Energy transformations in the birth of neutron stars

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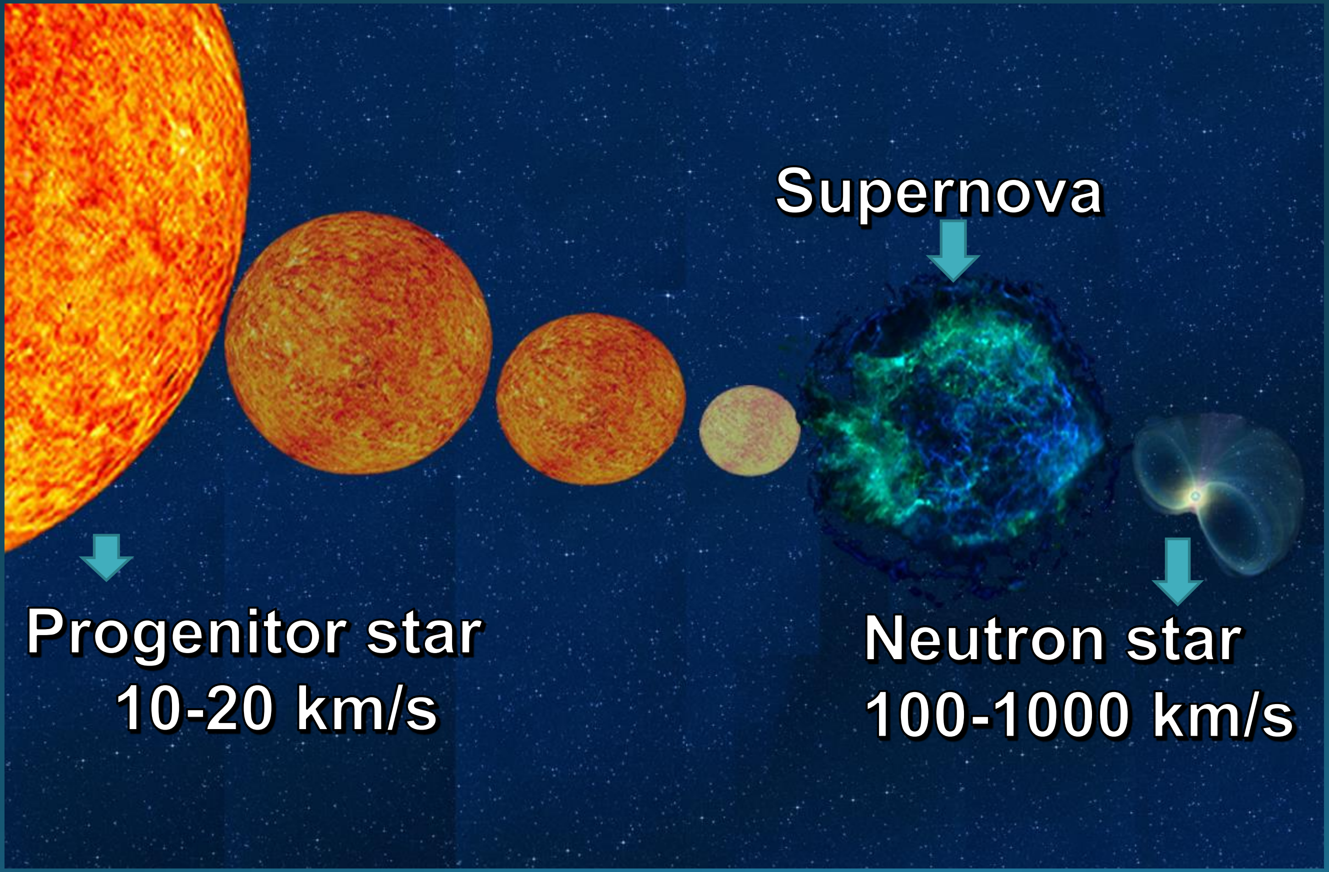
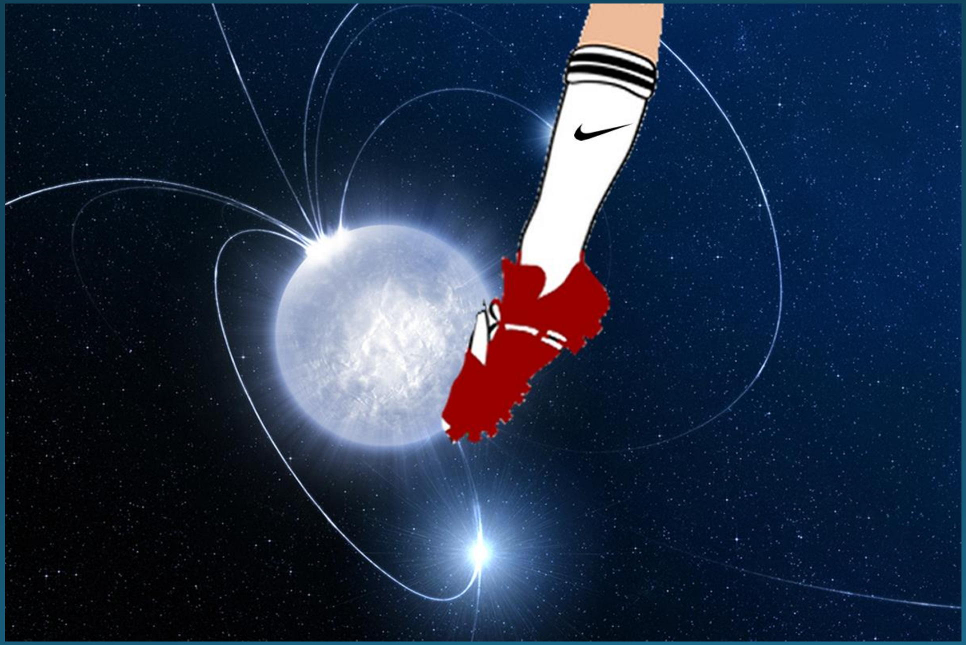
A long-standing problem:

