



# Refinements of the non-linear lattice model for the BESSY storage ring



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## Linear Lattice My Tracking Code Refinements of the Model

Fringe Fields of:

- Dipoles
- Quadrupoles
- Sextupoles

Additional Functions included in the Sextupole Magnets  
Small Adjustments of Harmonic Sextupoles

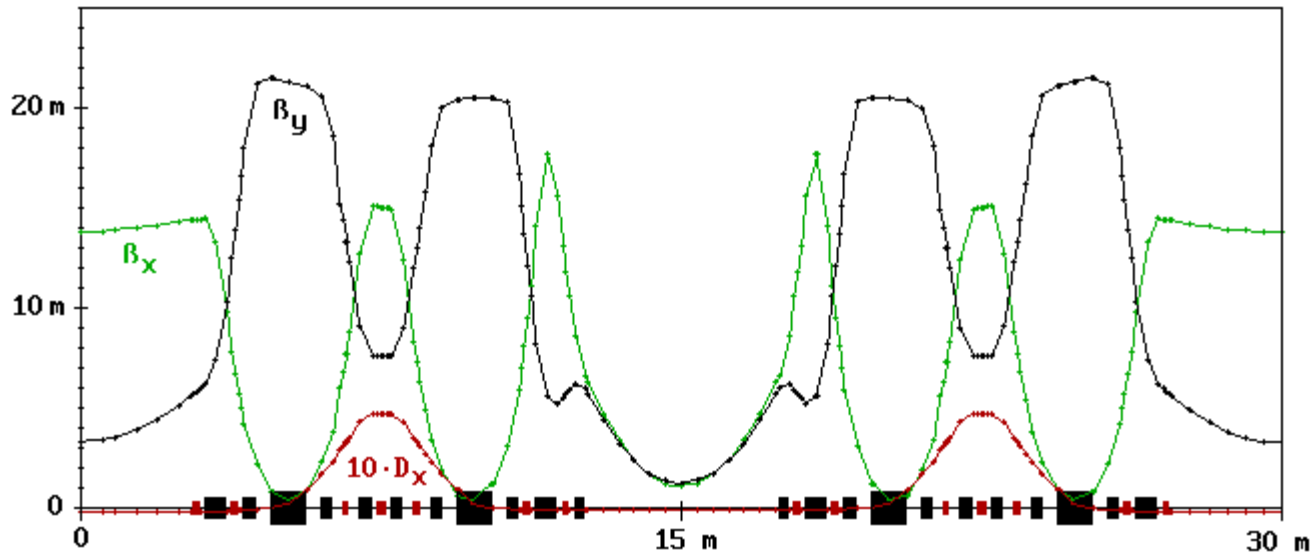
## Conclusions



# Linear Lattice



Linear (and linearly coupled) from orbit response matrix analysis



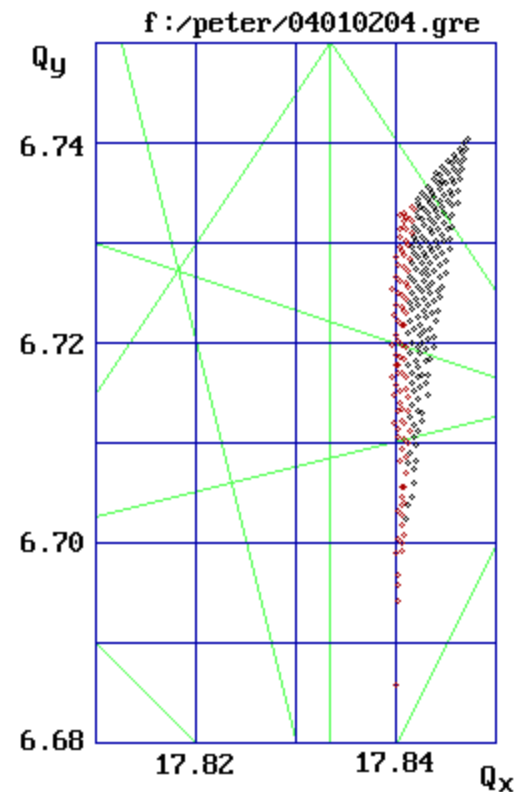
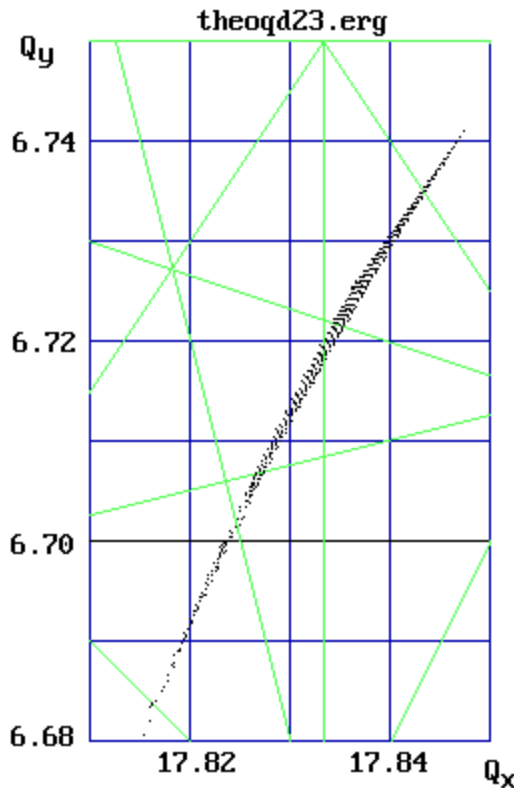
Optics functions of 1/8 of the storage ring as a result of a response matrix analysis



# My Tracking Code



- Small angle approximation
- symplectic, 4th order integrator (Laskar, Robutel, thesis L. Nadolski)
