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Properties of Infinite Nuclear Matter and Neutron Star Matter with the Gogny Interaction

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Outline

- The maximum mass and radius of a Neutron Star depends on the Equation of State (EOS) used
- The Gogny force is a finite range force and its ability to predict bulk properties of Infinite Nuclear Matter (INM) is relatively unexplored
- 10 Gogny type forces were examined and their bulk properties were evaluated in a Hartree-Fock framework
- The forces were applied to Neutron Star Matter to produce maximum masses and radii

Many Body Forces
Single Particle Energy

We find the total single particle energy within a Hartree-Fock mean field approximation to the many body problem using the two-body potential, $\hat{V}$:

$$\epsilon_{k_1 \sigma_1 \tau_1} = \langle k_1\ \sigma_1\ \tau_1 | \hat{T} | k_1\ \sigma_1\ \tau_1 \rangle + \frac{1}{(2\pi)^3} \sum_{\sigma_2} \sum_{\tau_2} \int_0^{k_F} d^3k_2\ \frac{1}{2} \langle k_1\ \sigma_1\ \tau_1,\ k_2\ \sigma_2\ \tau_2 | \hat{V} | k_1\ \sigma_1\ \tau_1,\ k_2\ \sigma_2\ \tau_2 \rangle$$

Where k_F is the Fermi momentum

$$k_F = \left(3\pi^2 \rho\right)^{\frac{1}{3}}$$

and ρ is the number density of nucleons in nuclear matter

Many Body Forces
Energy Per Particle

To find the total energy of the system we now integrate over all k_1:

$$E_{Total} = \frac{1}{(2\pi)^3} \sum_{\sigma_1} \sum_{\tau_1} \int_0^{k_F} d^3k_1 \, \Big[\langle k_1 \; \sigma_1 \; \tau_1 | \, \hat{T} \, | k_1 \; \sigma_1 \; \tau_1 \rangle$$

$$+ \frac{1}{(2\pi)^3} \sum_{\sigma_2} \sum_{\tau_2} \int_0^{k_F} d^3k_2 \, \frac{1}{2} \langle k_1 \; \sigma_1 \; \tau_1, \; k_2 \; \sigma_2 \; \tau_2 | \, \hat{V} \, | k_1 \; \sigma_1 \; \tau_1, \; k_2 \; \sigma_2 \; \tau_2 \rangle \Big]$$

- Now we are able to examine a two-body interaction, $\hat{V}$
  - Many two body interactions exist, e.g Skyrme
  - This work focusses on the Gogny type interaction

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The Gogny Interaction

Beyond Zero Range

The Gogny Force

$$V_{NN}(r) = \sum_{i=1}^{2} \left[W_i + B_i P_\sigma - H_i P_\tau - M_i P_{\sigma\tau}\right] e^{\frac{-\vec{r}^2}{\mu_i^2}} + \sum_{i=1}^{2} t_{0i} \left(1 + x_{0i} P_\sigma\right) \rho^{\alpha_i} \left(\vec{r}\right) \delta\left(\vec{r}\right)$$

- Many forces, such as Skyrme forces, are zero range forces
- Gogny force has a density dependent zero range component as well as finite range
- The operator P_σ (P_τ) is the spin (isospin) exchange operator

$$\langle \sigma_1, \sigma_2 | P_\sigma | \sigma_1, \sigma_2 \rangle = \langle \sigma_1, \sigma_2 | \sigma_2, \sigma_1 \rangle = \delta_{\sigma_1 \sigma_2}$$

- Remaining 14 variables are free parameters used to define the force
  - Fit to properties of finite nuclei
  - Only 10 appear so far in literature compared to >100 for Skyrme
  - Application to Neutron Star Matter not well explored

