Inhomogeneous structures of neutron star crust and mechanical properties

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Matter of neutron stars

Density

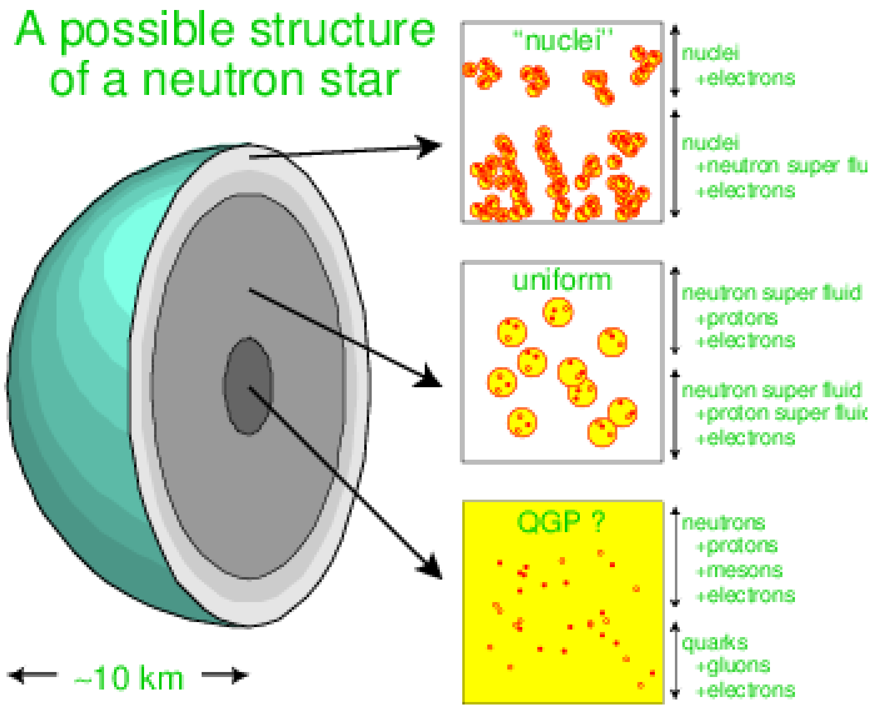
$\approx 0 \cdots \approx 10\rho_0$

Composition

- Nucleons + leptons
- . . . + mesons, hyperons
- quarks + gluons + leptons

Structure & correlation

- uniform
- crystal
- pasta
- amorphous
- pairing
- …..



Properties of neutron star matter

Compressibility (Equation of state).

→ Masses and radii of neutron stars
← Hadron-quark phase transition
← Hyperon mixture and/or meson condensation
← Effects of mixed phase

Rigidity (shear modulus)

→ Torsional oscillation might be directly measured as QPO
→ Mountain height
→ Starquakes

Our mean field studies of nuclear matter

- (inhomogeneous) structures of nuclear matter
- Eos
- **Useful for studies of mechanical strength (rigidity)**

RMF + Thomas-Fermi model

Lagrangian

$$L = L_N + L_M + L_e,$$

$$L_N = \bar{\Psi}\left[i\gamma^\mu \partial_\mu - m_N^* - g_{\omega N}\gamma^\mu \omega_\mu - g_{\rho N}\gamma^\mu \vec{\tau}\vec{b}_\mu - e\frac{1+\tau_3}{2}\gamma^\mu V_\mu \right]\Psi$$

$$L_M = \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{1}{2}m_\sigma^2\sigma^2 - U(\sigma) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{R}_\mu\vec{R}^\mu,$$

$$L_e = -\frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \bar{\Psi}_e\left[i\gamma^\mu\partial_\mu - m_e + e\gamma^\mu V_\mu \right]\Psi_e, \qquad (F_{\mu\nu} \equiv \partial_\mu F_\nu - \partial_\nu F_\mu)$$

$$m_N^* = m_N - g_{\sigma N}\sigma, \qquad U(\sigma) = \frac{1}{3}bm_N(g_{\sigma N}\sigma)^3 + \frac{1}{4}c(g_{\sigma N}\sigma)^4$$

Nucleons interact with each other via coupling with σ, ω, ρ mesons. Simple but feasible!

