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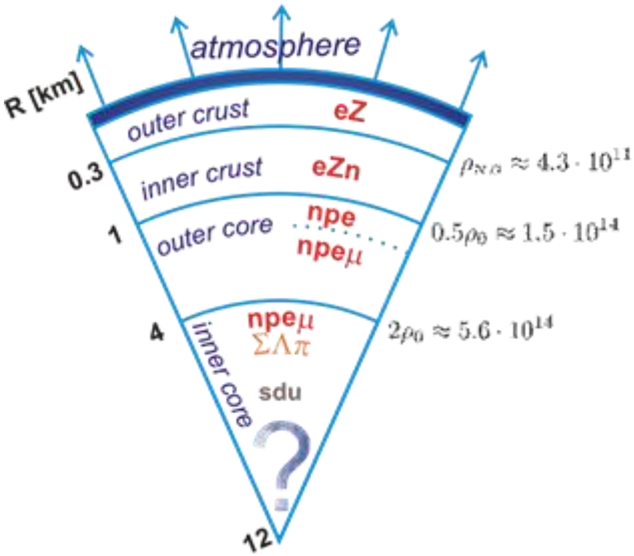
Transport coefficients of nuclear matter in neutron star cores in the BHF framework

PRC **88**, 065803 (2013)

The Structure and Signals of Neutron Stars, from Birth to Death
Firenze, Italy, 24-28 March, 2014

Introduction. Kinetic coefficients in NS cores



Non-superfluid beta-stable npeμ matter

Kinetic coefficients due to particle collisions

$$\kappa = \kappa_{e\mu}[ee, e\mu, ep] + \kappa_n[nn, np]$$

$$\kappa_{\rm p} \ll \kappa_{\rm n}$$

Electromagnetic part: $\kappa_{e\mu},\ \eta_{e\mu}$

Shternin, Yakovlev, 2007,2008

Present talk: nucleon sector

$$\kappa_{\rm n},\ \eta_{\rm n}$$

Strongly interacting multicomponent Fermi-liquid

Important: many-body effects in scattering

Controversial results by different groups

Plan

- **Thermal conductivity and shear viscosity in multicomponent Fermi-liquid**
- **Nucleon interaction in Brueckner-Hartree-Fock approximation**
- **Results**
- **Comparison with other works**
- **Conclusions**

Kinetic coefficients in multi-component Fermi-liquid: Formalism

$$\kappa = \sum_c \frac{\pi^2 k_B^2 T n_c \tau_c^\kappa}{3 m_c^*}; \qquad \eta = \sum_c \frac{n_c p_{\mathrm{F}c}^2 \tau_c^\eta}{5 m_c^*}$$

Perturbation $\nabla T,\ V_{\alpha\beta} = \frac{1}{2}\left(\frac{\partial V_\alpha}{\partial x_\beta} + \frac{\partial V_\beta}{\partial x_\alpha}\right)$ ⇨ **Deviation of the distribution function** $F = f_{eq} - \Phi \frac{\partial f_{eq}}{\partial \epsilon}$

Kinetic equation ***(linearized)***

$$\left.\begin{array}{ll} \kappa: & (\epsilon_1 - \mu_1)\mathbf{v}_1 \frac{\nabla T}{T} \\ \eta: & \left(v_{1\alpha} p_{1\beta} - \frac{1}{3}\delta_{\alpha\beta} v_1 p_1\right) V_{\alpha\beta} \end{array}\right\} \frac{\partial f_1}{\partial \epsilon_1} = \sum_i I_{ci}(12;1'2'), \quad c,\ i = \mathrm{n},\ \mathrm{p}$$

Boltzmann collision integral

$$I_{ci} = \frac{1}{(1+\delta_{ci})k_B T} \sum_{\sigma_{1'}\sigma_2\sigma_{2'}} \int\!\!\int\!\!\int \frac{\mathrm{d}\boldsymbol{p}_{1'}\mathrm{d}\boldsymbol{p}_2\mathrm{d}\boldsymbol{p}_{2'}}{(2\pi\hbar)^9} w_{ci}(12;1'2') f_1 f_2 (1-f_{1'})(1-f_{2'}) \left(\Phi_{1'} + \Phi_{2'} - \Phi_1 - \Phi_2\right)$$

Transition probability $\sum_{\mathrm{spins}} w_{ci}(12|1'2') = 4\frac{(2\pi)^4}{\hbar}\delta(\epsilon_1 + \epsilon_2 - \epsilon_1' - \epsilon_2')\delta(\boldsymbol{P} - \boldsymbol{P}')\mathcal{Q}_{ci}$

Solution: $\Phi^{\kappa,\eta} \to \tau_c^{\kappa,\eta} \to \kappa,\ \eta$

Input: $m^*,\ \mathcal{Q}$ **on the Fermi surface**

Variational solution

2x2 algebraic system $\sum_{i=n,p} \nu_{ci}\tau_i = 1$

$$\kappa = \sum_c \frac{\pi^2 k_B^2 T n_c \tau_c^\kappa}{3 m_c^*}; \quad \eta = \sum_c \frac{n_c p_{Fc}^2 \tau_c^\eta}{5 m_c^*}$$

$$\nu_{ci}^{(\kappa)} = \frac{64 m_c^* m_i^{*2} (k_B T)^2}{5 m_N^2 \hbar^3} S_{\kappa ci}, \quad \nu_{ci}^{(\eta)} = \frac{16 m_c^* m_i^{*2} (k_B T)^2}{3 m_N^2 \hbar^3} S_{\eta ci}.$$

