Frequency Map Analysis and Hilbert Transform for experimental resonance investigation at Elettra

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Outline

Keywords: nonlinearity, resonance
diffusion rate
decoherence, tune-shift with amplitude
phase space

Simulations

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- **Tracking**

Single particle tracking code over a grid of initial conditions with (p_x=0, p_y=0) and for 10.000 turns.
Harmonic and chromatic sexts. included. Optical asymmetries included. Longitudinal motion neglected.

- **Frequency map**

The NAFF algorithm provides a tune accuracy of $1/N^4$, N > 1000 turns $\Rightarrow$ $\Delta\nu_{accuracy} << \Delta\nu_{nonlinear}$

The diffusion rate was defined over 2 consecutive sets of 1000 turns for each tracked particle, giving the stability colour scale for the map (from ---- unstable/chaotic to ---- stable motion)

- **Results**

Non-systematic, high order (4^{th}, 5^{th}, 6^{th}) resonances. Uncoupled and coupling resonances.

Experimental strategy

Measurements

- TMBF kick (anti-damping mode) ⇒ coherent bunch oscillations around the reference orbit
- BPM (feedback dedicated) ⇒ transverse position z(N) of the bunch centroid
- Nonlinearity ⇒ at different kicks (amplitudes) we have different tunes
- Measurement setting: SB mode with I < 0.3 mA, 0.9 GeV & 2.0 GeV
  Beam stable, centered orbit

Data analysis

↓ Choose a nominal working point

↓ Excite the bunch at different amplitudes

↓ Apply the NAFF algorithm to the first 1000 turns after the kick to calculate the tune for each amplitude:

mapping the physical plane into the tune diagram

calculate the diffusion rate parameter

calculate the nonlinear coefficients (I)

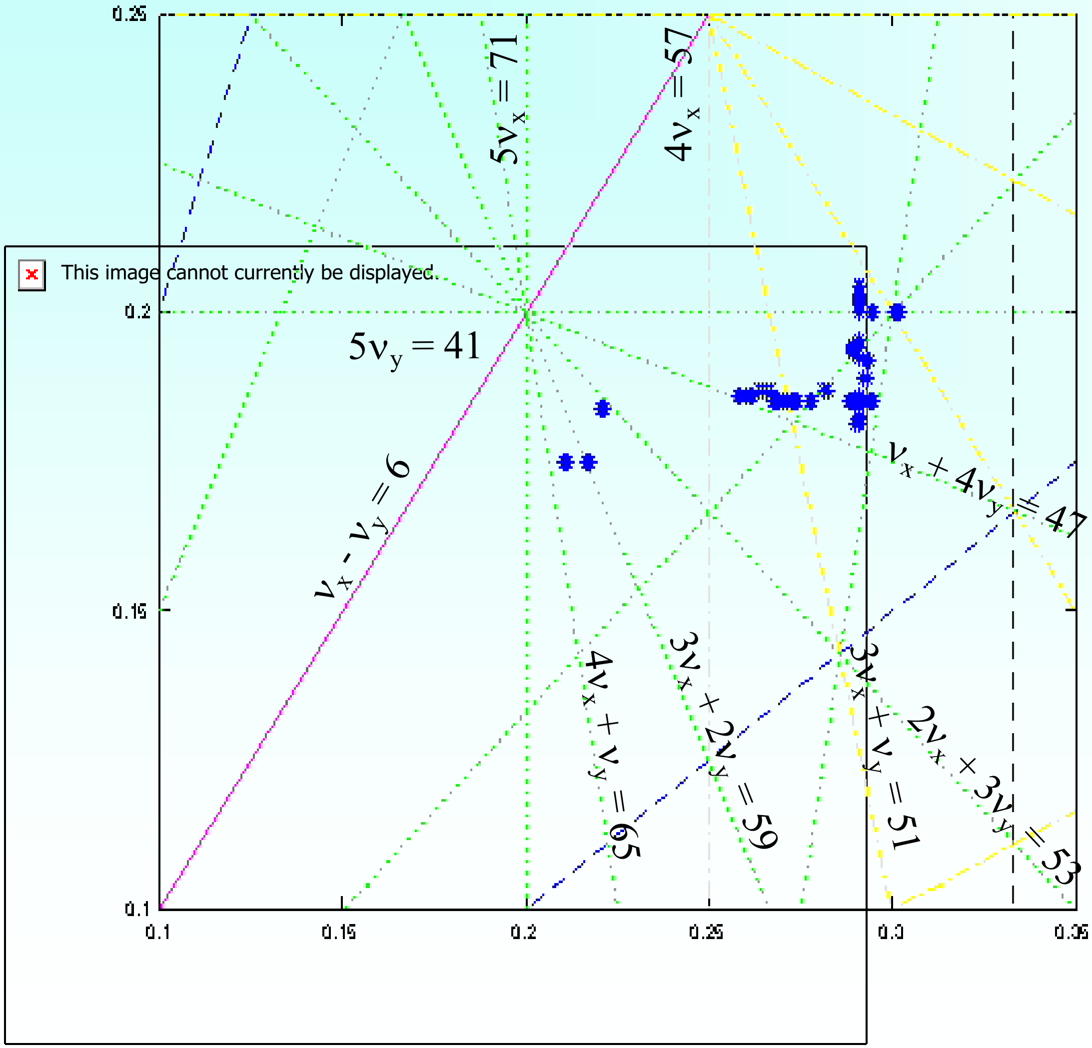
↓ Fit the centroid collapse due to nonlinear decoherence (theoretical model):

