Studies on Lattice Calibration With Frequency Analysis of Betatron Motion

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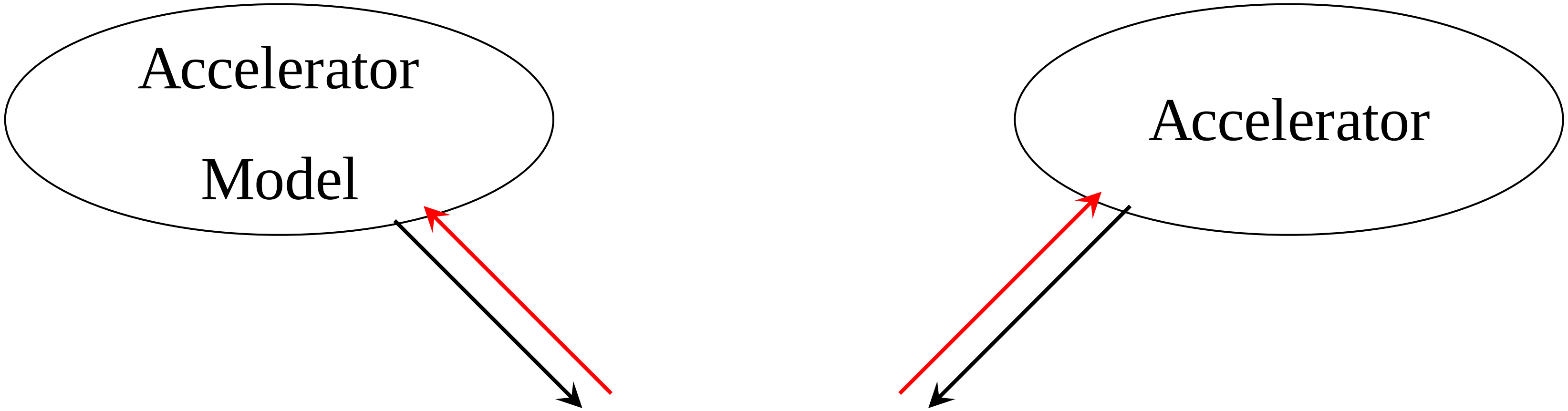
FMA workshop, Orsay, LURE, 1st and 2nd April 2004

- Introduction
  - NAFF
  - perturbative theory of betatron motion
  - SVD fit of lattice parameters

- DIAMOND Spectral Lines Analysis
  - Linear Model
    - β – beating
    - linear coupling
  - Nonlinear Model

Real Lattice to Model Comparison



- Closed Orbit Response Matrix (LOCO–like)
- Frequency Map Analysis
- Frequency Analysis of Betatron Motion (resonant driving terms)

Frequency Analysis of Betatron Motion and Lattice Model Reconstruction (1)

Accelerator Model

- tracking data at all BPMs
- spectral lines from model (NAFF)
- build a vector of Fourier coefficients

Accelerator

- beam data at all BPMs
- spectral lines from BPMs signals (NAFF)
- build a vector of Fourier coefficients

$$\overline{A} = \left(a_1^{(1)} \quad \ldots \quad a_{NBPM}^{(1)} \quad \varphi_1^{(1)} \quad \ldots \quad \varphi_{NBPM}^{(1)} \quad a_1^{(2)} \quad \ldots \quad a_{NBPM}^{(2)} \quad \varphi_1^{(2)} \quad \ldots \quad \varphi_{NBPM}^{(2)} \quad \ldots \right)$$

Define the distance between the two vector of Fourier coefficients

$$\chi^2 = \sum_k \left(A_{Model}(j) - A_{Measured}(j) \right)^2$$

Frequency Analysis of Betatron Motion and Lattice Model Reconstruction (2)

Least Square Fit (SVD) of accelerator parameters θ

to minimize the distance χ^2 of the two Fourier coefficients vectors

- Compute the "Sensitivity Matrix" M $\quad \Delta\bar{A} = M\bar{\theta}$
- Use SVD to invert the matrix M $\quad M = U^T W V$
- Get the fitted parameters $\quad \bar{\theta} = (V^T W^{-1} U)\,\Delta\bar{A}$

MODEL → TRACKING → NAFF →

Define the vector of Fourier Coefficients – Define the parameters to be fitted

SVD → CALIBRATED MODEL

DIAMOND Layout

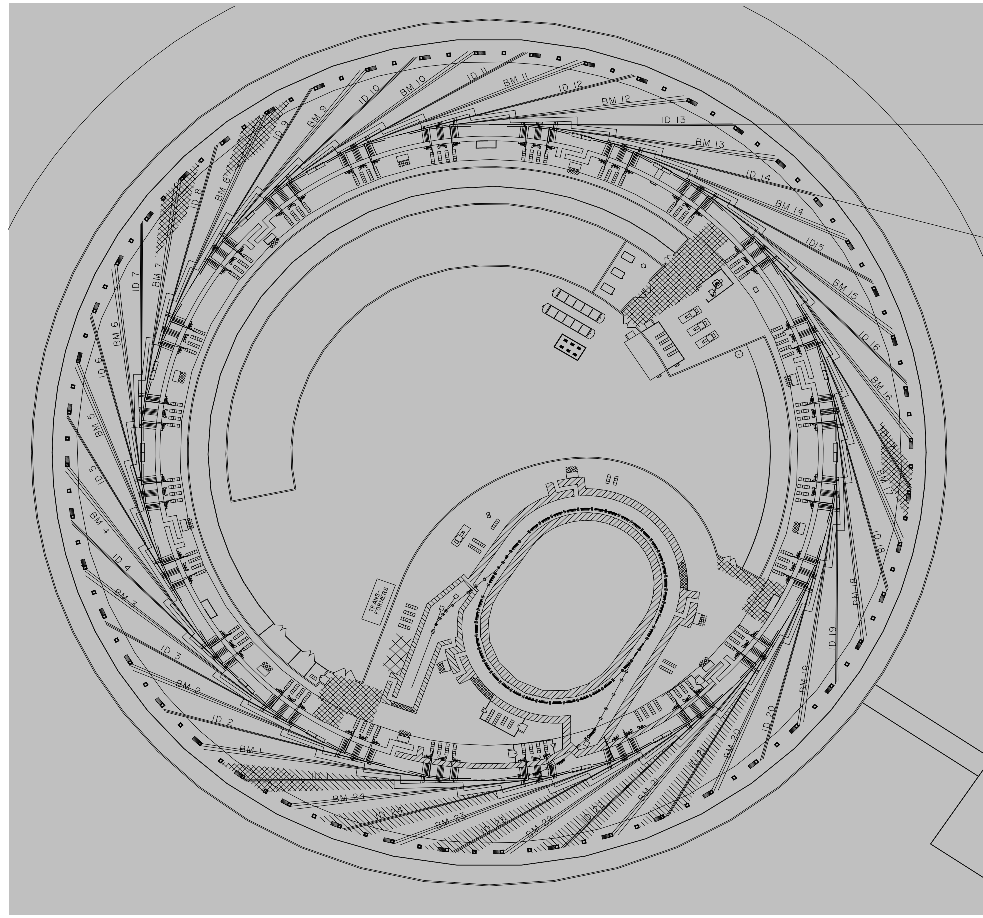
Main parameters:

100 MeV Linac

3 GeV Booster (158.4 m)

3 GeV Storage Ring (561.6 m)

- 24 cell DBA lattice
- 2 + 1 SC RF cavities
- 18 straight for ID (5 m)
- 6 long straights (8 m)