All particles on Fermi surface

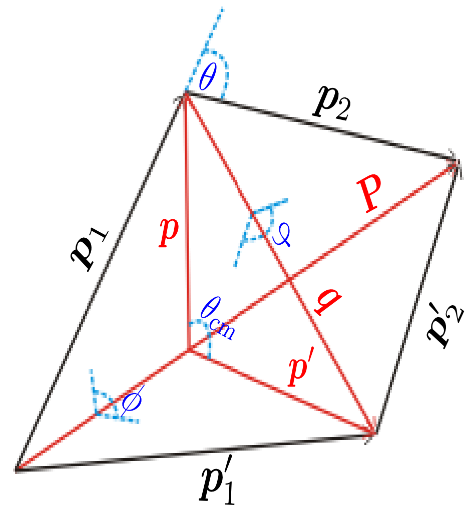
Effective transport cross-sections

$$S_{\kappa cc} = \frac{m_N^2}{128\pi^2\hbar^4 p_c^3} \int_0^{2p_c} \mathrm{d}P \int_0^{q_m(P)} \mathrm{d}q \frac{(4p_c^2 - P^2)}{\sqrt{q_m^2 - q^2}} \mathcal{Q}_{cc},$$

$$S_{\kappa ci} = \frac{m_N^2}{128\pi^2\hbar^4 p_c^3} \int_{|p_c-p_i|}^{p_c+p_i} \mathrm{d}P \int_0^{q_m(P)} \mathrm{d}q \frac{(4p_c^2 + q^2)}{\sqrt{q_m^2 - q^2}} \mathcal{Q}_{ci}, \quad c \neq i,$$

$$S_{\eta cc} = \frac{3m_N^2}{128\pi^2\hbar^4 p_c^5} \int_0^{2p_c} \mathrm{d}P \int_0^{q_m(P)} \mathrm{d}q \frac{q^2(4p_c^2 - P^2 - q^2)}{\sqrt{q_m^2 - q^2}} \mathcal{Q}_{cc},$$

$$S_{\eta ci} = \frac{3m_N^2}{128\pi^2\hbar^4 p_c^5} \int_{|p_c-p_i|}^{p_c+p_i} \mathrm{d}P \int_0^{q_m(P)} \mathrm{d}q \frac{q^2(4p_c^2 - q^2)}{\sqrt{q_m^2 - q^2}} \mathcal{Q}_{ci}, \quad c \neq i,$$



Two angles fix all momenta

$$\frac{1}{\tau_i} = \frac{1}{\tau_i}\left[\langle w(12|1'2')\beta(\theta,\phi)\rangle\right]$$

Compare:
Scattering cross-sections:

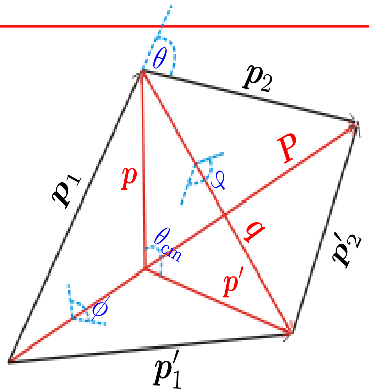
$$\frac{d\sigma_{\alpha\beta}}{d\Omega} = \frac{m_{\alpha\beta}^{*2}}{16\pi^2\hbar^4}\mathcal{Q}_{\alpha\beta}$$

Accuracy (*aposteriori*)

$$\kappa_{exact}/\kappa_{var} \approx 1.2$$
$$\eta_{exact}/\eta_{var} \approx 1.05$$

Approximate estimates

Pure neutron matter: $\tau_{\rm n} = \frac{1}{\nu_{\rm nn}}$

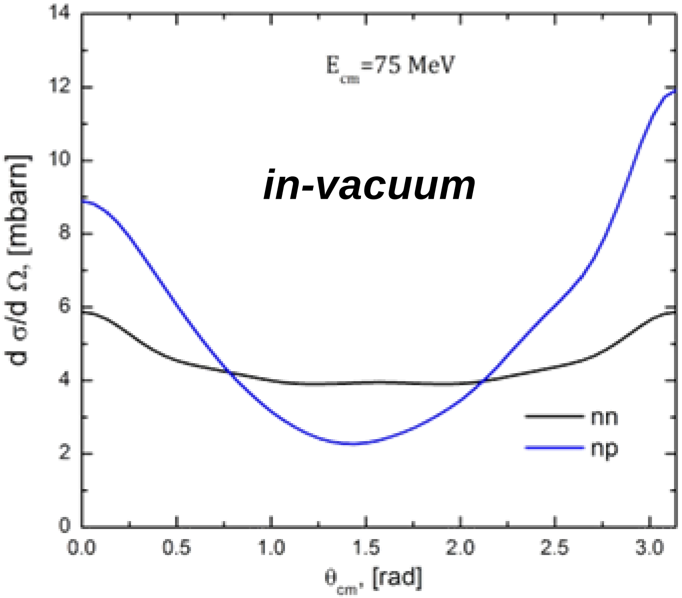


1. kinematics: $\theta \sim \pi \ (P \approx 0)$

2. nn cross-section is flat $\frac{\mathrm{d}\sigma_{\rm nn}}{\mathrm{d}\Omega} \rightarrow \frac{\sigma_{\rm tot}}{2\pi}$

$$\frac{d\sigma_{\rm nn}}{d\Omega} = \frac{m_{\rm n}^{*2}}{16\pi^2\hbar^4}\mathcal{Q}_{\rm nn}$$

$$\frac{1}{\tau_{\rm n}} = \frac{1}{\tau_{\rm n}}\left[\langle \mathcal{Q}_{\rm nn}\beta(\theta,\phi)\rangle\right]$$



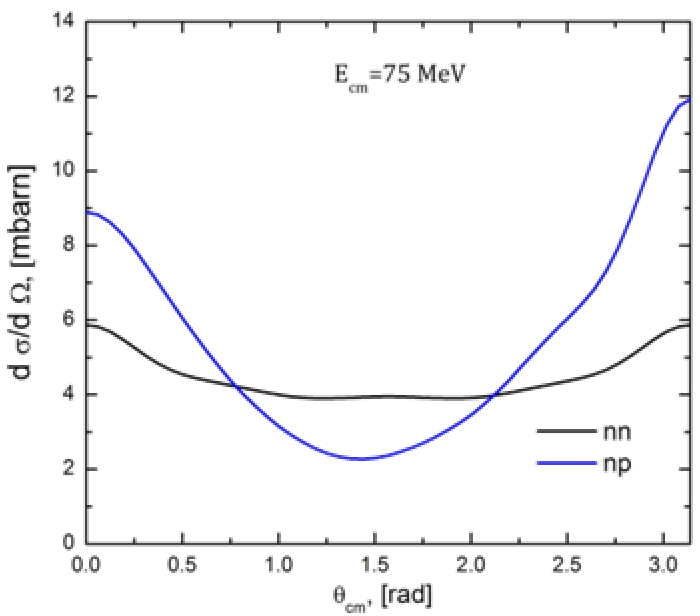
Finally: simple relations

$$\kappa_{\rm Var} = \frac{5}{3}\frac{p_F^3}{m^2}\frac{1}{T}\left[32\sigma_{\rm tot}\left(E_{\rm cm} = \frac{p_F^2}{m}\right)\right]^{-1}$$

$$\eta_{\rm Var} = \frac{p_F^5}{16\pi^2 m^2}\frac{1}{(k_B T)^2}\left[\sigma_{\rm tot}\left(E_{\rm cm} = \frac{p_F^2}{m}\right)\right]^{-1}$$

