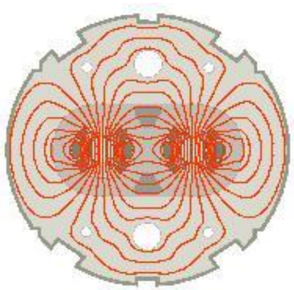
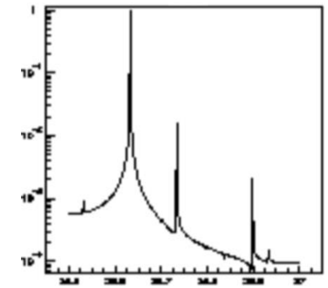
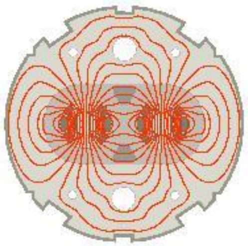


Nonlinear Correction Steering of the LHC using Harmonic Analysis



- References
- Theory – General Overview
- Resonance-Line Definition
- Model-Measurement Scheme
- (Order by Order Technique)
- New Developments
- List of Possibilities
- List of Limits

- Linear Coupling Compensation
- Sextupole Resonances
- Exotic Applications
- Commissioning of the LHC



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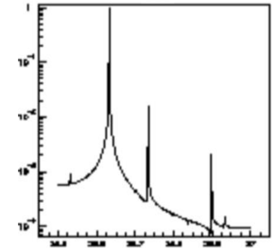
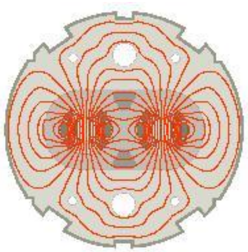
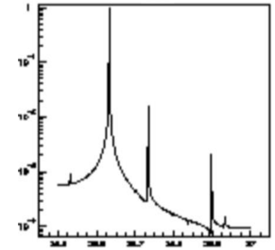


- 1988 - J. Bengtsson, "Non-linear transverse dynamics for storage rings with applications to the Low-Energy Antiproton Ring (LEAR) at CERN", Cern Yellow Report 88-05.
- 1992 - J. Laskar, C. Froeschlé and A. Celletti, "The measure of chaos by the numerical analysis of the fundamental frequencies. Application to the standard mapping" , Physica D **56**, pp. 253—269.
- 1998 - R. Bartolini and F. Schmidt, "Normal Form via Tracking or Beam Data", Part. Accel. 59, pp. 93-106.
- 2003 - R. Tomás, "Direct Measurement of Resonance Driving Terms in the Super Proton Synchrotron (SPS) of CERN using Beam Position Monitors", Cern PhD Thesis.
- 2004 – P. Urschütz, "Measurement and Compensation of Betatron Resonances at the CERN PS Booster Synchrotron", Cern PhD Thesis.



Theory – General Overview



I. Equivalence of Spectral Lines and Terms in the Generating Function

II. Term Comparison and Sextupole Resonances

Resonance-Line Definition

Model Measuring Scheme

In case the motion is regular it can be decomposed into a series of spectral lines [14] the frequencies of which are the fundamental tunes ν_x and ν_y and their linear combinations. For the horizontal particle coordinates (x, p_x) transformed into the linearly normalised Courant-Snyder variables [15]$(\hat{x}, \hat{p}_x)$, we get after N turns:

$$\hat{x}(N) - i\hat{p}_x(N) = \sum_{j=1}^{\infty} a_j e^{i[2\pi(m_j\nu_x+n_j\nu_y)N+\psi_j]} \quad m_j, n_j \in \mathbf{Z}, \tag{1}$$

where the a_j and ψ_j are amplitude and phase of the corresponding spectral line.

3 Generating Function versus Data Spectrum

In the following we will consider the four dimensional case of coupled horizontal and vertical motion. We consider this to be minimal as the coupling resonance are mandatory to understand the dynamics of a real accelerator. On the other hand we did not consider the full six dimensional case so as to avoid the formalism to become too complicated. We like to stress, however, that the formalism as well as all numerical tools can be fully applied to the general case of six dimensions provided 6D particle data are available.

The 4D particle coordinates $\mathbf{x}' = (x, p_x, y, p_y)$, after one turn in the machine, are related to the initial coordinates $\mathbf{x}$ by the mapping:

$$\mathbf{x}' = \mathbf{M}(\mathbf{x}) \tag{2}$$

The perturbative theory for the maps gives a powerful tool to parametrise the nonlinear contents of the particle motion in terms of resonances and detuning terms. The initial map $\mathbf{M}$ is reduced to a simpler map $\mathbf{U}$ by means of a symplectic change of coordinates following the scheme

$$\begin{tikzcd} \mathbf{x} \arrow[r, "\mathbf{M}"] \arrow[d, "\mathbf{\Phi}^{-1}"'] & \mathbf{x}' \arrow[d, "\mathbf{\Phi}^{-1}"] \\ \zeta \arrow[r, "\mathbf{U}"'] & \zeta' \end{tikzcd} \tag{3}$$

The goal of the transformation $\mathbf{\Phi}^{-1}$ is to perform a change of variable towards the action-angle variables leaving in the transformed map $\mathbf{U}$ only action dependent terms (detuning terms), the so called non resonant Normal Form. In the case of the resonant Normal Form angle dependent terms are left in the Hamiltonian. The Normal Form transformation is accomplished via an order–by–order procedure in the perturbative parameter which in our case is the distance from the origin of the phase space. The transformation $\mathbf{\Phi}$ and the map $\mathbf{U}$ may be expressed as Lie operators with generating function F and H respectively:

$$\mathbf{\Phi} = e^{:F:} \quad , \quad \mathbf{\Phi}^{-1} = e^{-:F:} \quad , \quad \mathbf{U} = e^{:H:} \quad . \tag{4}$$

In the lowest order, the linear case, the action angle variables $(J_x, \phi_x, J_y, \phi_y)$ are simply related to the Courant–Snyder variables $(\hat{x}, \hat{p}_x, \hat{y}, \hat{p}_y)$ by the formula (z stands for x or y):

$$\begin{aligned} \hat{z} &= \sqrt{2J_z}\cos(\phi_z + \phi_{z0}) \\ \hat{p}_z &= -\sqrt{2J_z}\sin(\phi_z + \phi_{z0}), \end{aligned} \tag{5}$$

where ϕ_{z_0} is the initial phase. It is convenient to express the linearly normalised variables in the so called resonance basis $\mathbf{h} = (h_x^+, h_x^-, h_y^+, h_y^-)$ defined by the relations:

$$h_z^\pm = \hat{z} \pm i\hat{p}_z = \sqrt{2J_z}e^{\mp i(\phi_z+\phi_{z0})}. \tag{6}$$

The transformation to the new set of canonical coordinates $\zeta = (\zeta_x^+, \zeta_x^-, \zeta_y^+, \zeta_y^-)$ which brings the map into the Normal Form is expressed as a Lie series:

$$\zeta = e^{-:F_r:}\mathbf{h} \tag{7}$$

where

$$\zeta_z^\pm = \sqrt{2I_z}e^{\mp i(\psi_z+\psi_{z0})} \tag{8}$$

and $(I_x, \psi_x, I_y, \psi_y)$ are the nonlinear action angle variables. In the resonance basis, F_r can be written as a sum of homogeneous polynomials of the variables ζ as

$$F_r = \sum_{jklm} f_{jklm}\zeta_x^{+j}\zeta_x^{-k}\zeta_y^{+l}\zeta_y^{-m}. \tag{9}$$

Introducing the $\zeta_z^\pm$ of Eq. 8, we arrive at:

$$F_r = \sum_{jklm} f_{jklm}(2I_x)^{\frac{j+k}{2}}(2I_y)^{\frac{l+m}{2}}e^{-i[(j-k)(\psi_x+\psi_{x0})+(l-m)(\psi_y+\psi_{y0})]}. \tag{10}$$

