Linkup of non-rotating neutron-star and outer-Schwarzschild metrics

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Introduction

- neutron stars (NSs) – compact objects with the metrics significantly departed from the flat, Euclidean
- inside the NS body, the description of metrics (metric tensor) can be derived from the model of NS internal structure
- the metrics of neighbouring empty space is described by the outer Schwarzschild (SCH) metrics in the case of non-rotating NS
- both NS and outer-space metrics should be linked up in the surface of NS
- i.e. the components of metric tensor should be the continuous functions of radial distance, r, (the non-rotating object)
- the linkup appears to be a non-trivial problem, we deal with in this talk

Continuous linkup: requirements

- ${}^{N}g_{ij}$ – metric tensor in the NS body
- ${}^{e}g_{ij}$ – metric tensor in the outer empty space
- mathematically, the linkup, in the distance of the outer radius, R_{out}, means:

$$
\begin{aligned}
{}^{N}g_{ij}(R_{out}) &= {}^{e}g_{ij}(R_{out}), \\
\left[\frac{d\left({}^{N}g_{ij}\right)}{dr}\right]_{r=R_{out}} &= \left[\frac{d\left({}^{e}g_{ij}\right)}{dr}\right]_{r=R_{out}},
\end{aligned}
$$

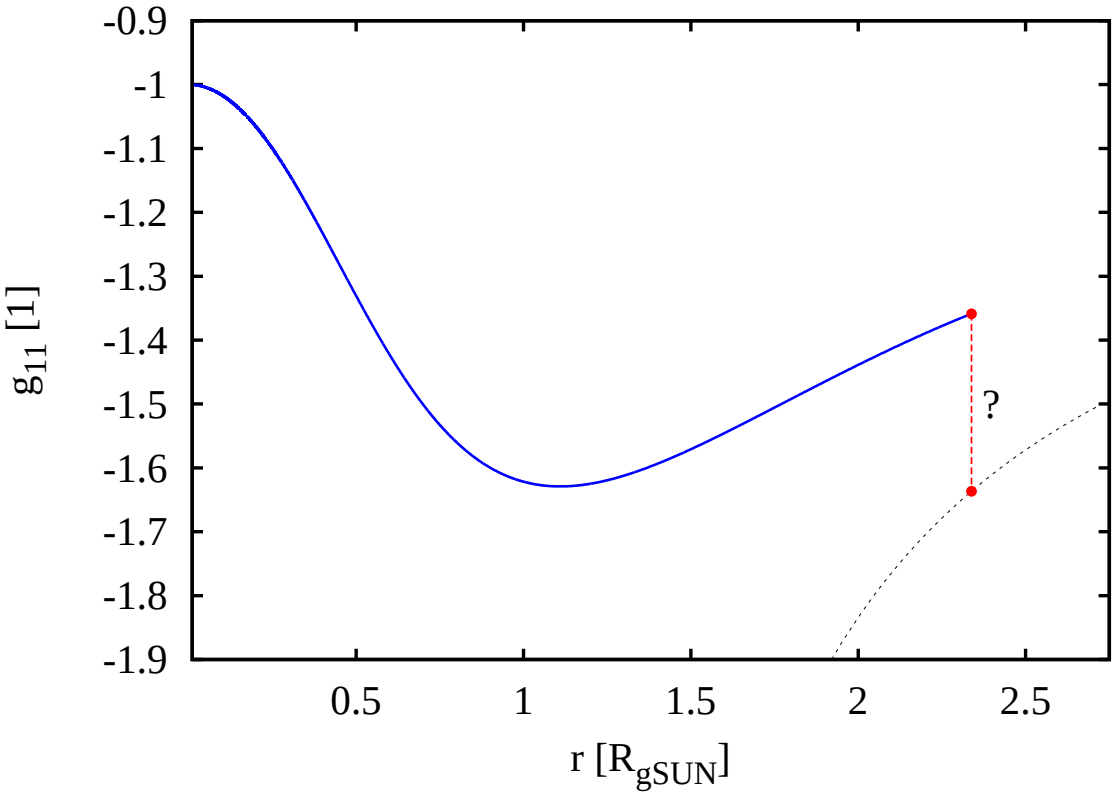
- for every combination of i and j
- since we are going to deal with a new problem, it is, methodically, reasonable to consider the simplest model of NS
- so, we consider the traditional Oppenheimer-Volkoff model of non-rotating NS consisting solely of cold Fermi-Dirac neutron gas (i.e. the mono-atomic gas)

