New Approach to an Effective Density Dependence
within an Advanced Relativistic Mean-Field Theory

Kristian Petrík
Štefan Gmuca

MOTIVATION

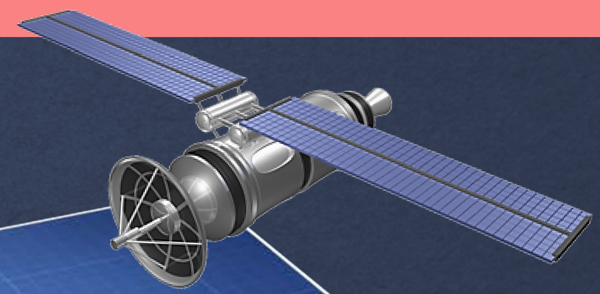
! MASSIVE PULSARS OBSERVED

- *masses around* ***2Msol***
- *precise measurements*

P. B. Demorest et al., ***Nature*** *467, 1081 (2010)*
J. Antoniadis et al., ***Science*** *340, 448 (2013)*

? HYPERON PUZZLE

- ***hyperons*** *generally* ***soften*** *the equation of state*
- ***many models*** *with exotic matter (hyperons, kaon condensates, …)* ***are ruled out***



! DENSITY DEPENDENCE

- ***not exactly known*** *for a nuclear interaction*
- *huge gap between* ***compact stars*** *and* ***nuclei***

? EFFECTIVE MODELS

- ***simplicity*** *vs* ***fundamentality***
- *generally* ***better results*** *but with a "****black box****"–like understanding*

EFFECTIVE FIELD THEORY

- ***Ab initio*** *approaches*
  - *microscopic treatment*
  - *parameter free EoS predictions*
  - *based on precise NN interaction*

- ***Phenomenological*** *approaches*
  - *relatively easy calculations*
  - *very good results*
  - *description of systems not reachable by ab initio models*

Equation of state

EFFECTIVE FIELD THEORY

- *Ab initio approaches*
  - *microscopic treatment*
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  - *based on precise NN interaction*

