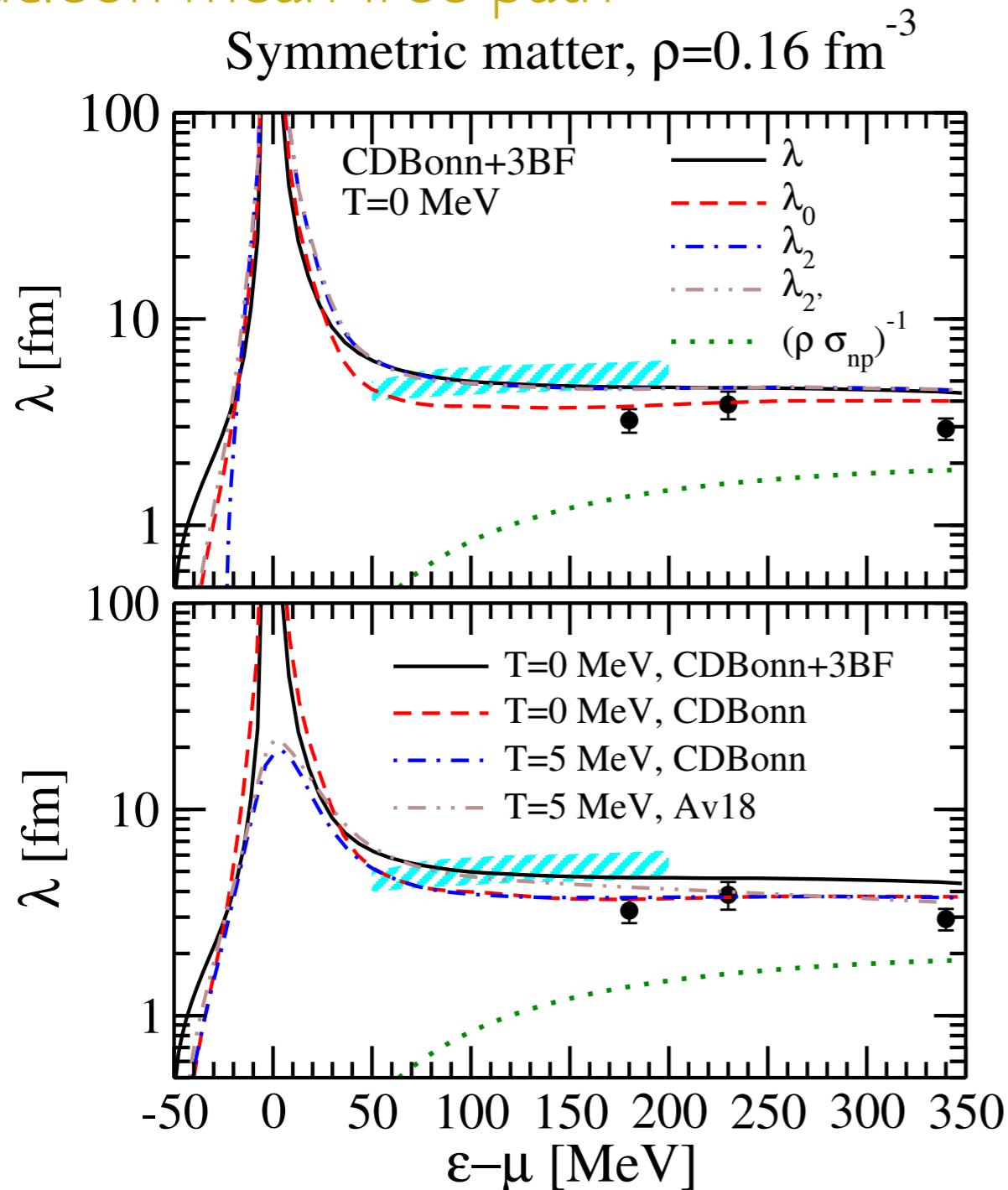


Experimental constrains



# Transport properties