calculate the nonlinear coefficients (II)

↓ Check the tune resonance conditions:

apply the Hilbert transform to observe the phase space and to investigate periodic structures

Present Scenario



The experimental points intercept some of the 4th and 5th order resonances predicted by the frequency map simulation.

The real existence of these non-systematic resonances can be checked through the measure of:

- diffusion rate
- tune-shift with amplitude
- transverse phase space

Nonlinear coefficients have been measured and compared with the simulation results to verify the "reality" of the nonlinear model for Elettra.

Measurements

Nonlinear magnetic components in the machine induce nonlinearity in the particle motion. It is described by a dominant 2nd order perturbation, giving:

2nd order tune-shift with amplitude

$$\Delta\nu_x = 2\, J_x\, c_{11} + J_y\, c_{12}$$
$$\Delta\nu_y = 2\, J_y\, c_{22} + J_x\, c_{12}$$

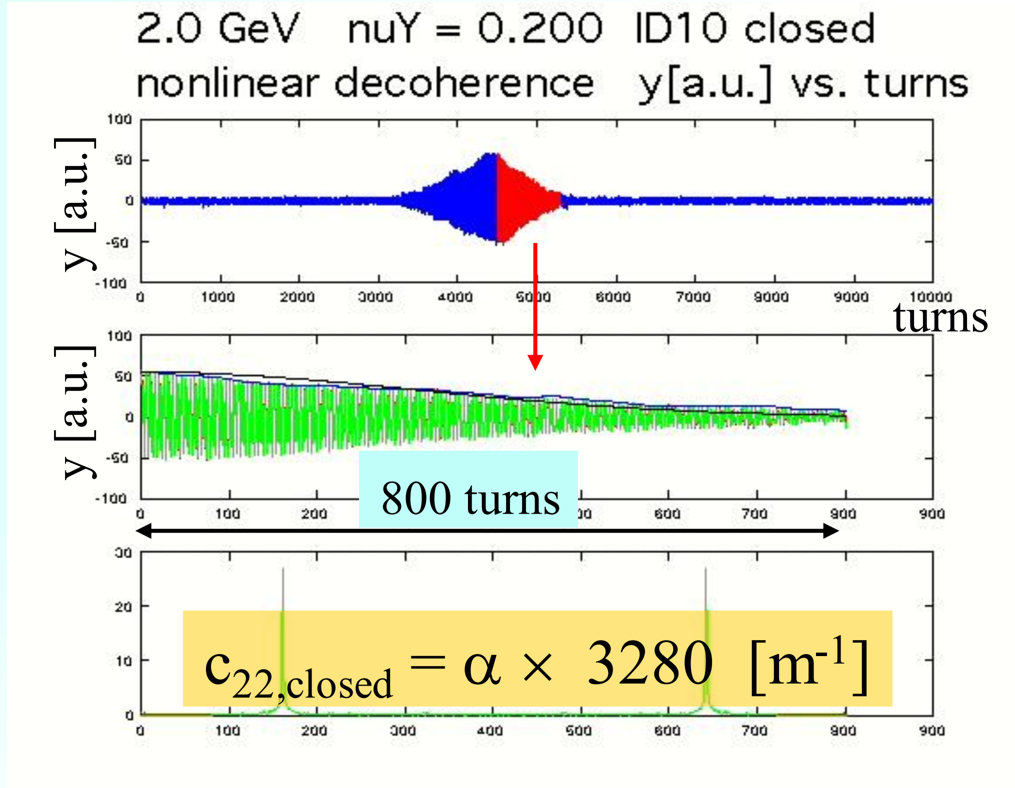
nonlinear decoherence (ref.[1],[2])