The transformation from the new action–angle variables to the linearly normalised variables is given by:

$$\mathbf{h} = e^{:F_r:}\zeta = \zeta + [F_r, \zeta] + \frac{1}{2}[F_r, [F_r, \zeta]] + \cdots \tag{11}$$

where $[F_r, \zeta]$ denotes the Poisson bracket of F_r and ζ. To the first order the transformation to the h_x^- reads:

$$h_x^- \approx \zeta_x^- + [F_r, \zeta_x^-] = \zeta_x^- - 2i\sum_{jklm} jf_{jklm}\zeta_x^{+j-1}\zeta_x^{-k}\zeta_y^{+l}\zeta_y^{-m} \tag{12}$$

where we have used the property of the Poisson bracket:

$$[\zeta_x^{+j}, \zeta_x^-] = -2ij\zeta_x^{+j-1}. \tag{13}$$

The evolution of the variable in Normal Form after N turns is given by:

$$\zeta_x^-(N) = \sqrt{2I_x}e^{i(2\pi\nu_x N+\psi_{x0})}, \tag{14}$$

where ν_x is the horizontal tune of the particle including the amplitude dependent detuning and ψ_{x0} is the horizontal initial phase. We can therefore obtain the evolution of the linearly normalised horizontal variable in the form:

$$\begin{aligned} h_x^-(N) &= \sqrt{2I_x}e^{i(2\pi\nu_x N+\psi_{x0})} \\ &-2i\sum_{jklm} jf_{jklm}(2I_x)^{\frac{j+k-1}{2}}(2I_y)^{\frac{l+m}{2}}e^{i[(1-j+k)(2\pi\nu_x N+\psi_{x0})+(m-l)(2\pi\nu_y N+\psi_{y0})]}. \end{aligned} \tag{15}$$

This expression is equivalent to the spectral decomposition of Eq. 1. The motion appears as a superposition of spectral lines given by the tune (first row) and the contribution from the resonant terms (second row).

The expression in Eq. 15 can be compared with the generating function (Eq. 10) in order to determine the coefficients f_{jklm} term by term. To this aim we rewrite the Eq. 15 as

$$h_x^-(N) = \sum_{jklm} HSL_{jklm}e^{2\pi i[(1-j+k)\nu_x+(m-l)\nu_y]N}. \tag{16}$$

where the complex Fourier coefficients of the horizontal spectral line is indicated by HSL_{jklm} with amplitude $|HSL_{jklm}|$ and phase $PHSL_{jklm}$. Analogously VSL_{jklm} will indicate the complex Fourier coefficient of the vertical motion. The following table compares the amplitudes and phases of the spectral lines with those of the generating function coefficients:

$$\mathbf{x}' = \mathbf{M}(\mathbf{x})$$

$$\begin{tikzcd}
\mathbf{x} \arrow[r, "\mathbf{M}"] \arrow[d, "\mathbf{\Phi}^{-1}"'] & \mathbf{x}' \arrow[d, "\mathbf{\Phi}^{-1}"] \\
\zeta \arrow[r, "\mathbf{U}"'] & \zeta'
\end{tikzcd}$$

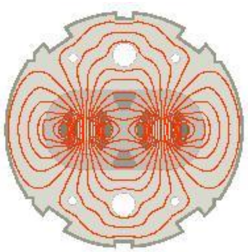
$$\zeta = e^{-:F_r:}\mathbf{h}$$

$$\mathbf{\Phi} = e^{:F:} \quad , \quad \mathbf{\Phi}^{-1} = e^{-:F:} \quad , \quad \mathbf{U} = e^{:H:}$$

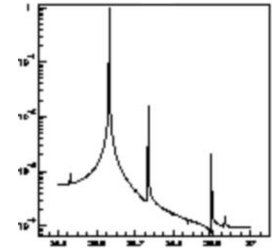
$$F_r = \sum_{jklm} f_{jklm}(2I_x)^{\frac{j+k}{2}}(2I_y)^{\frac{l+m}{2}} e^{-i[(j-k)(\psi_x+\psi_{x_0})+(l-m)(\psi_y+\psi_{y_0})]}.$$

$$\begin{aligned}
h_x^-(N) &= \sqrt{2I_x}e^{i(2\pi\nu_x N+\psi_{x_0})} \\
&-2i\sum_{jklm} jf_{jklm}(2I_x)^{\frac{j+k-1}{2}}(2I_y)^{\frac{l+m}{2}} e^{i[(1-j+k)(2\pi\nu_x N+\psi_{x_0})+(m-l)(2\pi\nu_y N+\psi_{y_0})]}.
\end{aligned}$$

$$h_x^-(N) = \sum_{jklm} HSL_{jklm} e^{2\pi i[(1-j+k)\nu_x+(m-l)\nu_y]N}.$$



Theory – General Overview



I. Equivalence of Spectral Lines and Terms in the Generating Function

II. Term Comparison and Sextupole Resonances

Resonance-Line Definition

Model Measuring Scheme

Table 1: Relation between spectral lines and coefficients of the generating function

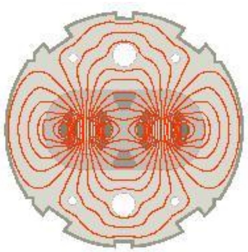
	Generating Function	Spectral Line
Amplitude	$\|f_{jklm}\|$	$\|HSL_{jklm}\| = 2 \cdot j \cdot (2I_x)^{\frac{j+k-1}{2}} (2I_y)^{\frac{l+m}{2}} \|f_{jklm}\|$ $\|VSL_{jklm}\| = 2 \cdot l \cdot (2I_x)^{\frac{j+k}{2}} (2I_y)^{\frac{l+m-1}{2}} \|f_{jklm}\|$
Phase	ϕ_{jklm}	$PHSL_{jklm} = \phi_{jklm} + (1-j+k)\psi_{x0} - (l-m)\psi_{y0} - \frac{\pi}{2}$ $PVSL_{jklm} = \phi_{jklm} - (j-k)\psi_{x0} + (1-l+m)\psi_{y0} - \frac{\pi}{2}$

Table 3: Resonances and spectral lines due to the normal sextupole term x^3

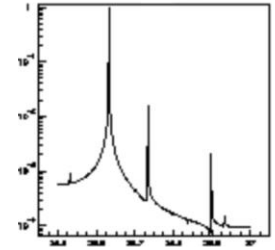
D.T.	Resonance	$j+k$	j	Lines Hor.	$l+m$	l	Lines Ver.
h_{3000}	(3,0)	3	3	(-2,0)	0	0	-
h_{2100}	(1,0)	3	2	(0,0)	0	0	-
h_{1200}	(1,0)	3	1	(2,0)	0	0	-
h_{0300}	(3,0)	3	0	-	0	0	-

Table 4: Resonances and spectral lines due to the normal sextupole term xy^2

D.T.	Resonance	$j+k$	j	Lines Hor.	$l+m$	l	Lines Ver.
h_{1020}	(1,2)	1	1	(0,-2)	2	2	(1,-1)
h_{1011}	(1,0)	1	1	(0,0)	2	1	(1,1)
h_{1002}	(1,-2)	1	1	(0,2)	2	0	-
h_{0120}	(1,-2)	1	0	-	2	2	(-1,-1)
h_{0111}	(1,0)	1	0	-	2	1	(-1,1)
h_{0102}	(1,2)	1	0	-	2	0	-



Theory – General Overview



I. Equivalence of Spectral Lines and Terms in the Generating Function

II. Term Comparison and Sextupole Resonances

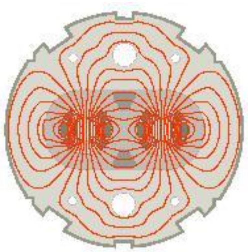
Resonance-Line Definition

Model Measuring Scheme

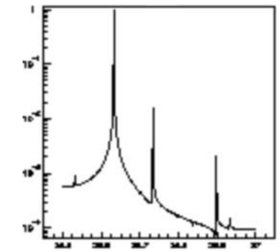
For each resonance (n,m) there is a spectrum line in the horizontal and vertical plane respectively:

- Horizontal Plane $(-[n-1], -m)$
- Vertical Plane $(-n, -[m-1])$

For example the (3,0) resonance appears as a (-2,0) line, the (1,2) resonance is found as (0,2) line, both in the horizontal spectrum. The skew resonance (0,3) is not found in the horizontal spectrum, but appears as (0,-2) in the vertical spectrum.