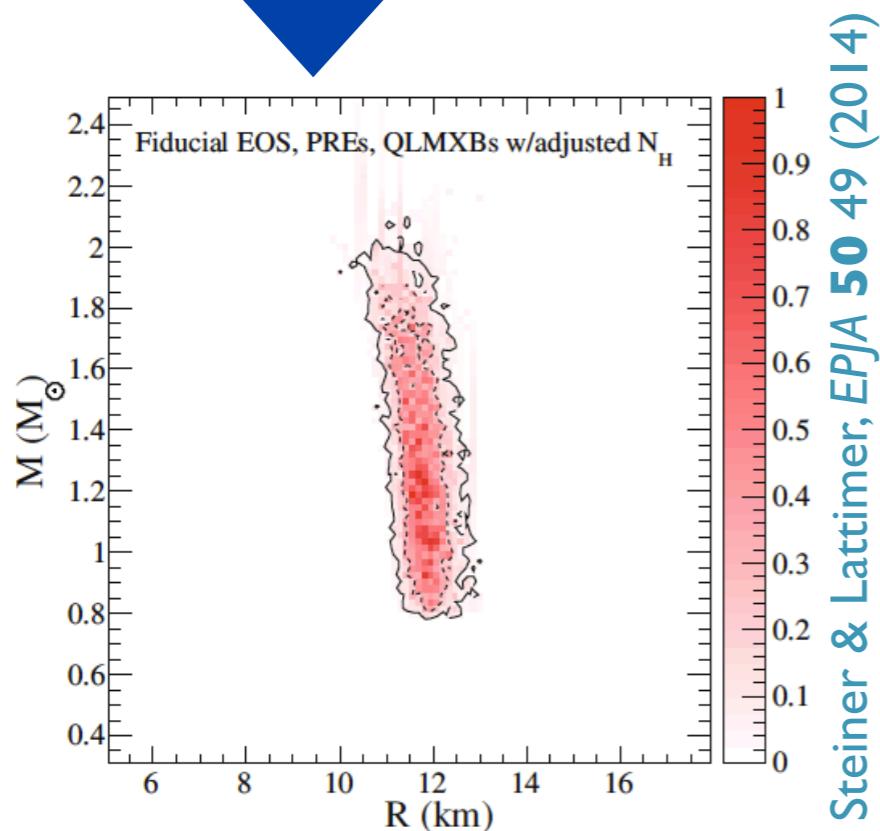
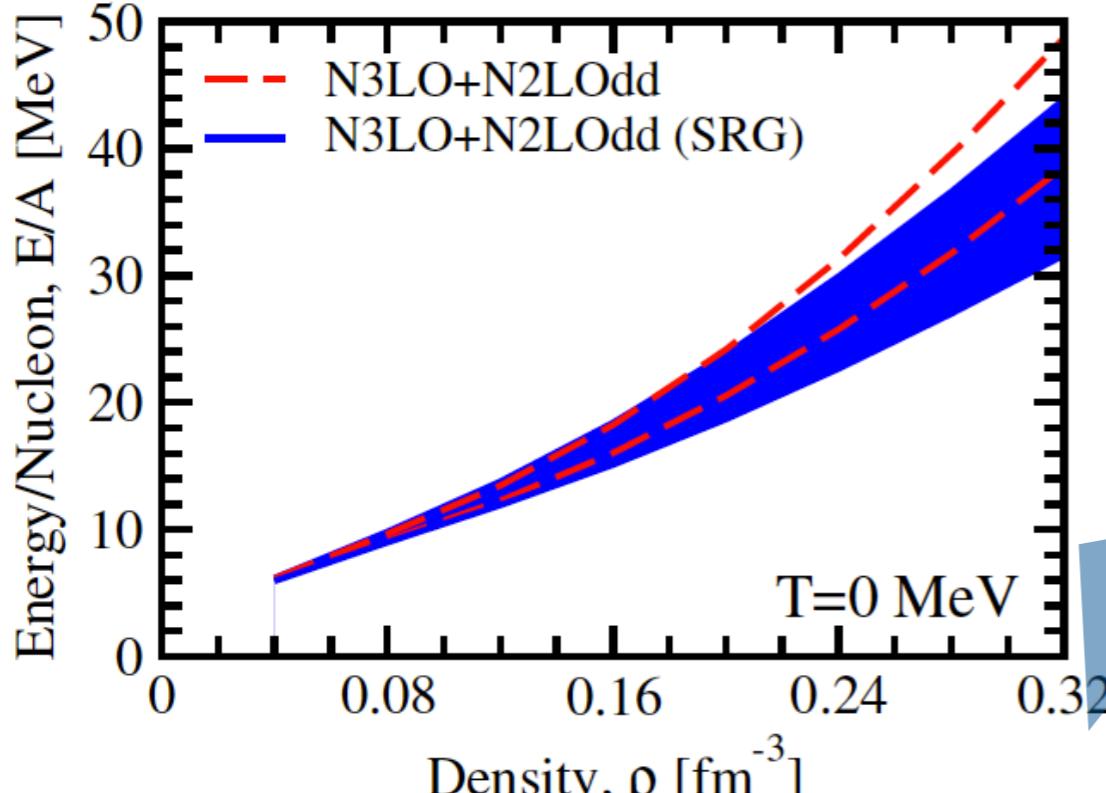
## Nucleon mean-free path