From $\partial_\mu\left[\partial \mathrm{L}/\partial(\partial_\mu\phi)\right] - \partial \mathrm{L}/\partial\phi = 0,$

$(\phi = \sigma, \omega_\mu, R_\mu, V_\mu, \Psi),$

$$-\nabla^2\sigma(\mathbf{r}) + m_\sigma^2\sigma(\mathbf{r}) = g_{\sigma N}(\rho_n^{(s)}(\mathbf{r}) + \rho_p^{(s)}(\mathbf{r})) - \frac{dU}{d\sigma}(\mathbf{r}),$$

$$-\nabla^2\omega_0(\mathbf{r}) + m_\omega^2\omega_0(\mathbf{r}) = g_{\omega N}(\rho_p(\mathbf{r}) + \rho_n(\mathbf{r})),$$

$$-\nabla^2 R_0(\mathbf{r}) + m_\rho^2 R_0(\mathbf{r}) = g_{\rho N}(\rho_p(\mathbf{r}) - \rho_n(\mathbf{r})),$$

$$\nabla^2 V_C(\mathbf{r}) = 4\pi e^2\rho_{\mathrm{ch}}(\mathbf{r}),$$

For Fermions, we employ Thomas-Fermi approx. with finite T

$$f_{i=n,p}(\mathbf{r};\mathbf{p},\mu_i) = \left(1 + \exp\left[\left(\sqrt{p^2 + m_i^*(\mathbf{r})^2} - \sqrt{p_{Fi}(\mathbf{r})^2 + m_i^*(\mathbf{r})^2}\right)/T\right]\right)^{-1},$$

$$f_e(\mathbf{r};\mathbf{p},\mu_e) = \left(1 + \exp\left[\left(p - (\mu_e - V_C(\mathbf{r}))\right)/T\right]\right)^{-1},$$

$$\rho_{i=p,n,e,\nu}(\mathbf{r}) = 2\int_0^\infty \frac{d^3p}{(2\pi)^3} f_i(\mathbf{r};\mathbf{p},\mu_i),$$

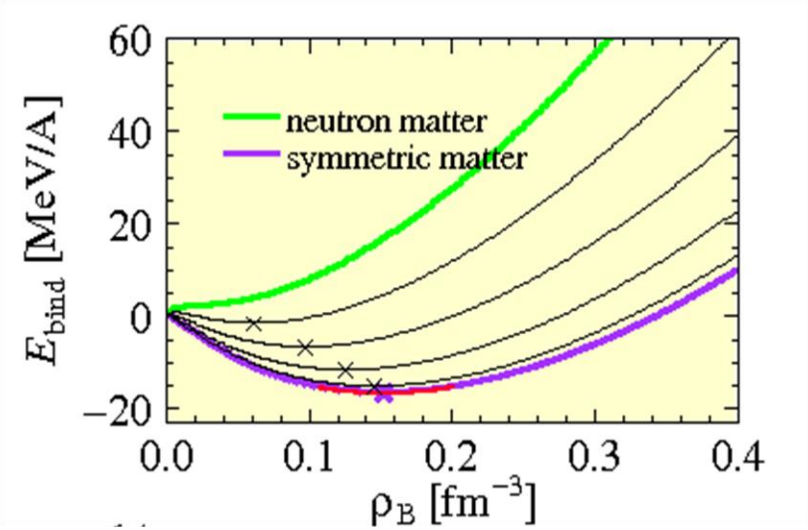
$$\mu_n = \sqrt{p_{Fn}(\mathbf{r})^2 + m_N^*(\mathbf{r})^2} + g_{\omega N}\omega_0(\mathbf{r}) - g_{\rho N}R_0(\mathbf{r}), \qquad \mu_n = \mu_p + \mu_e,$$

$$\mu_p = \sqrt{p_{Fp}(\mathbf{r})^2 + m_N^*(\mathbf{r})^2} + g_{\omega N}\omega_0(\mathbf{r}) + g_{\rho N}R_0(\mathbf{r}) - V_C(\mathbf{r}),$$

