Equation of State for Neutron Stars based on a microscopic energy density functional: From the outer crust to the core

B.K. Sharma[a], M. Centelles[a], X. Viñas[a], M. Baldo[b], and G.F. Burgio[b]

[a]Departament d'Estructura i Constituents de la Matèria and Institut de Ciències del Cosmos, Universitat de Barcelona, Barcelona, Spain
[b]Instituto Nazionale di Fisica Nucleare Sezione di Catania and Dipartamento di Fisica and Astronomia, Universitá di Catania, Catania, Italy

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Structure and Signals of Neutron Stars, from Birth to Death

Introduction

- A convergent effort of experimental and theoretical nuclear physics has been developed along several years to determine the structure and properties of Neutron Stars.
- The interpretation of signals coming from the astrophysical observations on the process and phenomena that occur in Neutron Stars requieres reliable theoretical inputs.
- It is of great interest to develop a unified theory which is able to describe on a microscopic level the overall structure of Neutron Stars, from the crust to the outer core.
- Recently, a new Energy Density Fuctional called **BCPM** has been developed by some of us. It is based on the nuclear matter EOS derived in the Brueckner-Hartree-Fock scheme plus a phenomenological part to describe the nuclear surface.
- We employ this EOS and the corresponding BCPM functional to describe the whole NS structure and compare with the results obtained with the few other methods that encompass the whole Neutron Stars structure.

The EOS of Nuclear Matter

The EOS is calculated with good accuracy in the Brueckner two-hole-line approximation with the continous choice for the single-particle potential.

In the BHF approximation the energy per nucleon is

$$\frac{E}{A} = \frac{3}{5}\frac{k_F^2}{2m} + \frac{1}{2\rho}\sum_{k,k'<k_F} \left\langle kk' \middle| G[\rho; e(k) + e(k')] \middle| kk' \right\rangle_a .$$

$$G[\rho;\omega] = V + \sum_{k_a,k_b} V \frac{|k_a k_b > Q < k_a k_b|}{\omega - e(k_a) - e(k_b)} G[\rho;\omega] \, ,$$

$$e(k) = e(k;\rho) = \frac{k^2}{2m} + U(k;\rho), \quad U(k;\rho) = \mathrm{Re} \sum_{k'<k_F} \left\langle kk' \middle| G[\rho; e(k)+e(k')] \middle| kk' \right\rangle_a ,$$

In this calculation we have used the v_{18} Argonne potential as the two-body interaction. To reproduce the correct saturation point, three-body forces based on the so-called Urbana model are added in the calculation

Fitting the EoS

The symmetric (s) and neutron (n) matter EoS are fitted with polynomials P_s and P_n of the total density ρ

$$P_s(\rho) = \sum_{k=1}^{5} a_k^{(n)} (\rho/\rho_0)^k$$

$$P_n(\rho) = \sum_{k=1}^{5} b_k^{(n)} (\rho/\rho_{0n})^k$$

with $\rho_0 = 0.16$ fm^{-3} and $\rho_{0n} = 0.155$fm^{-3}

- Can be used up to $\rho = 0.625$ fm^{-3}
- The interpolating polynomial for symmetric matter has been constrained to have a minimum around the energy $E/A = -16 MeV$ and Fermi momentum $k_F = 1.36 fm^{-1}$, i.e. $\rho_0 = 0.16$ fm^{-3}.
- Integer powers of the density (unlike expansions in k_F)

Fitting the EoS, results

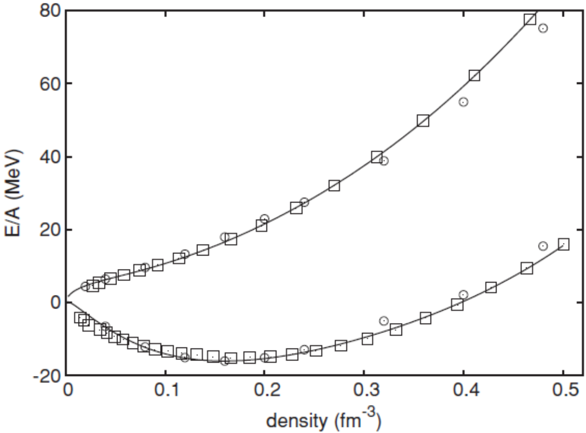


Figure 1. EOS of symmetric and neutron matter obtained by the microscopic calculation (squares) and the corresponding polynomial fits (solid lines). For comparison the microscopic EOS of [26] are also displayed by open circles.