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Equation of state

DIRAC
RELATIVISTIC

BRUECKNER
MEDIUM EFFECTS

HARTREE
DIRECT TERMS

FOCK
EXCHANGE TERMS

CORRELATIONS
EVERYTHING ELSE

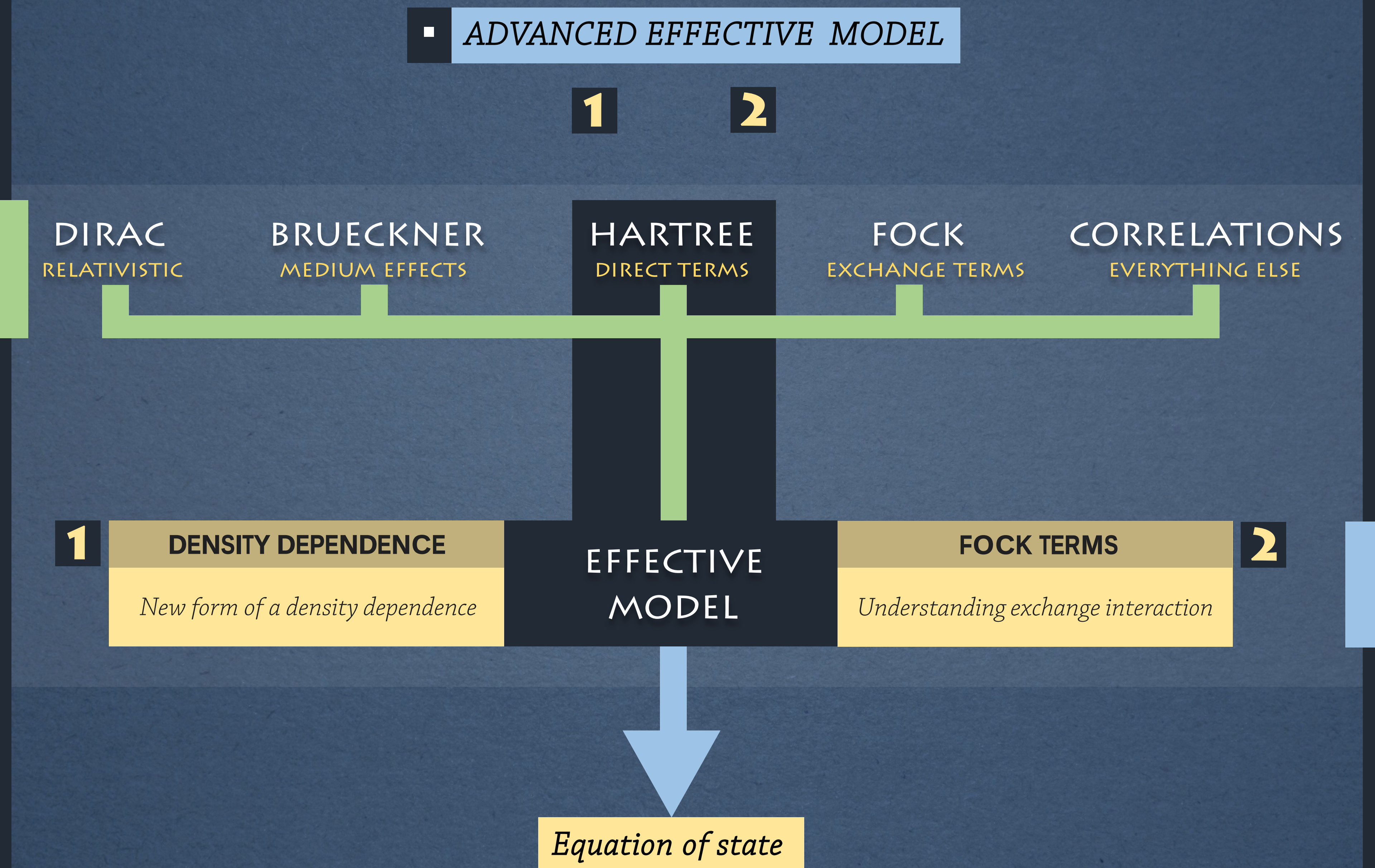
EFFECTIVE MODEL

Black Box

Everything is hidden in parameters

Equation of state

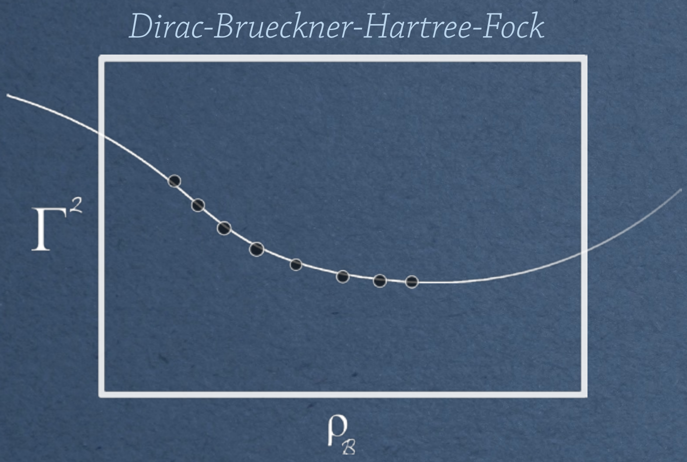
EFFECTIVE FIELD THEORY



2 parametric functions

- *Typical choice of a density dependence*

$$\Gamma_i(\rho_{\mathcal{B}}) = a_i \left[\frac{1 + b_i(\rho_{\mathcal{B}}/\rho_{sat} + d_i)^2}{1 + c_i(\rho_{\mathcal{B}}/\rho_{sat} + e_i)^2} \right],$$

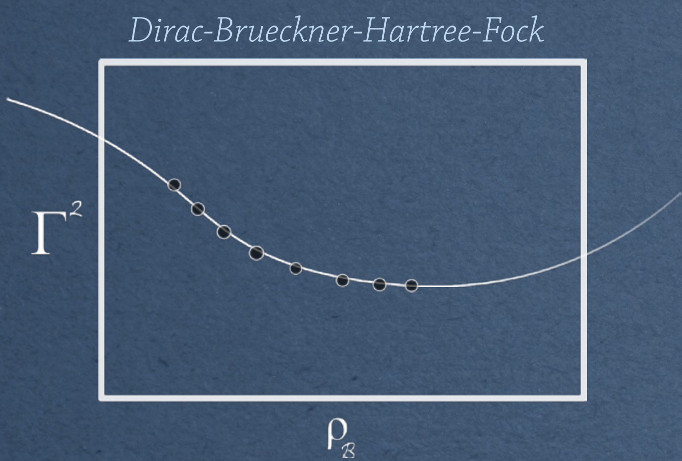


- *ambiguities*
- *no stable extrapolations*
- *too many free parameters*

2 parametric functions

- *Typical choice of a density dependence*

$$\Gamma_i(\rho_B) = a_i \left[\frac{1 + b_i(\rho_B/\rho_{sat} + d_i)^2}{1 + c_i(\rho_B/\rho_{sat} + e_i)^2} \right],$$



- *Other functional forms*

4–parametric class

$$
\begin{aligned}
{}^{A}\Gamma_i^{\bullet\bullet}(x) &= a_i \left[\frac{c_i + (x + d_i)^2}{e_i + (x)^2} \right], \\
{}^{B}\Gamma_i^{\bullet\bullet}(x) &= a_i \left[\frac{c_i + (x + d_i)^{1/2}}{e_i + (x)^{1/2}} \right],
\end{aligned}
$$

3–parametric class

$$
\begin{aligned}
{}^{C}\Gamma_i^{\circ\bullet}(x) &= a_i \left[\frac{1 + (x + d_i)^2}{(x + c_i)^2} \right], \\
{}^{D}\Gamma_i^{\circ\bullet}(x) &= a_i \left[\frac{1 + (x + d_i)^2}{c_i + (x)^2} \right], \\
{}^{E}\Gamma_i^{\circ\bullet}(x) &= a_i \left[\frac{1 + (x + d_i)^2}{c_i + (x + c_i)^2} \right],
\end{aligned}
$$