Continuous linkup: requirements

- in the case of spherical symmetry: $g_{ij} = 0$ for $i \neq j$
- as well, ${}^{N}g_{22} =^{e} g_{22}$ and ${}^{N}g_{33} =^{e} g_{33}$
- therefore, we need to deal only with the linkup of radial component g_{11} and time component g_{44}
- there were established the auxiliary functions λ and ν related to g_{11} and g_{44} as $g_{11} = -\exp(\lambda)$ and $g_{44} = \exp(\nu)$
- by Oppenheimer and Volkoff, we replace λ with u according to relation $u = r(1 - e^{-\lambda})/2$
- a model of NS structure and its mass can be obtained via numerical integration of appropriate equations
- the input values of Fermi impulse, p_{Fo}, and values u_o and ν_o of functions u and ν have to be given for the starting radial distance r_o

Example of ad hoc linkup

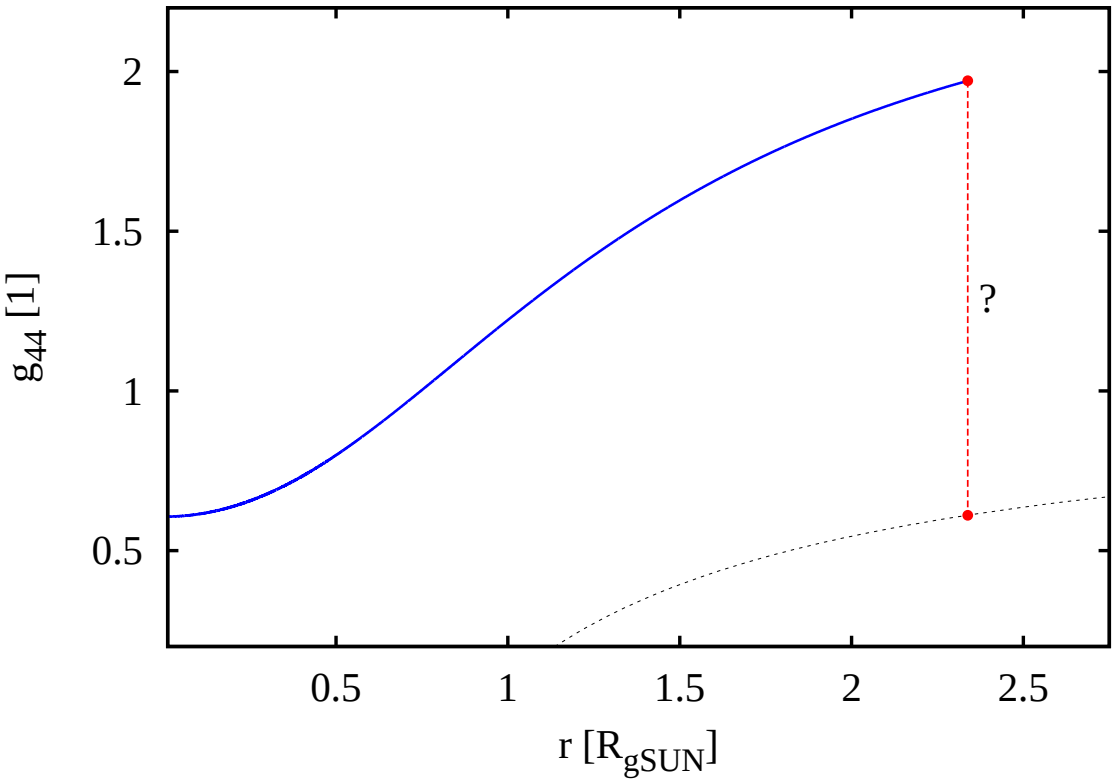
g_{11}:



...a displacement of ${}^{N}g_{11}$ and ${}^{e}g_{11}$ in R_{out}

Example of ad hoc linkup

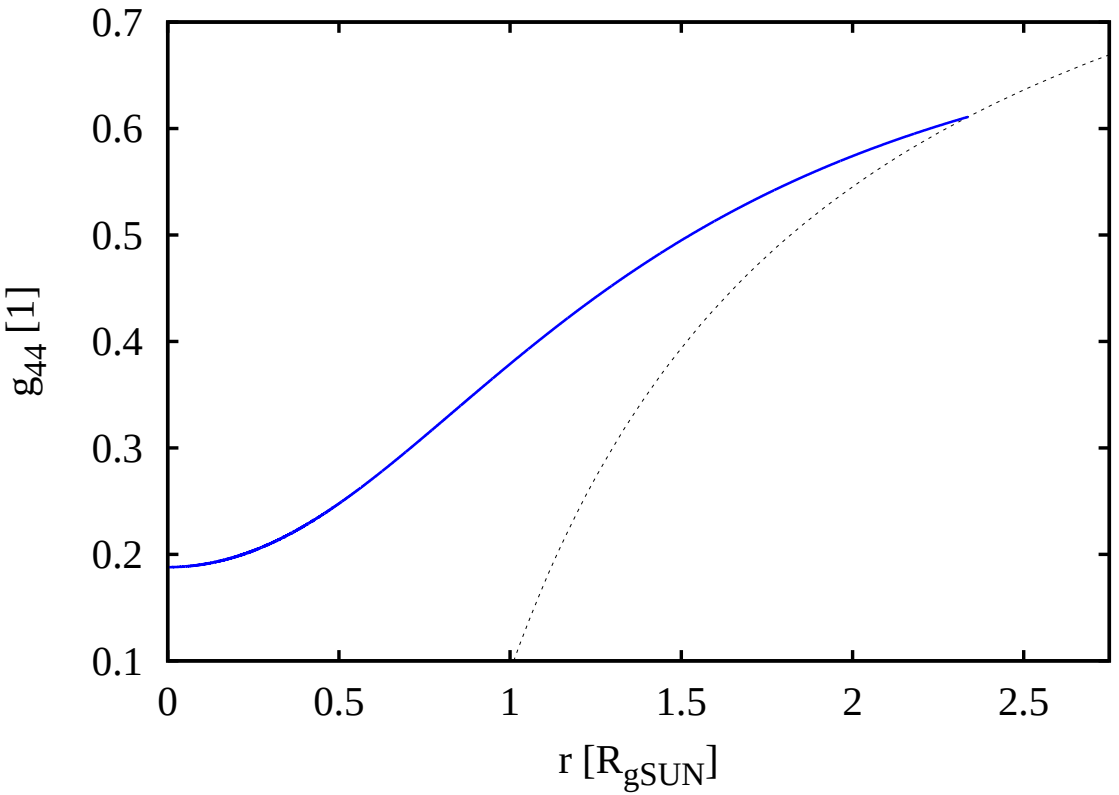
g_{44}:



...a displacement of ${}^{N}g_{11}$ and ${}^{e}g_{11}$ in R_{out}

Example of ad hoc linkup

g_{44} with the displacement removed setting the appropriate ν_o (via an iteration):