Commissioning end 2006

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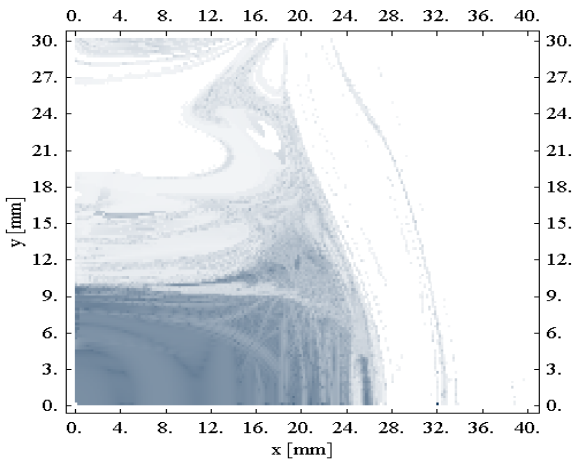
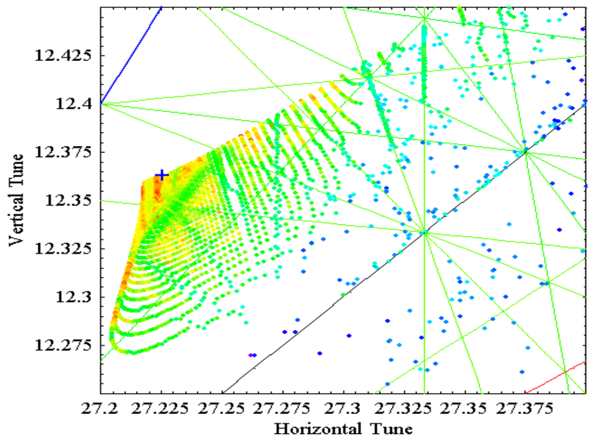
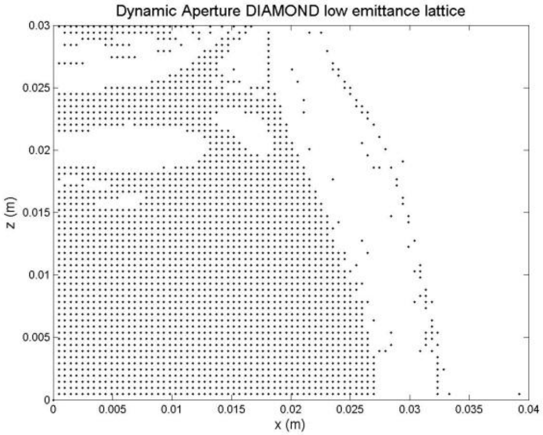
A comparison on tracking between MAD and AT: FMA and DA for DIAMOND

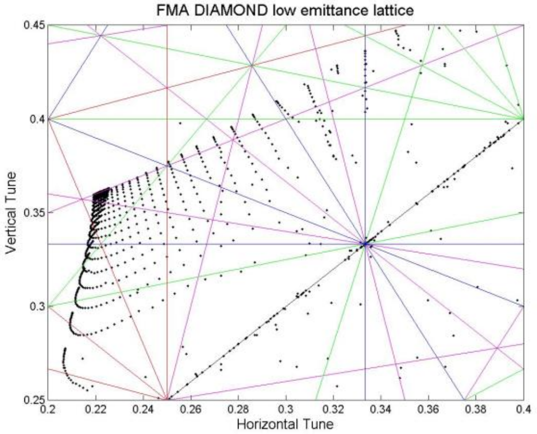




- MAD provided tracking data
- FMA built with Mathematica:

H.L. Owen and J.K. Jones, AP–SR–REP–072, (2002)





- AT provided tracking data
- FMA built with MATLAB

NAFF algorithm – J. Laskar (1988)
(Numerical Analysis of Fundamental Frequencies)

Given the quasi–periodic time series of the particle orbit ($x(n)$; $p_x(n)$),

- Find the main lines with the previous technique for tune measurement

  $\Rightarrow$ ν_1 frequency, a_1 amplitude, ϕ_1 phase;

- build the harmonic time series

$$z_1(n) = a_1 e^{i\phi_1} e^{2\pi i \nu_1 n}$$

- subtract form the original signal
- analyze again the new signal $z(n) - z_1(n)$ obtained

The decomposition $z(n) = \sum_{k=1}^{n} a_k e^{i\phi_k} e^{2\pi i \nu_k n}$ **allows the**

Measurement of Resonant driving terms of non linear resonances

Frequency Analysis of Non Linear Betatron Motion

A.Ando (1984), J. Bengtsson (1988), R.Bartolini-F. Schmidt (1998)

The quasi periodic decomposition of the orbit

$$x(n)-ip_x(n)=\sum_{k=1}^{n} c_k e^{2\pi i \nu_k n} \qquad c_k = a_k e^{i\phi_k}$$

can be compared to the perturbative expansion of the non linear betatron motion

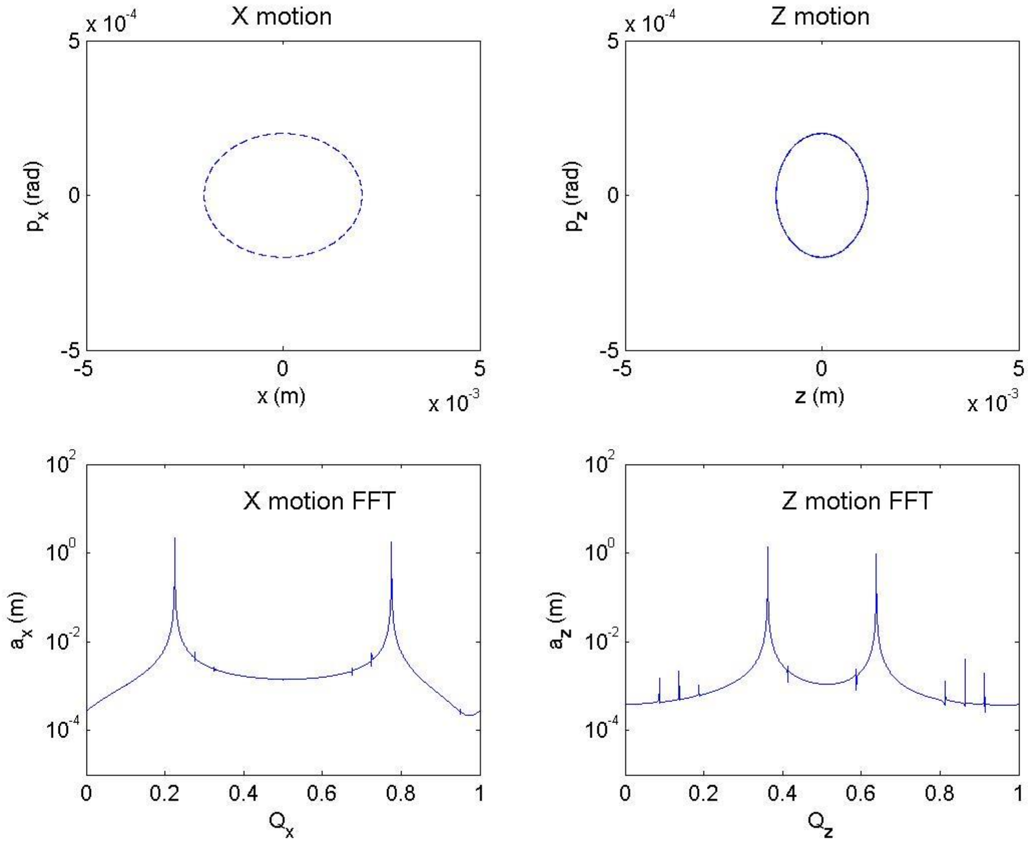
$$x(n)-ip_x(n)=\sqrt{2I_x}e^{i(2\pi Q_x n+\psi_0)} +$$

$$-2i\sum_{jklm} js_{jklm}(2I_x)^{\frac{j+k-1}{2}}(2I_y)^{\frac{l+m}{2}}e^{i\left[(1-j+k)(2\pi Q_x n+\psi_{x0})+(m-l)(2\pi Q_y n+\psi_{y0})\right]}$$