- includes the known physical aperture limitations (ID-vacuum chambers)



Frequency map as calculated by the course model (left) and measured (right).



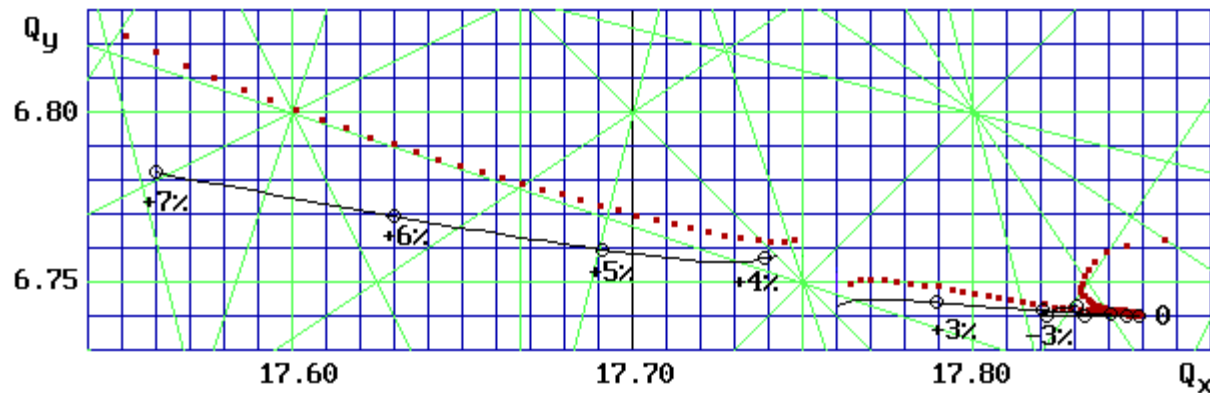
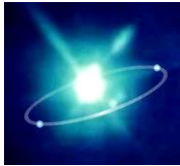
## Conditions for the frequency map calculation:

- chromaticity:  $\xi_x = +3.31$  and  $\xi_y = 3.64$
- with nominal harmonic sextupole settings
- 128 turns tracked
- fundamental frequencies determined with Hanning window

## Conditions for the tune calculation as a function of momentum:

- chromaticity:  $\xi_x = -0.19$  and  $\xi_y = 0.00$
- with nominal harmonic sextupole settings
- 256 turns tracked
- fundamental frequencies determined with Hanning window

Small skew gradient errors are included, sextupoles are modeled as 13 equal kicks.