The BCPM functional

In the spirit of the LDA it is proposed to use the previous fit in finite nuclei just replacing the nuclear matter density ρ by the finite nuclei one $\rho(\vec{r})$.

The energy of a finite nucleus is given by

$$E = T_0 + E_{int}^{\infty} + E_{int}^{FR} + E^{s.o.} + E_C + E_{pair}.$$

where

$$E_{int}^{\infty}[\rho_p, \rho_n] = \int d\vec{r}\left[P_s(\rho)(1-\beta^2) + P_n(\rho)\beta^2\right]\rho$$

with $\rho(\vec{r}) = \rho_n(\vec{r}) + \rho_p(\vec{r})$ and $\beta(\vec{r}) = (\rho_n(\vec{r}) - \rho_p(\vec{r}))/\rho(\vec{r})$
and

$$E_{int}^{FR}[\rho_n, \rho_p] = \frac{1}{2}\sum_{t,t'}\iint d\vec{r}d\vec{r'}\rho_t(\vec{r})V_{t,t'}e^{-(\vec{r}-\vec{r'})^2/r_{0\,tt}{}^2}\rho_{t'}(\vec{r'})$$

with

$$V_{n,n} = V_{p,p} = V_L = \frac{2\tilde{b}_1}{\pi^{3/2}r_{0L}^3\rho_0} \quad V_{n,p} = V_{p,n} = V_U = \frac{4a_1 - 2\tilde{b}_1}{\pi^{3/2}r_{0U}^3\rho_0}$$

r_{0L} and r_{0U} are free parameters to be fitted using finite nuclei data

Polynomial fit to realistic EoS to produce a function of ρ.
Invoke LDA to obtain an EDF for finite nuclei (+ some cooking)

M.Baldo et al, Phys. Rev. C87 064305 (2013)
Barcelona, Catania, Paris, Madrid

Requirements

- Integer powers of the density (beyond mean field)
- Mass table quality for binding energies and radii (for astrophysical applications !)
- Reasonable description of
  - Quadrupole and octupole deformation
  - Fission / moments of inertia
  - Giant resonances
  - Crust of neutron stars in TF approach

The Outer Crust, Formalism

In the outer crust the energy per baryon at given average density $\rho_B = A/V$ is given by $\varepsilon(A, Z, \rho_B) = \varepsilon_n + \varepsilon_{elec} + \varepsilon_l$, where the nuclear part reads

$$\varepsilon_n(A, Z) = \frac{M(A, Z)}{A} = \frac{(A - Z)m_n + Zm_p - B(A, Z)}{A}$$

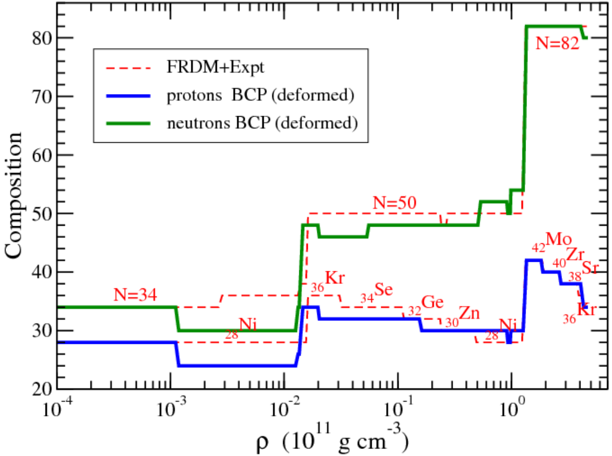
. The electronic and lattice contributions are given by

$$\varepsilon_{elec} = \frac{m_e^4}{8\pi^2 \rho_B} \left[x_e \left(2x_e^2 + 1 \right) \sqrt{x_e^2 + 1} - \ln \left(x_e + \sqrt{x_e^2 + 1} \right) \right]$$