Why neutron stars exhibit space velocities well above those of their progenitor stars?



One thing seems certain:

Neutron stars acquire their observed space velocity by a kick at birth!



Three aspects seem to be feasible “during” some stage of the birth process of NS:

1. NS can have rotational periods of order of ms
2. NS can have magnetic fields of order of 10^{15}-10^{16} G
3. NS can experience MRI

Lai and Qian 1998; Nardi and Zuluaga 2001; Lai et al. 2001; Kusenko and Segre 1999; Lambiase 2005; Barkovich et al. 2004; Fuller et al. 2003; Spruit 2008; Kishimoto 2011

The idea of this talk:

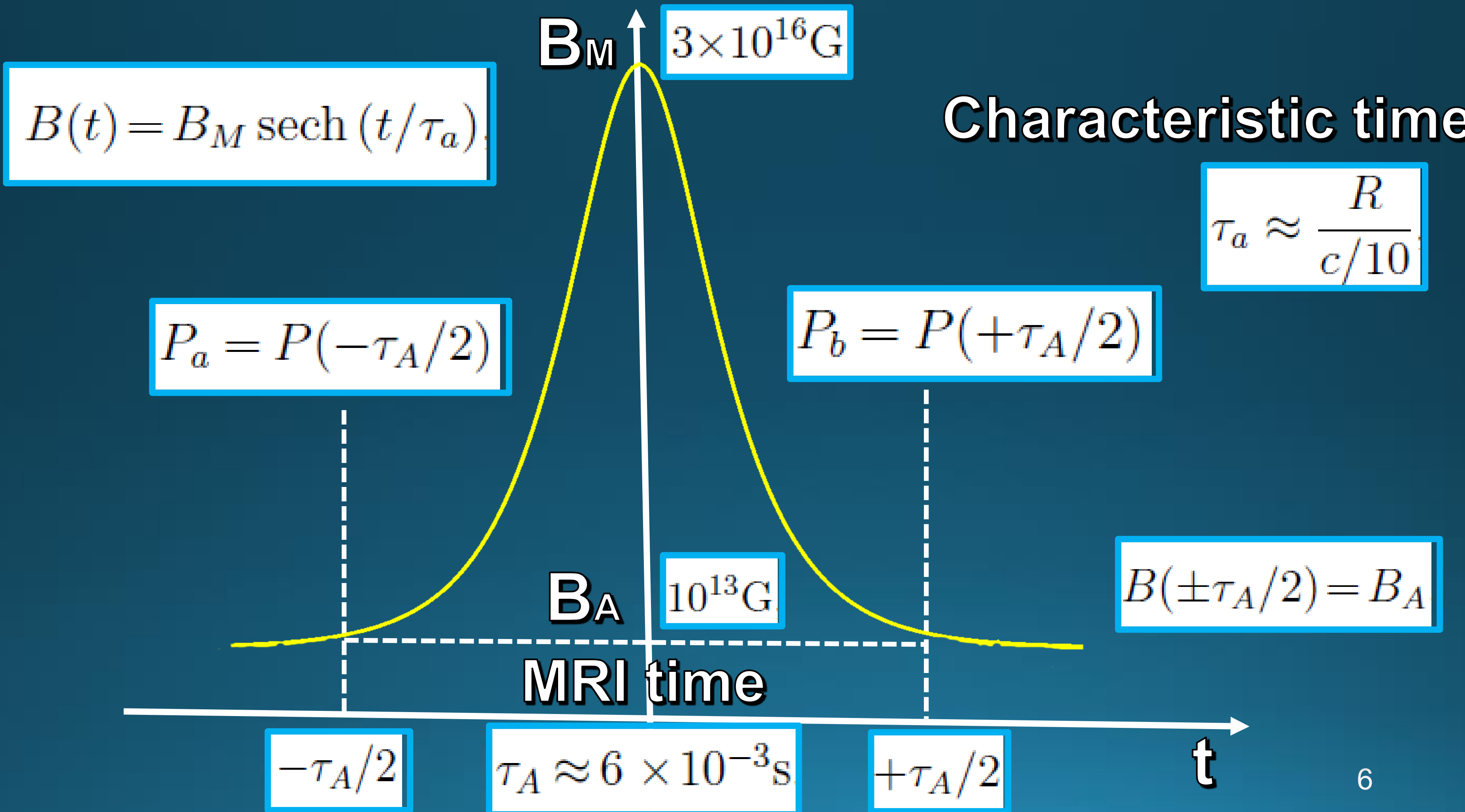
Newly-born neutron stars can be subject to magnetorotational instabilities (MRI). If during MRI neutron stars experience a rapid conversion of their rotational energy into kinetic and radiative energies then we can find an explanation for the observed large velocities of neutron stars

$$\boxed{P_{\mathrm{rad}} + \frac{d}{dt}\left(\frac{M\mathrm{v}^2}{2}\right) = -\frac{d}{dt}\left(\frac{I\Omega^2}{2}\right)}$$

R. Heras, 2012, Birth accelerations of neutron stars, Proc. IAU, Symp. 291, 399 [arXiv:1302.1275].

R. Heras, 2012, Pulsars are Born as Magnetars, ASP Conf. Ser., 466, 253 [arXiv:1302.1278].

Simple model for the magnetic field during the evolving of a MRI:



The MRI time:

τ_A is the time during which the MRI evolves

Using $B(t) = B_M \,\mathrm{sech}\,(t/\tau_a)$ and $B(\pm\tau_A/2) = B_A$

we obtain $B_A = B_M \,\mathrm{sech}\,[\tau_A/(2\tau_a)]$ or equivalently

$\tau_A = 2\tau_a \,\mathrm{asech}(B_A/B_M)$ In particular if $\tau_a \approx \frac{R}{c/10}$

then $.0027\ \mathrm{s} \lesssim \tau_A \lesssim .0058\ \mathrm{s}$ for $B_M = 3 \times 10^{16}\mathrm{G}$

and $10^{13}\mathrm{G} \leq B_A \leq 10^{15}\mathrm{G}$

The radiated power:

μ-B relationship

$$\mu(t) = B(t) R^3$$

Magnetic field

$$B(t) = B_M \operatorname{sech}(t/\tau_a)$$

Magnetic moment

$$\mu(t) = B_M R^3 \operatorname{sech}(t/\tau_a)$$

Larmor formula

$$\mathrm{P}_{\mathrm{rad}}(t) = 2\ddot{\mu}(t)^2/(3c^3)$$

$$\mathrm{P}_{\mathrm{rad}} = \frac{2B_M^2 R^6}{3c^3\tau_a^4} \operatorname{sech}\left(\frac{t}{\tau_a}\right)^2 \left[2\tanh\left(\frac{t}{\tau_a}\right)^2 - 1\right]^2$$

Radiated power

Birth energy conversion:

When the radiated power and

$$\frac{d}{dt}\left(\frac{I\Omega^2}{2}\right) = \frac{4}{5}\pi^2 M R^2 \frac{d}{dt}\left(\frac{1}{P^2}\right)$$

are substituted into

$$\mathrm{P}_{\mathrm{rad}} + \frac{d}{dt}\left(\frac{M\mathrm{v}^2}{2}\right) = -\frac{d}{dt}\left(\frac{I\Omega^2}{2}\right)$$

we get the instantaneous energy conversion

$$\frac{2B_M^2 R^6}{3c^3\tau_a^4}\operatorname{sech}\left(\frac{t}{\tau_a}\right)^2\left[2\tanh\left(\frac{t}{\tau_a}\right)^2 - 1\right]^2 + \frac{d}{dt}\left(\frac{M\mathrm{v}^2}{2}\right) = -\frac{4}{5}\pi^2 M R^2 \frac{d}{dt}\left(\frac{1}{P^2}\right)$$

Energy conversion during a birth MRI:

Integration of the instantaneous radiated power over the inteval $-\tau_A/2 \leq t \leq \tau_A/2$ yields

$$\frac{4}{45}\frac{R^6(B_M^2-B_A^2)^{1/2}(7B_M^4+12B_A^4-4B_M^2B_A^2)}{B_M^3c^3\tau_a^3} + \frac{M\mathrm{v}^2}{2}$$

Radiative energy

Kinetic energy

$$= \frac{4\pi^2 M R^2}{5}\left(\frac{1}{P_a^2}-\frac{1}{P_b^2}\right)$$

Rotational energy

$\mathrm{v}(-\tau_A/2)=0$

$\mathrm{v}=\mathrm{v}(\tau_A/2)$

$P_a = P(-\tau_A/2)$

$P_b = P(+\tau_A/2)$

Nice reduction of the energy conversion

If $\boxed{B_M \gg B_A}$ and $\boxed{\tau_a \approx \frac{R}{c/10}}$ then the energy conversion reads

$$\boxed{\frac{7R^3B_M^2}{11250} + \frac{M\mathrm{v}^2}{2} = \frac{4\pi^2 M R^2}{5}\left(\frac{1}{P_a^2} - \frac{1}{P_b^2}\right)}$$

This implies the formula for the kick velocity

$$\boxed{\mathrm{v}_{\mathrm{kick}} = \sqrt{\frac{8}{5}\pi^2 R^2 \left(\frac{1}{P_a^2} - \frac{1}{P_b^2}\right) - \frac{7R^3B_M^2}{5625M}}}$$