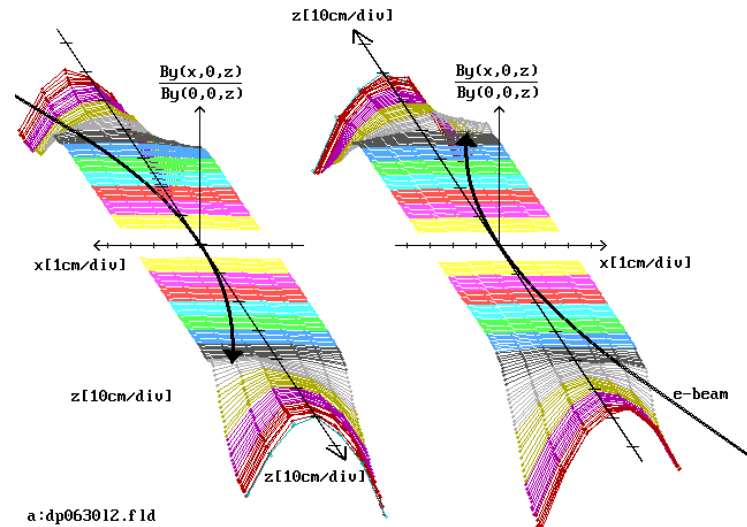
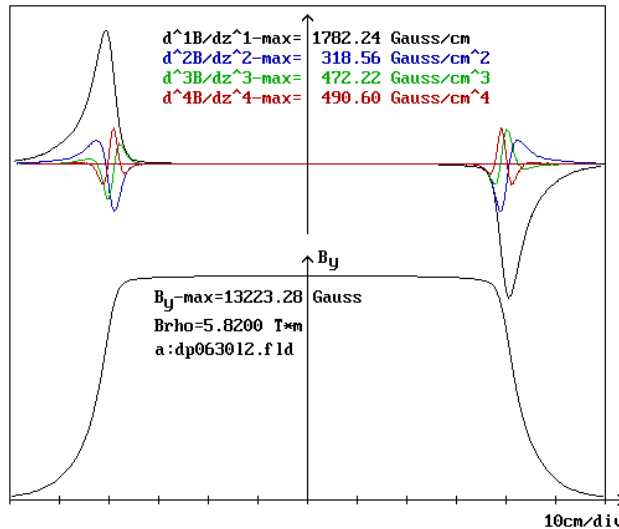


Momentum dependent tune shift. Measurement (red) and prediction with course model.