Each resonance driving term s_{jklm} contributes to the Fourier coefficient of a well precise spectral line

$$\nu(s_{jklm})=(1-j+k)Q_x+(m-l)Q_y$$

Spectral Lines for DIAMOND low emittance lattice (.2 mrad kick in both planes)



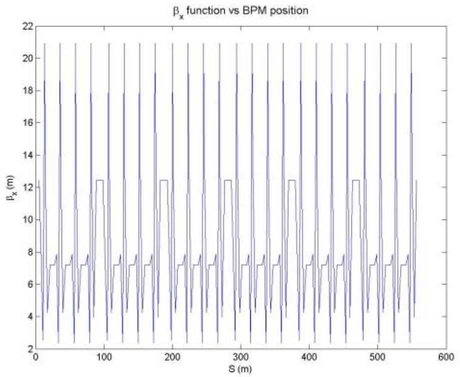
Spectral Lines detected with NAFF algorithm

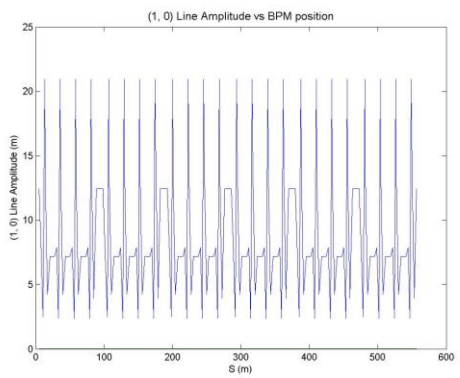
e.g. Horizontal:

- (1, 0) $1.10\ 10^{-3}$ horizontal tune
- (0, 2) $1.04\ 10^{-6}$ $Q_x - 2\,Q_z$
- (–3, 0) $2.21\ 10^{-7}$ $4\,Q_x$
- (–1, 2) $1.31\ 10^{-7}$ $2\,Q_x + 2\,Q_z$
- (–2, 0) $9.90\ 10^{-8}$ $3\,Q_x$
- (–1, 4) $2.08\ 10^{-8}$ $2\,Q_x + 4\,Q_z$

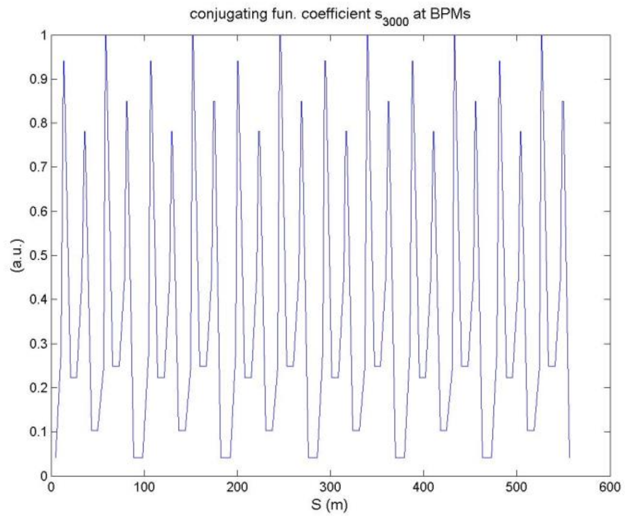
Amplitude of Spectral Lines for low emittance DIAMOND lattice computed at all the BPMs

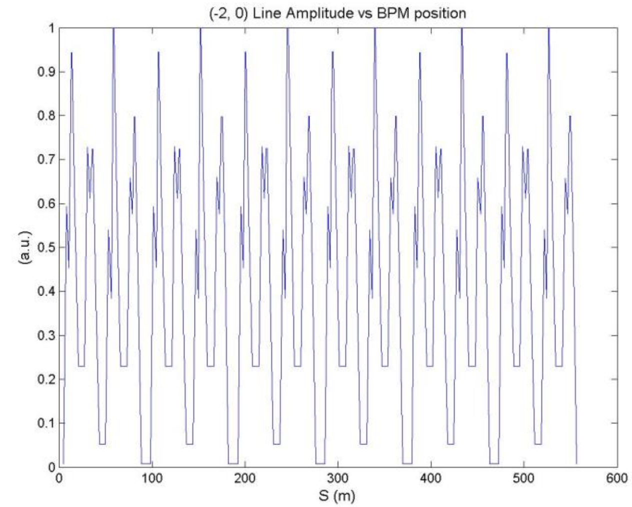
Main spectral line (Tune Q_x)





(-2, 0) spectral line: resonance driving term h_{3000} ($3Q_x = p$) at all BPMs





- The amplitude of the tune spectral line replicates the β functions
- The amplitude of the (– 2, 0) show that third order resonance is well compensated within one superperiod. Some residual is left every two cells ($5\pi/2$ phase advance)

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Example: DIAMOND with random misalignments (100 μm r.m.s) in chromatic sextupoles to generate linear coupling

The coupled linear motion in each plane can be written in terms of the coupling matrix

$$C = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}$$

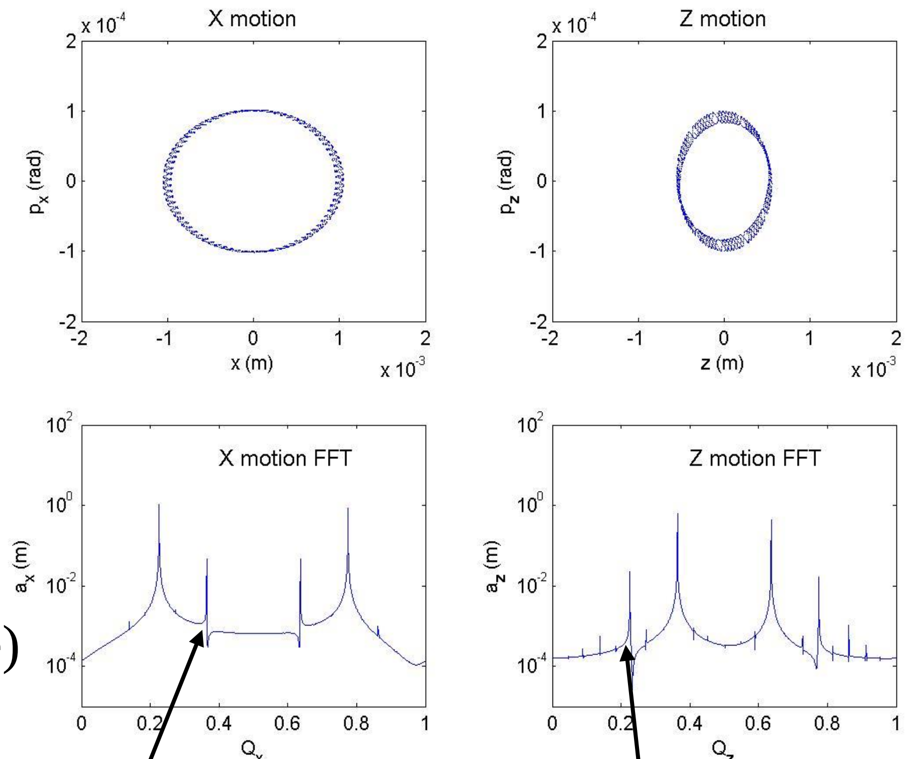
e.g. for the horizontal motion

$$\zeta_x = x - ip_x = a_1 e^{i(\phi_u + \delta_u)} + a_2 e^{-i(\phi_u + \delta_u)} + $$
$$ + a_3 e^{i(\phi_v + \delta_v)} + a_4 e^{-i(\phi_v + \delta_v)}$$

