

# SU(3) symmetric hypernuclear matter and related stellar properties



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## Motivation:

- PSR J1614-2230, PSR J0348+0432 ( $2 M_{\odot}$ ) require very stiff EOS at large densities
- Heavy ion collisions point in opposite direction for  $\rho < 5\rho_0$
- Hyperons are energetically favored, but soften the EoS
- A strange vector meson mediating the hyperon-hyperon interaction increases the maximum mass, just pushing away the hyperon threshold  
(**S.Weissenborn, D.Chatterjee, J.Schaffner-Bielich, Nucl. Phys. A 881, 62 (2012); Phys. Rev. C 85, 065802 (2012)**)
- Meson-hyperon coupling constants are unknown: how to parametrize them in a less handwaving way as possible?

**S.R. Beane et al, NPA 794, 62 (2007) :  $U_{Y(N)}$  (LQCD)**

**M. Oertel et al, PRC 85, 055806 (2012):**

$U_{\Lambda} = -29.6$  to  $-26.8$  MeV (attractive)

$U_{\Sigma} = +16.8$  MeV to  $+73.0$  MeV (repulsive)

$U_{\Xi} = -24.5$  MeV to  $-15.3$  MeV (attractive)

**Usually,**

$U_{\Lambda} = -28$  MeV - well known

$U_{\Sigma} = +30$  MeV,  $U_{\Xi} = -18$  MeV - present huge uncertainties

**YY potentials - even worst scenario:**

**S.Balberg and A. Gal, NPA 625, 435 (1997):  $U_{\Lambda\Lambda}(\rho_0) = -40$  MeV**

**K. Nakazawa, NPA 835, 207 (2010):  $U_{\Lambda\Lambda}(\rho_0) = -10$  MeV**

# Formalism

$$\begin{aligned}\mathcal{L}_{QHD} = & \sum_B \bar{\psi}_B [\gamma^\mu (i\partial_\mu - g_{BB\omega}\omega_\mu - g_{BB\phi}\phi_\mu - g_{BB\rho}\frac{1}{2}\vec{\tau} \cdot \vec{\rho}_\mu) - (m_B - g_{BB\sigma}\sigma)] \psi_B \\ & + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_s^2\sigma^2) - U(\sigma) - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2}m_v^2\omega_\mu\omega^\mu - \frac{1}{4}\Phi^{\mu\nu}\Phi_{\mu\nu} + \\ & + \frac{1}{2}m_\phi^2\phi_\mu\phi^\mu + \frac{1}{2}m_\rho^2\vec{\rho}_\mu \cdot \vec{\rho}^\mu - \frac{1}{4}\mathbf{P}^{\mu\nu} \cdot \mathbf{P}_{\mu\nu}\end{aligned}$$

$$U(\sigma) = \frac{1}{3!}\kappa\sigma^3 + \frac{1}{4!}\lambda\sigma^4 \quad (1)$$

$$\mathcal{L}_{lep} = \sum_l \bar{\psi}_l [i\gamma^\mu\partial_\mu - m_l] \psi_l \quad (2)$$

$\phi$  - strange vector meson field

	GM1	GM3	NL3
$g_{NN\sigma}$	8.910	8.175	10.217
$g_{NN\omega}$	10.610	8.712	12.868
$g_{NN\phi}$	0.0	0.0	0.0
$g_{NN\rho}$	8.196	8.259	8.948
$\kappa/M_N$	0.005894	0.017318	0.0041014
$\lambda$	-0.006426	-0.014526	-0.015921
$n_0$ ( $fm^{-3}$ )	0.153	0.153	0.148
$M^*/M$	0.70	0.78	0.60
$K$ (MeV)	300	240	272
$S_0$ (MeV)	32.5	32.5	37.4
$L$ (MeV)	94	90	118
$B/A$ (MeV)	-16.3	-16.3	-16.3

MFA  $\rightarrow$  EOS for  $\sigma\omega\rho$  and  $\sigma\omega\rho\phi$  models (GM1LM and GM3LM)

$$U_{N(N)} = g_{NN\omega}\omega_0 - g_{NN\sigma}\sigma_0 \quad U_{Y(N)} = g_{YY\omega}\omega_0 + g_{YY\phi}\phi_0 - g_{YY\sigma}\sigma_0$$

$U_\Lambda = -28$  MeV - well known - the only potential we fix

$g_{BBmeson}$  written in terms of  $g_{NNmeson}$ , which are defined by bulk matter

## Glendenning conjecture (GC)

$$\frac{g_{YY\sigma}}{g_{NN\sigma}} = 0.7, \quad \frac{g_{YY\omega}}{g_{NN\omega}} = \chi_\omega \quad \frac{g_{YY\rho}}{g_{NN\rho}} = \frac{I_{3B}}{I_{3N}}\chi_\rho, \quad (3)$$

the  $\rho$  meson always couples to the isospin projection  $I_3$

the value of  $\chi_\rho$  is completely arbitrary

$$\chi_\omega = \chi_\rho = 0.783 \text{ (GM1)}, 0.8 \text{ (GM3)}, 0.772 \text{ (NL3)}$$

so that  $U_\Lambda = -28 \text{ MeV}$ .

## SU(3) symmetry group approach

We impose that Yukawa type interaction  $\mathcal{L}_{YUK} = -g(\bar{\psi}_B\psi_B)M$  is invariant under SU(3) transformations.