The two components of the Kick velocity:

$$\mathrm{v_{kick}} = \sqrt{\mathrm{v}^2_{[\mathrm{rot}]} - \mathrm{v}^2_{[\mathrm{rad}]}}$$

$V_{[rot]}$ originated by the birth rotation change

$$\mathrm{v_{[rot]}} = \sqrt{\frac{8}{5}\pi^2 R^2 \left(\frac{1}{P_a^2} - \frac{1}{P_b^2}\right)}$$

$V_{[rad]}$ originated by the birth radiation change

$$\mathrm{v_{[rad]}} = \sqrt{\frac{7R^3 B_M^2}{5625M}}$$

radiation reaction

Crab Pulsar

If during a birth MRI the period increased from $\boxed{P_a = .019 \text{ s}}$ to $\boxed{P_b = .019153 \text{ s}}$ and the magnetic field increased from $\boxed{B_A = 10^{13} - 10^{14}\text{G}}$ to $\boxed{B_M = 3 \times 10^{16}\text{G}}$ and subsequently decreased to $\boxed{B_A = 10^{13} - 10^{14}\text{G}}$ then Crab acquired the kick velocity $\boxed{\text{v}_{\text{kick}} \approx 172 \text{ km/s}}$ which implies the observed transverse velocity $\boxed{\text{v}_{\perp} = 141 \text{ km/s}}$,

$$\boxed{\text{v}_{[\text{rot}]} \approx 264 \text{ km/s and v}_{[\text{rad}]} \approx 200 \text{ km/s}}$$

Magnetar J1809-1943

If during a birth MRI the period increased from $P_a = .020 \text{ s}$ to $P_b = .020304 \text{ s}$ and the magnetic field increased from $B_A = 10^{15}\text{G}$ to $B_M = 3 \times 10^{16} \text{ G}$ and subsequently decreased to $B_A = 10^{15}\text{G}$ then J1809-1943 acquired the kick velocity $\text{v}_{\text{kick}} \approx 278 \text{ km/s}$ which implies the observed transverse velocity $\text{v}_{\perp} \approx 229 \text{ km/s}$

$$\text{v}_{[\text{rot}]} \approx 342 \text{ km/s and } \text{v}_{[\text{rad}]} \approx 200 \text{ km/s}$$

MSP B1257+12

If during a birth MRI the period increased from $P_a = .0062065$ s to $P_b = .006218$ s and the magnetic field increased from $B_A = 10^{13}$G to $B_M = 3 \times 10^{16}$G and subsequently decreased to $B_A = 10^{13}$G then B1257+12 acquired the kick velocity $\mathrm{v_{kick}} \approx 334$ km/s which implies the observed transverse velocity $v_\perp \approx 273$ km/s

$\mathrm{v_{[rot]}} \approx 389$ km/s and $\mathrm{v_{[rad]}} \approx 200$ km/s

Pulsar B1508+55

If during a birth MRI the period increased from $P_a = .01$ s to $P_b = .010481$ s and the magnetic field increased from $B_A = 10^{13}$G to $B_M = 3 \times 10^{16}$G and subsequently decreased to $B_A = 10^{13}$G then B1508+55 acquired the kick velocity $\mathrm{v_{kick}} \approx 1173$ km/s which implies the observed transverse velocity $\mathrm{v}_\perp \approx 962$ km/s

$\mathrm{v_{[rot]}} \approx 1189$ km/s $\mathrm{v_{[rad]}} \approx 200$ km/s

Conclusion:

Newly-born neutron stars can be subject to magnetorotational instabilities in which a rapid conversion of rotational energy into kinetic and radiative energies can occur.

If during the evolving of these instabilities, newly-born neutron stars exhibit periods of ms and reach magnetic fields of order of 10^{16} G then the observed large velocity of these stars can be explained

Thanks for your time!