a_3 and a_4 depend linearly on c_{ij}

- two frequencies (the H tune and V tune)
- no detuning with amplitude

0.1 mrad kick in both planes



Tune Z in horizontal motion

Tune X in vertical motion

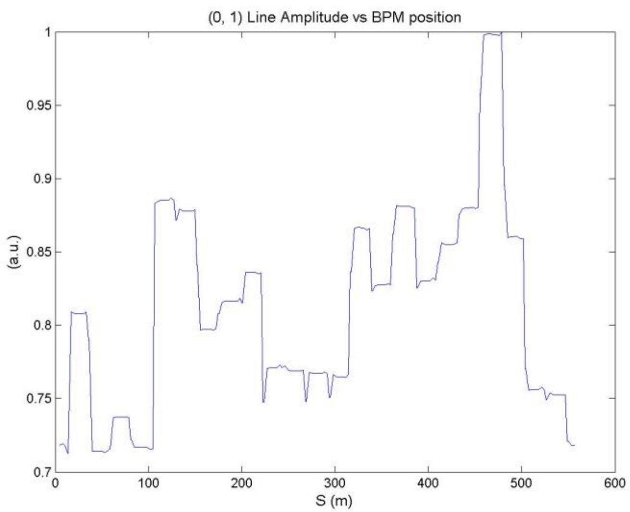
(0,1) spectral line for low emittance DIAMOND lattice computed at all the BPMs (V misalignment errors added to chromatic sextupoles)

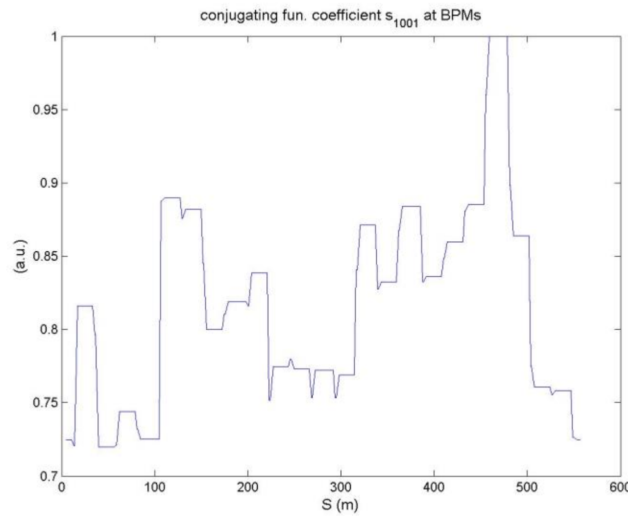
$$h_{1001p} = \frac{1}{2\pi}\int_0^{2\pi R} a_2(s)\left(\frac{\beta_x}{2}\right)^{1/2}\left(\frac{\beta_z}{2}\right)^{1/2} e^{i(W_x(s)-W_z(s))-i\frac{ps}{R}}ds$$

$$s_{1001p} = \frac{ih_{1001p}}{Q_x - Q_z + p} \qquad s_{1001}(s) = \frac{1}{2\pi R}\sum_{p=-\infty}^{\infty} s_{1001p}e^{ips/R}$$

The resonance driving term h_{1001} contributes to the (0, 1) spectral line in horizontal motion

from analytical formula





from spectral line

The amplitude of the (0, 1) spectral line replicates well the s dependence of the difference resonance $Q_x - Q_z$ driving term

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DIAMOND Spectral Lines Analysis

- Horizontal Misalignment of sextupoles (β – beating)
- Vertical Misalignment of sextupoles (linear coupling)
- Gradient errors in sextupoles (non linear resonances)

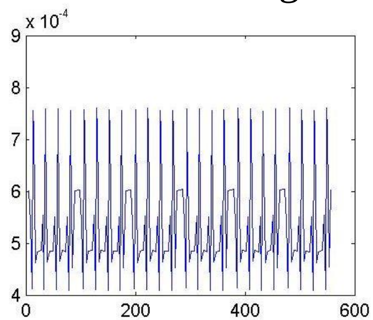
Horizontal misalignment of a set of 24 sextupoles with 100 μm rms (β - beating correction)

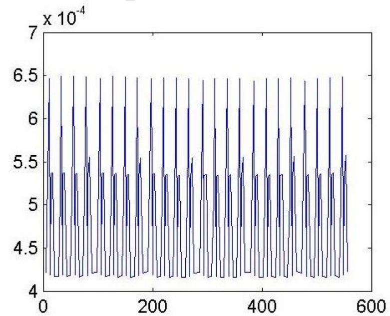
The generated normal quadrupole components introduce a β - beating.

- we build the vector of Fourier coefficients of the horizontal and vertical tune line
- we use the horizontal misalignments as fit parameters

H tune line

(no misalignments)



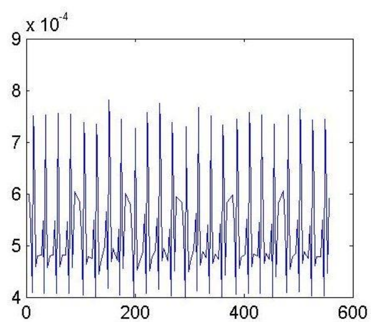


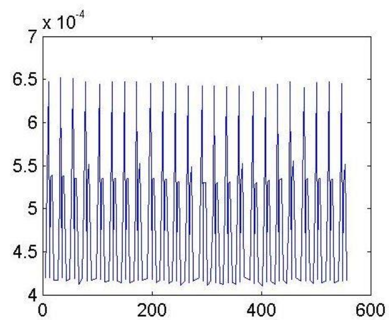
V tune line

(no misalignments)

H tune line

with misalignments





V tune line

with misalignments

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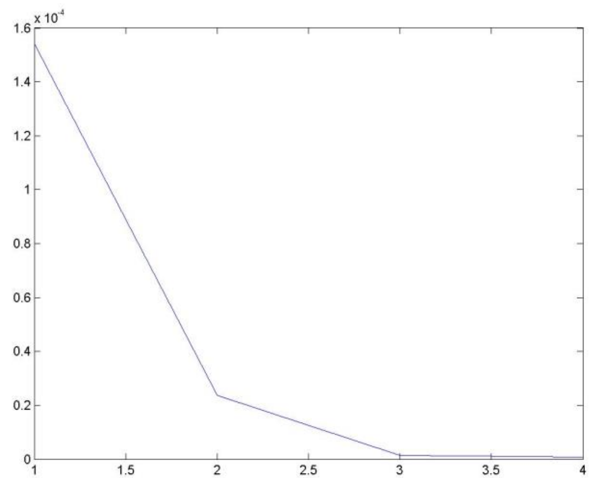
SVD on sextupoles horizontal misalignments

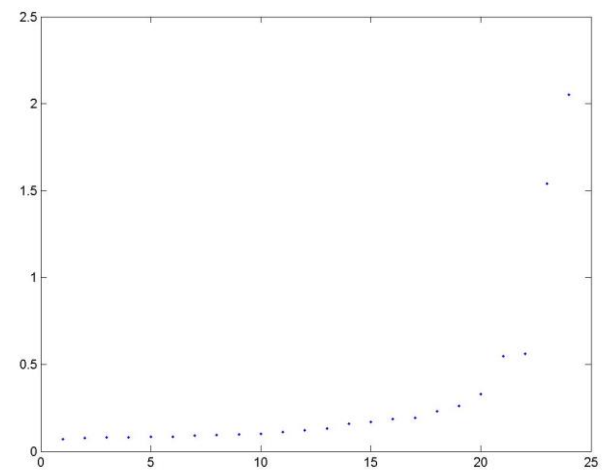
We build the vector $\overline{A} = \left(a_1^{H(1,0)} \quad \ldots \quad a_{NBPM}^{H(1,0)} \quad a_1^{V(0,1)} \quad \ldots \quad a_{NBPM}^{V(0,1)} \right)$

containing the amplitude of the tune lines in the two planes at all BPMs

We minimize the sum $\chi^2 = \sum_j \left(A_{Model}(j) - A_{Measured}(j) \right)^2$