Agreement is not satisfactory



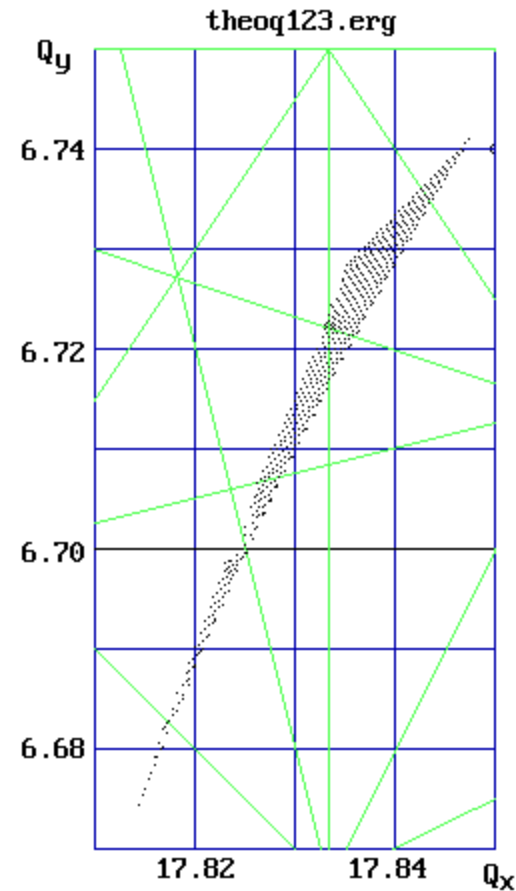
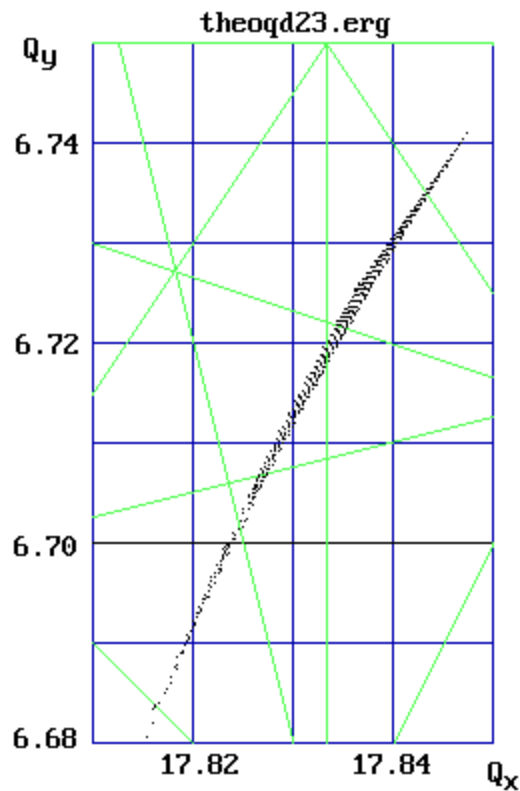
# Refinements: Dipole Fringe Field



## Analysis of 33 measured dipole field maps:

- 12% reduction of the vertical focussing
- integrated sextupole component entrance  $+0.03\text{m}^{-2}$  exit edge  $+0.11\text{m}^{-2}$
- integrated octupole component entrance  $+5\text{m}^{-3}$  exit edge  $-13\text{m}^{-3}$

Result: Calculated sextupole currents are in better agreement with actual settings and important as soon as amplitude dependent tune shifts are small...



Modification of the calculated frequency map if the dipole fringe field is included (right).



# Refinements: Quadrupole Fringe Field



$$x^f = x \pm \frac{b_2}{12(1+\delta)} \{x^3 + 3y^2x\}$$

$$p_x^f = p_x \pm \frac{b_2}{4(1+\delta)} \{2xyp_y - x^2p_x - y^2p_x\}$$

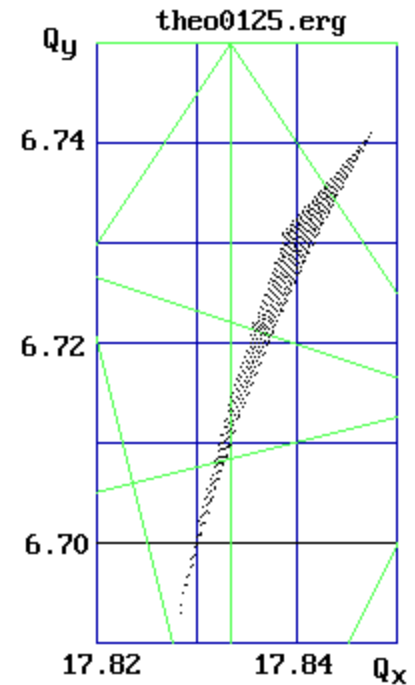
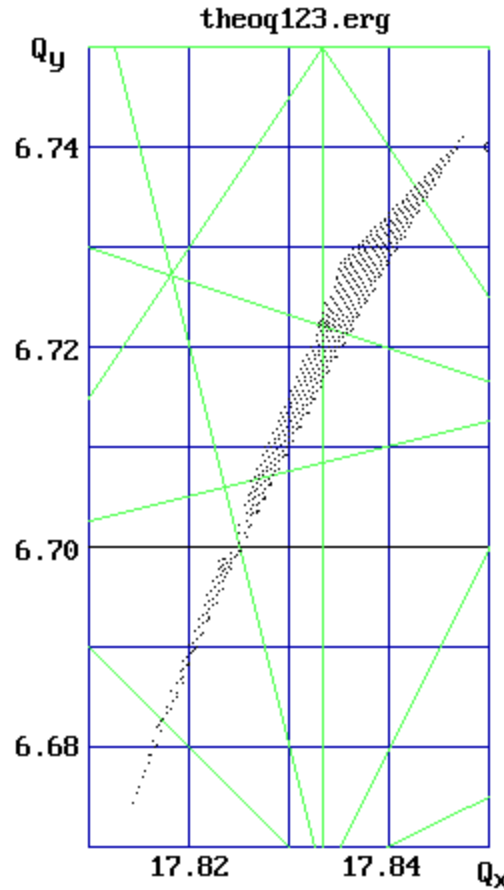
$$y^f = y \mp \frac{b_2}{12(1+\delta)} \{y^3 + 3x^2y\}$$