To preserve the unitary symmetry,  $(\bar{\psi}_B\psi_B)$  must transform as:

IR{8} when  $M$  belongs to IR{8},    IR{1} when  $M$  belongs to IR{1}

Speiser method:

$$\mathcal{L}_{YUK} = -(g_1^8 c^1 + g_2^8 c^2)(\bar{\psi}_B\psi_B)M, \quad \text{for the mesons belonging to IR}\{8\}, \quad (4)$$

$$\mathcal{L}_{YUK} = -(g^1)M, \quad \text{for the mesons belonging to IR}\{1\}, \quad (5)$$

$c^1, c^2 =$  SU(3) Clebsch-Gordan (CG) coefficients of the symmetric and antisymmetric coupling respectively.

$$g_8 = [\sqrt{30}/40g_1^8 + (\sqrt{6}/24)g_2^8], \quad \alpha = (\sqrt{6}/24)(g_2^8/g_8), \quad (6)$$

$$\begin{aligned} g_{NN\rho} &= g_{8v}, & g_{\Sigma\Sigma\rho} &= 2g_{8v}\alpha_v, & g_{\Xi\Xi\rho} &= -g_{8v}(1 - 2\alpha_v), \\ g_{NN\omega_8} &= \frac{1}{3}g_{8v}\sqrt{3}(4\alpha_v - 1), & g_{\Sigma\Sigma\omega_8} &= \frac{2}{3}g_{8v}\sqrt{3}(1 - \alpha_v), \\ g_{\Xi\Xi\omega_8} &= -\frac{1}{3}\sqrt{3}g_{8v}(1 + 2\alpha_v), & g_{\Lambda\Lambda\omega_8} &= -\frac{2}{3}g_{8v}\sqrt{3}(1 - \alpha_v), \\ g_{\Lambda\Lambda\rho} &= 0, & g_{NN\phi_1} &= g_{\Sigma\Sigma\phi_1} = g_{\Lambda\Lambda\phi_1} = g_{\Xi\Xi\phi_1} = g_{1v}, \end{aligned} \quad (7)$$

$$\begin{aligned} g_{NN\sigma_8} &= \frac{1}{3}g_{8s}\sqrt{3}(4\alpha_s - 1), & g_{\Sigma\Sigma\sigma_8} &= \frac{2}{3}g_{8s}\sqrt{3}(1 - \alpha_s), \\ g_{\Xi\Xi\sigma_8} &= -\frac{1}{3}\sqrt{3}g_{8s}(1 + 2\alpha_s), & g_{\Lambda\Lambda\sigma_8} &= -\frac{2}{3}g_{8s}\sqrt{3}(1 - \alpha_s), \\ g_{NN\sigma_1} &= g_{\Sigma\Sigma\sigma_1} = g_{\Lambda\Lambda\sigma_1} = g_{\Xi\Xi\sigma_1} = g_{1s}, \end{aligned} \quad (8)$$



**C.Dover, A.Gal: Prog. Part. Nucl. Phys. 12, 171 (1984)**

The observed  $\omega$  and  $\phi$  mesons are not the theoretical  $\omega_8$  and  $\phi_1$  ones, but a mixture of them:

$$\begin{aligned}g_{NN\omega} &= \cos \theta_v g_{1v} + \sin \theta_v \frac{1}{3} \sqrt{3} g_{8v} (4\alpha_v - 1), \\g_{\Sigma\Sigma\omega} &= \cos \theta_v g_{1v} + \sin \theta_v \frac{2}{3} \sqrt{3} g_{8v} (1 - \alpha_v), \\g_{\Lambda\Lambda\omega} &= \cos \theta_v g_{1v} - \sin \theta_v \frac{2}{3} \sqrt{3} g_{8v} (1 - \alpha_v), \\g_{\Xi\Xi\omega} &= \cos \theta_v g_{1v} - \sin \theta_v \frac{2}{3} \sqrt{3} g_{8v} (1 + 2\alpha_v).\end{aligned}\tag{9}$$

- We are left with 6 free parameters:  $z_v = (g_{8v}/g_{1v})$ ,  $\theta_v$ ,  $\alpha_v$ ,  
 $z_s = (g_{8s}/g_{1s})$ ,  $\theta_s$  and  $\alpha_s$ .
- For the vector mesons, we use the hybrid SU(6) symmetry group:  
 $z_v = \sqrt{6}$ ,  $\theta_v = 35.264$ ,
- For the scalar mesons, we consider  $\theta_s = 35.254$  (ideal mixing approximation),  
 $z_s = \frac{8}{9}\sqrt{6}$  (near SU(6) symmetry)
- We relate the two left parameters  $\alpha_s$  to  $\alpha_v$  forcing the  $U_\Lambda$  potential to be equal to -28 MeV

$$\frac{g_{\Lambda\Lambda\sigma}}{g_{NN\sigma}} = \frac{10 + 6\alpha_s}{13 + 12\alpha_s}, \quad \frac{g_{\Sigma\Sigma\sigma}}{g_{NN\sigma}} = \frac{22 - 6\alpha_s}{13 + 12\alpha_s}, \quad \frac{g_{\Xi\Xi\sigma}}{g_{NN\sigma}} = \frac{13 - 6\alpha_s}{13 + 12\alpha_s}. \quad (10)$$