$$x_{cm}(N) = x_0 e^{-(N/N_c)^2} \cos(2\pi N \nu_z + \phi)$$

$$N_c \cong (\sqrt{\beta_z \varepsilon_z})(2\pi\, |c_{ii}| z_0\ )^{-1}$$

The orbit diffusion due to resonances or chaotic motion can be estimated through the diffusion rate parameter:

$$D = \log_{10}\{[\ (\nu_x^{(2)} - \nu_x^{(1)})^2 + (\nu_y^{(2)} - \nu_y^{(1)})^2]^{1/2}\}$$

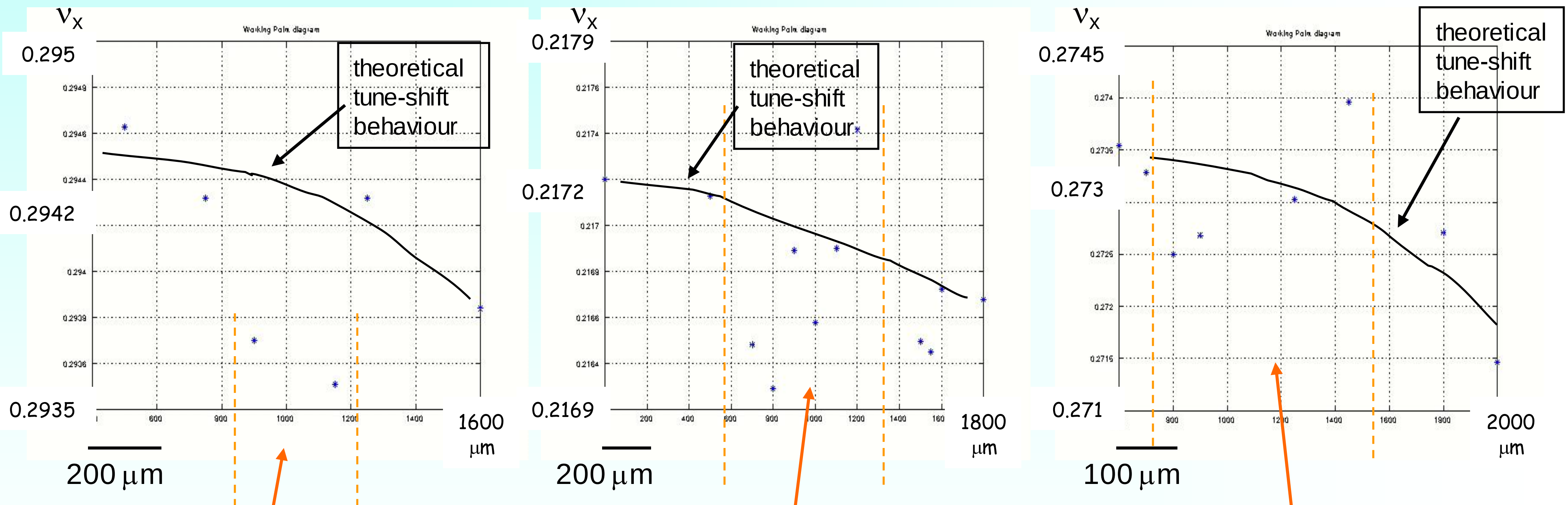


from transverse oscillations:

- diffusion rate
- tune/amplitude dependence $\Rightarrow c_{ij}$
- decoherence $\Rightarrow c_{ij}$

Non-regular frequency map

Irregular orbits in the phase space traduce themselves in a non-regular frequency map behaviour. This can reveal the influence of a resonance on the particle motion.