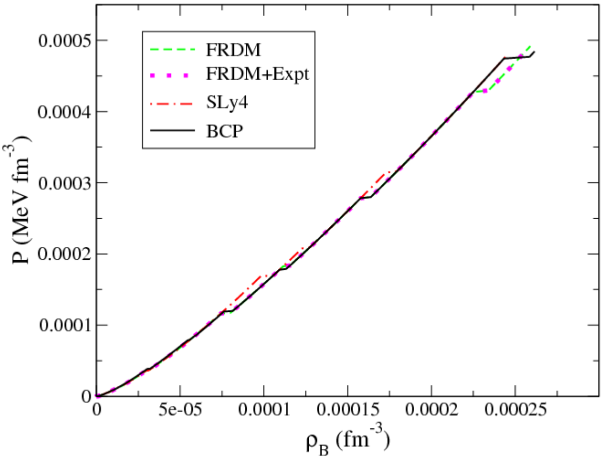
$$\varepsilon_l = -C_l \frac{Z^2}{A^{4/3}} m_e x_e \quad (C_l = 3.40665 \times 10^{-3})$$

with $x_e = k_{F,e}/m_e$ and $k_{F,e} = \left(3\pi^2 \rho_e\right)^{1/3}$.
It is assumed that thermal, hydrostatic and chemical equilibrium is reached at each layer of the outer crust. Only electronic and lattice terms contribute to the pressure: $P = P_{elec} + P_l$.
At fixed pressure the quantity to be minimized is the chemical potential $\mu(A, Z, P) = \varepsilon(A, Z, \rho_B) + \frac{P}{\rho_B}$

The Outer Crust, Composition and EOS



The Outer Crust, EOS



The Inner Crust, Self-consistent Thomas-Fermi approach

The total energy of an esemble of neutrons, protons and electrons in a Wigner-Seitz cell of volume V_c is given by:

$$E = \int dV \left[\mathcal{H}\left(\rho_n, \rho_p\right) + \mathcal{E}_{elec} + \mathcal{E}_{coul} - \frac{3}{4}\left(\frac{3}{\pi}\right)^{1/3} e^2 \left({\rho_p}^{4/3} + {\rho_e}^{4/3}\right) + m_n\rho_n + m_p\rho_p \right]$$

where the nuclear energy density in the TF approach reads:

$$\mathcal{H}\left(\rho_n, \rho_p\right) = \frac{\hbar^2}{2m_n}\frac{3}{5}\left(3\pi^2\right)^{2/3}\rho_n^{5/3}(\mathbf{r}) + \frac{\hbar^2}{2m_p}\frac{3}{5}\left(3\pi^2\right)^{2/3}\rho_p^{5/3}(\mathbf{r}) + \mathcal{V}\left(\rho_n(\mathbf{r}), \rho_{\mathbf{p}}(\mathbf{r})\right)$$

The Coulomb energy density coming from the direct part of the proton-proton and electron-electron plus the proton-electron interaction is given by

$$\mathcal{E}_{coul} = \frac{1}{2}\left(\rho_p(\mathbf{r}) - \rho_e\right)\left(V_p(\mathbf{r}) - V_e(\mathbf{r})\right) = \frac{1}{2}\left(\rho_p(\mathbf{r}) - \rho_e\right)\int \frac{e^2}{|\mathbf{r} - \mathbf{r}'|}\left(\rho_p(\mathbf{r}') - \rho_e\right) d\mathbf{r}'.$$