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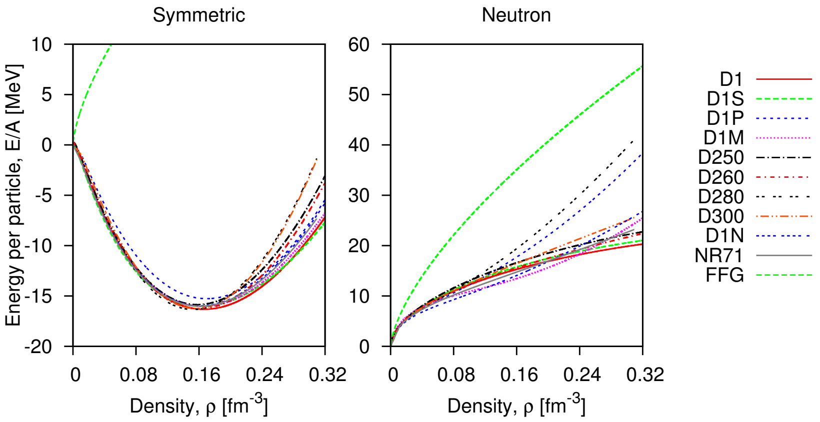
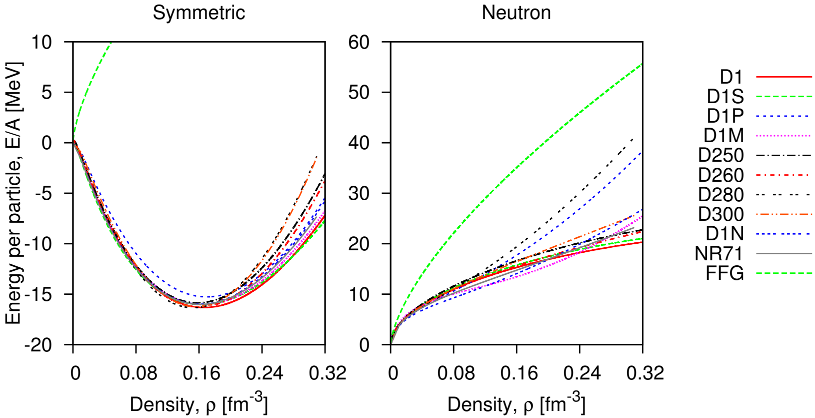
Energy Per Particle



Using the Gogny forces, plots were made for the energy per particle of both Symmetric Matter and Pure Neutron Matter

The Gogny Interaction

Energy Per Particle



Normal nuclear matter has a saturation density at around 0.16 nucleons per fm^3 and has an energy of approximately -16 MeV

We can now explore the isovector properties of these forces

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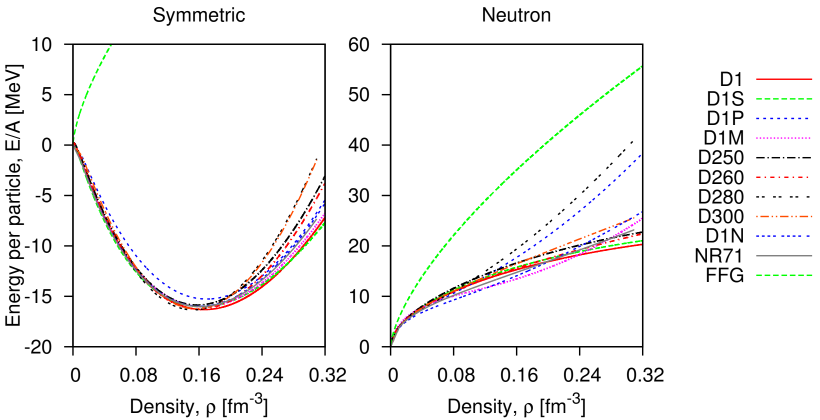

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The Gogny Interaction
Symmetry Energy and Slope

The Symmetry Energy is the energy difference between Symmetric Matter and Pure Neutron Matter

$$E_{Sym}(\rho) = \frac{1}{2!} \left. \frac{\delta^2 E(\rho, \beta)}{\delta \beta^2} \right|_{\beta=0}$$

- Influences ratio of protons to neutrons in a Neutron Star
- At saturation density it is in the region of 32 MeV

The Slope Parameter, L, is the gradient of the Symmetry Energy with respect to Density

$$L = 3\rho_0 \left. \frac{\delta E_{Sym}(\rho)}{\delta \rho} \right|_{\rho_0}$$

- Generally accepted to lay between 30 MeV and 100 MeV
- Influences how pressure changes with density (stiffness)