Effects of the proton fraction

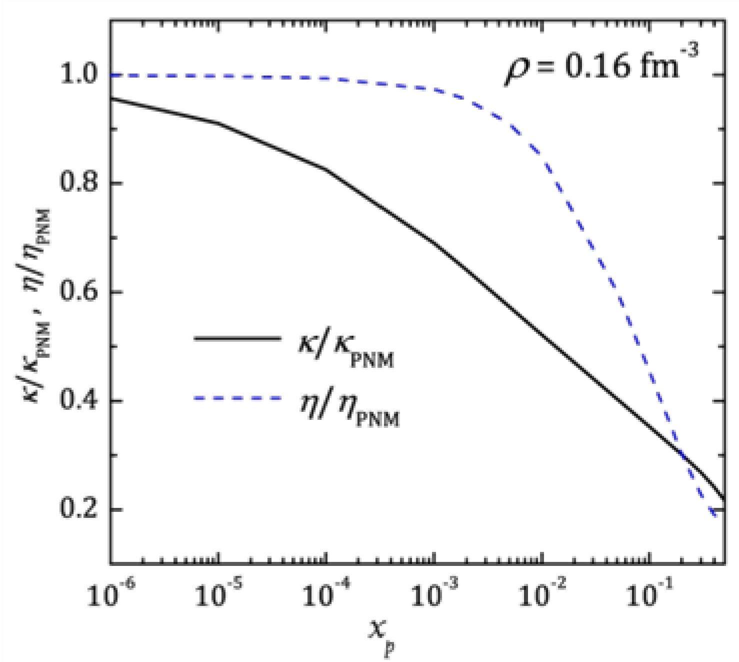
PNM result is inaccurate even at small proton fraction



$$\nu_{\mathrm{np}} \sim \nu_{\mathrm{nn}}$$

$$\kappa = \kappa_{\mathrm{n}} + \kappa_{\mathrm{p}}$$

$$\eta = \eta_{\mathrm{n}} + \eta_{\mathrm{p}}$$



Reasons

small-angle scattering
smaller c.m. energy

$$p_{F\mathrm{p}} \ll p_{F\mathrm{n}}$$

Higher cross-section
(T_z=0 isospin channel)

Interactions in-medium

$$\kappa = \sum_c \frac{\pi^2 k_B^2 T n_c \tau_c^{\kappa}}{3 m_c^*}; \quad \eta = \sum_c \frac{n_c p_{\mathrm{F}c}^2 \tau_c^{\eta}}{5 m_c^*}$$

$$\sum_{i=n,p} \nu_{ci} \tau_i = 1$$

$$\nu_{ci}^{(\kappa)} = \frac{64 m_c^* m_i^{*2} (k_B T)^2}{5 m_N^2 \hbar^3} S_{\kappa ci}, \qquad \nu_{ci}^{(\eta)} = \frac{16 m_c^* m_i^{*2} (k_B T)^2}{3 m_N^2 \hbar^3} S_{\eta ci}.$$

From many-body theory we need

$$m^*, \ \mathcal{Q}$$

4^{th} power of m* – the main effect?

Bethe-Brueckner-Goldstone equation

$$V \to G$$

All diagrams on two hole-line level are included

$$\langle p_1 p_2 | G^{\alpha\beta}(\omega) | p_3 p_4 \rangle = \langle p_1 p_2 | V^{\alpha\beta} | p_3 p_4 \rangle + \sum_{k_1, k_2} \langle p_1 p_2 | V^{\alpha\beta} | k_1 k_2 \rangle \frac{Q^{\alpha\beta}(k_1, k_2)}{\omega - \epsilon_\alpha(k_1) - \epsilon_\beta(k_2)} \langle k_1 k_2 | G^{\alpha\beta} | p_3 p_4 \rangle$$

$Q^{\alpha\beta}(k_1, k_2)$ **– Pauli operator**

$$\epsilon_\alpha(p) = \frac{p^2}{2m_\alpha} + U_\alpha(p)$$

Continuous choice of auxiliary single-particle potential

$$U_\alpha(p_1) = \sum_{\beta; p_2 < p_{F\beta}} \langle p_1 p_2 | G^{\alpha\beta}(\epsilon_1(p_1) + \epsilon_2(p_2)) | p_1 p_2 \rangle_A$$

minimizes the contribution from the three-hole line diagrams

BHF energy:

$$E = E_{kin} + \frac{1}{2} \sum_{\alpha\beta} \sum_{p_1 < p_{F\alpha}, p_2 < p_{F\beta}} \langle p_1 p_2 | G^{\alpha\beta}(\epsilon_1(p_1) + \epsilon_2(p_2)) | p_1 p_2 \rangle_A, \qquad \alpha, \beta = n, p$$

Three-nucleon interactions

$$V = V_{12} + V_{12}^{(3)}$$

Effective interaction (averaged over the third particle)

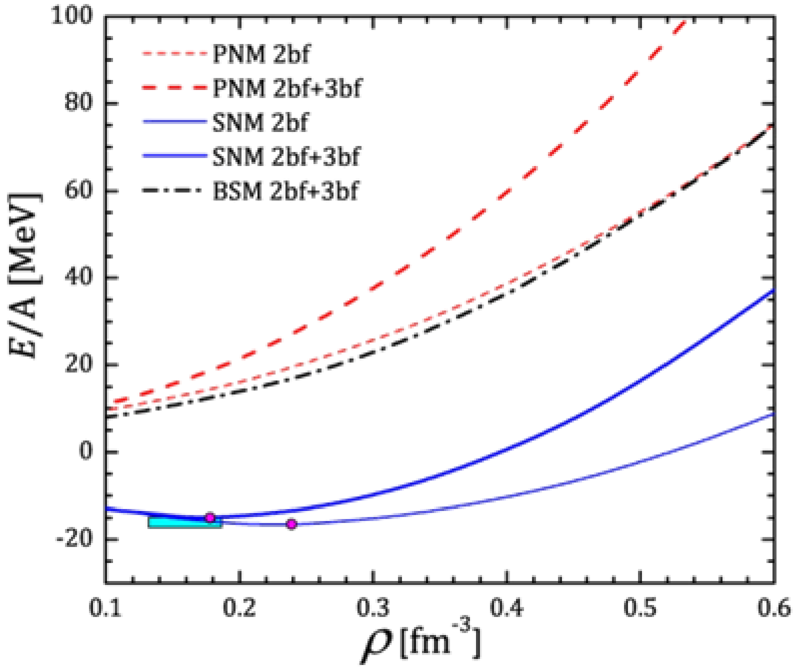
$$V_{12}^{(3)} = \rho \int \mathrm{d}\boldsymbol{r}_3 \, g^2(r_{13}) g^2(r_{23}) V_{123}$$