Theory – General Overview

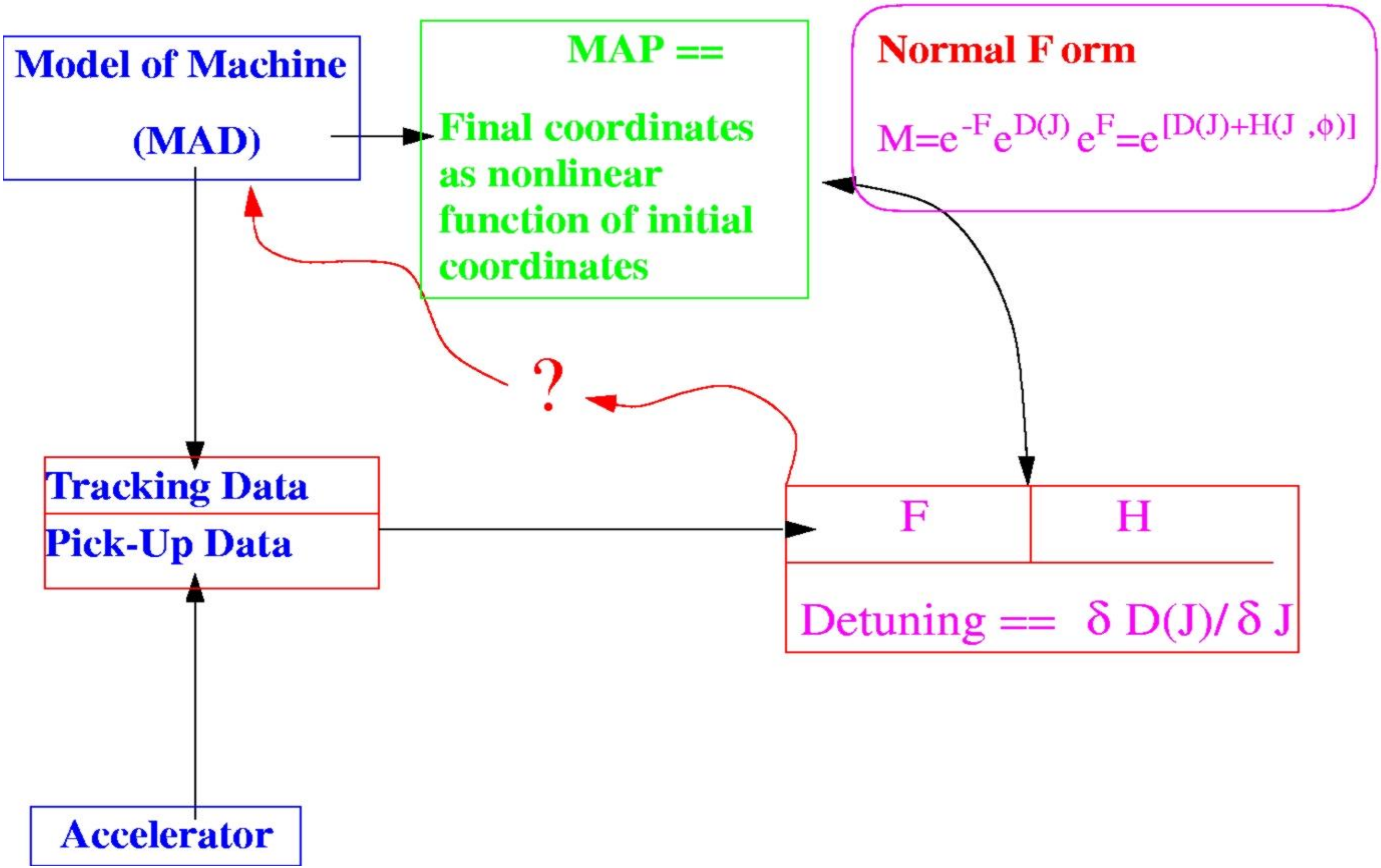


I. Equivalence of Spectral Lines and Terms in the Generating Function →

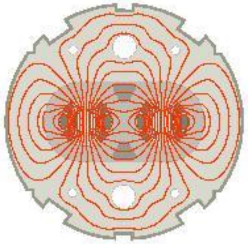
II. Term Comparison and Sextupole Resonances →

Resonance-Line Definition →

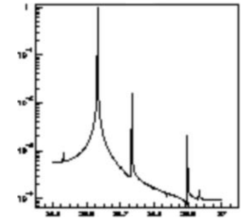
Model Measuring Scheme →



Model of Machine
(MAD)
MAP ==
Final coordinates as nonlinear function of initial coordinates
Normal F orm
$M=e^{-F}e^{D(J)}e^{F}=e^{[D(J)+H(J\ ,\phi)]}$
?
Tracking Data
Pick-Up Data
F
H
Detuning == $\delta D(J)/\delta J$
Accelerator

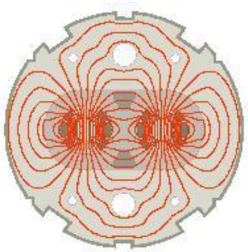


Order by Order Technique

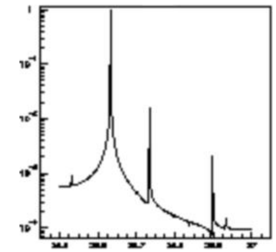


This techniques has to be operated in an order by order fashion

- First step is the transformation to the linearly normalised phase space. In the experiment this is realized by a second pick–up with 90° phase advance (see also later!).
- Then the first terms free of contribution with the same amplitude dependence are the third order terms which are due to sextupoles.
- If one has to worry about higher order multipole contributions to that resonance one has to measure these terms as a function of amplitude. The slope or change of slope will reveal the multipole at work. Reminder: This is an additional plus of the method, in principle we want just one value at small amplitude, where the lowest orders prevail.
- However, the fourth order terms are more tricky. These come from octupoles in first and sextupoles in second order of multipole strength. At first sight they seem indistinguishable. Yet, by acquiring the third order and applying to the linearly normalised coordinates the F_3 transformation $e^{-F_3} \circ (x + i\overline{x}')$ one gets rid of F_3 contributions to all orders.
- Then the fourth order terms are just due to octupoles.
- This can be taken to higher orders. It goes without saying, however, that one does not have to go through this exercise when the lower order multipoles are sufficiently weak.