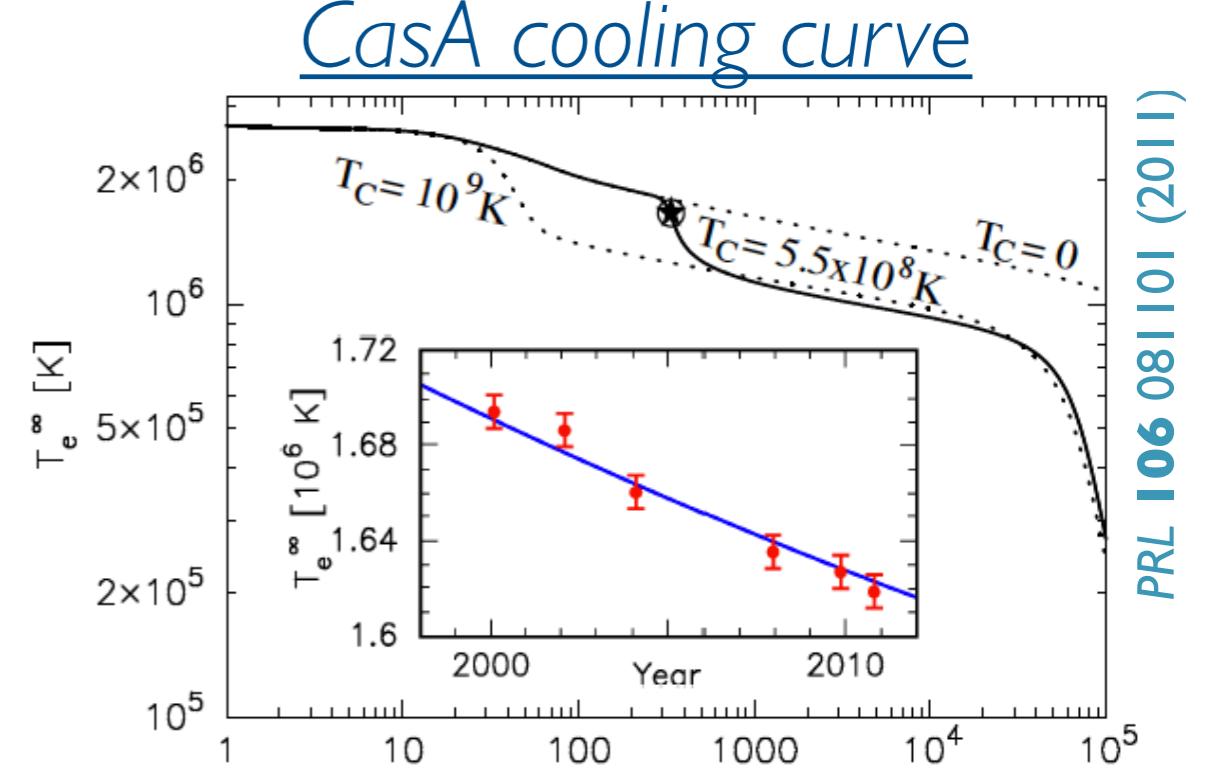
- Compatible with  $pA$  experiments
- Small model dependence