$g(r)$ **–** ***defect function***

Results. Equation of state

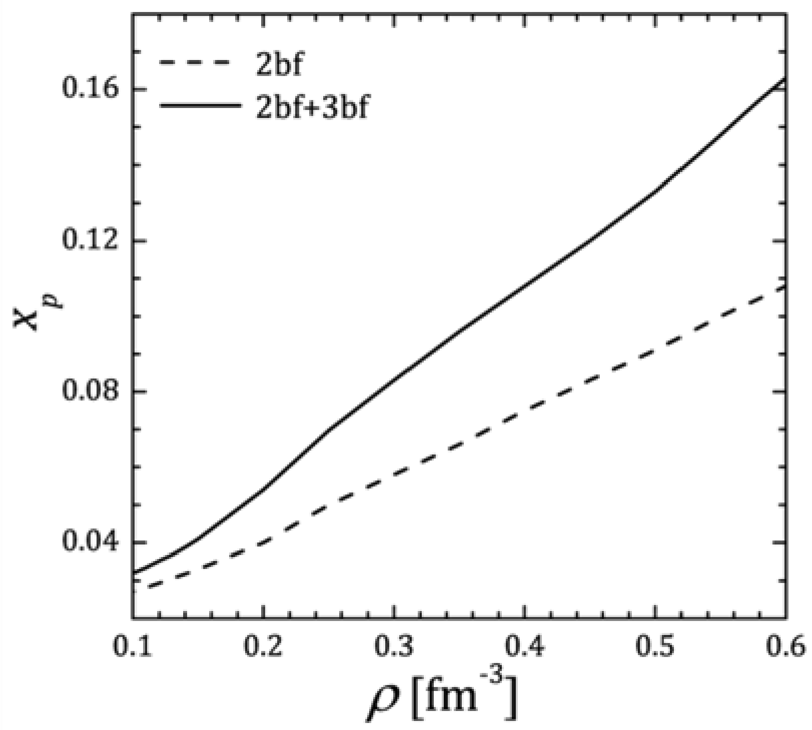
Argonne v18 + Urbana IX

Energy of the nuclear matter



3bf produce correct s.p. of SNM
EOS is stiffer

Proton fraction in beta-stable matter



$$E = E_0 + S_b(1 - 2x_p)^2$$

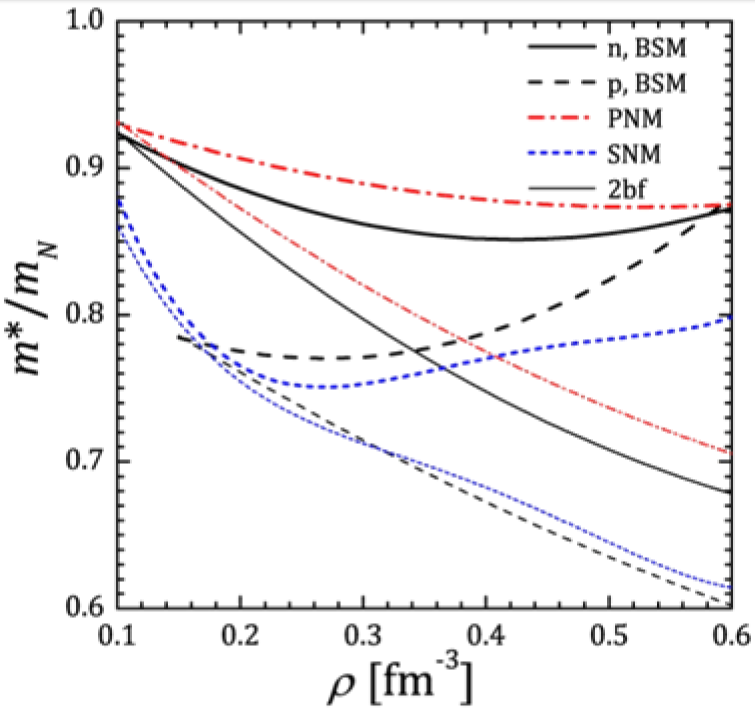
Results. Effective mass

Fermi-sea of quasi-particles $\epsilon(p)$ **interacting via** G

$$m^* = \left(\frac{1}{p} \frac{\mathrm{d}\epsilon(p)}{\mathrm{d}p} \right)^{-1}_{p=p_F}$$

2bf decrease effective masses
3bf increase

Effective masses at Fermi surface



NB: Fermi-liquid effects are not included

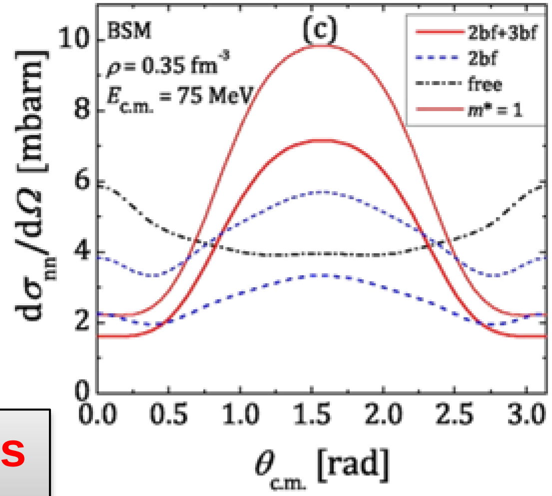
Results. In-medium cross-sections

"cross-section" at Fermi surface

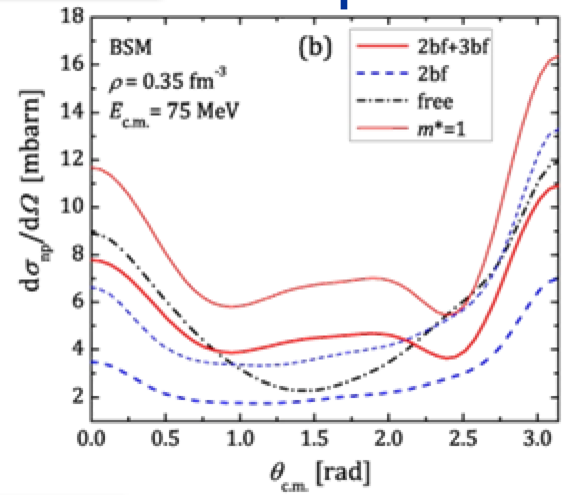
$$\frac{d\sigma_{\alpha\beta}}{d\Omega} = \frac{m^{*2}_{\alpha\beta}}{16\pi^2\hbar^4}\mathcal{Q}_{\alpha\beta}$$