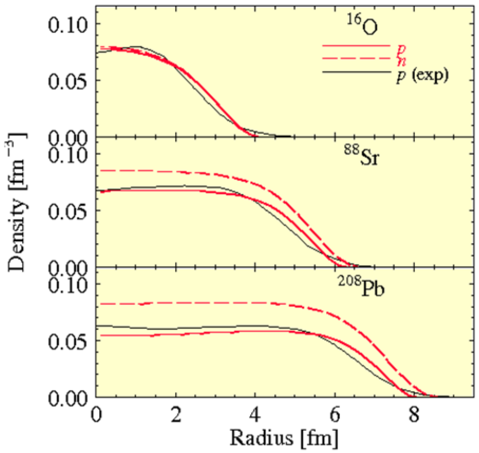
$$\int_V d^3r\left[\rho_p(\mathbf{r}) + \rho_n(\mathbf{r})\right] = \mathrm{const}, \qquad \int_V d^3r\,\rho_p(\mathbf{r}) = \int_V d^3r\,\rho_e(\mathbf{r}),$$

RMF + Thomas-Fermi model

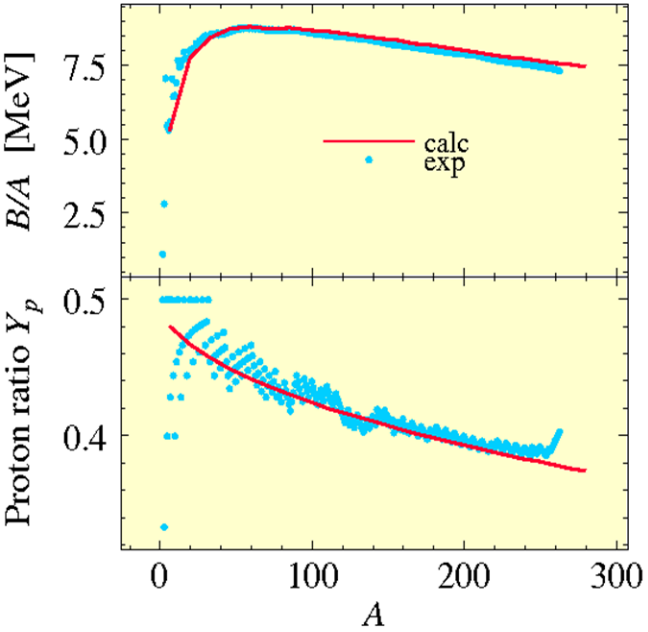
Nucleons interact with each other via coupling with σ, ω, ρ mesons. Simple but realistic enough.



Saturation property of symmetric nuclear matter : minimum energy $E/A \approx -16$ MeV at $\rho_B \approx 0.16\ \mathrm{fm}^{-3}$.



Binding energies, proton fractions, and density profiles of nuclei are well reproduced.



Numerical procedure

To **equilibrate** $\mu_i(r)$ **in** $\boldsymbol{r}$ and **among species** $\boldsymbol{i}$

$$\left.\begin{array}{ll}\text{remove } r\text{-dependence} & \mu_i(r) = \mu_i \\ \text{satisfy chemical balances} & \mu_n = \mu_p + \mu_e\end{array}\right\} \#$$

- Divide whole space into equivalent and neutral cubic cells with **periodic boundary conditions**
- Distribute fermions (p, n, e) **randomly** but $\int d^3r\, \rho_i(r) = \text{given}$
- Solve field equations for $\sigma(r), \omega_0(r), \rho_0(r), V_{\text{Coul}}(r)$
- Calculate local chemical potentials of fermions $\mu_i(r)$