assuming that the electrons are uniformly distributed in the cell

The Inner Crust, Variational equations

We perform a fully variational calculation of the energy in the WS cell under the constraints of given average density ρ_B in the WS cell of size R_c and charge neutrality in the cell. Taking functional derivatives respect to the neutron, proton and electron densities one finds

$$\frac{\delta \mathcal{H}(\rho_n, \rho_p)}{\delta \rho_n} + m_n - \mu_n = 0,$$

$$\frac{\delta \mathcal{H}(\rho_n, \rho_p)}{\delta \rho_p} + V_p(\mathbf{r}) - V_e(\mathbf{r}) - \left(\frac{3}{\pi}\right)^{1/3} e^2 \rho_p^{1/3}(\mathbf{r}) + m_p - \mu_p = 0,$$

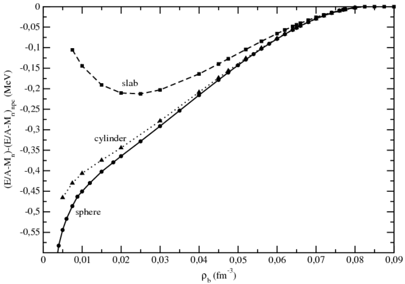
$$\sqrt{k_F^2 + m_e^2} - V_p(\mathbf{r}) + V_e(\mathbf{r}) - \left(\frac{3}{\pi}\right)^{1/3} e^2 \rho_e^{1/3} = \mu_e,$$

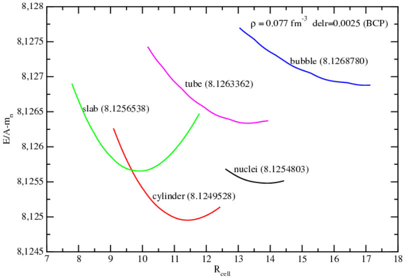
together with the β-equilibrium condition

$$\mu_e = m_n - m_p + \mu_n - \mu_p,$$

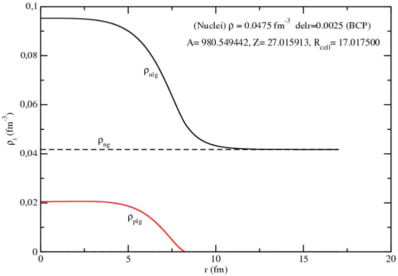
imposed by the the aforementioned constraints.