differential

nn



np

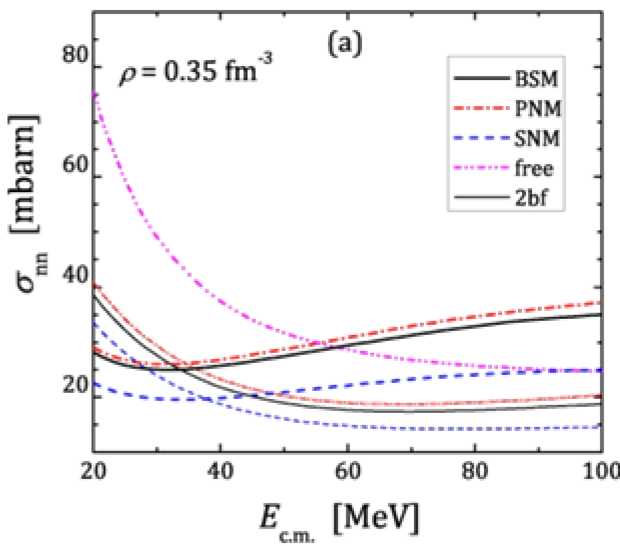


2bf decrease cross-sections
3bf increase

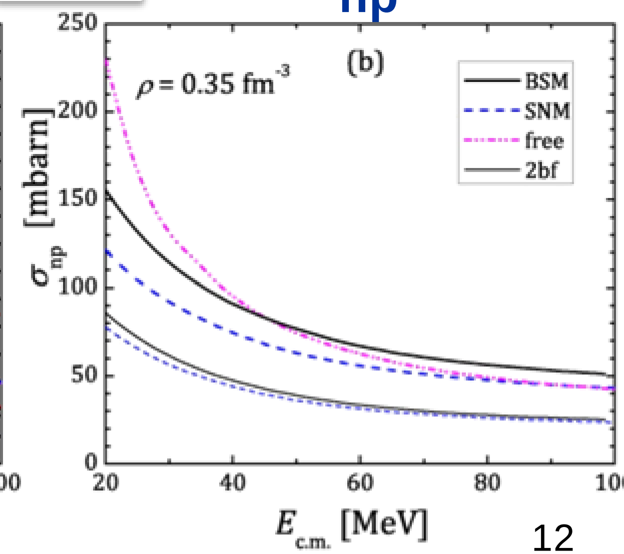
$$\sigma_{ab} = \frac{1}{1+\delta_{ab}}\int_{(4\pi)} \mathrm{d}\Omega \frac{\mathrm{d}\sigma_{ab}}{\mathrm{d}\Omega}$$

total

nn



np

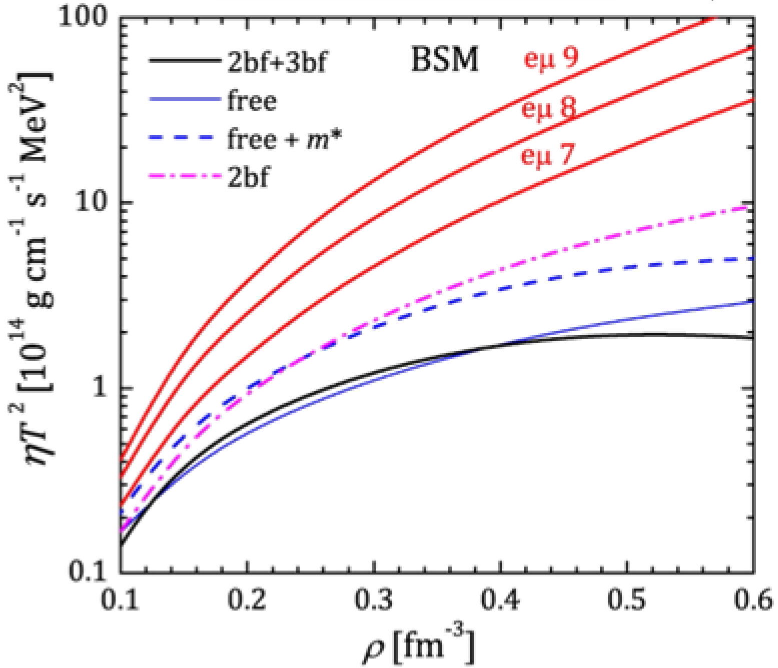


Results. Kinetic coefficients.