χ^2 as a function of the iteration number

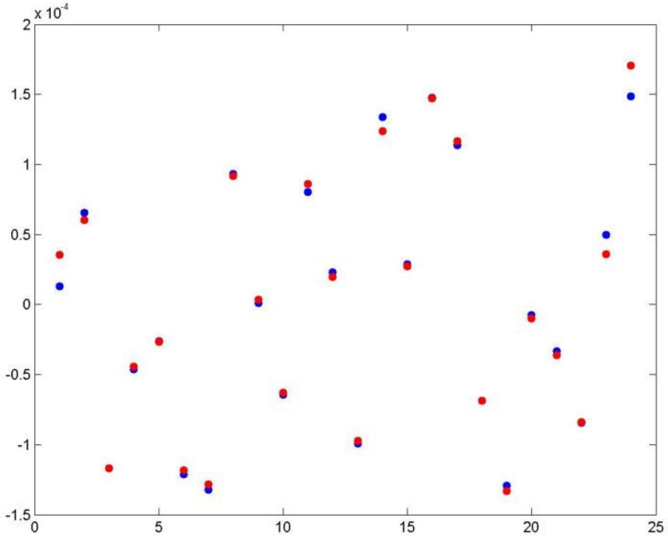


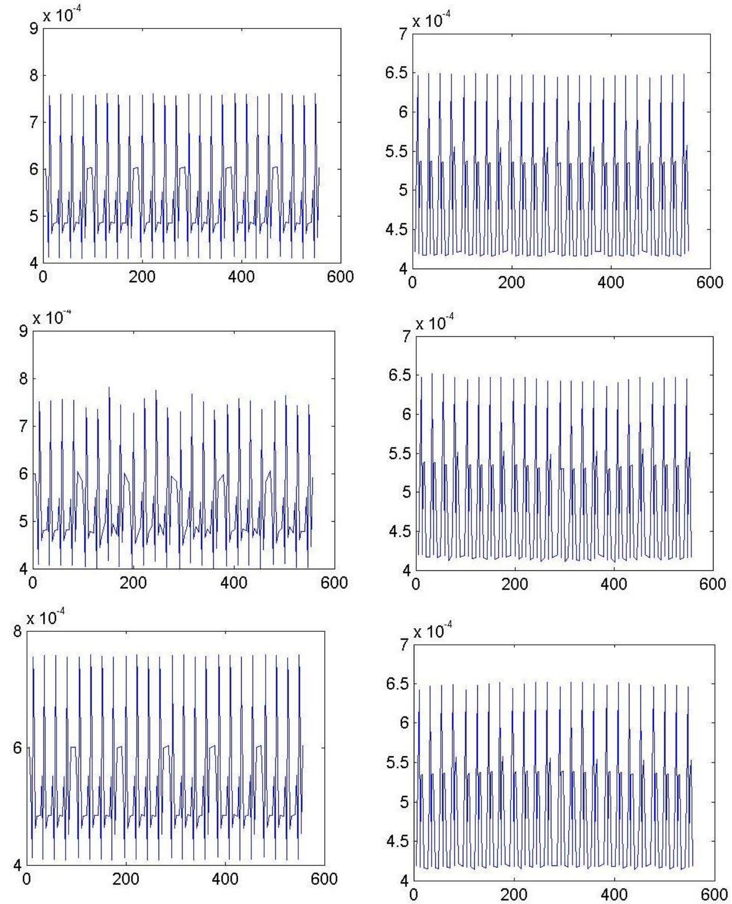
Example of SVD principal values

Fitted values for the 24 horizontal sextupole misalignments obtained from the SVD

Blu dots = assigned misalignments

Red dots = reconstructed misalignments





no misalignments

with misalignments

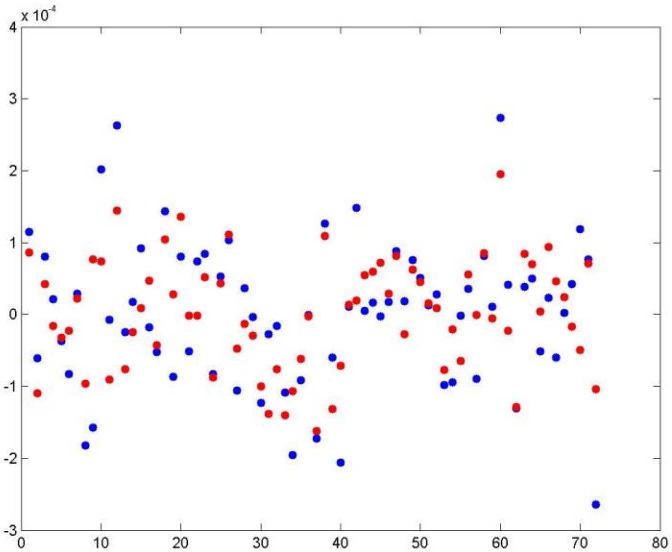
with misalignments and corrections

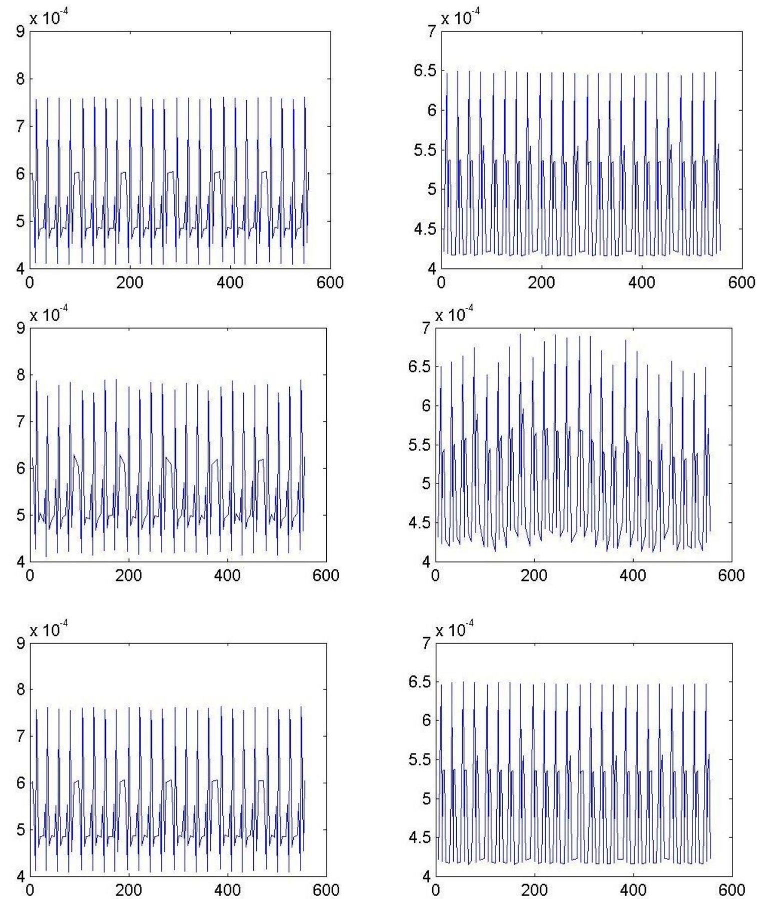
diamond

Fitted values for the 72 horizontal sextupole misalignments obtained from the SVD

Blu dots = assigned misalignments

Red dots = reconstructed misalignments





no

misalignments

with

misalignments

with

misalignments
and corrections

diamond

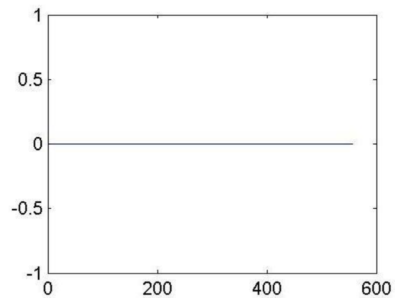
Vertical misalignment of a set of 24 sextupoles with 100 μm rms (linear coupling correction)

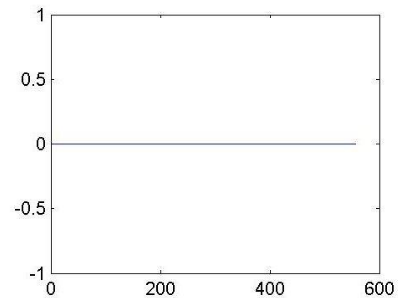
The generated skew quadrupole components introduce a linear coupling.