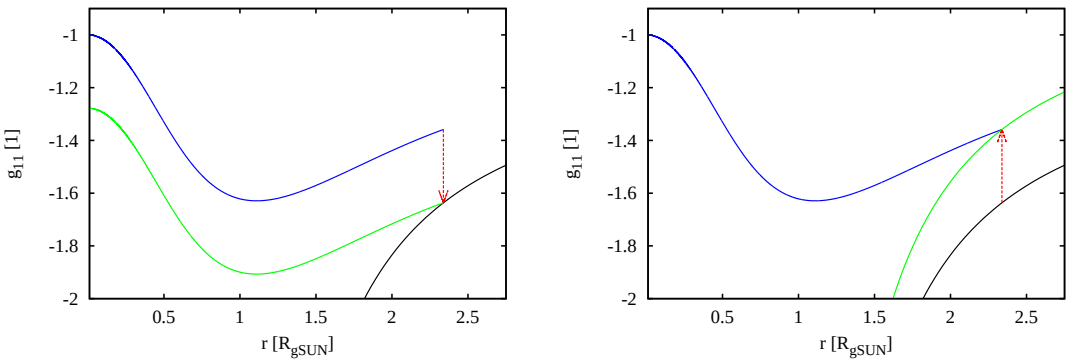


...but the derivatives of ${}^{N}g_{11}$ and ${}^{e}g_{11}$ in R_{out} still do not equal

Possibilities of continuous linkup

removing the displacement in g_{11}:

- (A) a suitable linkup can occur using **other equation of state** than was used in the Oppenheimer-Volkoff problem – improbable
- (B) behavior of ${}^{e}g_{11}$ (curve) can be „lifted” making an **alternative gauging** of the outer SCH solution of EFEs; this gauging implies the speed limit larger than c – problem
- (C) behavior of ${}^{N}g_{11}$ (right end of curve) can be modified to touch ${}^{e}g_{11}$ in R_{out} by modifying some constants describing the **spacetime** inside the NS
- in this contribution, possibility (C) is presented/discussed

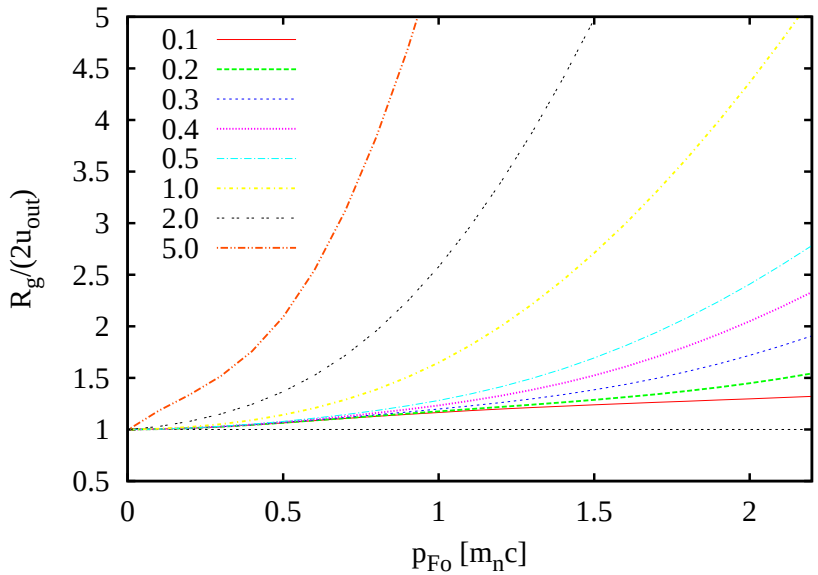


Search for the suitable p_{Fo} and u_o

- the displacement in function $g_{44} = g_{44}(r)$ between its parts for the NS and outer space was solved finding a suitable input constant ν_o
- can we solve also the analogous displacement in function $g_{11} = g_{11}(r)$ finding the suitable values of the other input constants, i.e. p_{Fo} and u_o?
- from the numerical integration, we obtain $u(R_{out}) \equiv u_{out}$ as well as the mass of the NS, M, which implies the Schwarzschild gravitational radius, R_g; specifically $R_g = 2GM/c^2$
- ${}^N g_{11}$ in $r = R_{out}$ written with the help of function u is ${}^N g_{11} = -1/(1 - 2u_{out}/R_{out})$ and corresponding outer SCH solution is ${}^e g_{11} = -1/(1 - R_g/R_{out})$
- both functions are equal if $2u_{out} = R_g$ or ratio $R_g/(2u_{out}) = 1$

Search for the suitable p_{Fo} and u_o

when we construct a set of models for a grid of all conventional input values and r_o (another free parameter)...