$$\frac{g_{\Lambda\Lambda\omega}}{g_{NN\omega}} = \frac{4 + 2\alpha_v}{5 + 4\alpha_v}, \quad \frac{g_{\Sigma\Sigma\omega}}{g_{NN\omega}} = \frac{8 - 2\alpha_v}{5 + 4\alpha_v}, \quad \frac{g_{\Xi\Xi\omega}}{g_{NN\omega}} = \frac{5 - 2\alpha_v}{5 + 4\alpha_v}, \quad (11)$$

$$\frac{g_{\Sigma\Sigma\rho}}{g_{NN\rho}} = 2\alpha_v, \quad \frac{g_{\Xi\Xi\rho}}{g_{NN\rho}} = -(1 - 2\alpha_v), \quad \frac{g_{\Lambda\Lambda\rho}}{g_{NN\rho}} = 0, \quad (12)$$

$$\begin{aligned} \frac{g_{NN\phi}}{g_{NN\omega}} &= \sqrt{2} \cdot \left( \frac{4\alpha_v - 4}{5 + 4\alpha_v} \right), & \frac{g_{\Lambda\Lambda\phi}}{g_{NN\omega}} &= \sqrt{2} \cdot \left( \frac{2\alpha_v - 5}{5 + 4\alpha_v} \right), \\ \frac{g_{\Sigma\Sigma\phi}}{g_{\Lambda\Lambda\omega}} &= \sqrt{2} \cdot \left( \frac{-2\alpha_v - 1}{5 + 4\alpha_v} \right), & \frac{g_{\Xi\Xi\phi}}{g_{NN\omega}} &= \sqrt{2} \cdot \left( \frac{-2\alpha_v - 4}{5 + 4\alpha_v} \right), \end{aligned} \quad (13)$$

$\alpha_v = 1$ , we recover the SU(6) parametrization for the vector mesons (the  $\omega$  meson couples to hypercharge, the  $\rho$  meson couples to isospin)

$\alpha \neq 1$ ,  $\phi$  meson couples to the nucleon in the  $\sigma\omega\rho\phi$  model.

We calculate  $\alpha_s$  such that  $U_\Lambda = -28$  MeV

We give arbitrary values to  $\alpha_v$  varying from 1 to 0

$U_\Sigma$  and  $U_\Xi$  are determined only by symmetry properties

$\alpha_v = 1 = \text{SU}(6)$  usual choice of couplings

$\alpha_v = 1$	$\alpha_s = 1.568$	$g_{NN\omega} = 10.610$	$U_\Sigma = +32$	$U_\Xi = +40$
$\alpha_v = 0.75$	$\alpha_s = 1.251$	$g_{NN\omega} = 10.610$	$U_\Sigma = +29$	$U_\Xi = +39$
$\alpha_v = 0.50$	$\alpha_s = 0.9007$	$g_{NN\omega} = 10.610$	$U_\Sigma = +19$	$U_\Xi = +33$
$\alpha_v = 0.25$	$\alpha_s = 0.5230$	$g_{NN\omega} = 10.610$	$U_\Sigma = +11$	$U_\Xi = +29$
$\alpha_v = 0.0$	$\alpha_s = 0.2859$	$g_{NN\omega} = 10.610$	$U_\Sigma = -2$	$U_\Xi = +22$
GC	-	$g_{NN\omega} = 10.610$	$U_\Sigma = -28$	$U_\Xi = -28$

Family of parametrizations and hyperon potential depths for GM1

$\alpha_v = 1$	$\alpha_s = 1.678$	$g_{NN\omega} = 8.712$	$U_{\Sigma} = +22$	$U_{\Xi} = +29$
$\alpha_v = 0.75$	$\alpha_s = 1.345$	$g_{NN\omega} = 8.712$	$U_{\Sigma} = +19$	$U_{\Xi} = +28$
$\alpha_v = 0.50$	$\alpha_s = 1.012$	$g_{NN\omega} = 8.712$	$U_{\Sigma} = +19$	$U_{\Xi} = +33$
$\alpha_v = 0.25$	$\alpha_s = 0.6889$	$g_{NN\omega} = 8.712$	$U_{\Sigma} = +8$	$U_{\Xi} = +23$
$\alpha_v = 0.0$	$\alpha_s = 0.3763$	$g_{NN\omega} = 8.712$	$U_{\Sigma} = +0.2$	$U_{\Xi} = +18$
GC	-	$g_{NN\omega} = 8.712$	$U_{\Sigma} = -28$	$U_{\Xi} = -28$