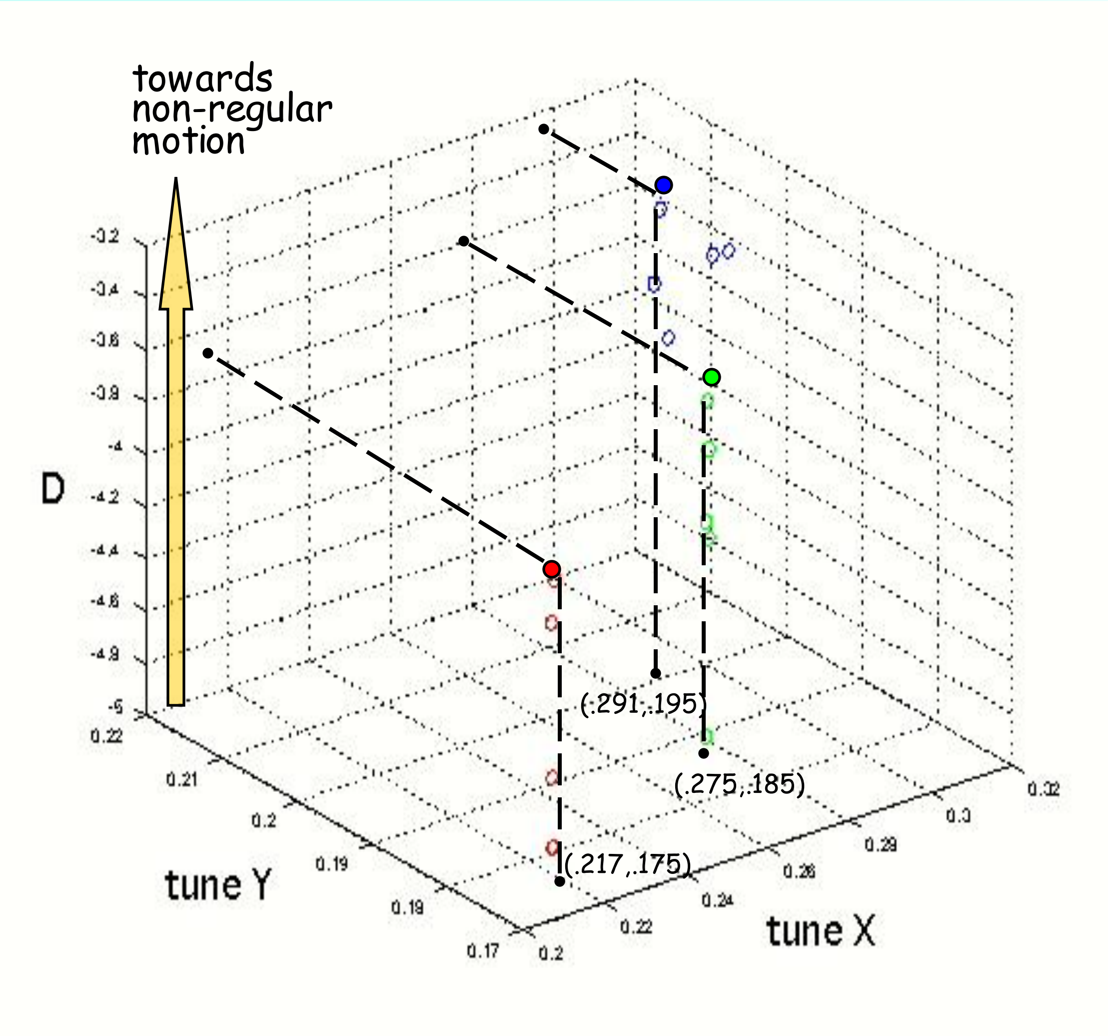


the working point (.294,.180) satisfies $\nu_x + 4\nu_y = 47$ for $\delta < 0.014$

the working point (.217,.170) satisfies $3\nu_x + 2\nu_y = 59$ for $\delta < 0.010$

the working point (.275,.185) satisfies $\nu_x + 4\nu_y = 47$ for $\delta < 0.013$