# Chiral EoS & pairing gaps

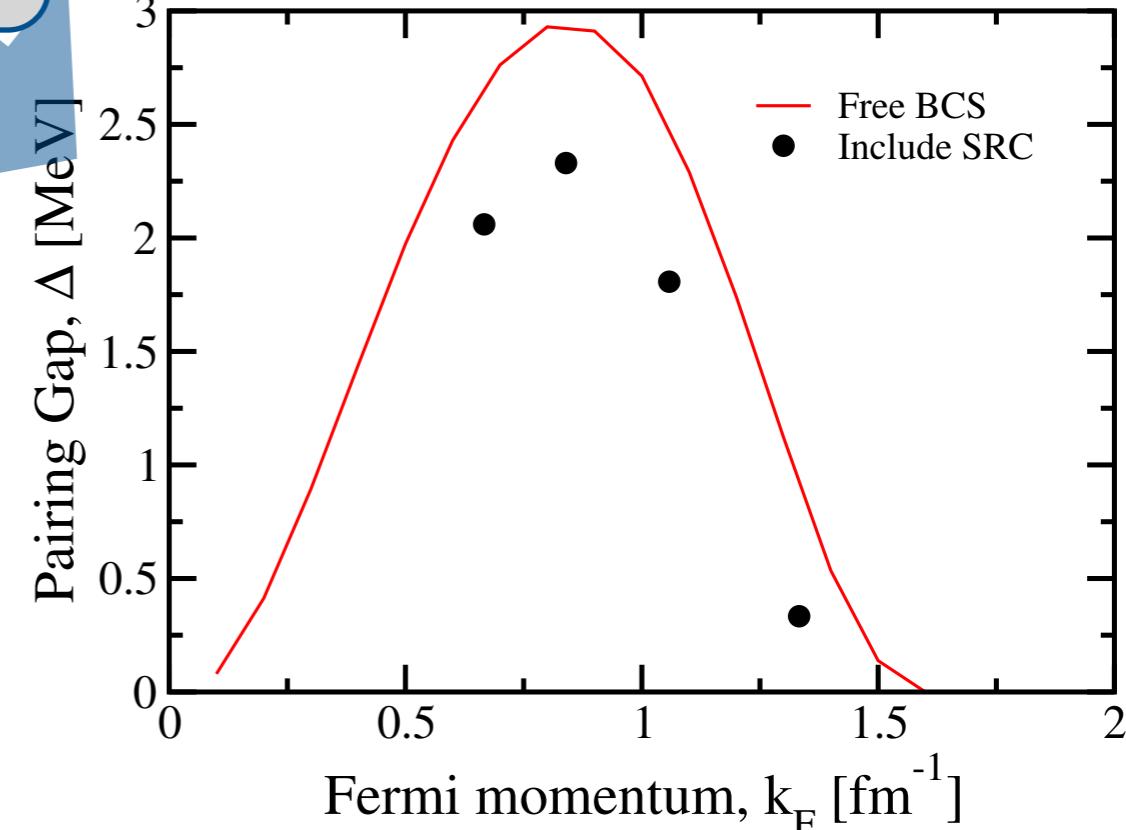
$N^3LO$  EoS with 3BFs