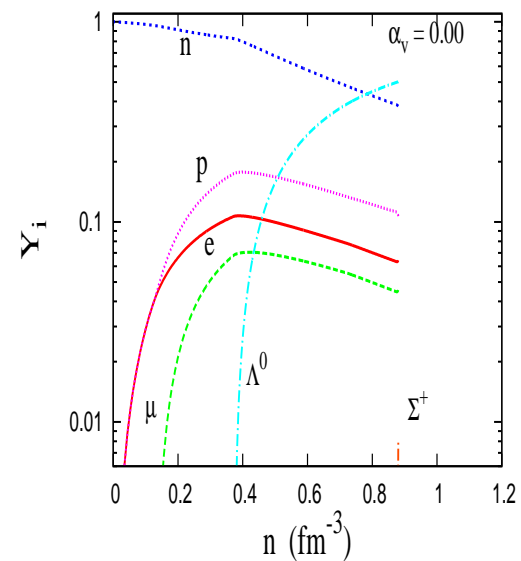
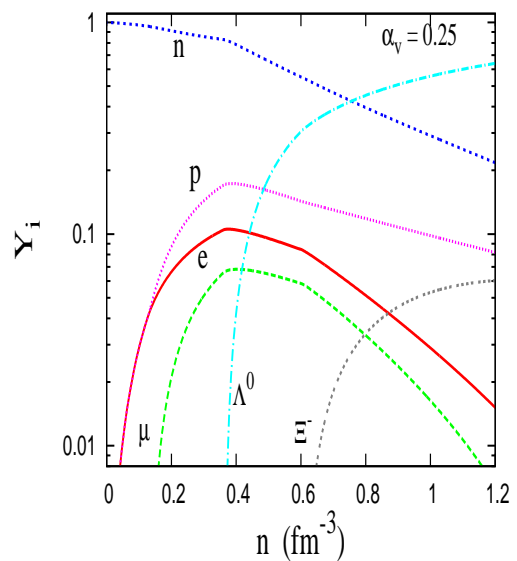
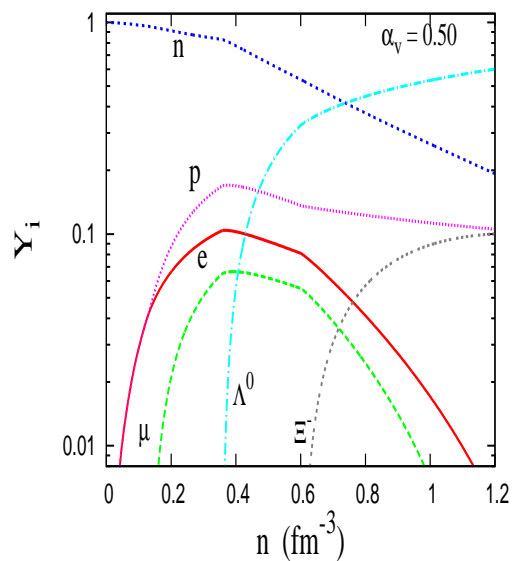
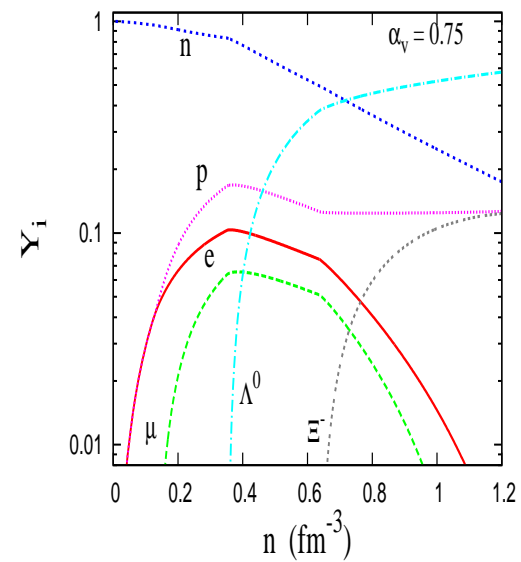
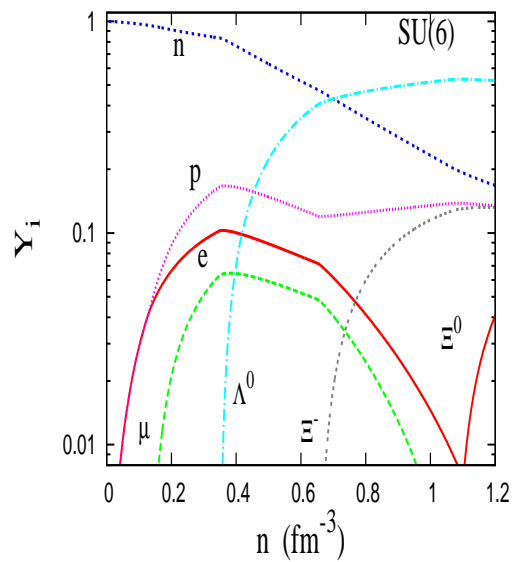
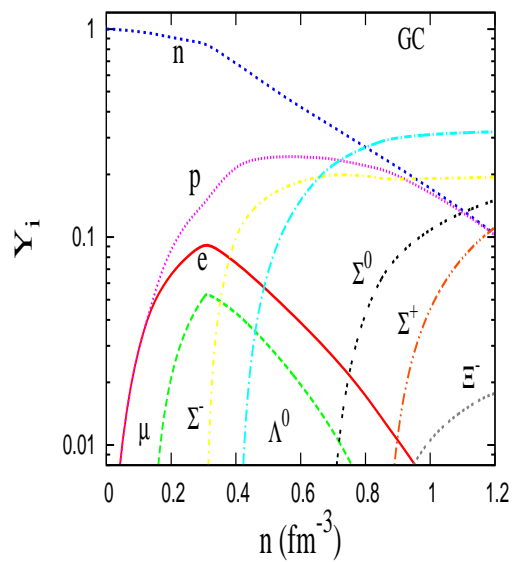
GM3 (top)      GM1LM (bottom)