$$\mu_i(r) = V_i(r) + \sqrt{{m_i}^2 + {p_F}_i(r)^2}$$

- Adjust densities $\rho_i(r)$ as

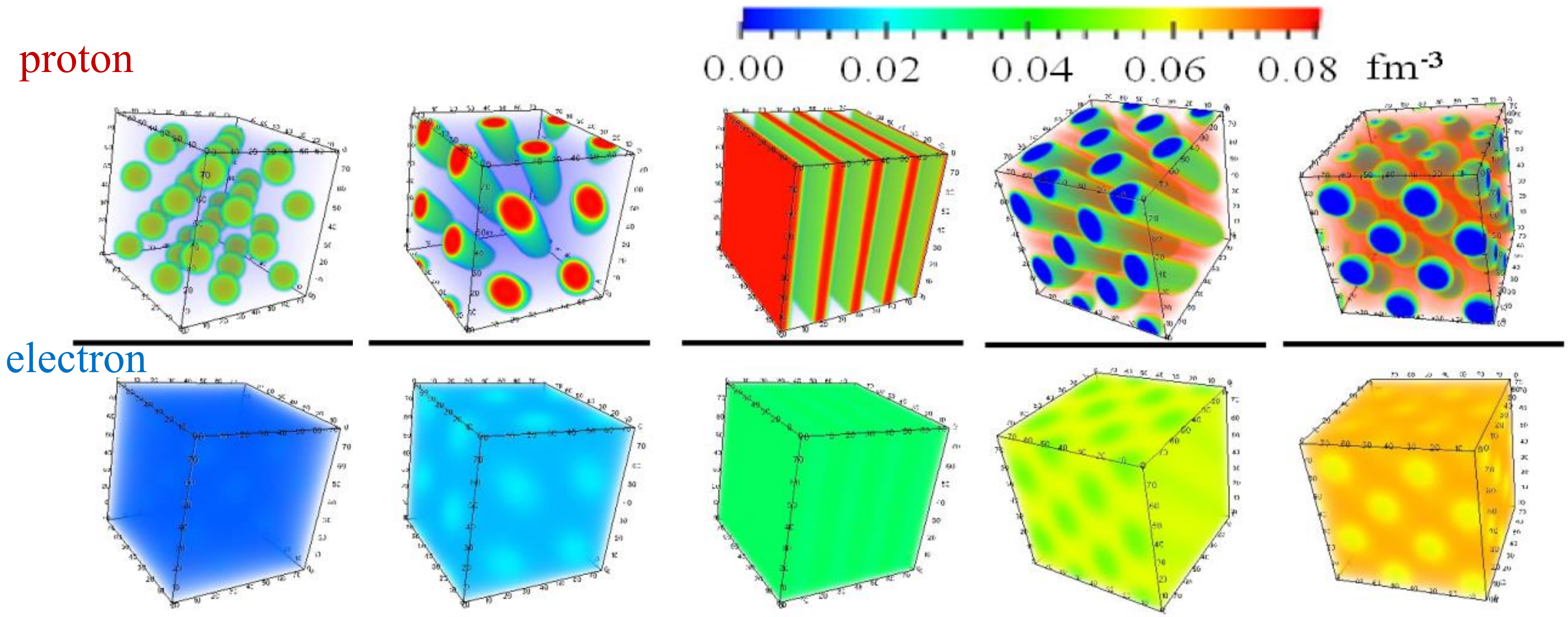
$$\mu_i(r_1) > \mu_i(r_2) \quad \rightarrow \quad \rho_i(r_1)\downarrow,\ \rho_i(r_2)\uparrow$$
$$\mu_n(r) > \mu_p(r) + \mu_e \quad \rightarrow \quad \rho_n(r)\downarrow,\ \rho_p(r)\uparrow$$

- repeat until #

Fully 3D RMF calculations

[Phys.Lett. B713 (2012) 284]