$$p_y^f = p_y \mp \frac{b_2}{4(1+\delta)} \{2xyp_x - y^2p_y - x^2p_y\}$$

$$\delta^f = \delta$$

$$l^f = l \pm \frac{b_2}{12(1+\delta)} \{y^3p_y - x^3p_x + 3x^2yp_y - 3y^2xp_x\}$$

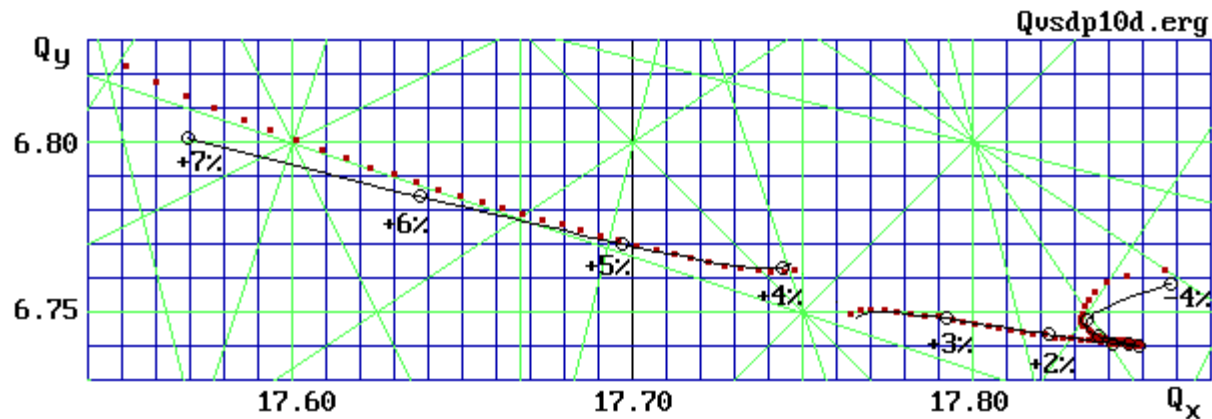
*E. Forest and many others*



Modification of the calculated frequency map due to the quadrupole fringe field (right).

Quadrupole fringe field important as soon as amplitude dependent tune shifts are small





Momentum dependent tune shift calculated taking dipole and quadrupole fringe fields into account.  
Measurement is shown in red.



## Refinements: Sextupole Magnets



Fringe Field

$$p_x^f = p_x \pm \frac{S}{24(1+\delta)} \{11x^2 y p_y + y^3 p_y - x^3 p_x - x p_x p_y^2\}$$

has very small impact

$$p_y^f = p_y \pm \frac{S}{24(1+\delta)} \{3x^3 p_y + 3xy^2 p_y - 3x^2 y p_x + 5y^3 p_x\}$$

Added Functions :