New Developments



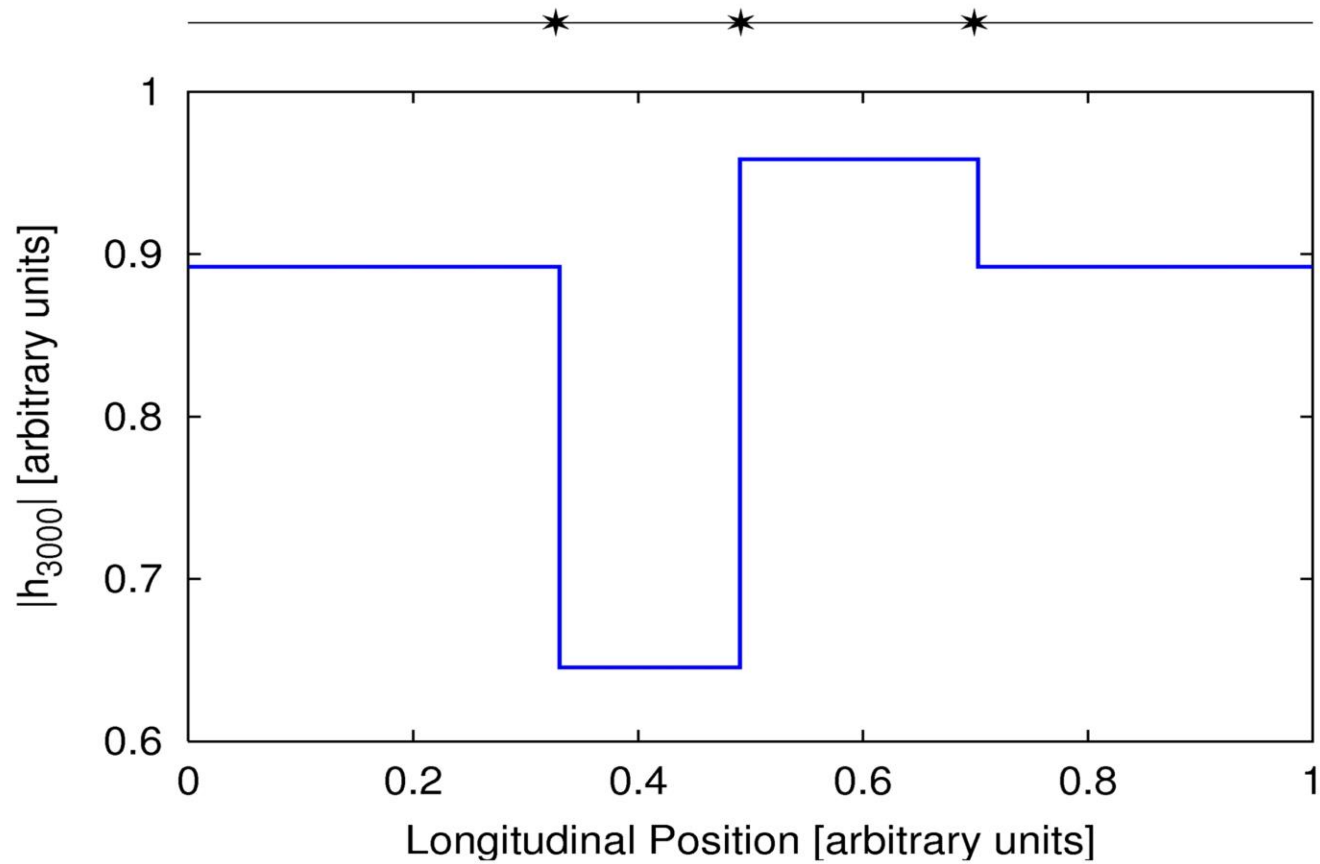
- Localization of Multipoles

- Effect of Particle Distribution → Signal reduces by **1/(m-1)** (→ derivation Rogelio's thesis)

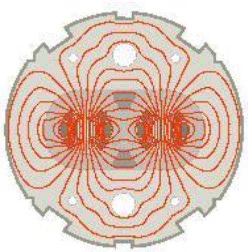
- AC-Dipole instead of Kicker (non-destructive!!)
→R. Tomás, "Normal Form of Particle Motion under the Influence of an AC Dipole", Phys. Rev. ST Accel. Beams, **5**, 54001 (2002).

Longitudinal variation of resonance terms

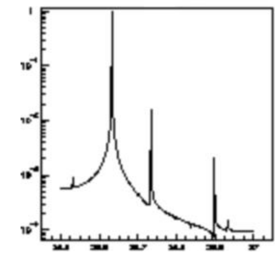
Tracking simulation of FODO lattice with 3 sextupoles:



$\Rightarrow$ Localisation of multipoles.



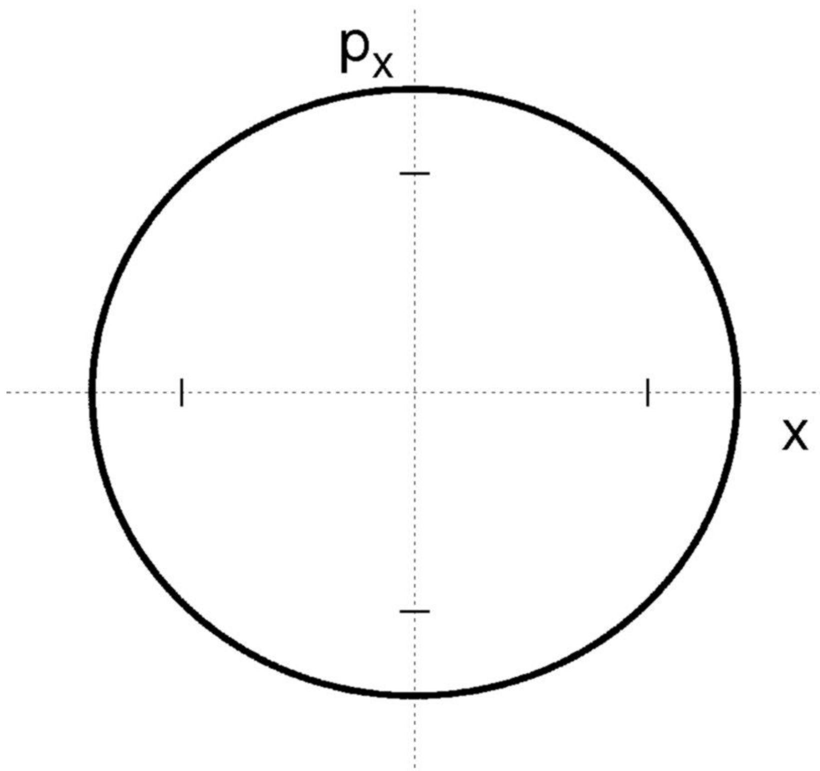
New Developments

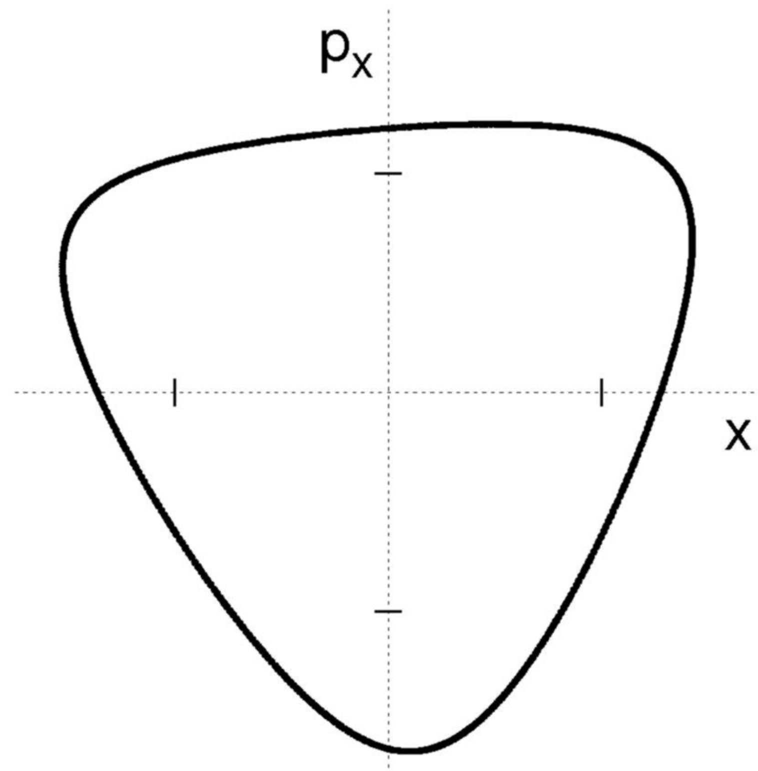


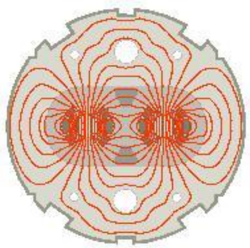
- Localization of Multipoles
- Effect of Particle Distribution ➔ Signal reduces by **1/(m-1)** (➔ derivation Rogelio's thesis)
- AC-Dipole instead of Kicker (non-destructive!!)

➔R. Tomás, "Normal Form of Particle Motion under the Influence of an AC Dipole", Phys. Rev. ST Accel. Beams, **5**, 54001 (2002).

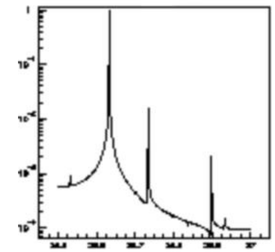
Linear motion   Non-linear motion





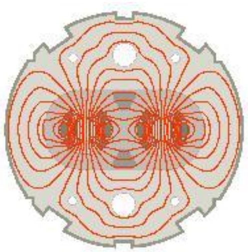


List of Possibilities

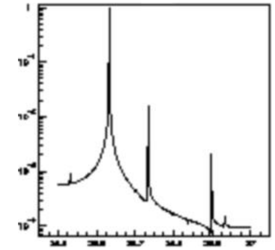


- Provide a tool for all linear and nonlinear corrections for the operation of accelerators.
- Techniques based on beam pick–up data.
- Measure longitudinal variation of linear and nonlinear quantities.
- Exploiting the spectral response of particle distributions.