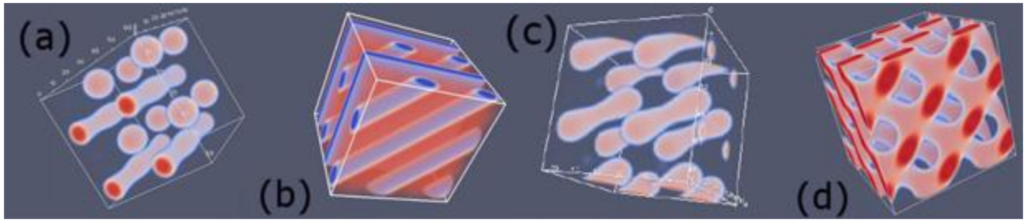
$Y_p = Z/A = 0.5$



“droplet”	“rod”	“slab”	“tube”	“bubble”
[fcc]	[honeycomb]		[honeycomb]	[fcc]
$\rho_B = 0.012\ \mathrm{fm}^{-3}$	$0.024\ \mathrm{fm}^{-3}$	$0.05\ \mathrm{fm}^{-3}$	$0.08\ \mathrm{fm}^{-3}$	$0.094\ \mathrm{fm}^{-3}$

Advantages of a three-dimensional calculation

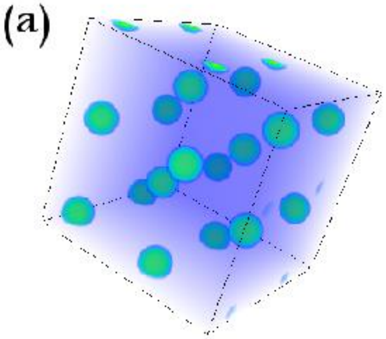
- Not only simple structures but any complex ones are taken into consideration in our new calculation.



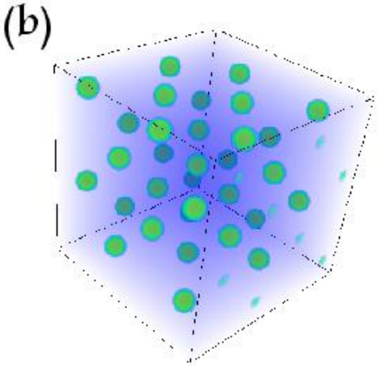
Mixture of **droplet** & **rod** | Mixture of **slab** & **tube** | **Dumbbell** | **Network**

Above structures are observed as excited states in our RMF calculation for symmetric ($Y_p = 0.5$) nuclear matter.
Typical pasta structures are found to be the ground states.

- Crystalline structures can be discussed. We have found that fcc lattice appears at higher densities of droplet phase.



bcc, $\rho = 0.01\ \mathrm{fm}^{-3}$



fcc, $\rho = 0.03\ \mathrm{fm}^{-3}$

Shear

[Ogata etal, PRA42(1990)4867]

$r_i \to r_k \delta_{ik} u$
$(\vec{r} \to \vec{r} u)$

Deformation = translation or rotation + isotropic compression + **shear**

Nothing essential

Conserves the volume. There are 6 kinds

6 kinds of shear deformations

D1: $u_{xx} = \frac{2\epsilon - \epsilon^2}{(1-\epsilon)^2}$, $u_{yy} = u_{zz} = -\epsilon$

D2: D1 with $x \to y, y \to z, z \to x$

D3: D1 with $x \to z, y \to x, z \to y$

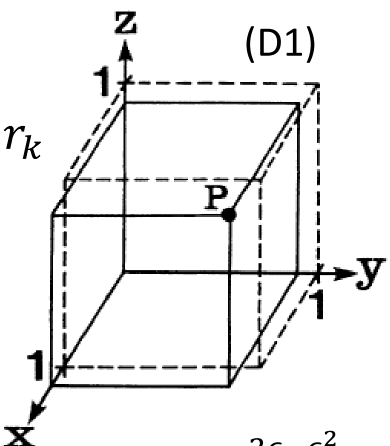
D4: $u_{xx} = \frac{\epsilon^2}{(1-\epsilon)^2}$, $u_{yz} = u_{zy} = \epsilon$