Diffusion rate



$$D = \log_{10}\{[(\nu_x^{(2)} - \nu_x^{(1)})^2 + (\nu_y^{(2)} - \nu_y^{(1)})^2]^{1/2}\}$$

The diffusion rate D has been calculated for 3 sets of measures in the vicinity of high order resonances, $5\nu_y=41$, $\nu_x+4\nu_y=47$ and $3\nu_x+2\nu_y=59$.

The frequency map shows the motion is still bounded by regular orbits (inner part of the dynamic aperture) so that the excursion of the diffusion rate is limited.

However, the evidence of diffusion peaks in corrispondence of the expected resonances is an index of their real existence in the machine.

Horizontal tune-shift with amplitude

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Vertical tune-shift with amplitude

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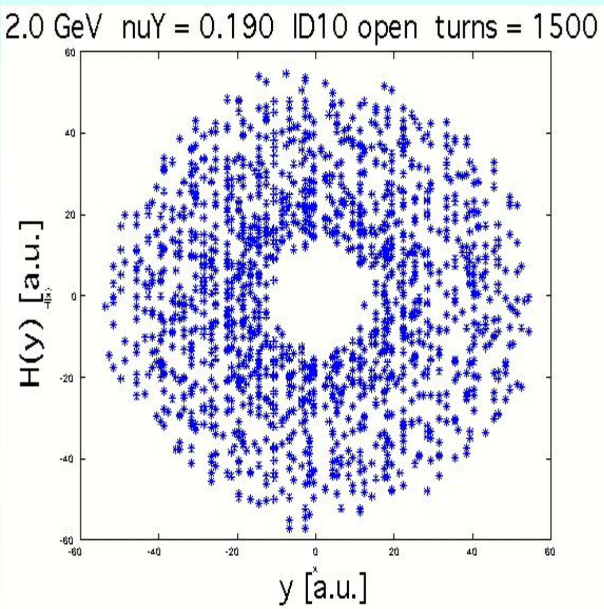
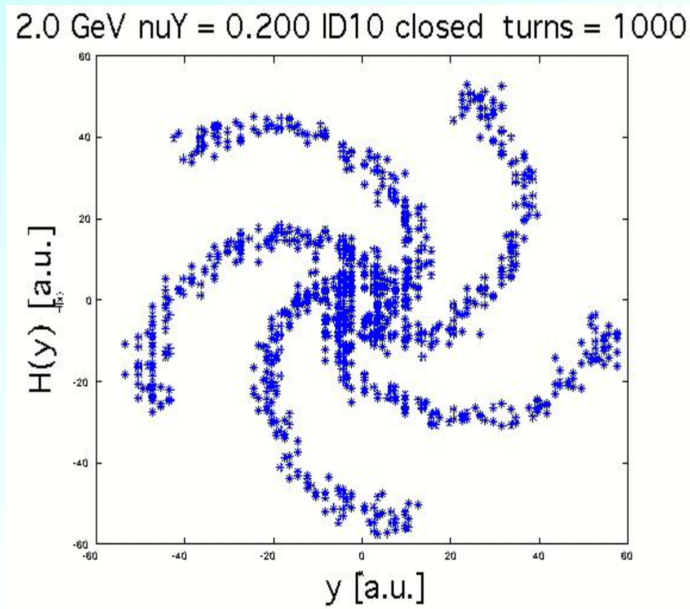
TMFB mulfunctioning corrupted the beam decoherence profile