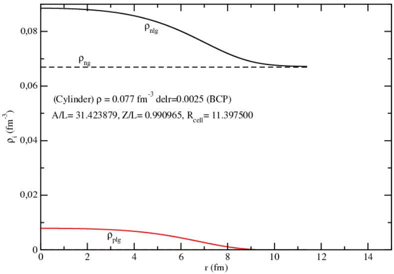
The Inner Crust, Energy per particle



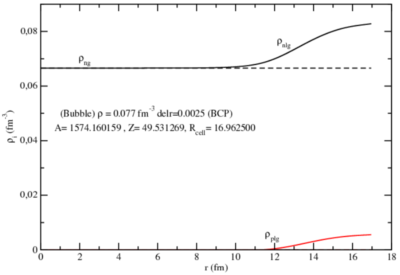


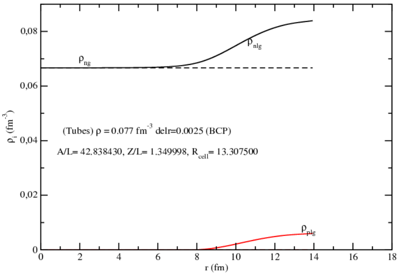
The Inner Crust, Density Profiles



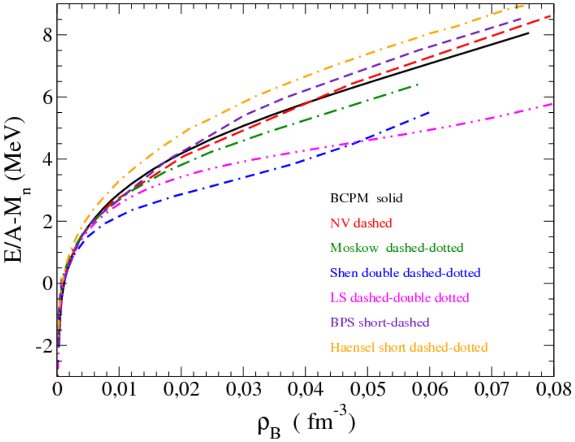


The Inner Crust, Density Profiles



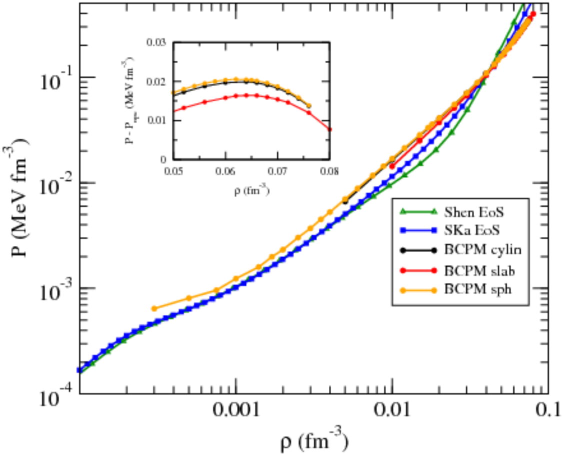


The Inner Crust, EOS



The Inner Crust, EOS

The total pressure is the sum of the pressure due to free neutrons, the electrons and a corrective term coming from the electron exchange $P = P_g + P^{el}_{free} + P^{el}_{exch}$



The Liquid Core, Basic Theory

Equation of State

$$\varepsilon(\rho_n, \rho_p, \rho_e, \rho_\mu) = (\rho_n m_n + \rho_p m_p) + (\rho_n + \rho_p)\frac{E}{A}(\rho_n, \rho_p)$$
$$+ \rho_\mu m_\mu + \frac{1}{2m_\mu}\frac{(3\pi^2\rho_\mu)^{5/3}}{5\pi^2} + \frac{(3\pi^2\rho_e)^{4/3}}{4\pi^2}$$

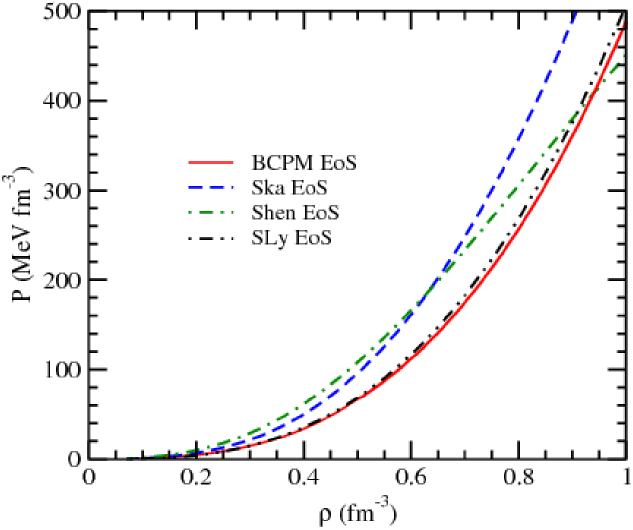
$$\mu_i = \frac{\partial\varepsilon}{\partial\rho_i}, \quad \mu_i = b_i\mu_n - q_i\mu_e, \quad \sum_i \rho_i q_i = 0$$

$$P(\rho) = \rho^2 \frac{d}{d\rho}\frac{\varepsilon(\rho_i(\rho))}{\rho} = \rho\frac{d\varepsilon}{d\rho} - \varepsilon = \rho\mu_n - \varepsilon$$