...for $p_{Fo} > 0$, always $R_g/(2u_{out}) > 1$; it means that the continuous linkup within the traditional concept is impossible, in principle

C: Alternative gauging of metrics

- the EFEs are the differential equations, which imply some integration constants
- one constant occurs in process of the derivation of outer SCH solution, where one gains equality $d\lambda/dr = -d\nu/dr$
- it yields $\lambda = K_\nu - \nu$, where K_ν is the integration constant
- it can be re-written with the help of constant K_c defined as $K_c^2 = \exp(K_\nu)$ and, then, the line element of outer SCH metrics is

$$ds^2 = -e^\lambda dr^2 - r^2 d\vartheta^2 - r^2 \sin^2\vartheta\, d\varphi^2 + e^\nu (K_c c)^2 dt^2,$$

- where we used, in purpose, the SI units

C: Alternative gauging of outer SCH solution

- if the speed limit acquires the universal value, c, then $K_c = 1$; otherwise $K_c \neq 1$
- to achieve the continuous linkup, we will assume that the velocity of light inside a material object (NS), c_m, is smaller than c
- hence, we have:

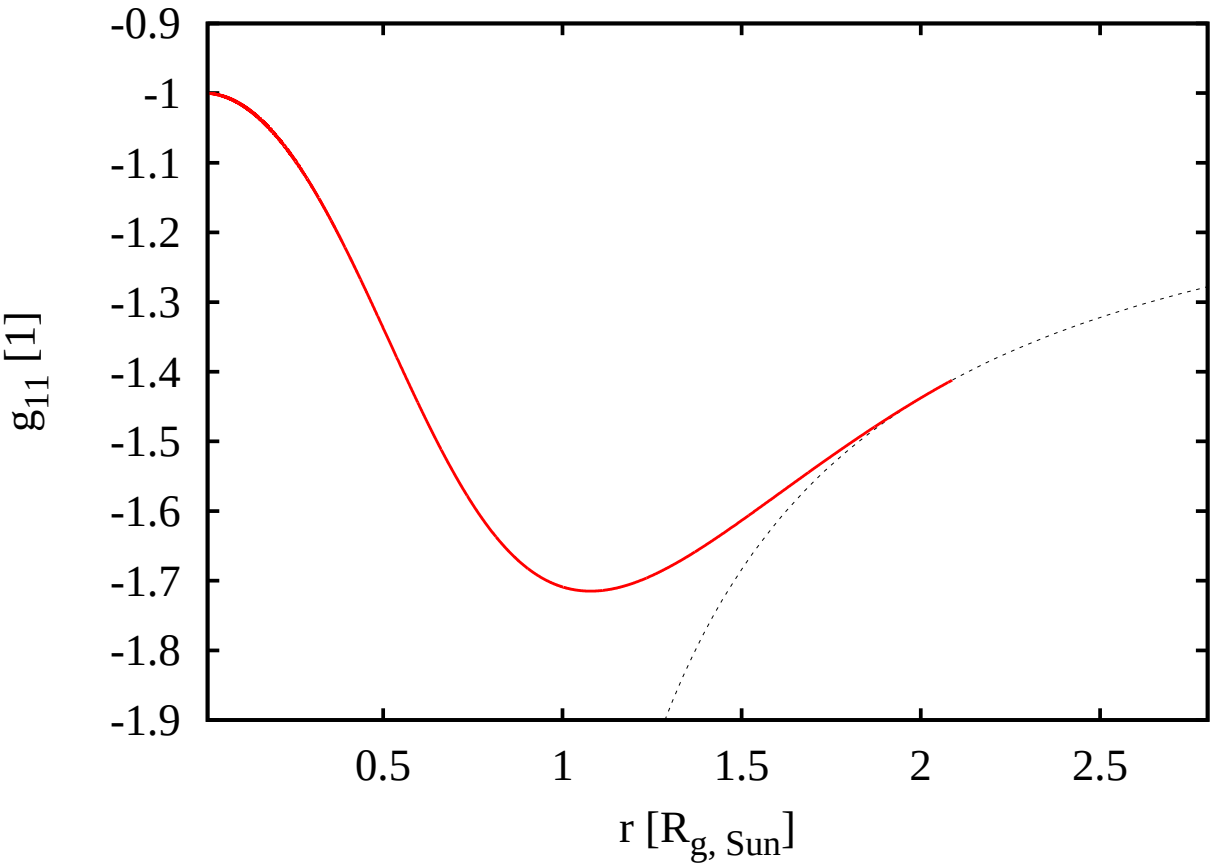
$$\text{es}: \qquad ds^2 = -e^{\lambda}\,dr^2 - r^2\,d\vartheta^2 - r^2\sin^2\vartheta\,d\varphi^2 + e^{\nu}(K_e c)^2 dt^2,$$