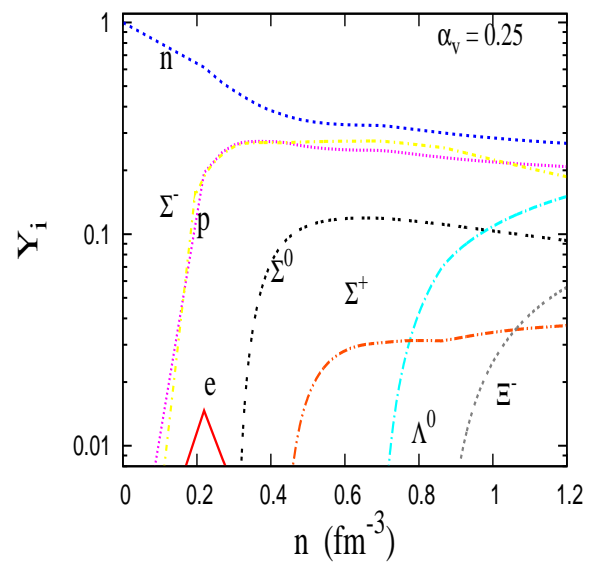
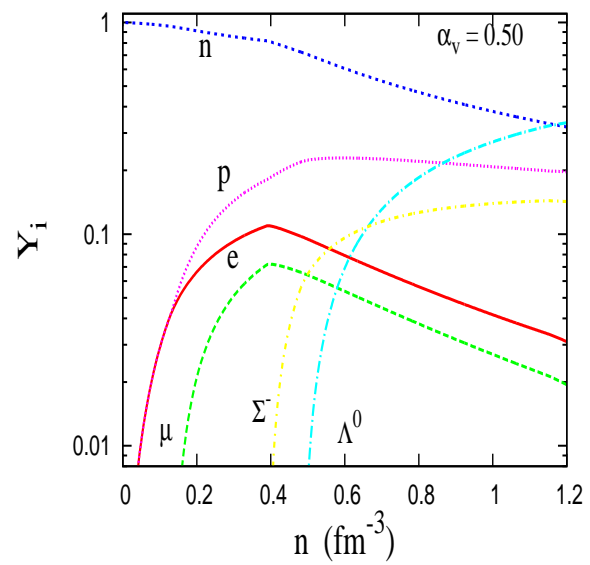
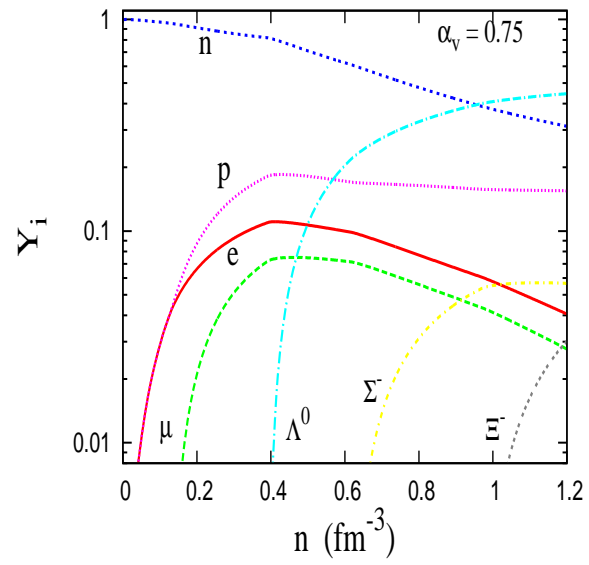
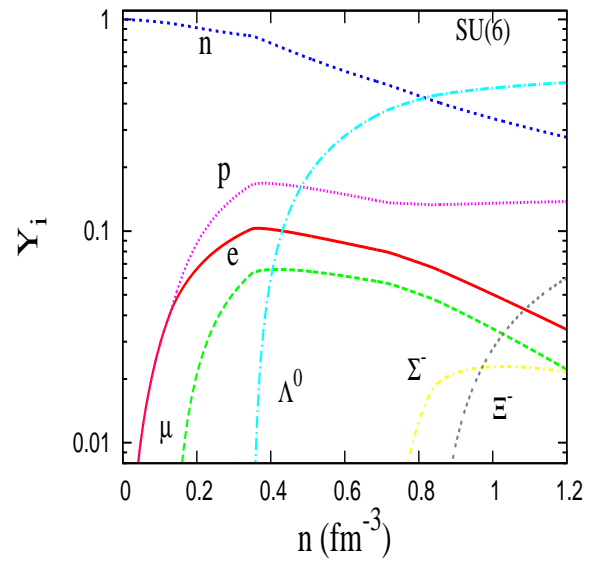
$\alpha_v = 1$	$\alpha_s = 1.568$	$g_{NN\omega} = 10.610$	$U_{\Sigma} = +32$	$U_{\Xi} = +40$
$\alpha_v = 0.75$	$\alpha_s = 1.231$	$g_{NN\omega} = 10.514$	$U_{\Sigma} = +25$	$U_{\Xi} = +39$
$\alpha_v = 0.50$	$\alpha_s = 0.8229$	$g_{NN\omega} = 10.133$	$U_{\Sigma} = -5$	$U_{\Xi} = +32$
$\alpha_v = 0.25$	$\alpha_s = 0.6889$	$g_{NN\omega} = 9.324$	$U_{\Sigma} = -157$	$U_{\Xi} = -40$