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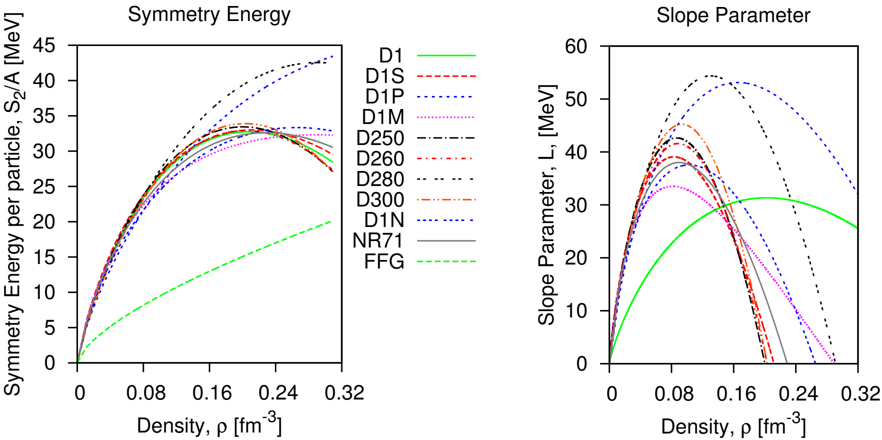

- Large divergence at the high density end
- D1P and D280 give the highest peaks in both graphs
  - These stiffer equations of state should provide higher mass neutron stars

The Gogny Interaction

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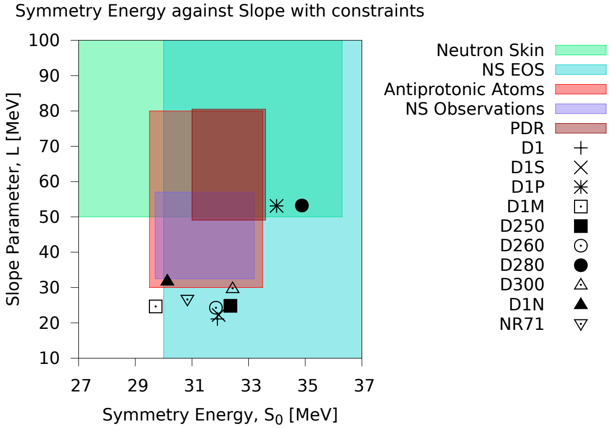

- Most forces have very low Slope
- Only D1P and D280 give high enough values
- We predicted these would give the largest Neutron Stars

Building a Neutron Star
The TOV Equations

To construct a Neutron Star from an EOS we use the Tolman Oppenheimer Volkoff equations:

$$\frac{dm(r)}{dr} = 4\pi r^2 \frac{\epsilon}{c^2}$$

$$\frac{dp(r)}{dr} = \frac{G\epsilon(r)m(r)}{c^2 r^2}\left[1 + \frac{p(r)}{\epsilon(r)}\right]\left[1 + \frac{4\pi r^3 p(r)}{m(r)c^2}\right]\left[1 - \frac{2Gm(r)}{c^2 r}\right]^{-1}$$

$$\rho(r) = \frac{\epsilon(r)}{c^2}$$

Pressure as a function of density was calculated from the Gogny EOS as

$$p(\rho) = \rho^2(r)\frac{\delta E(\rho)}{\delta\rho}$$

- We compiled a lookup table of pressures
- The table was then used to interpolate density as a function of pressure

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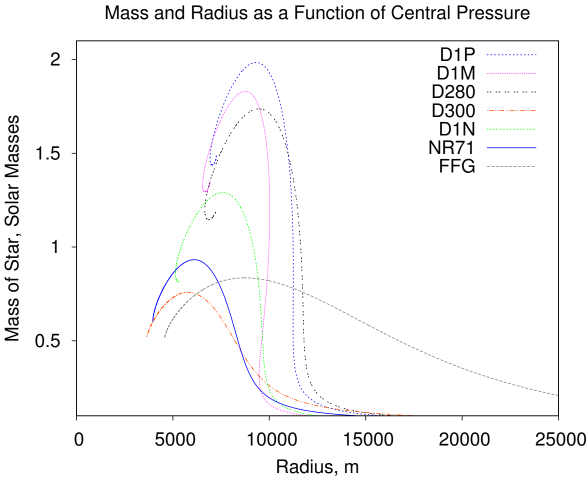
Building a Neutron Star

Pure Neutron Matter

- D1P gives largest maximum radius at 1.98 Solar Masses
- Not all Gogny forces provide stable configurations



Demorest et al. (2010) measured a 1.97(4) mass neutron star!

- Most, if not all, Gogny EOS are too soft to produce a Neutron Star in keeping with current observations
- Isovector properties of Gogny forces should be improved if they are to be applied to Neutron Star matter
- There may also be shortcomings when these forces are applied to neutron-rich nuclei

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- Maximum masses and radii will be given for each Gogny force
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Questions

Any Questions?