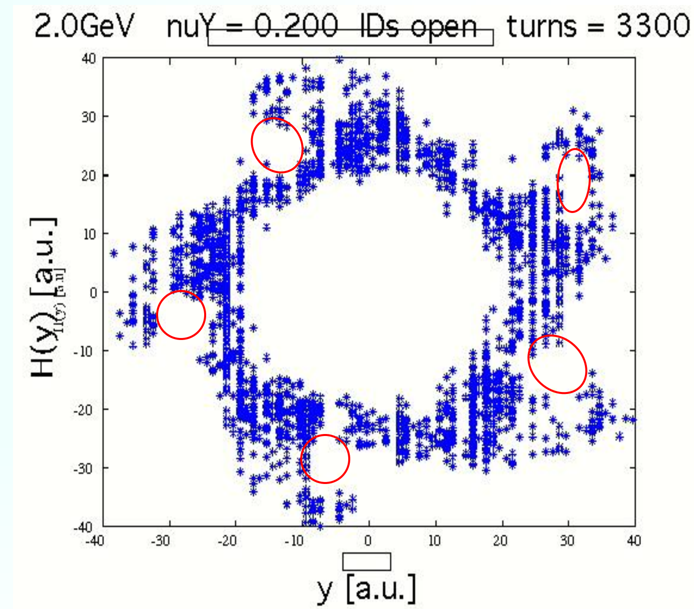
Hilbert Transform

$x = \{x_1, x_2, x_3, \ldots\ldots\ldots, x_n\}$ is a vector position of the bunch centroid, where $x_i \sim A_i \cos\phi_i$. The Hilbert transform (ref.[3]) of the position vector is its rotation of 90^o in the time domain. $x' \sim A\sin\phi$ is the divergence vector with the same amplitude of the position vector