$$\text{NS}: \qquad ds^2 = -e^{\lambda}\,dr^2 - r^2\,d\vartheta^2 - r^2\sin^2\vartheta\,d\varphi^2 + e^{\nu}(K_N c)^2 dt^2,$$

- where $K_e = 1$ and $K_N < 1$
- the appropriate value of constant K_N, relating c_m and c as $c_m = K_N c$, is found in the iteration

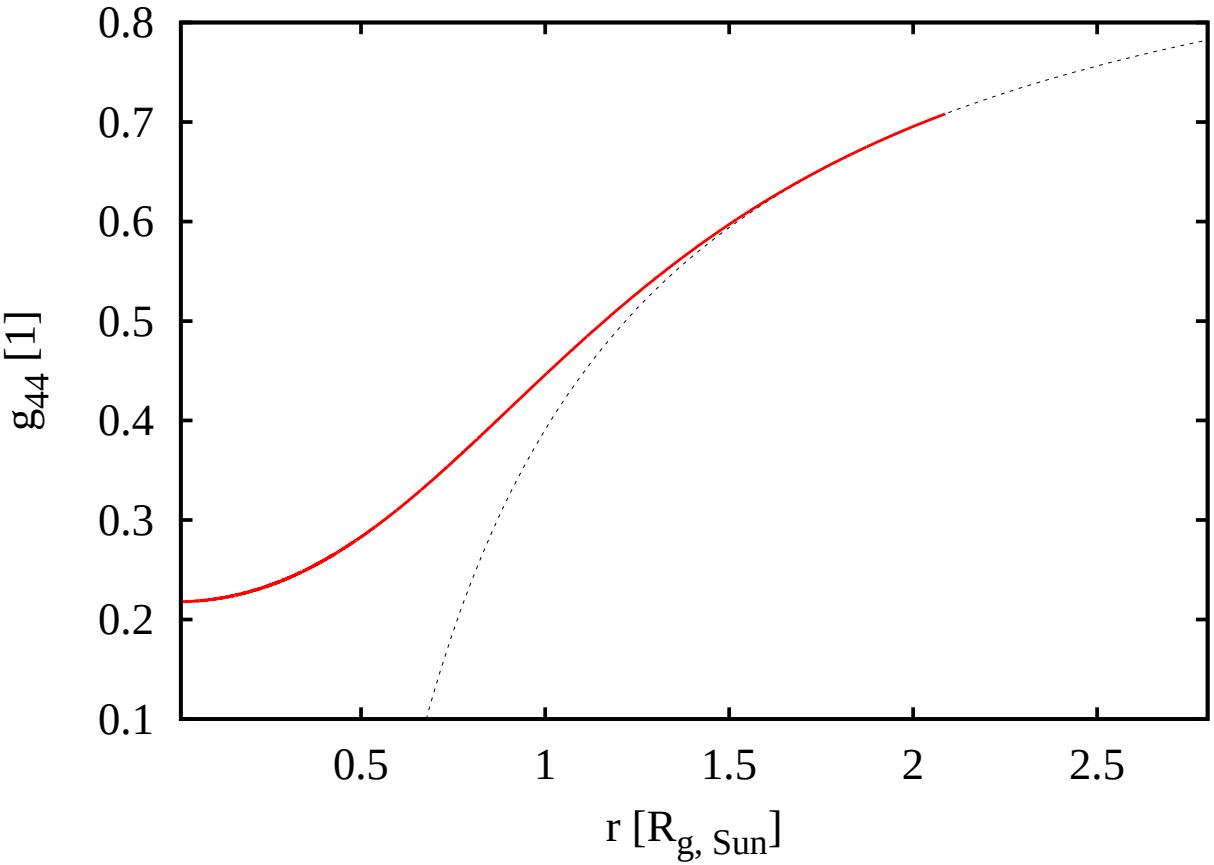
C: Example of continuous linkup in R_{out}

g_{11}:



C: Example of continuous linkup in R_{out}

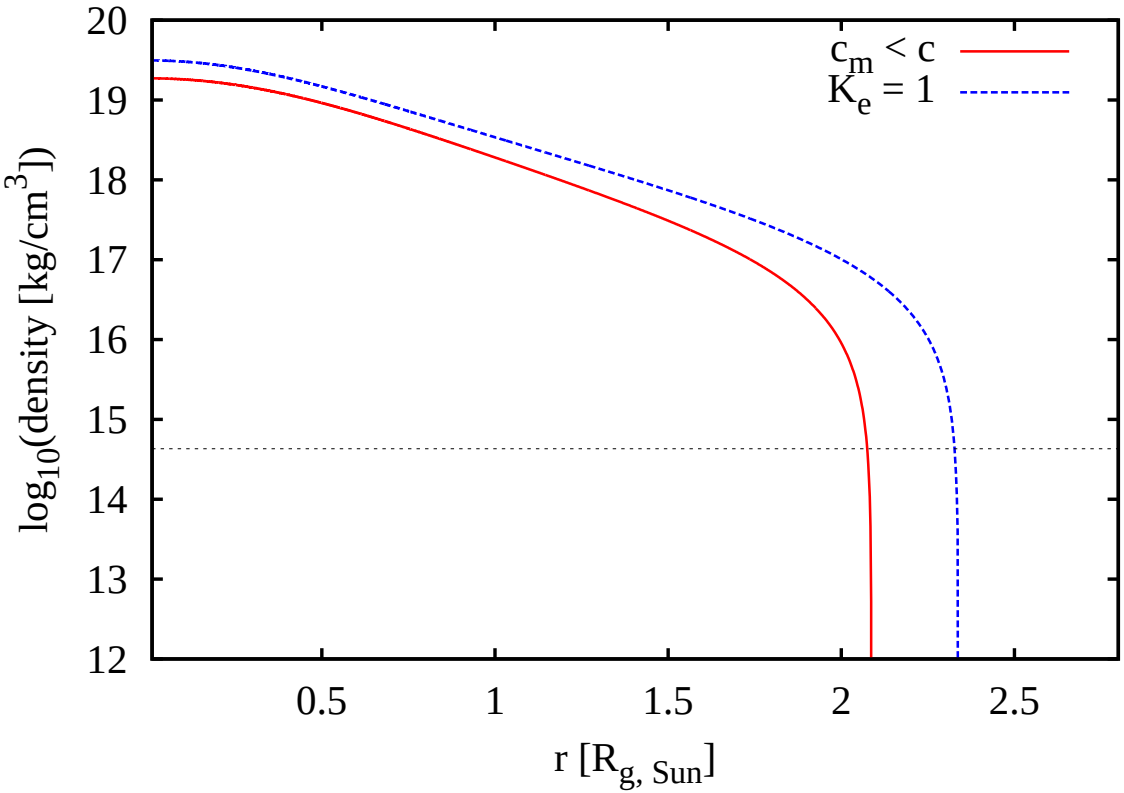
g_{44}:



...intriguing fact: also the corresponding derivatives appear to be equal

C: Example of continuous linkup in R_{out}

density (E/c_m^2):



$M(B) = 0.910\,\mathrm{M}_\odot$; $M(C) = 0.609\,\mathrm{M}_\odot$

Conclusion remarks

- in any realistic model of NS, it is necessary to pay a special attention to the metrics of NS: it must be the continuous function of all coordinates from the center of NS sphere up to the adjacent outer empty space
- the plausible way to achieve the continuity of metrics, at least in the case of non-rotating object, is the assumption of the velocity of light inside the NS body smaller than that in vacuum