$$x = \frac{\rho_B}{\rho_{sat}}$$

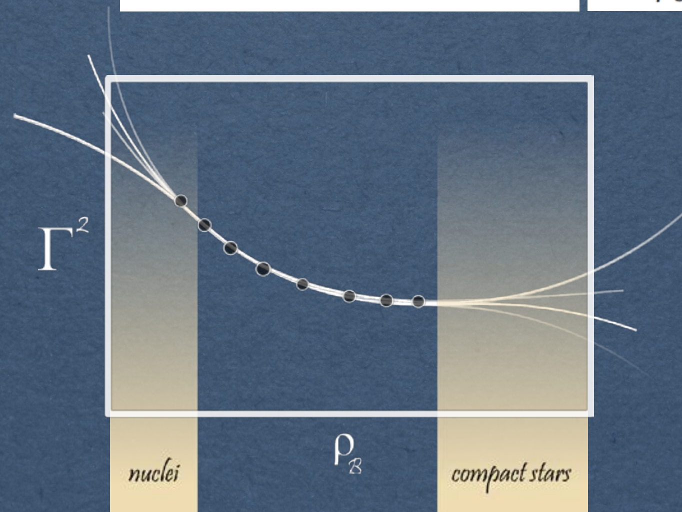
- *2 parametric class*

2–parametric class

$$
\begin{aligned}
{}^{I}\Gamma_i^{\circ\circ}(x) &= a_i \left[\frac{\beta_i + x^2}{b_i + x^2} \right], \\
{}^{II}\Gamma_i^{\circ\circ}(x) &= a_i \left[\frac{\beta_i + x^{3/2}}{b_i + x^{3/2}} \right], \\
{}^{I}\tilde{\Gamma}_i^{\circ\circ}(x) &= a_i \left[\frac{\beta_i + x^2}{b_i + x^2} \right]^{-1}, \\
{}^{II}\tilde{\Gamma}_i^{\circ\circ}(x) &= a_i \left[\frac{\beta_i + x^{3/2}}{b_i + x^{3/2}} \right]^{-1},
\end{aligned}
$$

$$
\begin{aligned}
{}^{I}\Gamma_\delta^{\circ\bullet}(x) &= a_\delta \left[\frac{\beta_\delta + (x + c_\delta)^2}{b_\delta + x^2} \right], \\
{}^{II}\Gamma_\delta^{\circ\bullet}(x) &= a_\delta \left[\frac{\beta_\delta + (x + c_\delta)^{3/2}}{b_\delta + x^{3/2}} \right].
\end{aligned}
$$

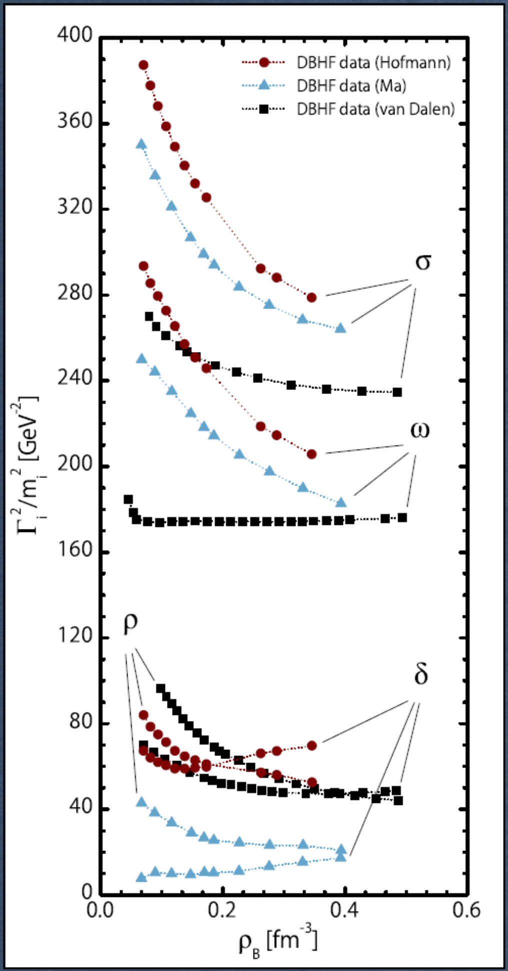
- *ambiguities*
- *no stable extrapolations*
- *too many free parameters*



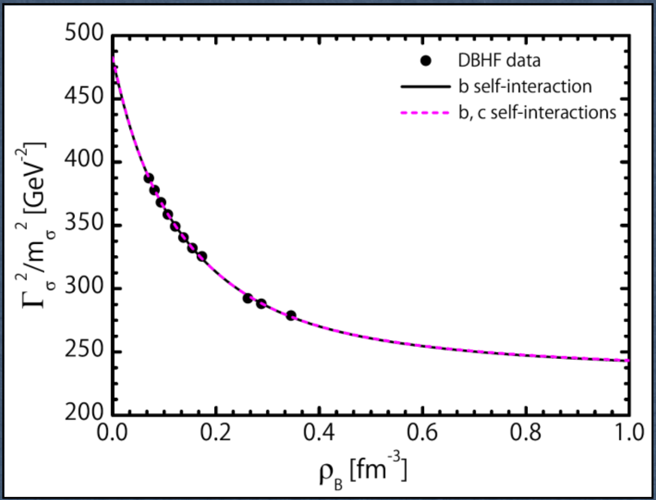
- *2+ parameters*
- *stable extrapolations*
- *great universality*