$\Rightarrow$ we can plot the transverse (x,x') phase space





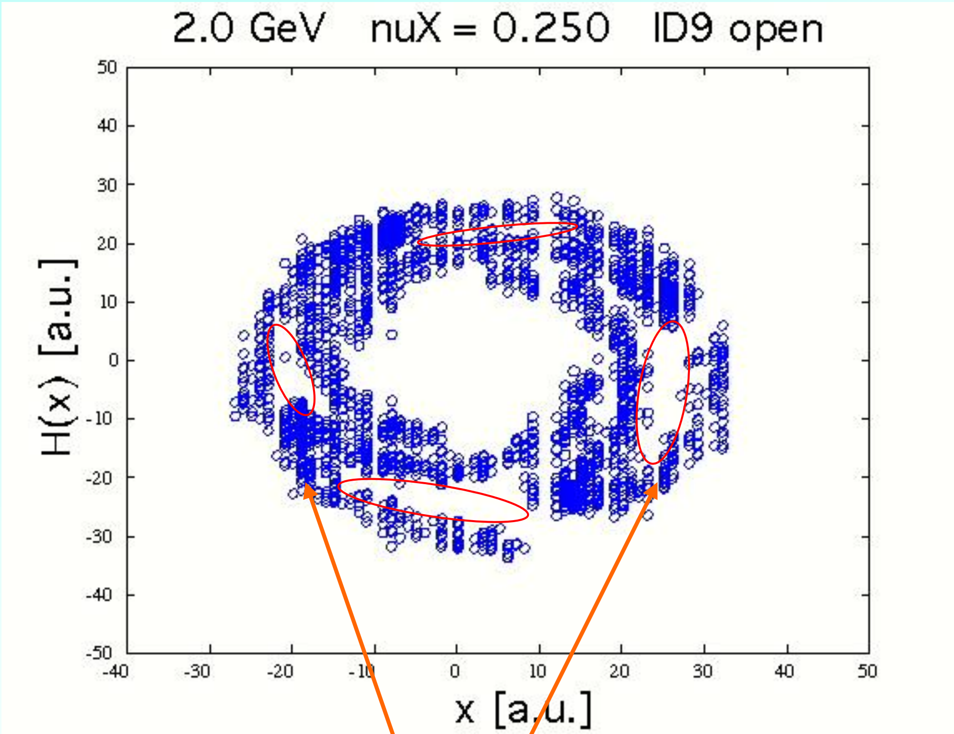


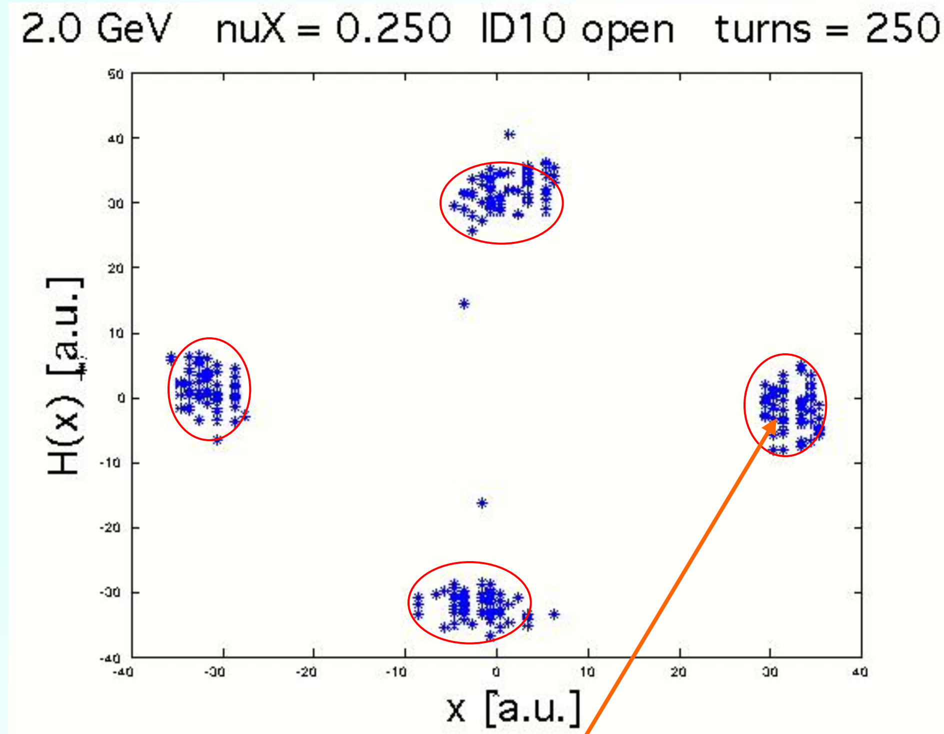
Vertical tune far away from a resonance condition.
The smearing of the ellipse is due to the bunch decoherence

Nonlinearity can shift the tune towards a resonance condition. Here $5\nu_y = 41$

At large amplitudes we can observe phase space distorsion and unstable motion. Here stochastic motion around the island region for $5\nu_y = 41$