GM3LM

$\alpha_v = 1$	$\alpha_s = 1.678$	$g_{NN\omega} = 8.712$	$U_{\Sigma} = +22$	$U_{\Xi} = +29$
$\alpha_v = 0.75$	$\alpha_s = 1.367$	$g_{NN\omega} = 8.633$	$U_{\Sigma} = +20$	$U_{\Xi} = +31$
$\alpha_v = 0.50$	$\alpha_s = 0.9281$	$g_{NN\omega} = 8.320$	$U_{\Sigma} = -2$	$U_{\Xi} = +25$
$\alpha_v = 0.25$	$\alpha_s = 0.2775$	$g_{NN\omega} = 7.182$	$U_{\Sigma} = -104$	$U_{\Xi} = -23$

# Results - GM1

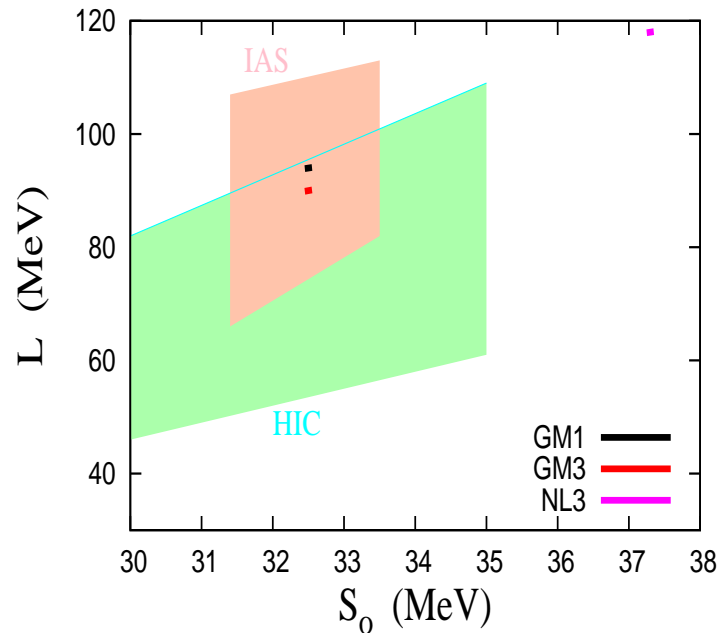




GM1LM

# Constraints

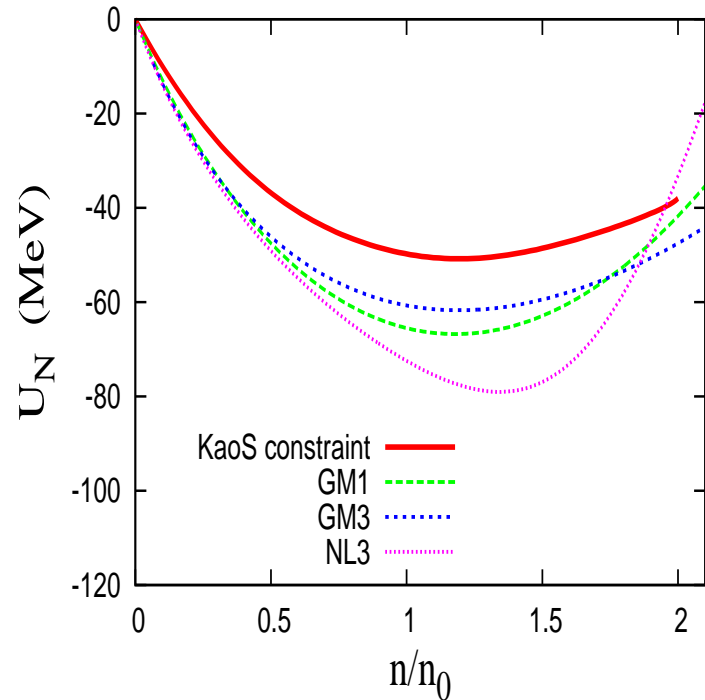
$$S(n) = S_0 - L\epsilon + \frac{1}{2}K_{sym}\epsilon^2 + O(\epsilon^3), \quad \epsilon = \frac{n_0 - n}{3n_0}, \quad L = 3n_0 \left. \frac{dS}{dn} \right|_{n_0} \quad (14)$$



**EC1:**  $S_0$  and  $L$  experimental values obtained for HIC and isobaric analog states (IAS)

**M.B.Tsang et al: Phys. Rev. C 86, 015803 (2012)**

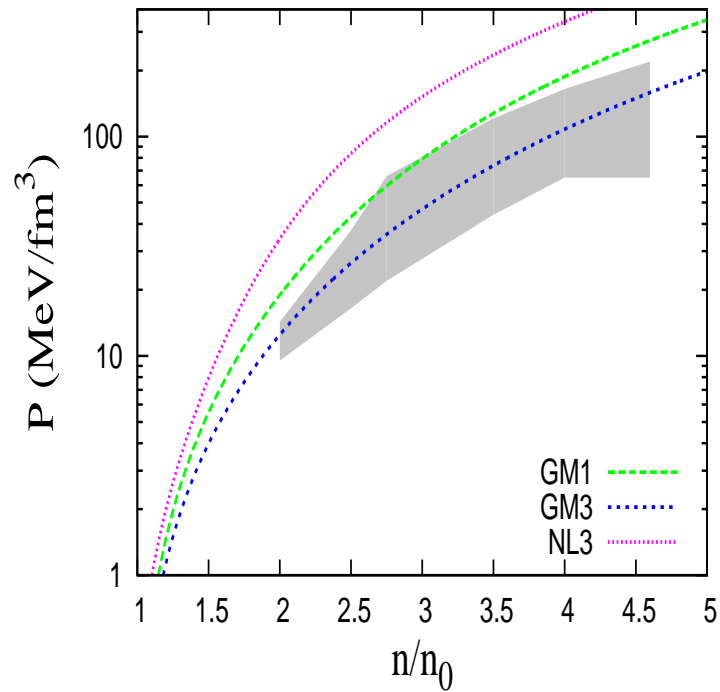




**EC2:** Experimental value of  $U_N$  from kaon production up to two times the saturation point ( $n_0 = 0.153 \text{ fm}^{-3}$ )