- we build the vector of Fourier coefficients of the (0, 1) line in the H plane
- we use the vertical misalignments as fit parameters

(0,1) line amplitude in H plane

(no misalignments)



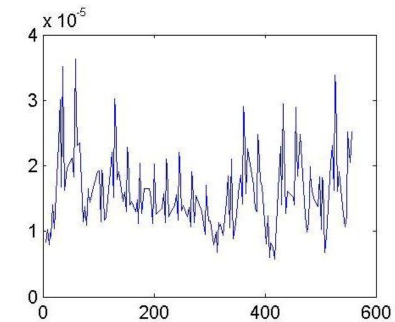


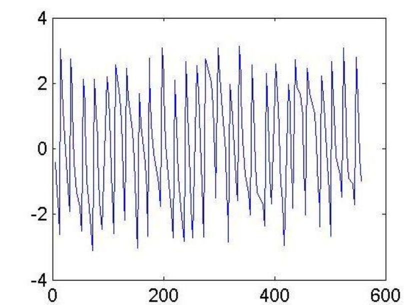
(0,1) line phase in H plane

(no misalignments)

(0,1) line amplitude in H plane

with misalignments





(0,1) line phase in H plane

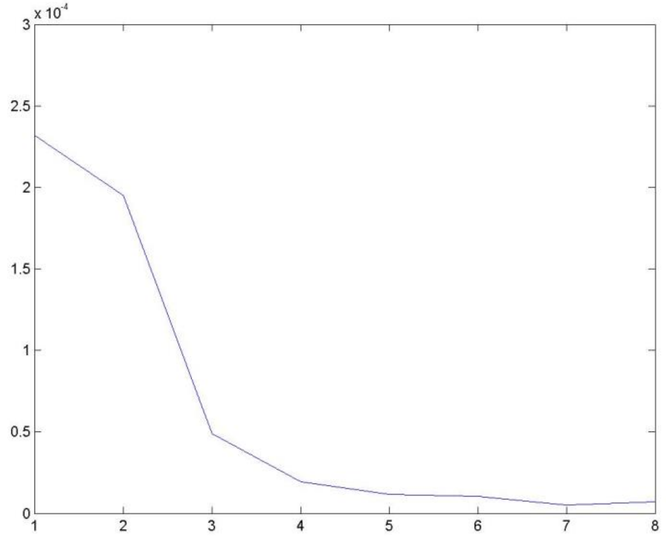
with misalignments

SVD on sextupole vertical misalignments

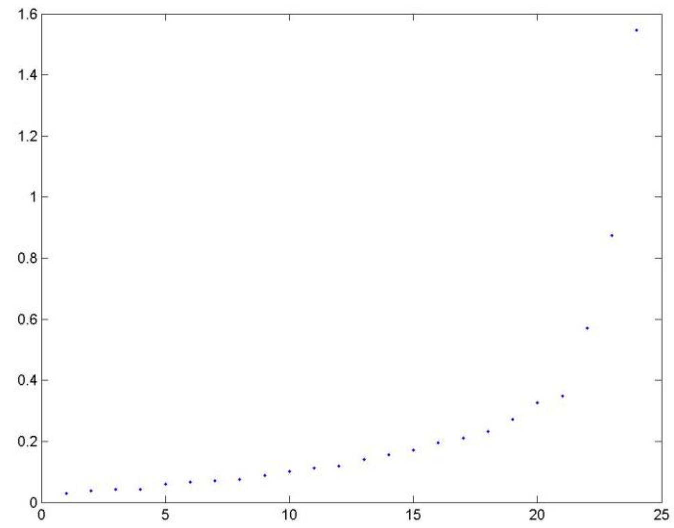
We build the vector $\overline{A} = \left(a_1^{H(0,1)} \quad \ldots \quad a_{NBPM}^{H(0,1)} \quad \phi_1^{H(0,1)} \quad \ldots \quad \phi_{NBPM}^{H(0,1)} \right)$

containing the amplitude and phase of the (0, 1) line in the H planes at all BPMs

We minimize the sum $\chi^2 = \sum_j \left(A_{Model}(j) - A_{Measured}(j) \right)^2$