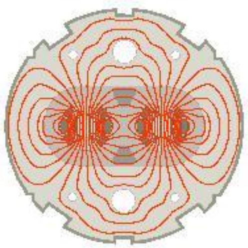
- Possible Measurements:
  - Phase Advance between Pickups
  - β–Beating
  - Linear Coupling
  - Chromaticity
  - Detuning versus Amplitude
  - Driving Terms of Resonances
  - Localisation of Multipoles
  - Full Non–Linear Model of the Accelerator



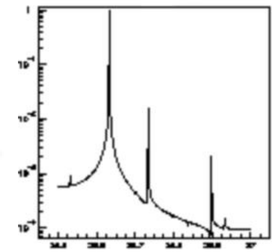
List of Limits



- Pairs of 90° pick-ups needed
- Method does not work near chaos (no surprise!)
- Detuning Terms are not directly accessible
- Higher order Terms need large Kicks, in particular due to **1/(m-1)** reduction due to decoherence (➔ AC-Dipole)
- High resolution pick-up System needed each Turn! Did not work at DESY, BNL, GSI(?) & FERMILAB! (➔ a PhD student made it work for PS booster)

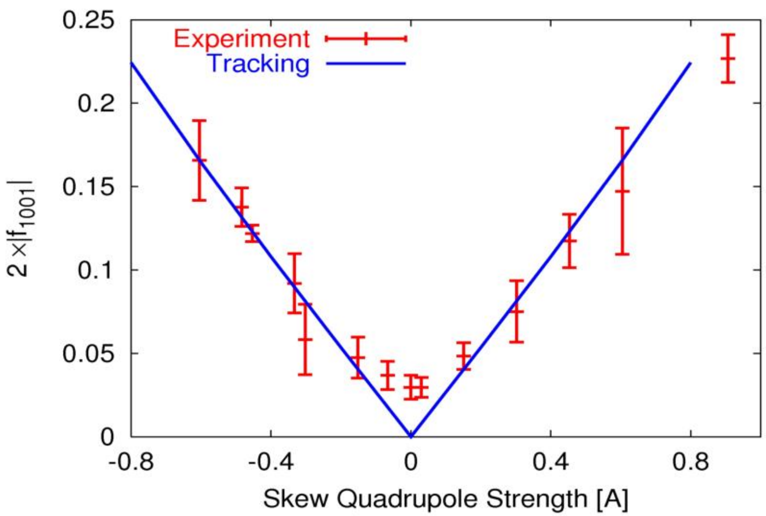


Linear Coupling Compensation



Linear Coupling in SPS (2001)

The coupling term $|f_{1001}|$ is plotted as function of the strength of the skew quadrupoles:



⇒Model and experiment are in excellent agreement.

⇒This shows that SPS is decoupled in this particular case.

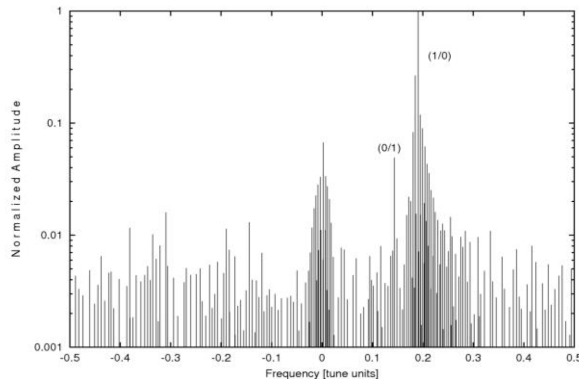
PS Booster

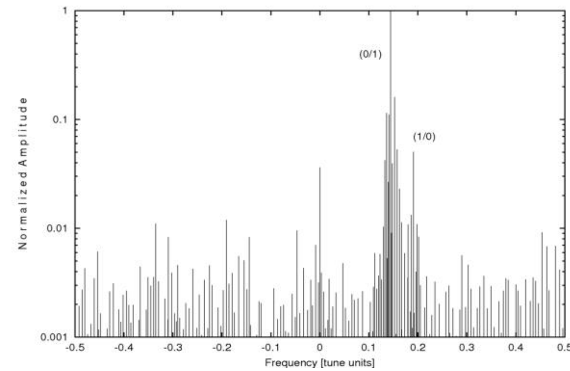
Results 2003 – linear coupling Q_x-Q_y=-1

Fourier spectra for the bare machine.

$|h_{1001}| = 7.1 \pm 0.1 * 10^{-3}$

$\psi_{1001} = 282.8^{o} \pm 5.2^{o}$



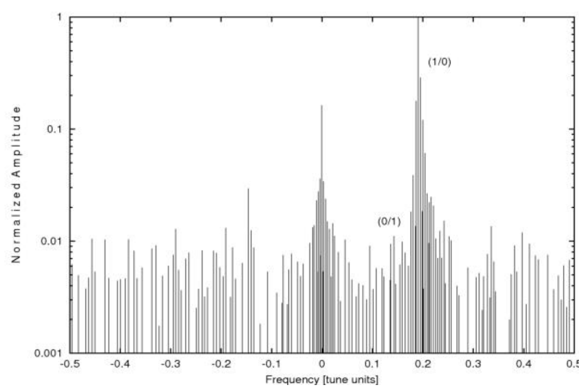


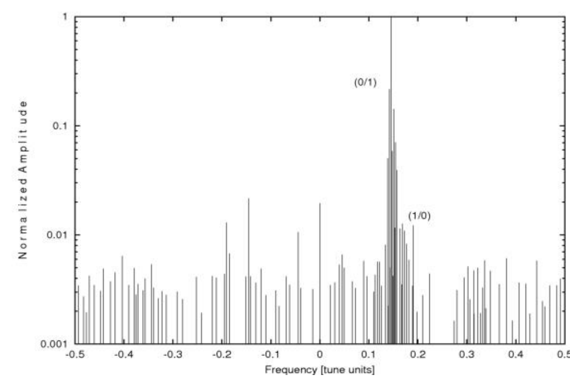
corresponding resonance spectral lines: (0/1) resp. (1/0)

Fourier spectra with calculated compen-sation currents on.

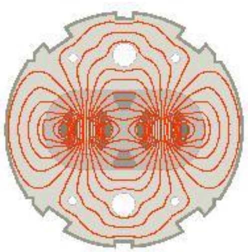
$I_{QSK210L3} = +3.6A$, $I_{QSK614L3} = +1.2A$

(QSK…skew quadrupoles)

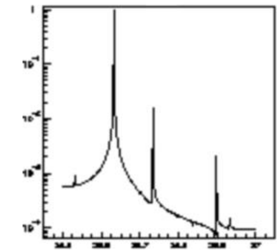




Fourier spectra of horizontal and vertical phase space



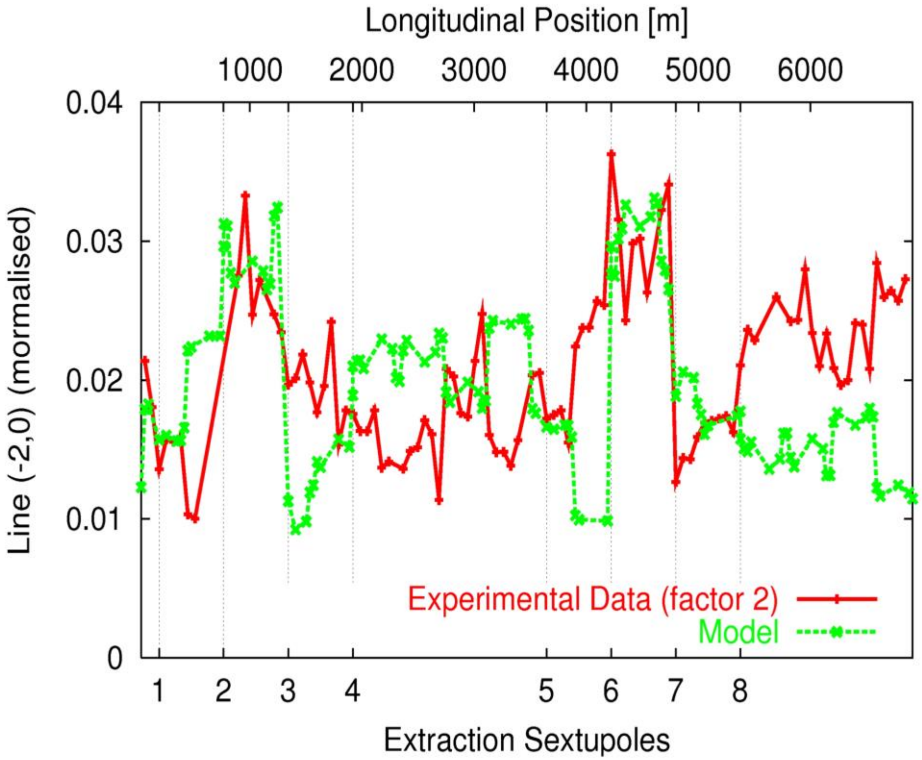
Sextupole Resonances



- SPS
  - Polarity Problem
  - Sextupole Failure
  - Measure Coefficient
  - Closed Orbit
- PS Booster