Horizontal and vertical steering,  
skew quadrupole component

Result of rotating  
coil measurements

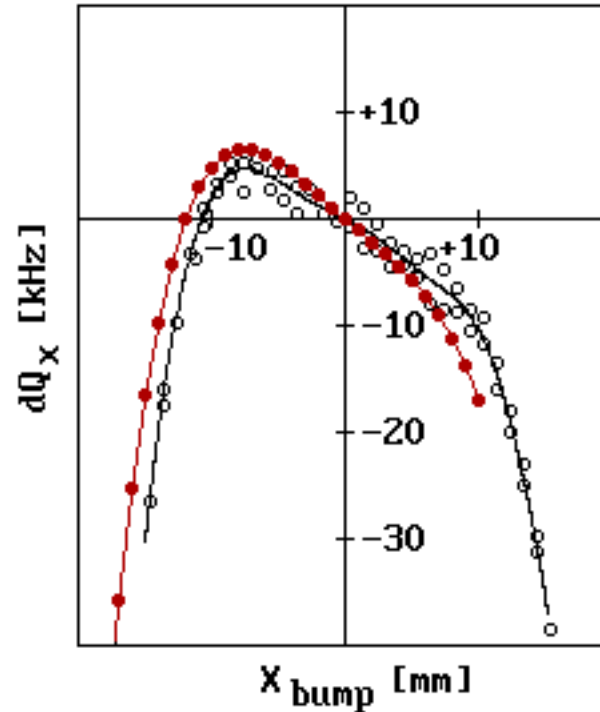
BESSY TB 209/97:

$b_3$ ,  $b_5$ , and  $a_5$  are  
noticeable

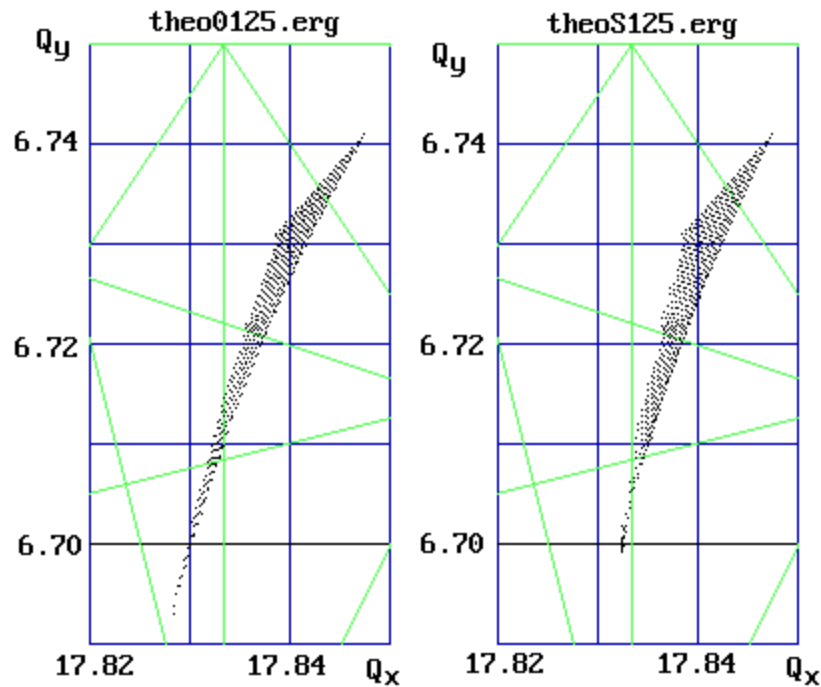
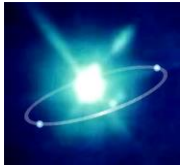
	Multipole	S1-sextupole (21 cm long)	S2-sextupole (16 cm long)
Vertical field $B_y$	$b_1 L/I$	$6.019 \cdot 10^{-3} \text{ Tm/A}$	$5.031 \cdot 10^{-3} \text{ Tm/A}$
	$b_3 L/I$	$6.6 \cdot 10^{-1} \text{ Tm}^{-1}/\text{A}$	$5 \cdot 10^{-1} \text{ Tm}^{-1}/\text{A}$
	$b_5 L/I$	$3.41 \cdot 10^2 \text{ Tm}^{-3}/\text{A}$	$2.67 \cdot 10^2 \text{ Tm}^{-3}/\text{A}$
	$b_7 L/I$	$3.10 \cdot 10^4 \text{ Tm}^{-5}/\text{A}$	$2.27 \cdot 10^4 \text{ Tm}^{-5}/\text{A}$
	$b_{11} L/I$	$-5.43 \cdot 10^9 \text{ Tm}^{-9}/\text{A}$	$-4.16 \cdot 10^9 \text{ Tm}^{-9}/\text{A}$
	$b_{13} L/I$	$-7.72 \cdot 10^{12} \text{ Tm}^{-11}/\text{A}$	$-6.20 \cdot 10^{12} \text{ Tm}^{-11}/\text{A}$
Horizontal field $B_x$	$a_1 L/I$	$-3.455 \cdot 10^{-3} \text{ Tm/A}$	$-2.883 \cdot 10^{-3} \text{ Tm/A}$
	$a_5 L/I$	$1.88 \cdot 10^2 \text{ Tm}^{-3}/\text{A}$	$1.46 \cdot 10^2 \text{ Tm}^{-3}/\text{A}$
	$a_7 L/I$	$-1.72 \cdot 10^4 \text{ Tm}^{-5}/\text{A}$	$-1.26 \cdot 10^4 \text{ Tm}^{-5}/\text{A}$