2 parametric functions

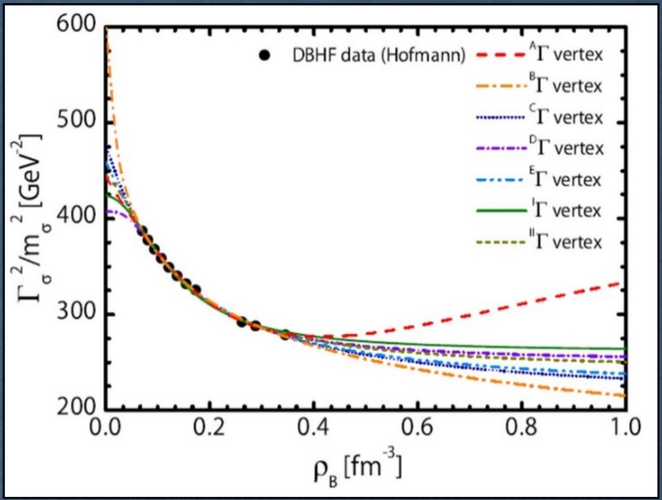
- *DBHF vertices*



- ***Single** density dependence*



- ***Several** density dependences*



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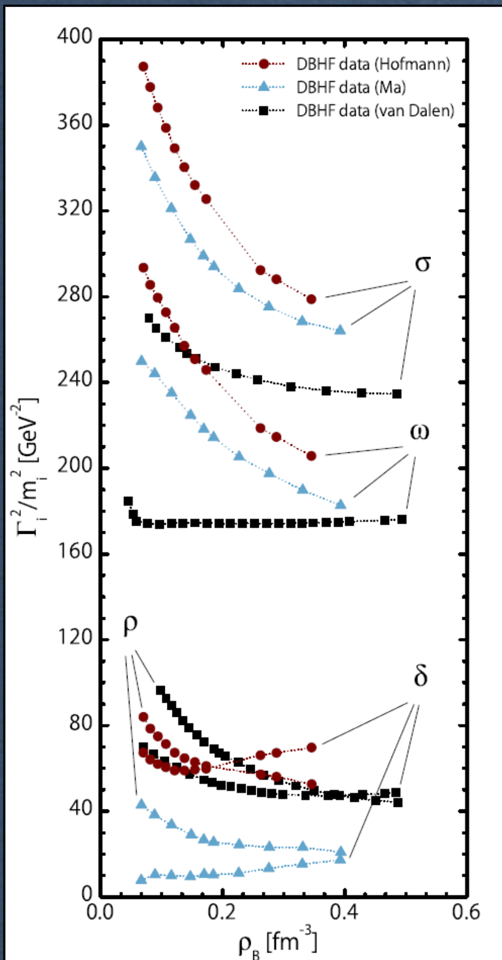
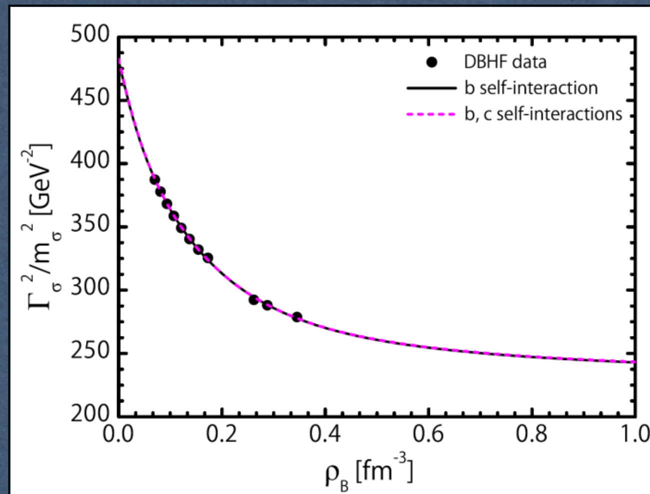
2 PARAMETRIC FUNCTIONS