Exact solutions are shown

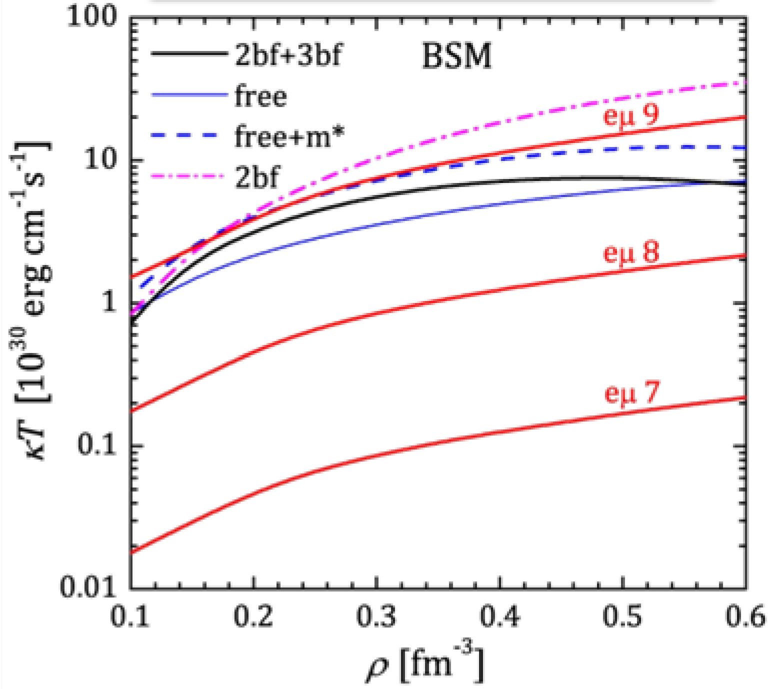
Shear viscosity

$\eta_{e\mu} \propto T^{-5/3}$



Thermal conductivity

$\kappa_{e\mu} \propto T^{0}$



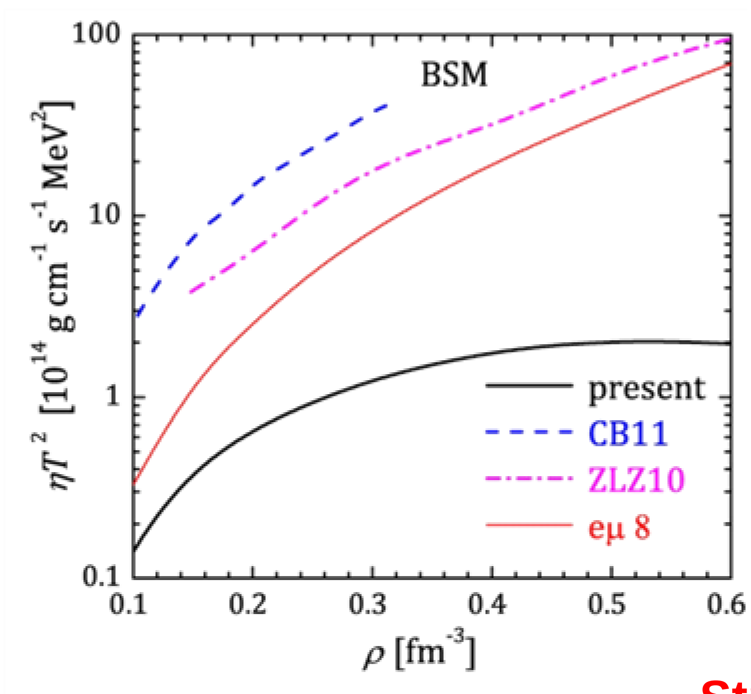
Urbana IX 3bf results are comparable with 'free-scattering'

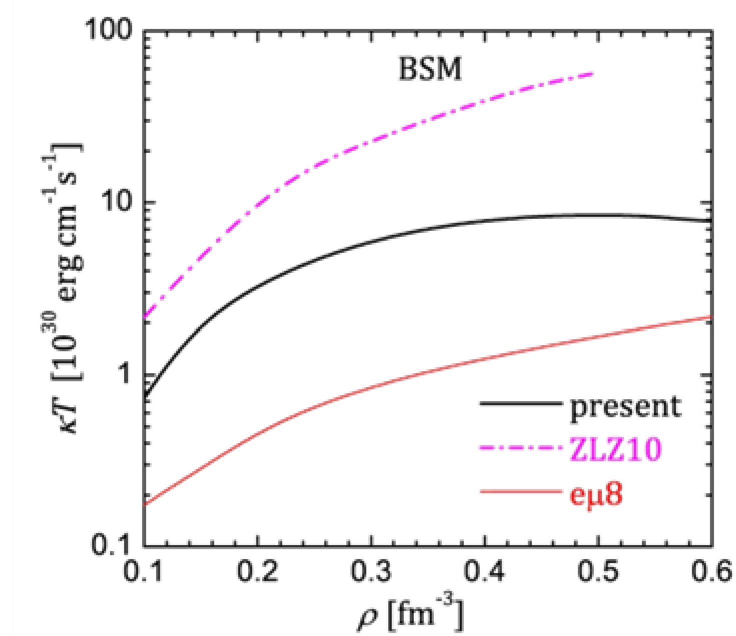
m* modification only is insufficient

Comparison with other works

Carbone&Benhar, J.Phys.Conf.Ser. 336, 012015 (2011)

Zhang, Lombardo, Zuo, PRC 82, 015805 (2010)





Strong disagreement!

CB11: CBF approach, no 3bf

ZLZ10: BHF approach, different 3bf, rearrangement in m*

Summary

- **Kinetic coefficients of neutrons in neutron star cores are calculated in the BHF framework with account for the effective three-nucleon interactions**

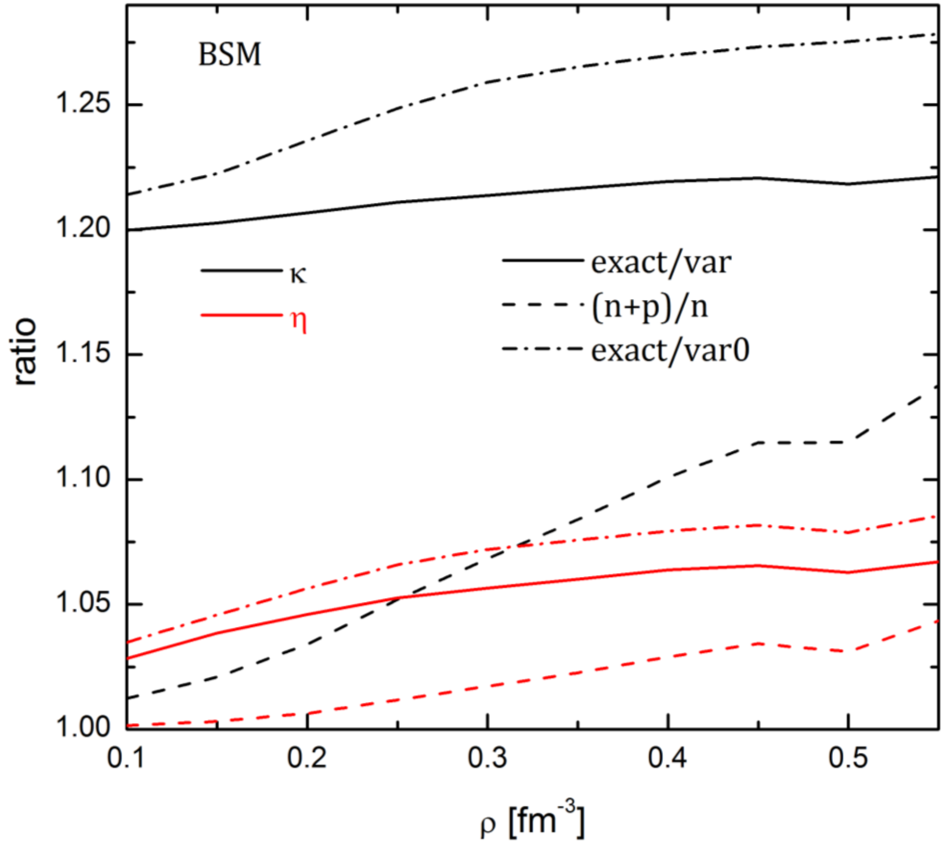
- **Both effective mass and the intermediate state blocking by Pauli principle are important**