D5: D4 with $x \to y, y \to z, z \to x$

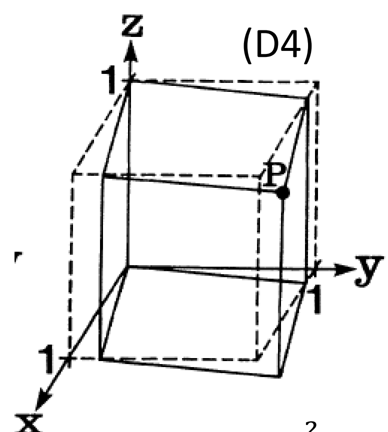
D6: D4 with $x \to z, y \to x, z \to y$

ϵ : infinitesimal

$r_i \to (\delta_{ik} + u_{ik})\, r_k$



$P = (1 + \frac{2\epsilon - \epsilon^2}{(1-\epsilon)^2}, 1 - \epsilon, 1 - \epsilon)$



$P = (1 + \frac{\epsilon^2}{1-\epsilon^2}, 1 + \epsilon, 1 + \epsilon)$

Calculation of shear modulus

How rigid against shear deformation

increment of free energy

$$\delta F = \frac{1}{2}\lambda u_{ii}^2 + \boldsymbol{\mu}\, u_{ik}u_{ik}$$

λ:volume elasticity

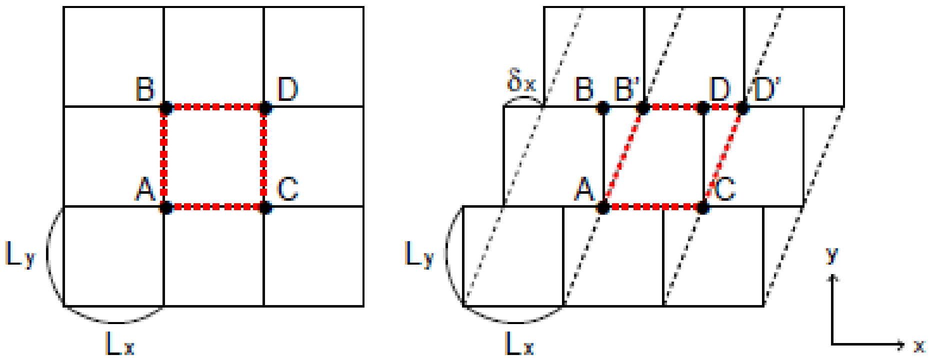
$\boldsymbol{\mu}$: shear modulus

How to calculate the shear modulus in our framework

- Prepare a ground state
- Give a small shear deformation without compression
- Calculate the curvature of the energy change against a shear deformation.
- Average the curvature among different shears

How to give a shear deformation

Deformed periodic boundary condition (DPBC)



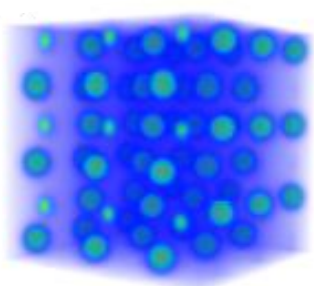
$$F(x \pm Lx, y) = F(x, y), \quad F(x, y \pm Ly) = F(x, y)$$