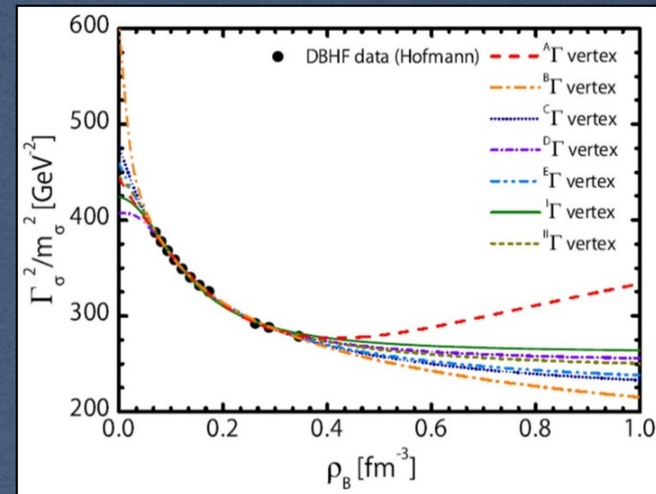
- *DBHF vertices*



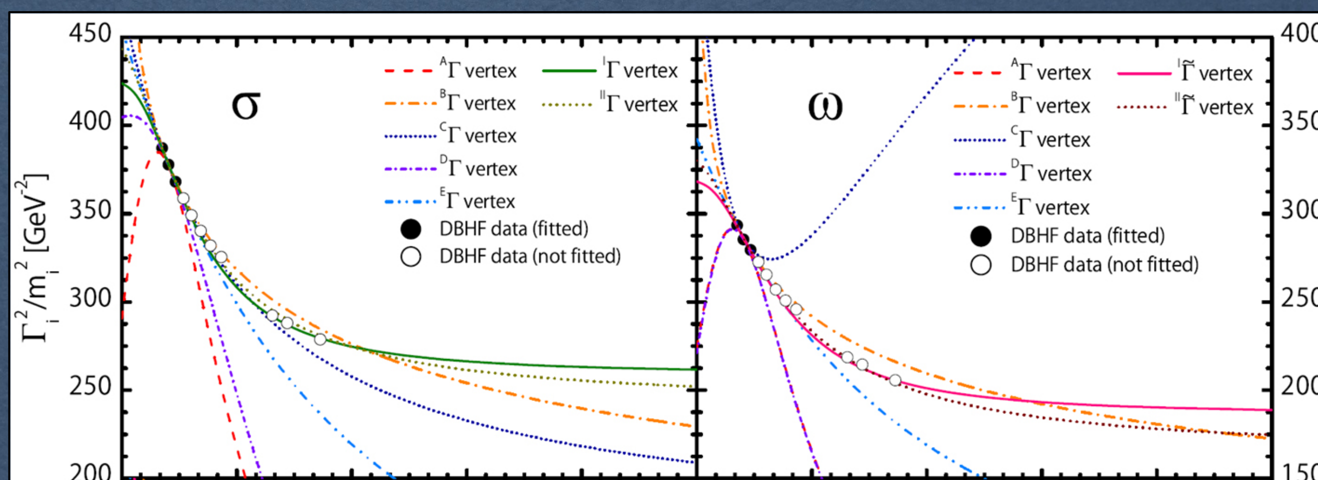
- ***Single** density dependence*



- ***Several** density dependences*



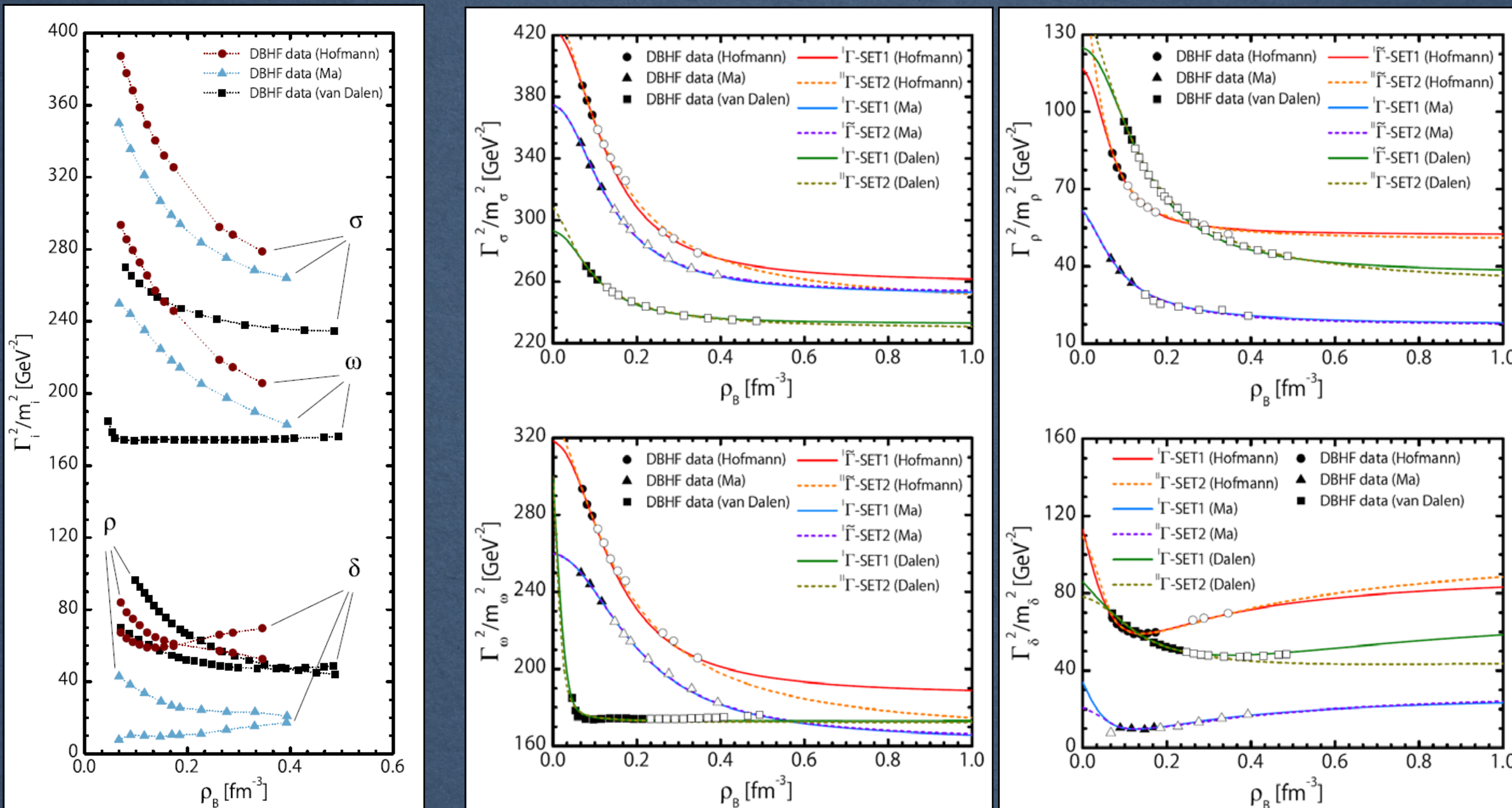
partial fits ▪



2 PARAMETRIC FUNCTIONS

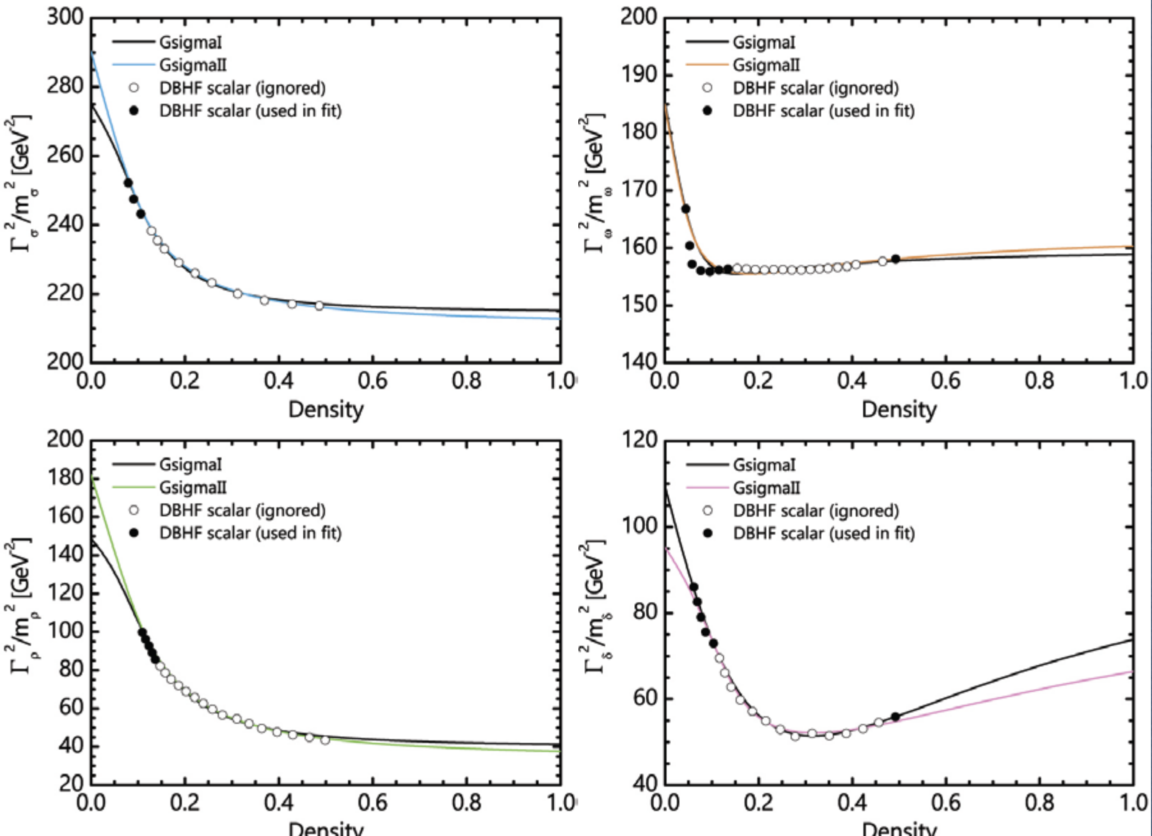
- *DBHF vertices*

partial fits



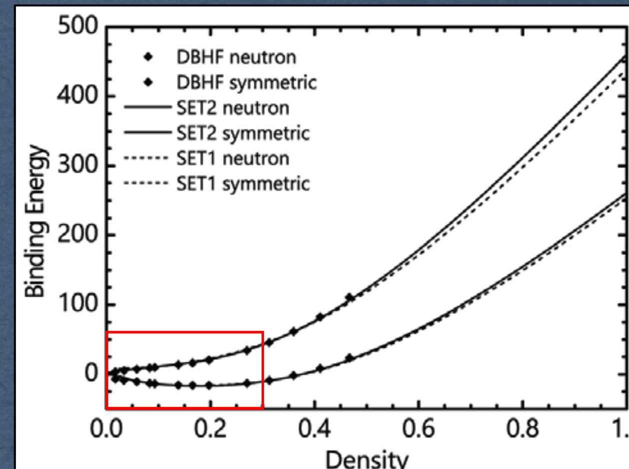