χ^2 as a function of the iteration number

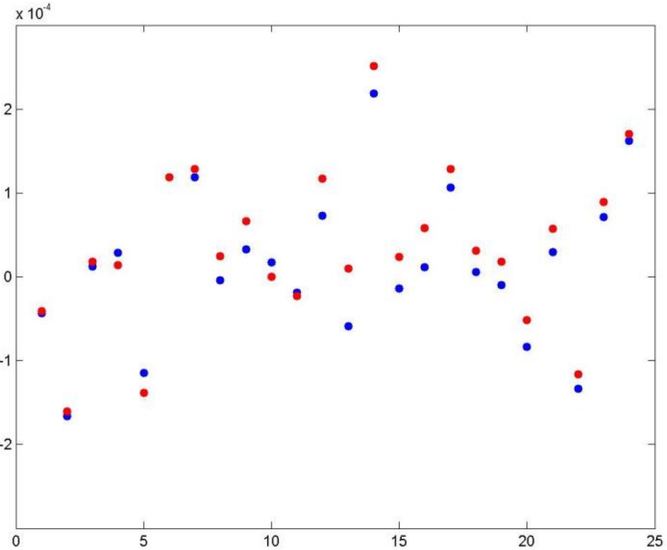


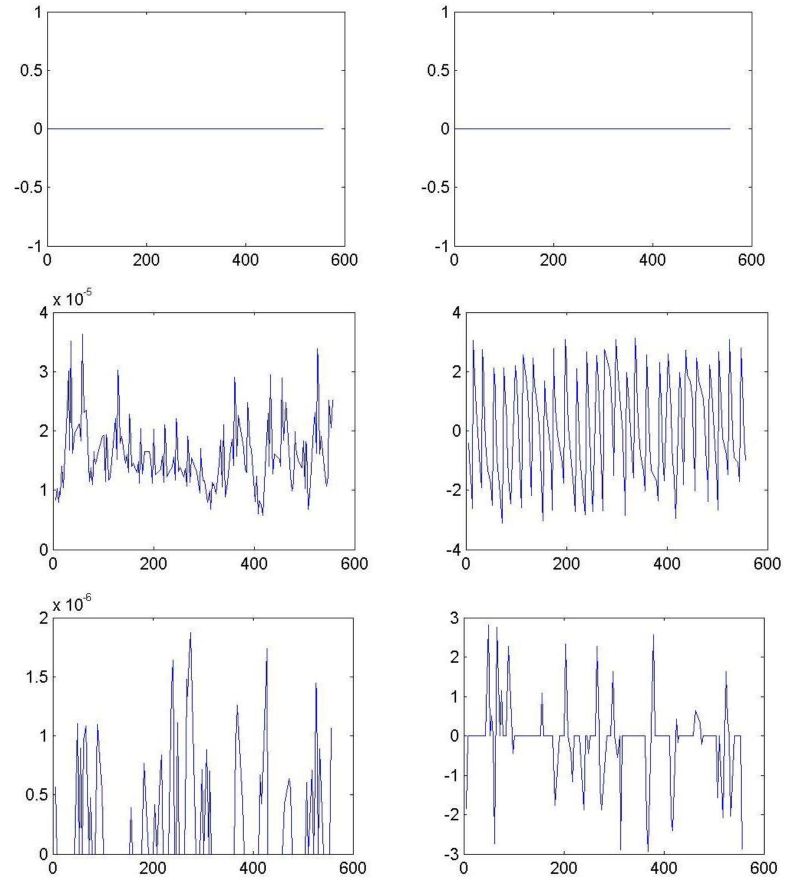
Example of SVD principal values

Fitted values for the 24 vertical sextupole misalignments obtained from SVD

Blu dots = assigned misalignments

Red dots = reconstructed misalignments





no misalignments

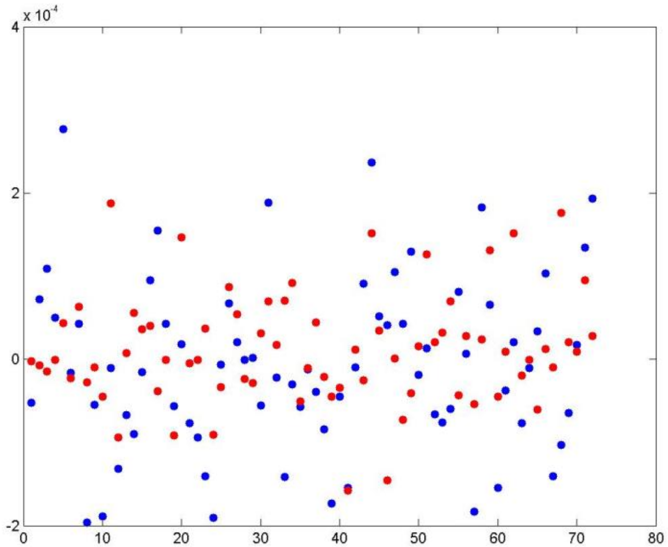
with misalignments

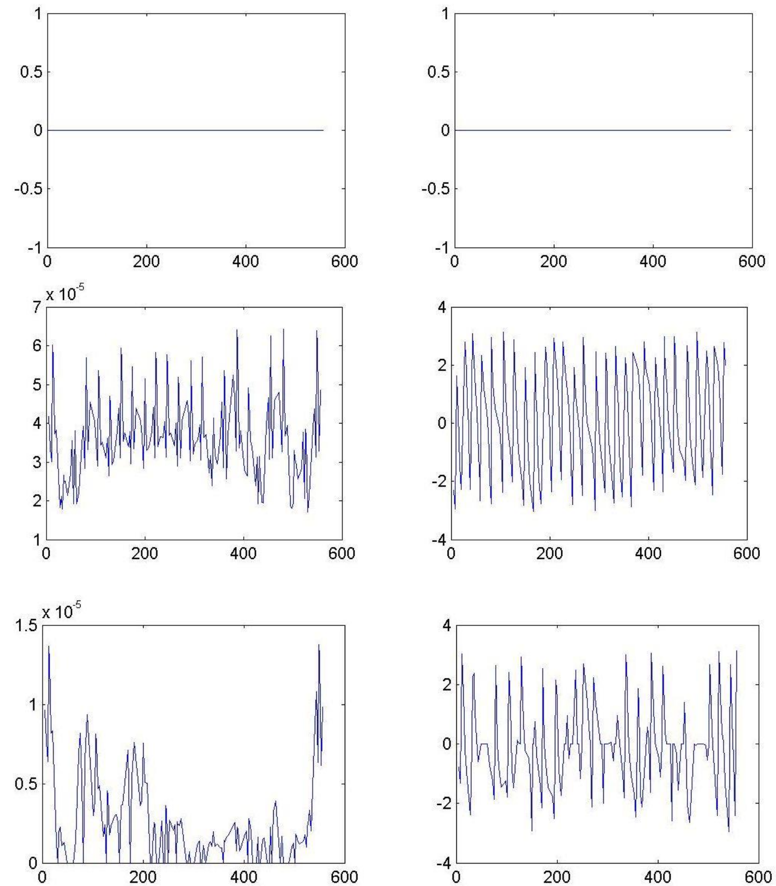
with misalignments and corrections

Fitted values for the 72 vertical sextupole misalignments obtained from SVD

Blu dots = assigned misalignments

Red dots = reconstructed misalignments





no misalignments

with misalignments

with misalignments and corrections

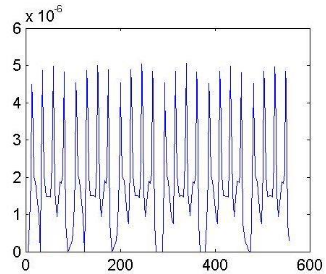
Sextupoles gradient errors applied to 24 sextupoles ($dK_2/K_2 = 5\%$)

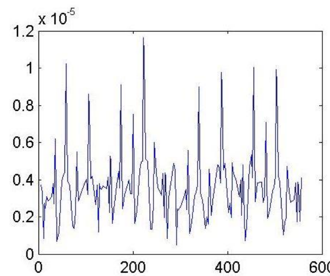
The sextupole gradient errors spoil the compensation of the third order resonances, e.g $3Q_x = p$ and $Q_x - 2Q_z = p$

- we build the vector of Fourier coefficients of the H(-2,0) and H(0,2) line
- we use the errors gradients as fit parameters

(0,2) line amplitude in H plane

(no gradient errors)



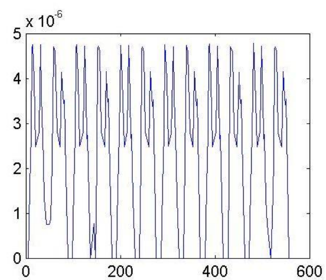


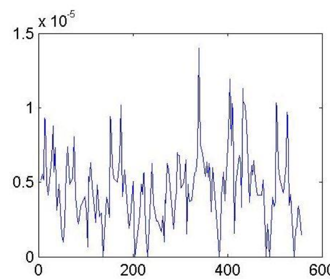
(0,2) line amplitude in H plane

with gradient errors

(-2,0) line amplitude in H plane

(no gradient errors)





(-2,0) line amplitude in H plane

with gradient errors

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SVD on sextupole gradient errors

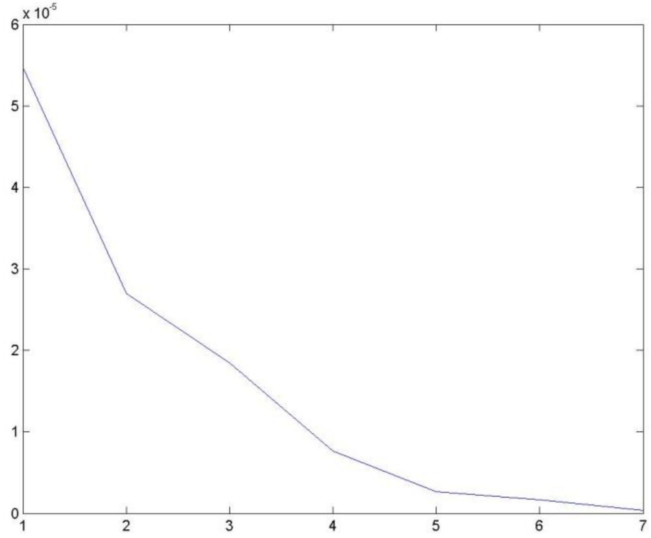
We build the vector $\overline{A} = \left(a_1^{H(-2,0)} \quad \ldots \quad a_{NBPM}^{H(-2,0)} \quad a_1^{H(0,2)} \quad \ldots \quad a_{NBPM}^{H(0,2)} \right)$

containing the amplitudes at all BPMs

- the (–2, 0) line in the H plane related to h_{3000}
- the (0, 2) line in the H plane related to h_{1002}