**I.Sagert et al: Phys. Rev. C 86, 045802 (2012)**

$$U_N = g_{NN\omega}\omega_0 - g_{NN\sigma}\sigma_0$$



**EC3:** Experimental determination of the pressure in symmetric nuclear matter

**P.Danielewicz et al: Science 298, 1592 (2002)**

**Experimental constraints point towards a soft EOS at low densities**

**PSR J1614-2230 and PSR J0348+0432 indicate that the EoS has to be stiff enough to reproduce  $2 M_{\odot}$  neutron stars**

**AC1:** mass of the PSR J0348+0432 -  $2.01 \pm 0.04 M_{\odot}$

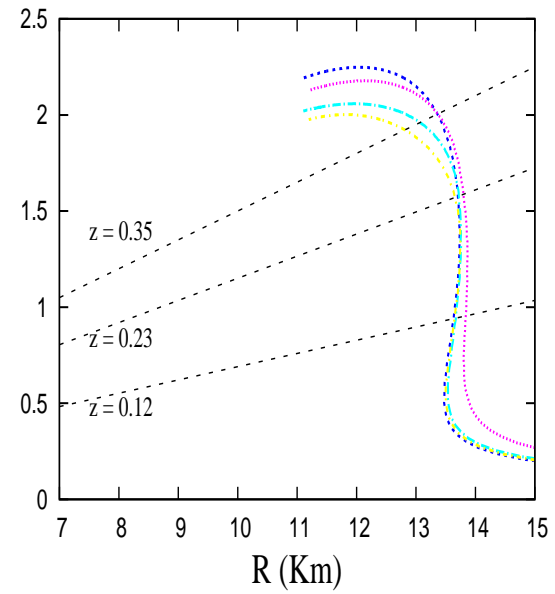
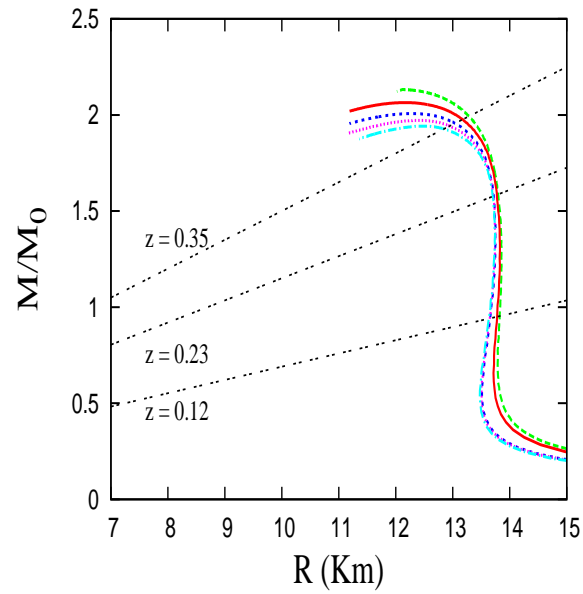
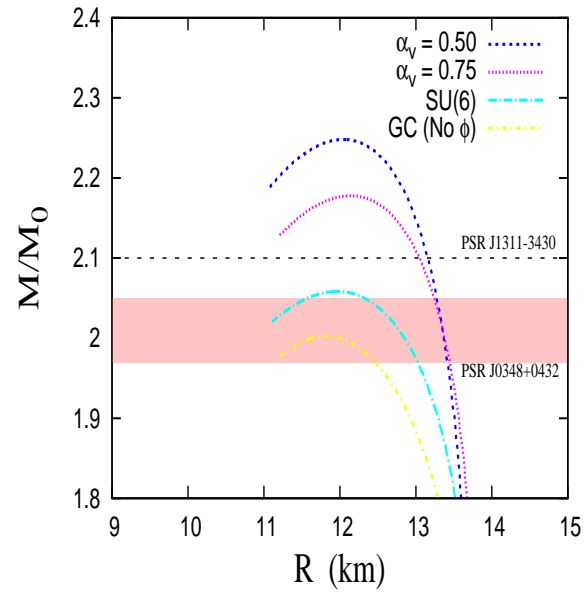
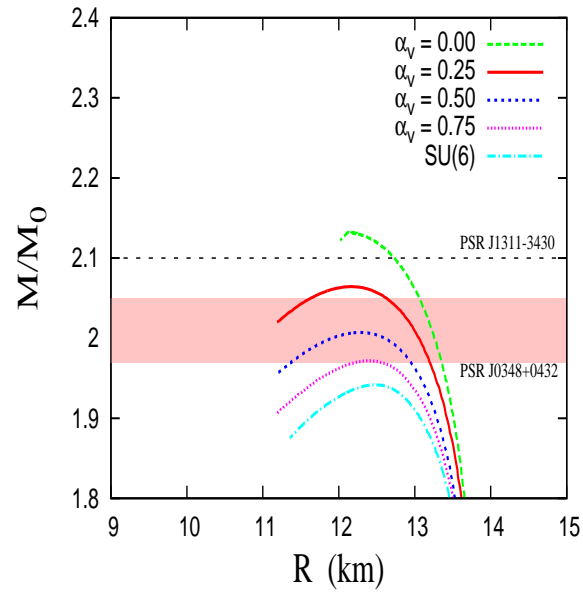
**J.Antoniadis et al: Science 340, 1233232 (2013)**