2 PARAMETRIC FUNCTIONS - RESULTS

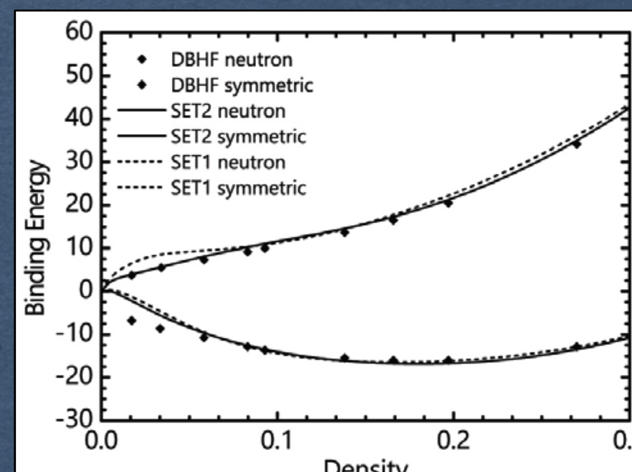
DBHF data | van Dalen *et al.*, *Eur. Phys. J. A* **31**, *29-42 (2007)*



- *DBHF*

E_B = -16.15
ρ_{sat} = 0.181
kF = 1.39





- *SET 1*

E_B = -16.4825
ρ_{sat} = 0.167
kF = 1.352
K = 205
a_S = 33.4

binding energy
saturation density
Fermi momentum
compression modulus
symmetry energy