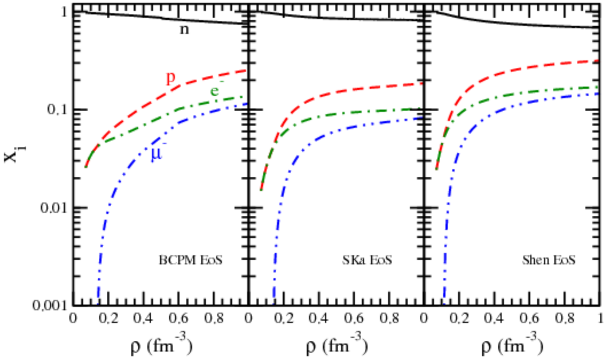
Tolman-Oppenheimer-Volkov equations

$$\frac{dP}{dr} = -G\,\frac{\epsilon m}{r^2}\left(1 + \frac{P}{\epsilon}\right)\left(1 + \frac{4\pi P r^3}{m}\right)\left(1 - \frac{2Gm}{r}\right)^{-1}$$
$$\frac{dm}{dr} = 4\pi r^2 \epsilon\,,$$

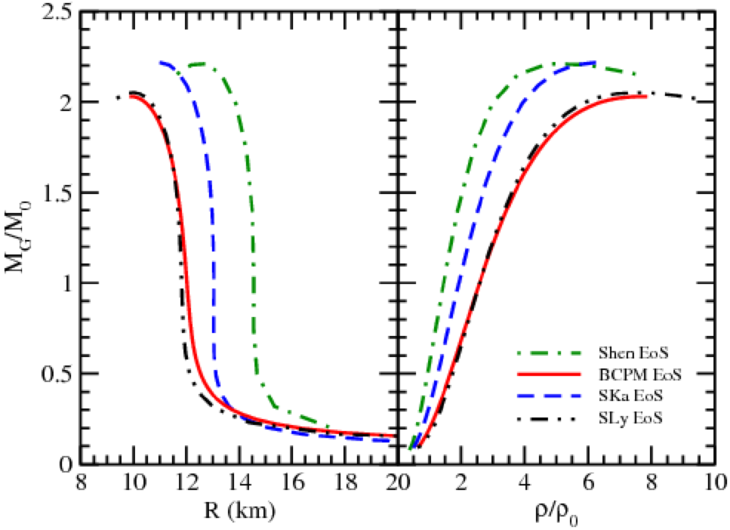
The inner Crust, EOS



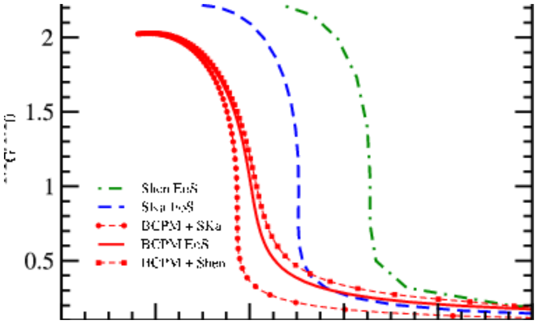
The Liquid core, Composition



Mass-Radius plot



Mass-Radius plot



Conclusions

- We have studied the Neutron Star structure based on an EOS derived within the Brueckner-Hartree-Fock scheme. This EOS is the bases of an accurated Energy Density Functional that was recently devised to reproduce nuclear binding energies and charge radii of finite nuclei.
- Using this Energy Density Functional we obtain:
  The composition of the outer crust is obtained from the masses of dinite nuclei derived within the deformed Hartree-Fock-Bogoliubov formalism.
  The structure and Equation of State of the inner crust calculated by means of a fully self-consistent Thomas-Fermi approximation which also allows to investigate pasta phases.
- The results are compared with other more phenomenological models that are also able to describe the whole Neutron Star structure.
- To emphasize the role of the crust we have performed calculations by employing the microscopic EOS to describe the core and joining it to the crust EOS from more phenomenological approaches
- This analysis shows that depending on the EOS of the crust chosen may appear sizeable effects on the Neutron Star radius. This fact points out the importance of using models based on the same physical framework both for the crust and the core.