**AC2:** the redshift measurements ( $z$ ) of two neutron stars:  $z = 0.35$  from EXO0748-676 - corresponds to  $M/R = 0.15 M_{\odot}/km$

**J.Cottam, F.Paerels, M.Mendez, Nature 420, 51 (2002)**

$z = 0.12$  to  $z = 0.23$  from 1E 1207.4-5209 - correspond to  $M/R = 0.069 M_{\odot}/km$  to  $M/R = 0.115 M_{\odot}/km$

**D.Sanwal et al: Astrophys. J. Lett. 574, 61 (2002)**



GM1 (left) - GM1LM (right)- AC1 (above) - AC2 (below)

**AC3:** from chiral effective theory, the radii of the canonical  $1.4M_{\odot}$  neutron star - 9.7-13.9 *Km*

**K.Hebeler et al: Phys. Rev. Lett. 105, 161102 (2010)**

Model	Min./Max. radii ( $1.4M_{\odot}$ ).	AC3
GM1,GM1LM	13.73/13.85 Km	OK
GM3,GM3LM	13.06/13.14 Km	OK
NL3	14.71 Km	Failed

Minimum and maximum radii of the canonical  $1.4M_{\odot}$  for different models and parametrizations

Model	$\alpha_v$	EC1	EC2	EC3	AC1	AC2	AC3
GM1	No hyp.	OK	OK	Failed	OK	OK	OK
GM1	SU(6)	OK	OK	-	Failed	OK	OK
GM1	0.75	OK	OK	-	OK	OK	OK
GM1	0.50	OK	OK	-	OK	OK	OK
GM1	0.25	OK	OK	-	OK	OK	OK
GM1	0.0	OK	OK	-	OK	OK	OK
GM1	GC	OK	OK	-	OK	OK	OK
GM1LM	SU(6)	OK	OK	-	OK	OK	OK
GM1LM	0.75	OK	OK	-	OK	OK	OK
GM1LM	0.50	OK	OK	-	OK	OK	OK
GM3	No hyp.	OK	OK	OK	OK	OK	OK
GM3	All/GC	OK	OK	-	Failed	OK	OK
GM3LM	All	OK	OK	-	Failed	OK	OK
NL3	No hyp.	Failed	Failed	Failed	OK	OK	Failed

# Conclusions

- When the strange vector potential is analysed, we see that less repulsive potentials produce stiffer EoS. This is due to the fact that the vector meson channel dominates at high densities (the hyperon potentials play only a secondary role).
- NL3 - describes nuclear matter properties very well, fails to describe dense asymmetric matter.
- Although we can predict very massive hyperonic stars with many of the investigated parametrizations, a maximum mass of  $2.06 M_{\odot}$  arises in order to describe the soft EoS at low densities regime.
- This constraint prevents us from explaining the mass of the speculative PSR J1311-3430 ( $2.7 M_{\odot}$ ) **R.Romani et al: *Astrophys. J. Lett.* 760, L36 (2012)** - our largest possible mass is  $2.25 M_{\odot}$  (GM1LM,  $\alpha_v = 0.5$ ).

- GM1 is a good parametrization to describe nuclear properties, this implies that the hyperon production is not only possible, but necessary to soften the EoS and reconcile theory and experience.
- Neutron stars radii are correlated with the symmetry energy slope: the lower the slope, the smaller the radius. The  $\omega - \rho$  interaction present in the FSU and IUFSU models can be added and its strength adjusted so that the symmetry energy slope can be reduced.
- FSU and IUFSU (low slopes) give low values for the radii of  $1.4 M_{\odot}$  stars but the maximum masses are too low.





Luiz L. Lopes and Debora P. Menezes, Phys. Rev. C 89, 025805 (2014); arXiv:1309.4173[nucl-th]

**Thank you**

To make sure that the nuclear matter properties are not affected, we reparametrize the  $g_{NN\omega}$ :

$$g_{NN\omega}\omega_0 \rightarrow \tilde{g}_{NN\omega}\omega_0 + g_{NN\phi}\phi, \quad (15)$$

left side is related to the  $\sigma\omega\phi$  model, right side to the  $\sigma\omega\rho\phi$  model

$$g_{NN\omega} \sum_B \frac{g_{BB\omega}}{m_v^2} n_B \equiv \tilde{g}_{NN\omega} \sum_B \frac{\tilde{g}_{BB\omega}}{m_v^2} n_B + g_{NN\phi} \sum_B \frac{g_{BB\phi}}{m_\phi^2} n_B. \quad (16)$$

$$\frac{g_{NN\omega}^2}{m_v^2} \equiv \frac{\tilde{g}_{NN\omega}^2}{m_v^2} + 2 \left( \frac{4\alpha_v - 4}{5 + 4\alpha_v} \right)^2 \frac{\tilde{g}_{NN\omega}^2}{m_\phi^2}. \quad (17)$$