- *SET 2*

E_B = -16.8913
ρ_{sat} = 0.180
kF = 1.386
K = 236
a_S = 35.0

2 MAPPING OF FOCK TERMS

- *Lagrangian density* for DHF model

$$\mathcal{L}_{\mathrm{HF}} = \mathcal{L}^{\mathrm{fermi}} + \mathcal{L}^{\mathrm{bose}} + \mathcal{L}^{\mathrm{int}},$$

$$\begin{aligned}
\mathcal{L}^{\mathrm{fermi}} = &\ \bar{\psi}\left[i\gamma^{\mu}\partial_{\mu} - m_N\right]\psi, \\
\mathcal{L}^{\mathrm{bose}} = &\ \frac{1}{2}\partial_{\mu}\hat{\sigma}\partial^{\mu}\hat{\sigma} - \frac{1}{2}m_{\sigma}^2\hat{\sigma}^2 - \frac{1}{4}\hat{\omega}_{\mu\nu}\hat{\omega}^{\mu\nu} + \frac{1}{2}m_{\omega}^2\hat{\omega}_{\mu}\hat{\omega}^{\mu} \\
& + \frac{1}{2}\partial_{\mu}\hat{\pi}\partial^{\mu}\hat{\pi} - \frac{1}{2}m_{\pi}^2\hat{\pi}^2 \\
\mathcal{L}^{\mathrm{int}} = &\ \bar{\psi}\left[g_{\sigma}\hat{\sigma} - g_{\omega}\hat{\omega}_{\mu}\gamma^{\mu} - \frac{f_{\pi}}{m_{\pi}}\gamma^5\gamma^{\mu}\partial_{\nu}\boldsymbol{\pi}\cdot\boldsymbol{\tau}\right]\psi,
\end{aligned}$$

- Nuclear *self energy*

$$\Sigma(k,k_F) = \Sigma^S(k,k_F) - \gamma^0\Sigma^0(k,k_F) + \boldsymbol{\gamma}\cdot\boldsymbol{k}\Sigma^V(k,k_F)$$

+

- *exchange* ***Fock*** *terms taken into account*
- ***pion*** *contributions* ***do not*** *vanish*
- *significant* ***enhancement*** *of the* ***maximum mass***
- *each coupling depends on a* ***particular density***

-

- ***density dependence*** *of Fock part is* ***not*** *so strong*
- *in medium effects still* ***not covered***

DIRAC
RELATIVISTIC

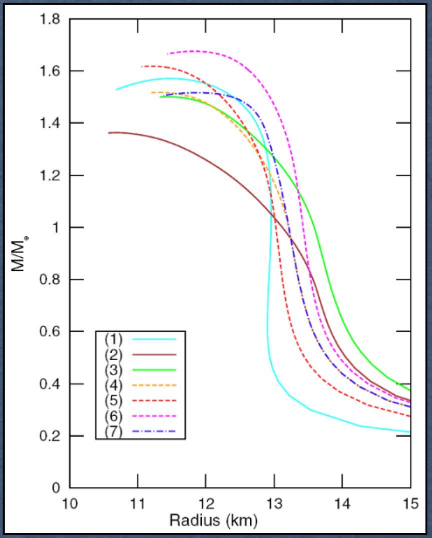
HARTREE
DIRECT TERMS

FOCK
EXCHANGE TERMS

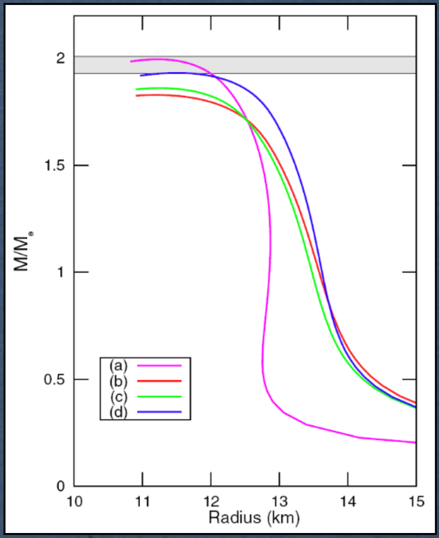
EFFECTIVE MODEL

Dark Gray Box

without Fock



with Fock



2 MAPPING OF FOCK TERMS

- *Nuclear* ***self energy***

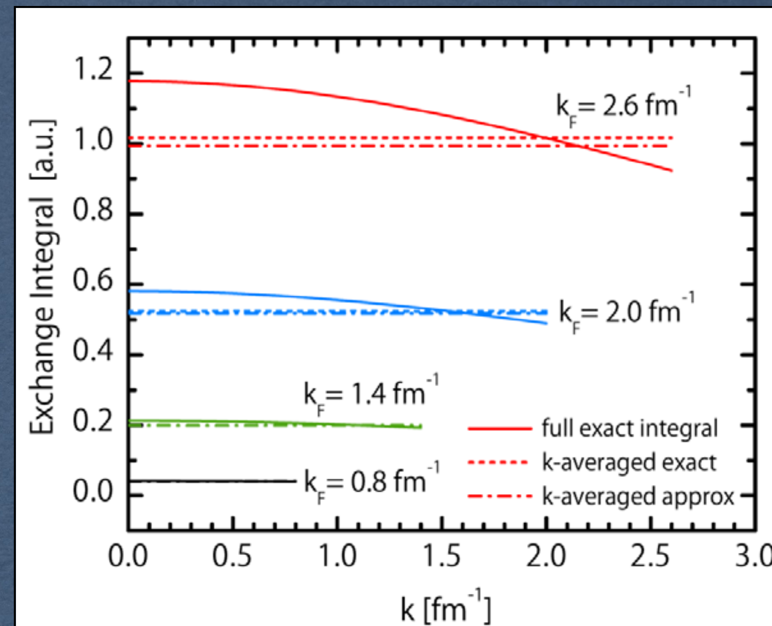
$$\Sigma(k,k_F)=\Sigma^s(k,k_F)-\gamma^0\Sigma^0(k,k_F)+\boldsymbol{\gamma}\cdot\boldsymbol{k}\Sigma^v(k,k_F)$$