Sextupolar driving terms in SPS (2000)

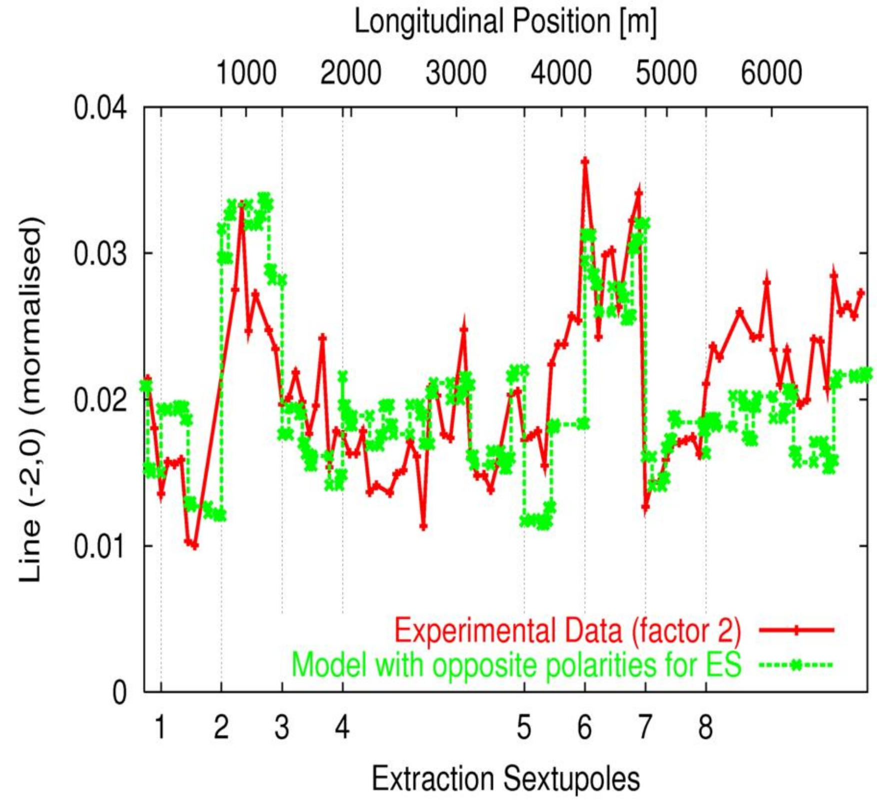
The resonance (3,0) introduces the spectral line (-2,0).



⇒ We have a problem!

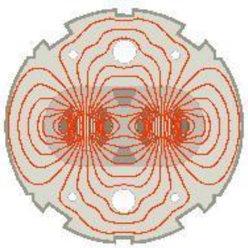
Solution

Change polarities of the extraction sextupoles?

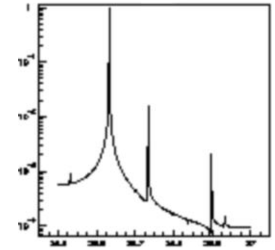


Hardware checks confirmed that these sextupoles had opposite polarities.

⇒ First success of this technique!



Sextupole Resonances



- SPS
  - Polarity Problem
  - Sextupole Failure
  - Measure Coefficient
  - Closed Orbit
- PS Booster

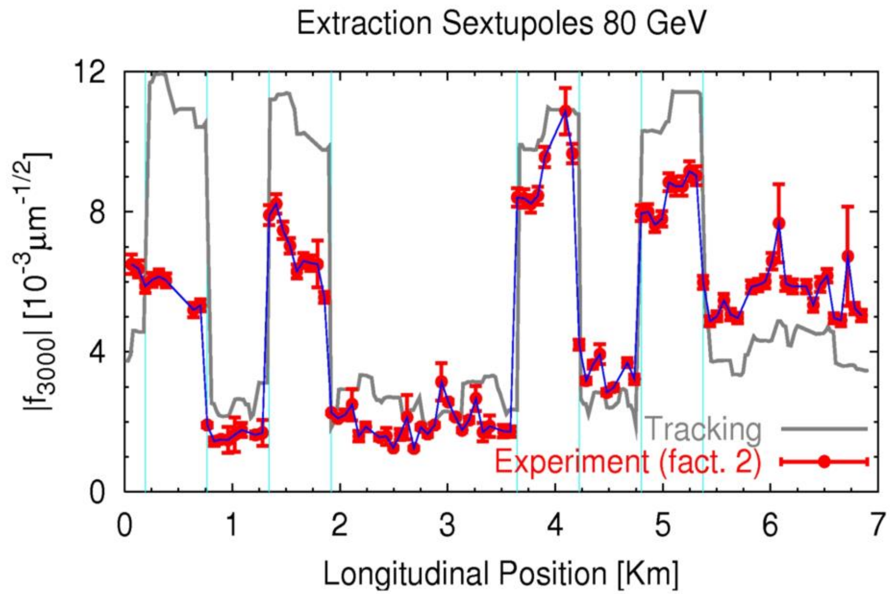


Figure 7.26: Amplitude of the term f_{3000} versus longitudinal position from experiment and tracking model for the SPS with extraction sextupoles powered to $(++++-----)$100 A at 80 GeV. The blue line is used to connect the experimental points. The vertical lines denote the position of the extraction sextupoles.

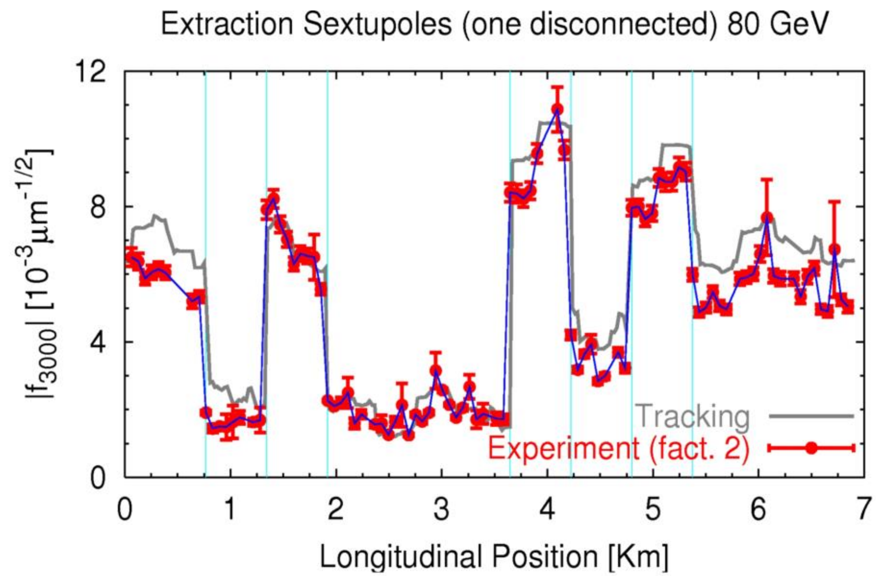
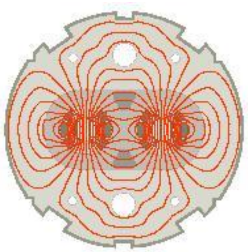
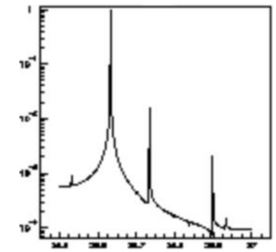


Figure 7.27: Amplitude of the term f_{3000} versus longitudinal position from experiment and tracking model for SPS with one sextupole disconnected $(+++-----)$ 100 A at 80 GeV. The blue line is used to connect the experimental points. The vertical lines denote the position of the extraction sextupoles. Note that there is one less sextupole with the polarity "+".