We minimize the sum

$$\chi^2 = \sum_j \left(A_{Model}(j) - A_{Measured}(j) \right)^2$$

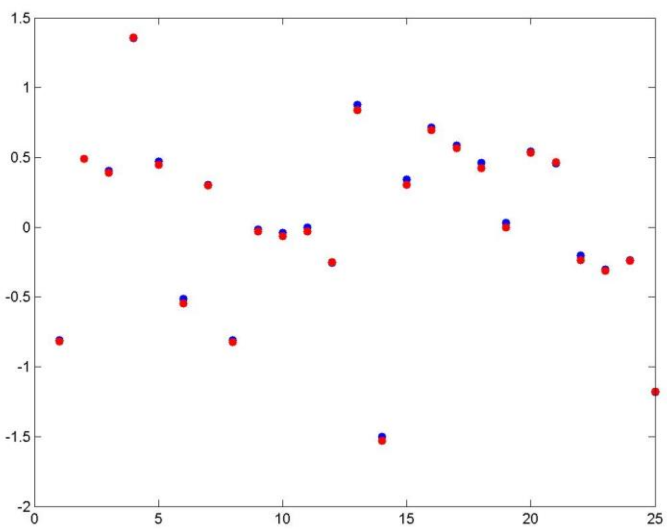


χ^2 as a function of the iteration number

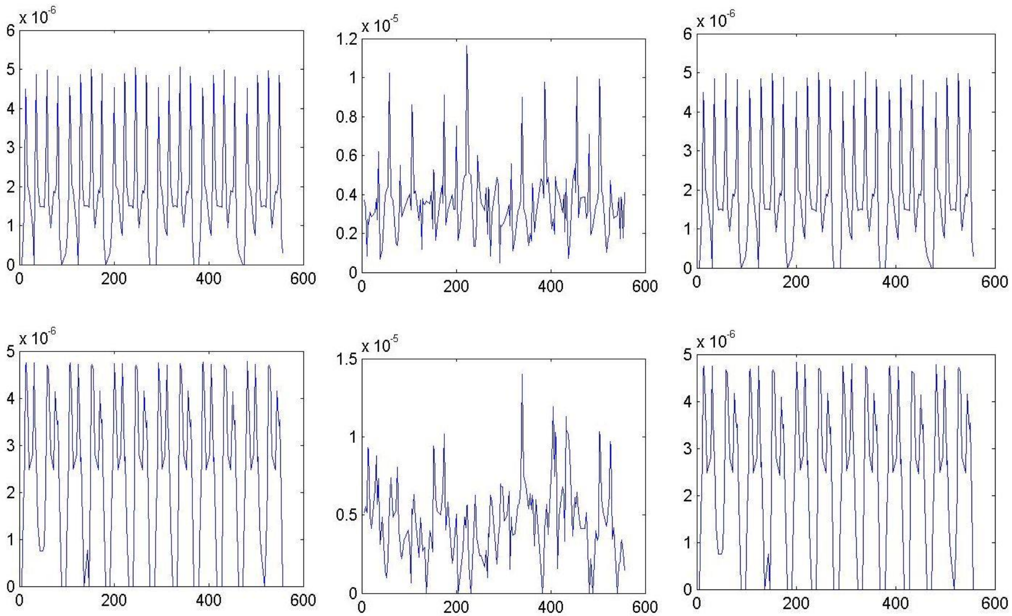
Fitted values for the 24 sextupoles gradients errors obtained from SVD

Blu dots = assigned misalignments

Red dots = reconstructed misalignments



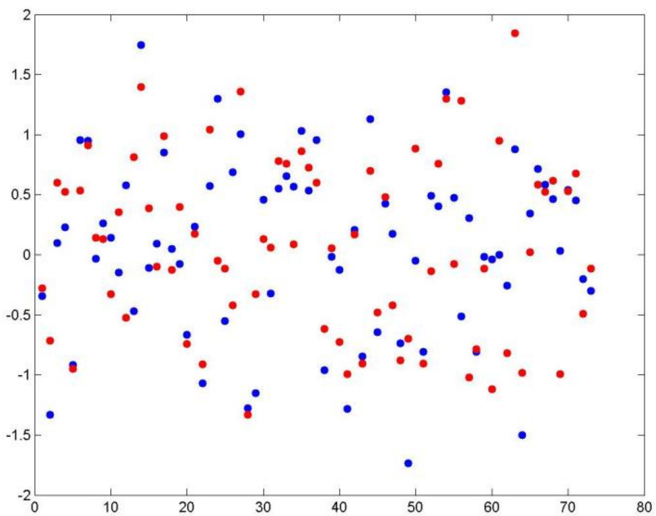
no gradient errors	with gradient errors	with gradient errors and corrections



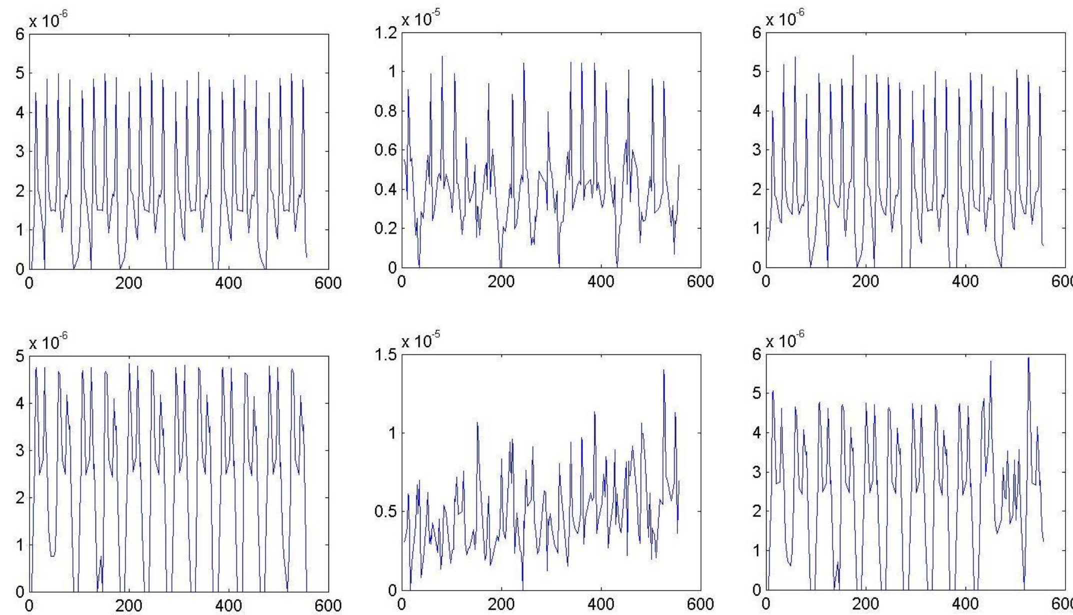
Fitted values for the 72 sextupoles gradients errors obtained from SVD

Blu dots = assigned misalignments

Red dots = reconstructed misalignments



no gradient errors | with gradient errors | with gradient errors and corrections



Conclusions and Ongoing Work

Can we use the spectral lines to recover the LINEAR and NON LINEAR machine model with a Least Square method?

- the SVD solution for fit is not unique
- select threshold for principal values helps
- use both amplitude and phase information

If decoherence is a problem it can be tackled with AC dipole techniques, many more spectral lines have to be identified…

Further studies on more complete models are ongoing