$$\Sigma^0(k,k_F)=-\frac{\gamma}{2\pi^2}\frac{g_\omega^2}{m_\omega^2}\int_0^{k_F}q^2dq \quad \text{Hartree}$$

$$\text{Fock} \quad -\frac{1}{(4\pi)^2k}\int_0^{k_F}qdq\left[g_\sigma^2\Theta_\sigma(k,q)+2g_\omega^2\Theta_\omega(k,q)-(\gamma-1)\left(\frac{f_\pi}{m_\pi}\right)^2m_\pi^2\Theta_\pi(k,q)\right],$$

- ***Fock*** *integral*

$$I_i(k,k_F)=\frac{1}{k}\int_0^{k_F}qdq\ln\frac{(k+q)^2+m_i^2}{(k-q)^2+m_i^2}.$$



- ***Expand*** *the exact integral in k and keep the terms up to k^2*
- ***Average*** *it over the* ***Fermi sphere***

Approximate k-averaged exchange integral

$$\bar{I}_i(k_F)=4\left(k_F-m_i\arctan\left(\frac{k_F}{m_i}\right)-\frac{k_F^5}{5(k_F^2+m_i^2)^2}\right).$$

- ***Fock*** *density dependence*

$$X(x)=3\left(x-\arctan(x)-\frac{x^5}{5(1+x^2)^2}\right)/x^3$$

2 Mapping of Fock terms

- *New density dependent* ***couplings***

  - ***Scalar*** *channel*

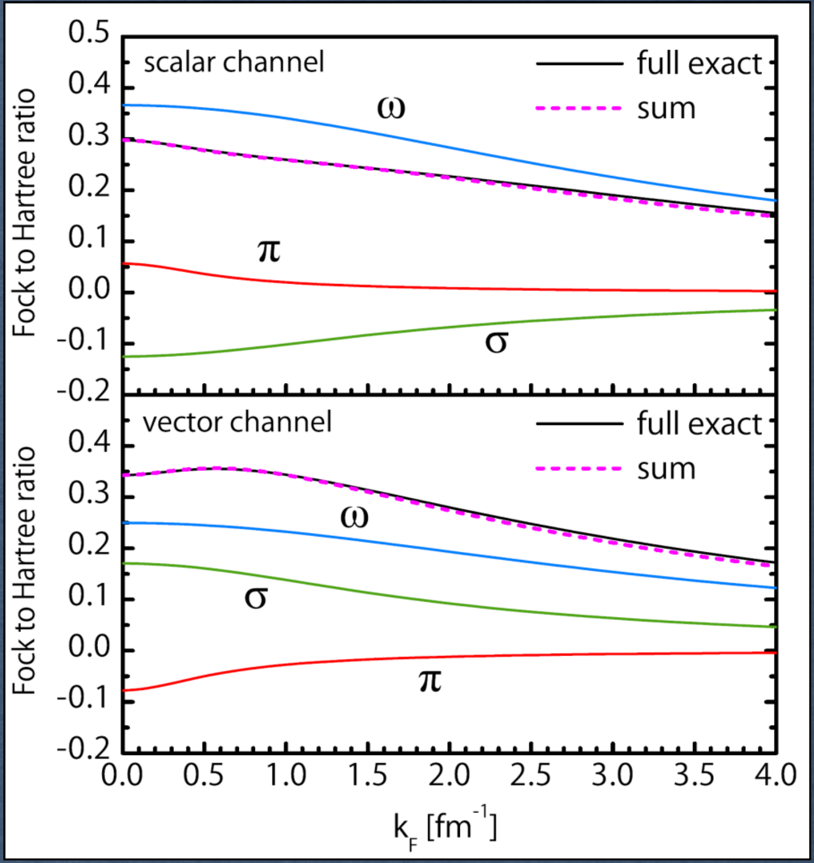
$$\frac{\Gamma_\sigma^2(k_F)}{m_\sigma^2} = \frac{g_\sigma^2}{m_\sigma^2} + \sum_{i=\{\sigma,\omega,\pi\}} b_i \frac{g_i^2}{m_i^2} X(k_F/m_i).$$

direct part ***exchange part***

mixing !

  - ***Vector*** *channel*

$$\frac{\Gamma_\omega^2(k_F)}{m_\omega^2} = \frac{g_\omega^2}{m_\omega^2} + \sum_{i=\{\sigma,\omega,\pi\}} a_i \frac{g_i^2}{m_i^2} X(k_F/m_i),$$



- *Dependence on a* ***specific*** *nuclear* ***density***

$$\Sigma^j(k_F) = \frac{\Gamma_j^2(k_F)}{m_j^2}\rho_j(k_F)$$

PERSPECTIVE

1 DENSITY DEPENDENCE

2 parametric density functions

2 FOCK TERMS

exchange interaction

- ***less ambiguous** DBHF data*
- ***2 parametric functions** with a dependence on **all** nuclear densities ?*
- ***hyperons** – scaling ?*

- ***in medium** correlations still play an important role*
- ***an improvement** through **non-linear** self interactions ?*