$$F(x \pm Lx, y) = F(x, y), \quad F(x, y \pm Ly) = F(x \mp \delta x, y)$$

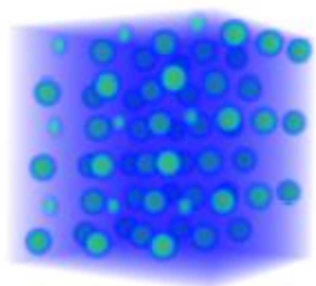
$$\epsilon = \frac{\delta x}{L_x}$$

Ground state under DPBC

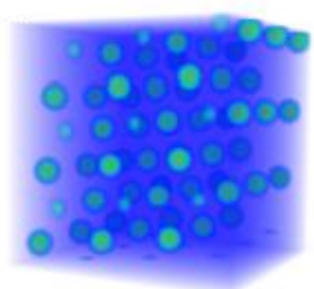
$\epsilon = 0$

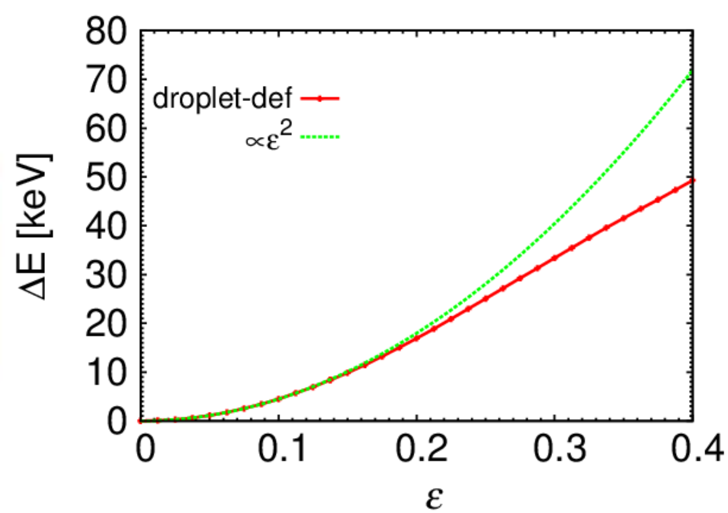


$\epsilon = 0.05$

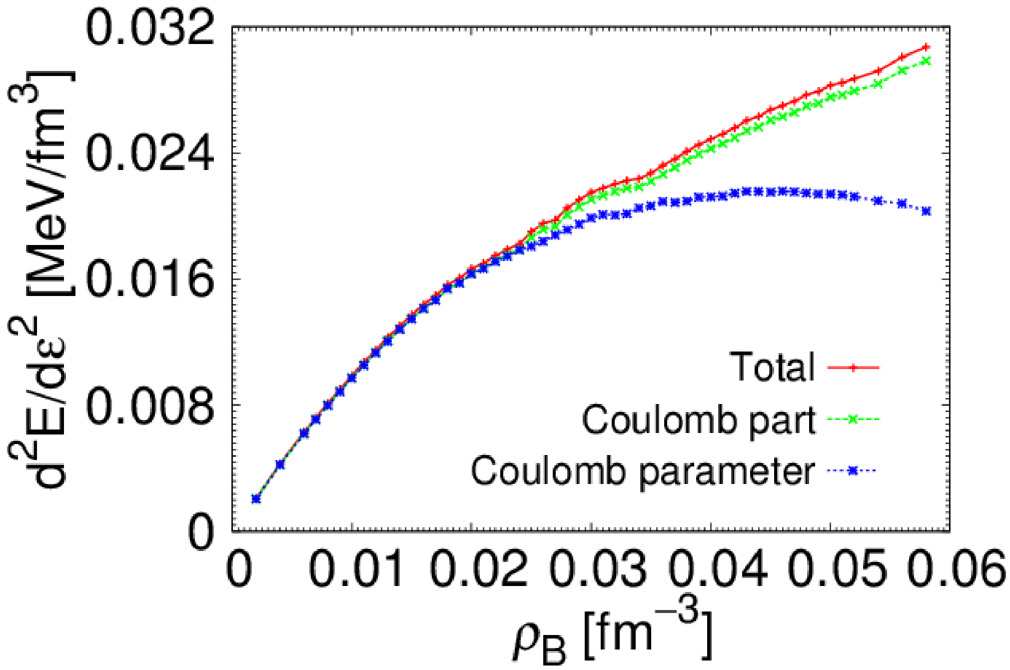


$\epsilon = 0.1$





Preliminary result for droplet



- Below $0.02\ \mathrm{fm}^{-3}$, we get shear modulus slightly smaller than the conventional study. → screening effects.
- Above $0.02\ \mathrm{fm}^{-3}$, our value is larger. →finite size effects.

Summary

RMF + Thomas-Fermi calculation for neutron-star matter
Inhomogeneous structures appear below saturation density

Shear modulus for droplets calculated by deformed periodic boundary conditions
→ screening effects and finite-size effects

Future plan:

Comparison of shear moduli for bcc and fcc lattices and rod