Chiral NN &  
3N forces

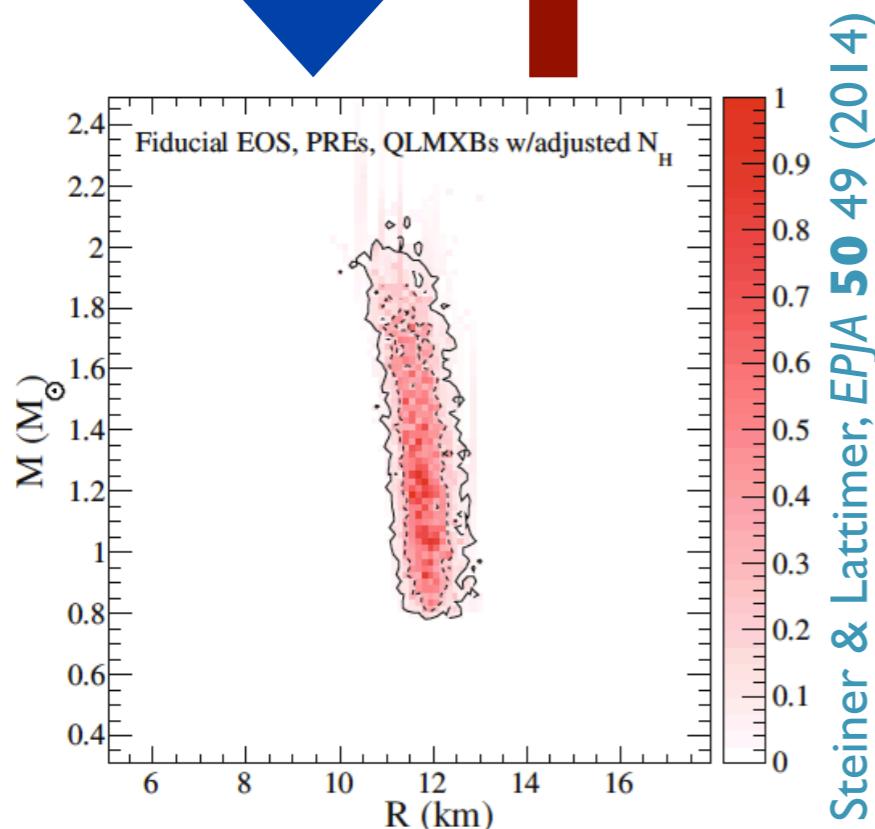
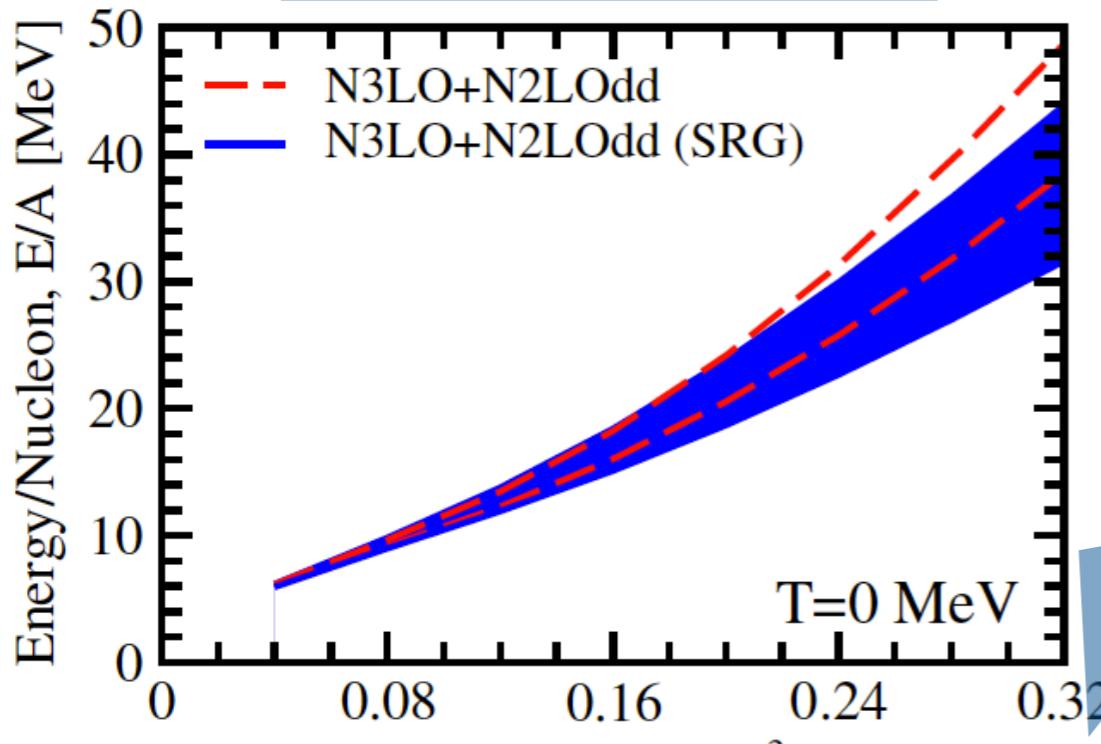