- **Three-nucleon forces can lead to lower kinetic coefficients**

- **Many-body effects in the present model do not result in significant change of kinetic coefficients**

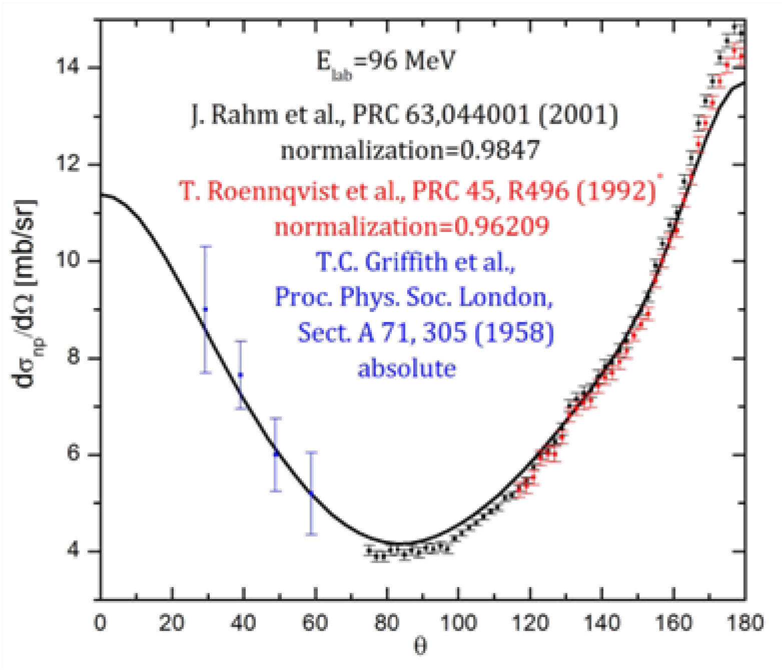
Results. Comparison with exact solution

Var0 – carriers are neutrons
Var – solution of 2x2 system

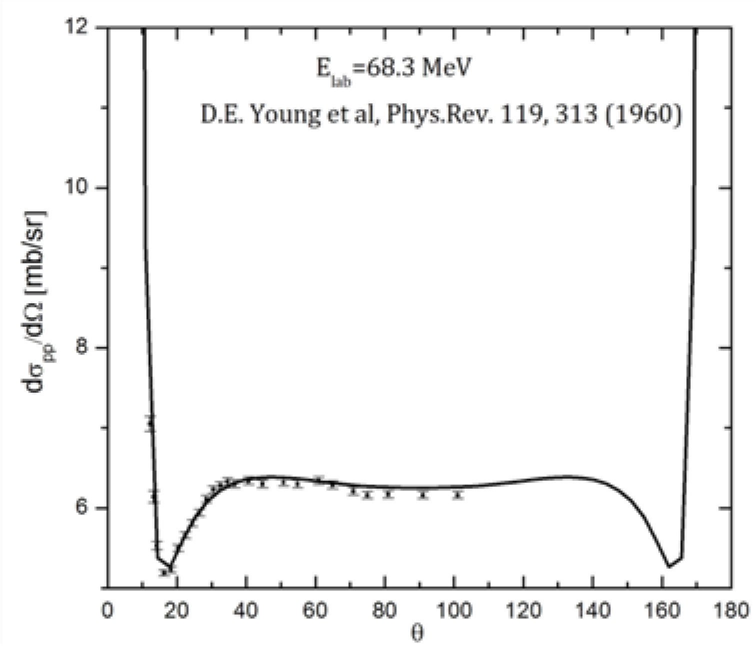


In-vacuum cross-sections

Comparison with experiments



pp cross-section with Coulomb interaction (Mott) included

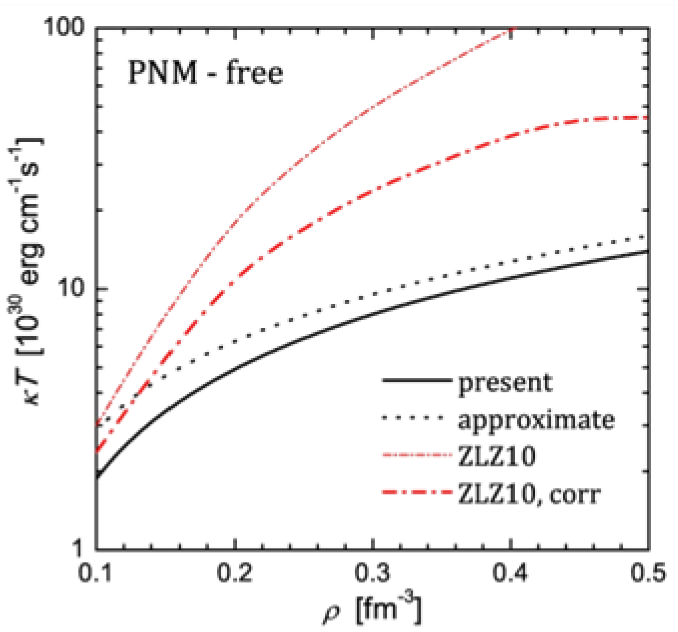


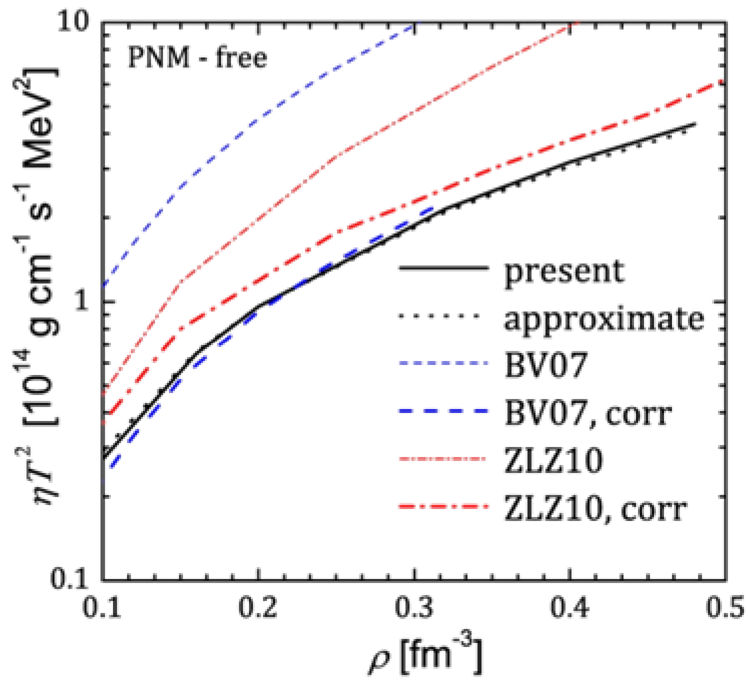
Results. Comparison with other works.

Approximation for the pure neutron matter

$$\kappa_{\mathrm{Var}} = \frac{5}{3}\frac{p_F^3}{m^2}\frac{1}{T}\left[32\sigma_{\mathrm{tot}}\left(E_{\mathrm{cm}} = \frac{p_F^2}{m}\right)\right]^{-1}$$

$$\eta_{\mathrm{Var}} = \frac{p_F^5}{16\pi^2 m^2}\frac{1}{(k_B T)^2}\left[\sigma_{\mathrm{tot}}\left(E_{\mathrm{cm}} = \frac{p_F^2}{m}\right)\right]^{-1}$$





Possible reasons

Baym, Pethick "Landau Fermi-Liquid Theory: concepts and applications, 1991"

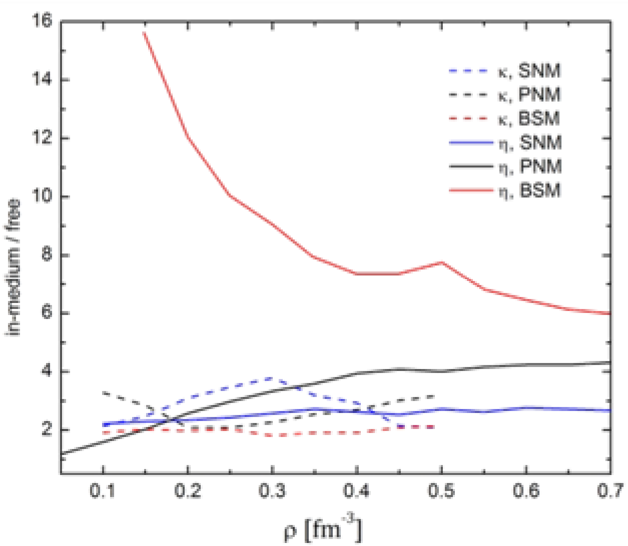
Characteristic relaxation time $\tau \equiv \frac{8\pi^4\hbar^6}{m^{*3}\langle W\rangle (k_B T)^2}$

Where $W = \frac{\pi}{\hbar}\sum'_{\sigma_2\sigma_3\sigma_4} |\langle 34|t|12\rangle|^2 = \pi \mathcal{Q}$