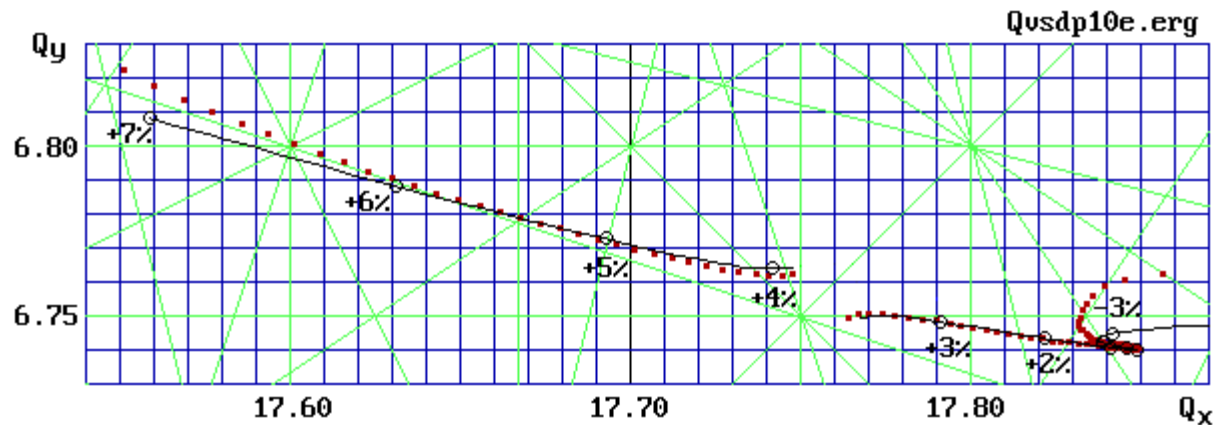
# Importance of the Decapole Component



Horizontal tune as a function of a horizontal closed orbit bump. Theoretical prediction is shown in red and experimental result in black. Harmonic sextupole magnets were turned off and the chromaticity was set to zero during the experiment.



Predictions if sextupole and decapole components are taken into account.