$N^3LO$  pairing gap: beyond BCS

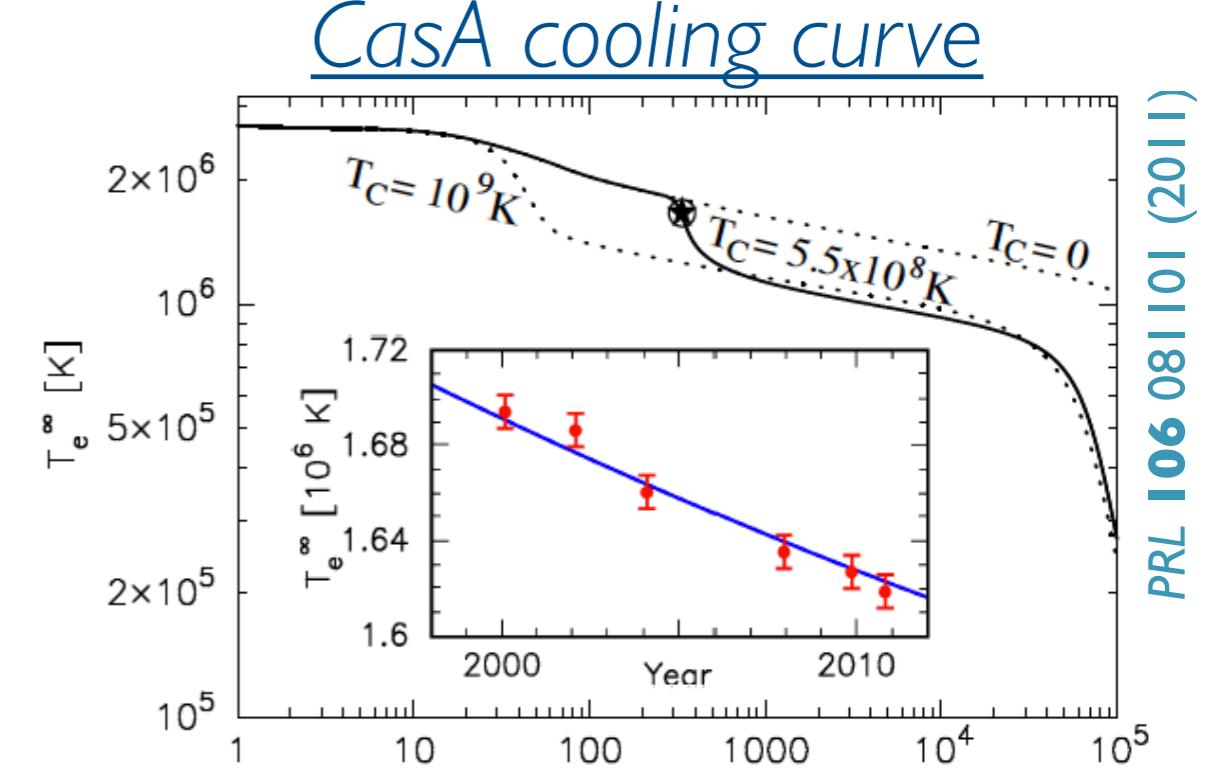


# Chiral EoS & pairing gaps

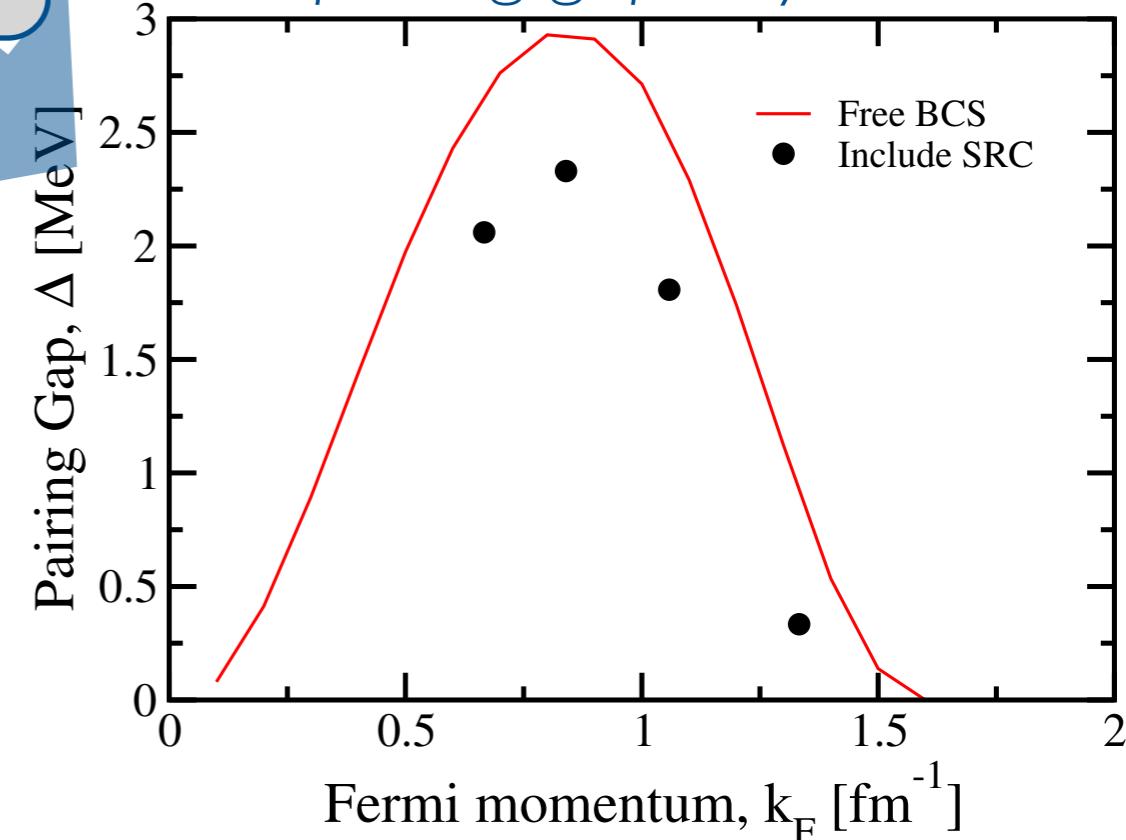
$N^3LO$  EoS with 3BFs



Chiral NN &  
3N forces



$N^3LO$  pairing gap: beyond BCS



# Conclusions

- Ab initio nuclear theory is expanding
- Neutron star physics is relevant for this endeavor
- Many-body forces can now be accessed
- Micro, macro, transport, pairing properties on-going
- Hopefully a coherent picture will come out!

Thank you!



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