Benhar, Valli: $$W = \frac{16\pi^2}{m^{*2}}\frac{d\sigma}{d\Omega}$$

Factor of π

Zhang et al. : strange behavior of shear viscosity in BSM



Kinematics

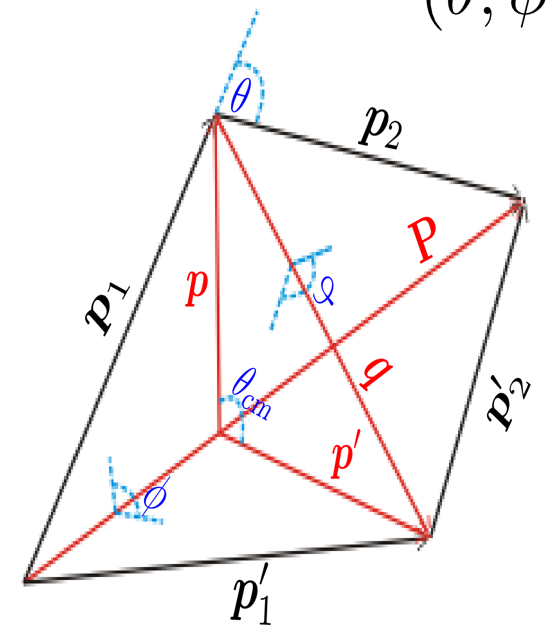
$$1 + 2 \to 1' + 2'$$

Abrikosov-Khalatnikov angles

$$(\theta, \phi)$$

All particles are placed on the Fermi surface

$$p_1 = p_1' = p_{\mathrm{F}1};\ p_2 = p_2' = p_{\mathrm{F}2}$$



Total momentum $\boldsymbol{P} = \boldsymbol{p}_1 + \boldsymbol{p}_2 = \boldsymbol{p}_1' + \boldsymbol{p}_2'$

C.m. momenta

$$\boldsymbol{p} = \tfrac{1}{2}\left(\boldsymbol{p}_1 - \boldsymbol{p}_2\right)$$

$$\boldsymbol{p}' = \tfrac{1}{2}\left(\boldsymbol{p}_1' - \boldsymbol{p}_2'\right)$$

Transferred momentum $\boldsymbol{q} = \boldsymbol{p}_1' - \boldsymbol{p}_1$

$$\mathrm{d}\Omega_{1'}\Omega_2\Omega_{2'}\delta(\boldsymbol{p_1} + \boldsymbol{p_2} - \boldsymbol{p_1'} - \boldsymbol{p_2'}) \to \mathrm{d}\theta\mathrm{d}\phi\beta(\theta, \phi)$$

$$P^2 + 4p^2 = 2(p_{\mathrm{F}1}^2 + p_{\mathrm{F}2}^2)$$

$$\boldsymbol{P}\boldsymbol{p} = \boldsymbol{P}\boldsymbol{p}'$$

More convenient to use (q, φ) **or** (q, P)

Scattering is defined by averaged transition probabilities on the Fermi surface

$$\frac{1}{\tau_i} = \frac{1}{\tau_i}\left[\langle w(12|1'2')\beta(\theta, \phi)\rangle\right]$$