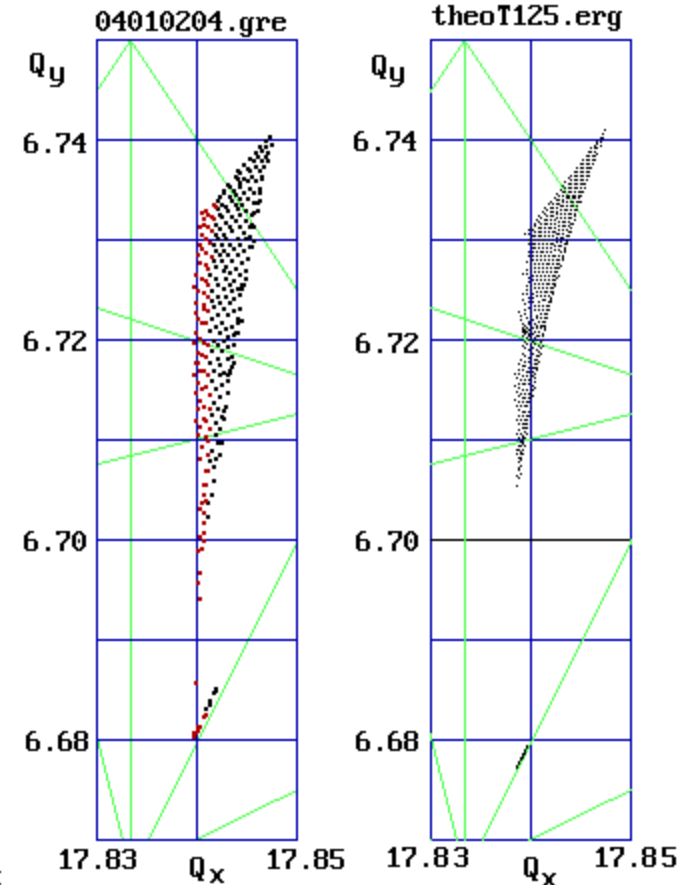
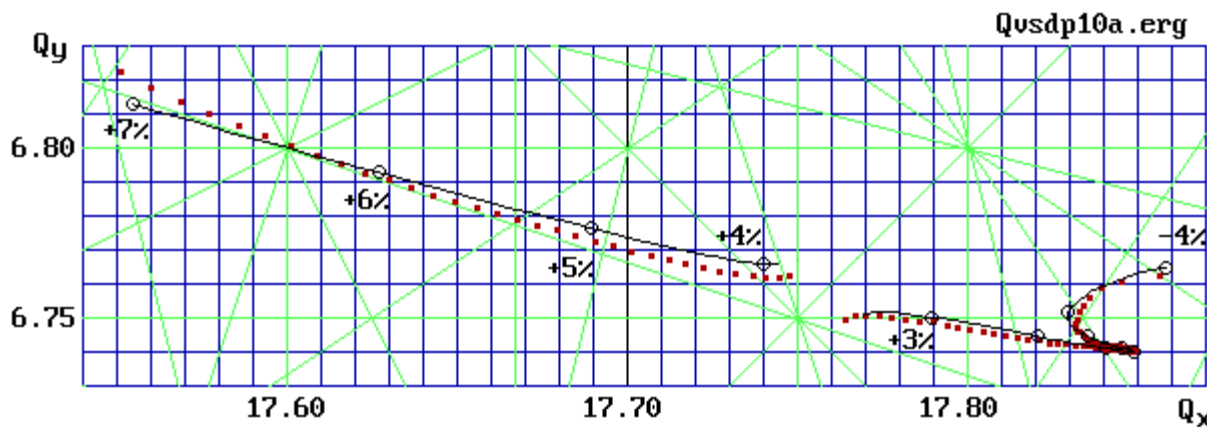




# Adjustments of the Harmonic Sextupole Strengths



Small corrections ( $\pm 1.2\%$ )  
give generally better  
agreement between  
experiment and theory:



State-of-the-art in non-linear modeling of the BESSY storage ring.



## Conclusions



Predictions with (symplectic) tracking codes based on the correct linearly coupled lattice model are in good agreement with experimental observations if:

- Realistic dipole fringe fields (at least up to 3<sup>rd</sup> order)
- Theoretical quadrupole fringe fields and
- Perturbations in the sextupole magnets  
are included.

Small corrections to harmonic sextupole fields could be real or just compensate for overlapping fringe fields of other magnets.

This deserves further studies for advanced lattice designs.

These points are less important for lattices without harmonic sextupole magnets (like the ALS).