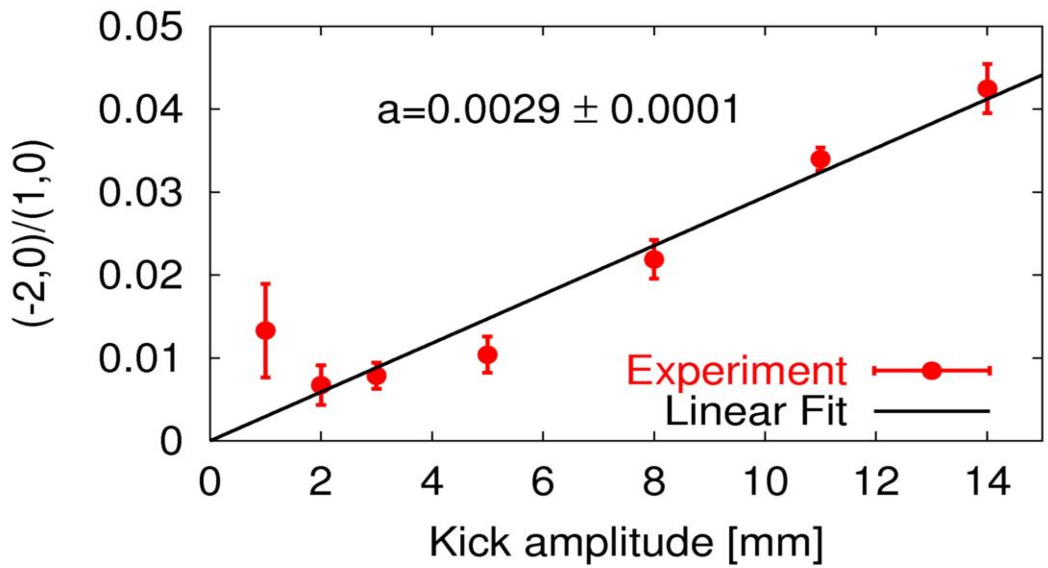
Sextupole Resonances



- SPS
  - Polarity Problem
  - Sextupole Failure
  - Measure Coefficient
  - Closed Orbit
- PS Booster

Line to Resonance conversion

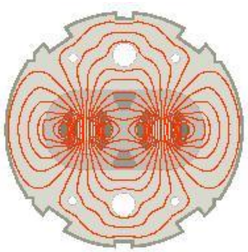
The resonance (3,0) is driven by the deformation term f_{3000} and produces the spectral line (-2,0).



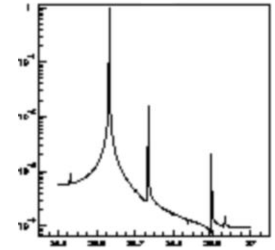
$$|f_{3000}| = \frac{1}{6} a \sqrt{\beta_e} \ [\sqrt{\mu m}]$$

$$\sqrt{\beta_e} = 10.6 \ [\sqrt{m}]$$

$\Rightarrow$ $|f_{3000}|$ is obtained around the ring by doing this fit for all the pick-ups.



Sextupole Resonances



- SPS
  - Polarity Problem
  - Sextupole Failure
  - Measure Coefficient
  - Closed Orbit
- PS Booster

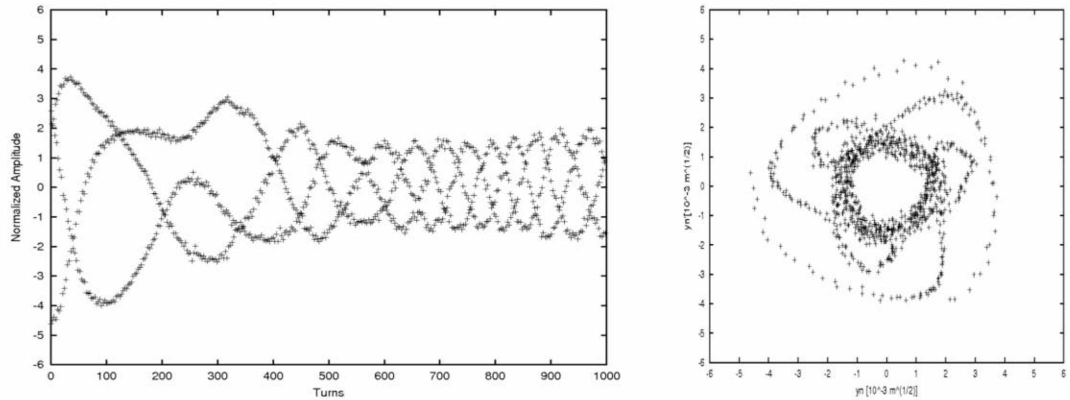
Results 2003 – systematic $3Q_y=16$

Situation for the bare machine.

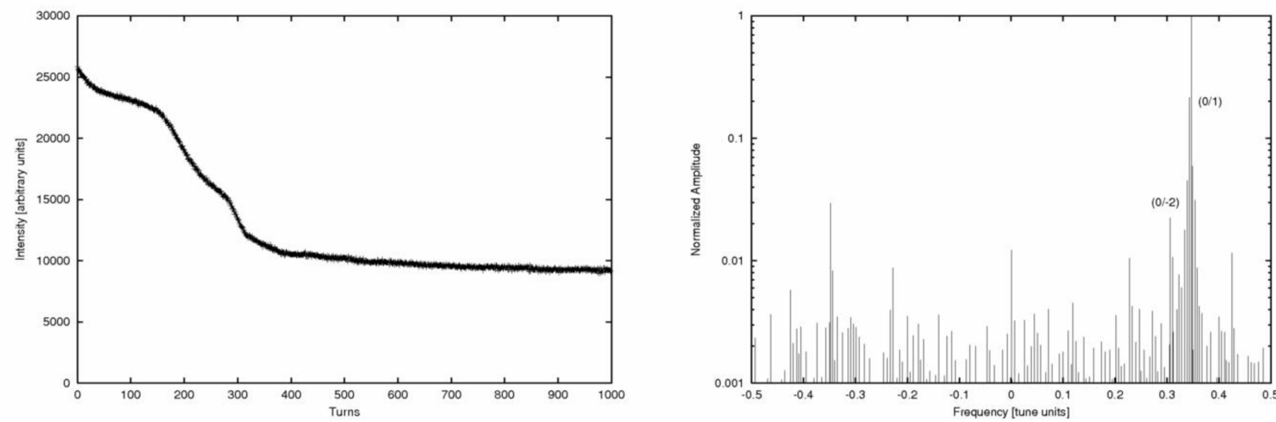
$|h_{0030}| = 9.0 \pm 0.6$*

$10^{-3}\ \mathrm{mm}^{-1/2}$

$\psi_{0030} = -21.4^{\circ} \pm 13.9^{\circ}$



Vertical beam position and phase space



Intensity curve and Fourier spectrum (res. line: (0/-2))

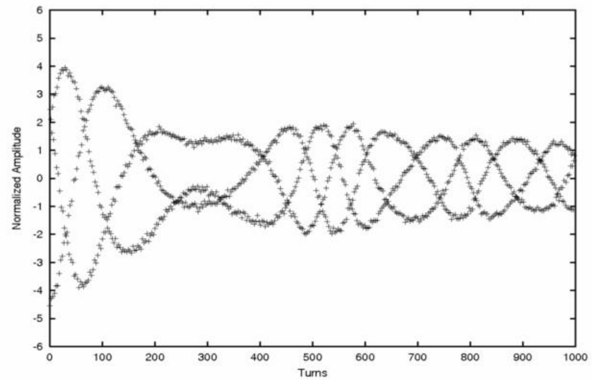
Results 2003 – $3Q_y=16$

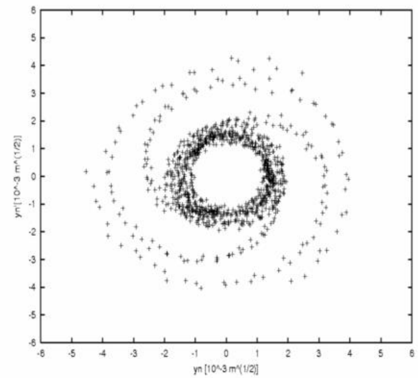
Resonance compensated with the calculated currents:

$I_{XSK2L4} = -12.3A$,

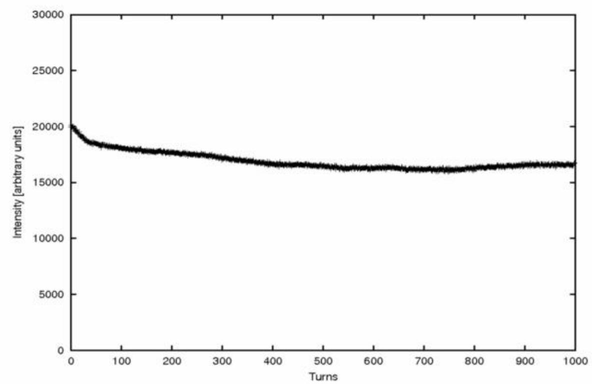
$I_{XSK9L1} = +15.3A$

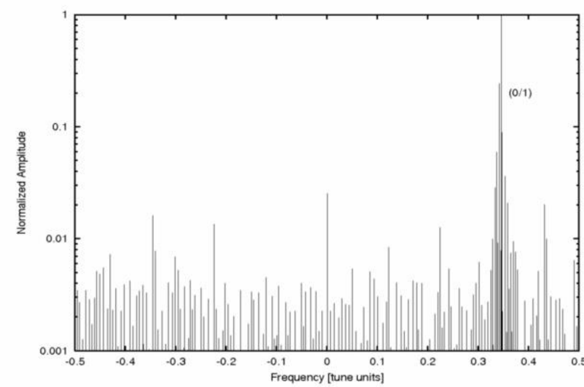
(XSK…skew sextupoles)



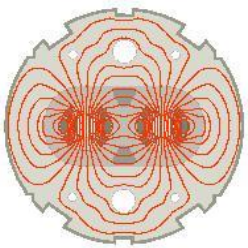


Vertical beam position and phase space

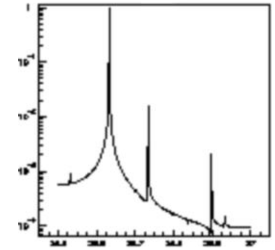




Intensity curve and Fourier spectrum

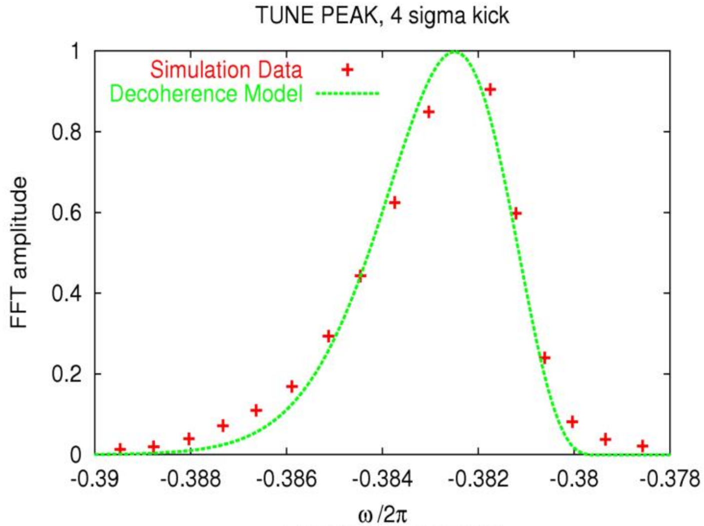


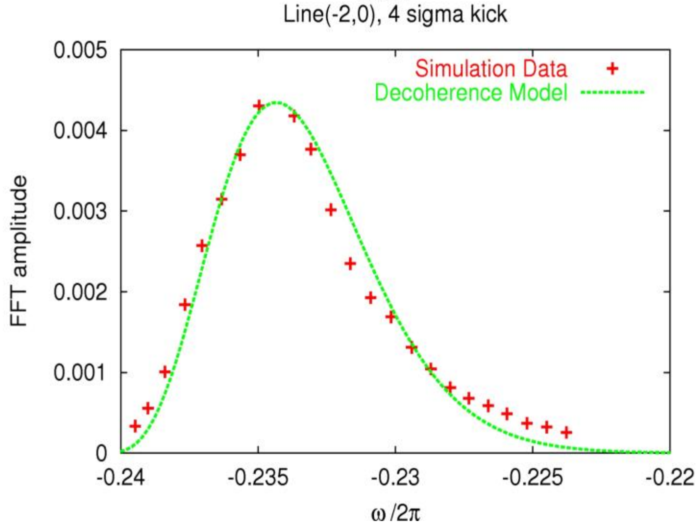
Exotic Applications



- Single 4σ Kick does everything!!
- AC-Dipole idea great → lacks proof of applicability!!
- Chromaticity Measurement (sign also!)
  In simulation works with simultaneous transverse and longitudinal kick → experiment remains sketchy

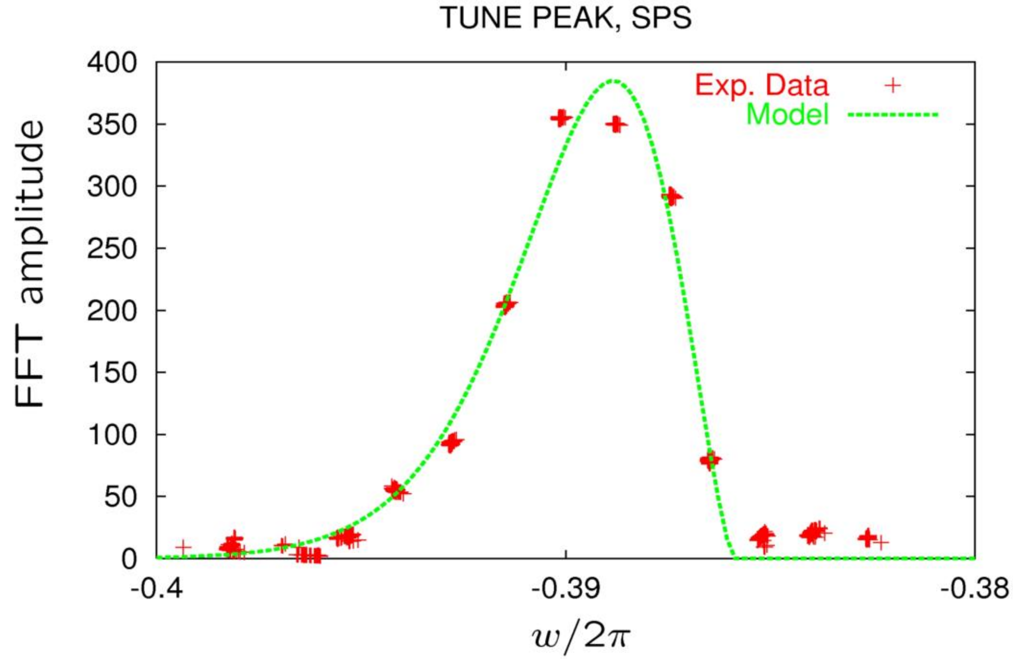
Model versus simulation





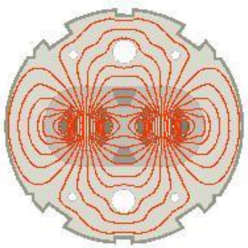
Application: SPS 120 GeV (2000)

Fitting tune line distribution:

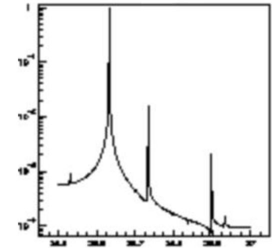


Observable	Fit result	Classic Meth.
Q_{x0}	$-0.38593 \pm 5 \cdot 10^{-5}$	-0.3859
$(\Delta Q_x)_{1\sigma_x}$	$(-3.6 \pm 0.1) \cdot 10^{-4}$	$-3.71 \cdot 10^{-4}$
$\sigma_x[mm/\sqrt{\beta_x}]$	0.266 ± 0.008	

$\Rightarrow$ Measurement of these observables from a single kick



Commissioning of the LHC



- **Polarity Checks & Correction Steering both Beam-based**
- **Mostly for Injection and b_3 Spool Piece Correction**
- **Higher Orders and Top Energy *not* excluded**
- **Need turn-by-turn BPM System on *Day 1*, which means low Failure Rate for all BPMs each Turn!!**