Phase space: 4th order resonance in the horizontal plane

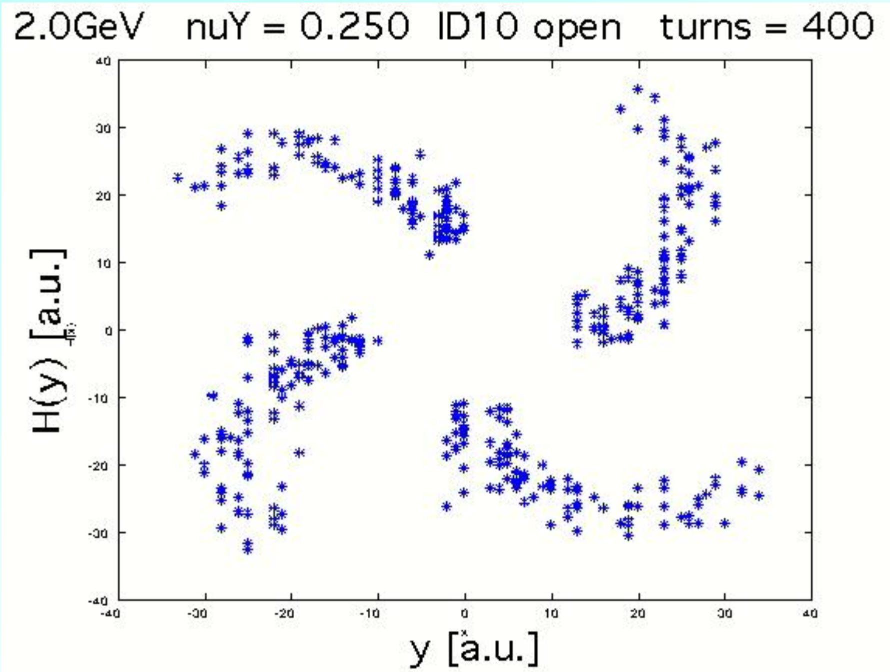


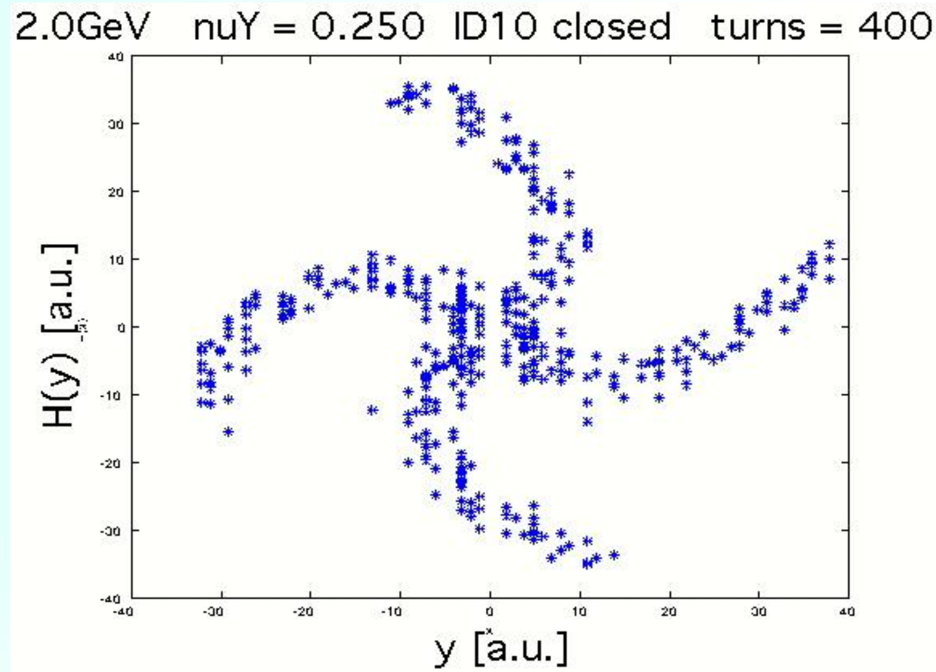


The island ring defines a region of boundary motion around itself

On-resonance condition.
The inherent betatron coupling generates smeared ellipses around the stable fixed points

Phase space: 4th order resonance in the vertical plane





$c_{22,open} = \alpha \times 5840 \; [m^{-1}]$

$c_{22,closed} = \alpha \times 10950 \; [m^{-1}]$

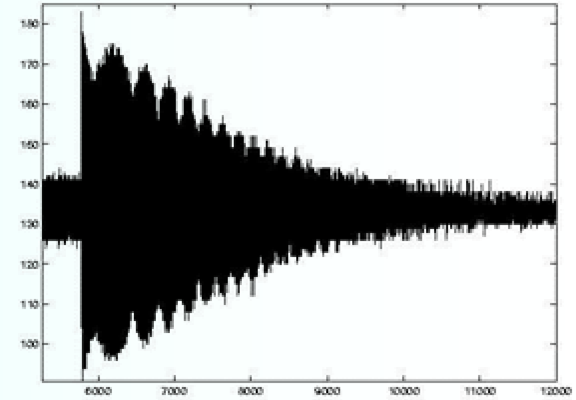
The phase space is plotted in the 2 cases of an insertion device open (left) and closed to the minimum gap (right). The resonance condition is still conserved but the nonlinear vertical coefficient is about doubled.

The FM analysis can be used to characterize the nonlinear components of an insertion device.

Solving problems

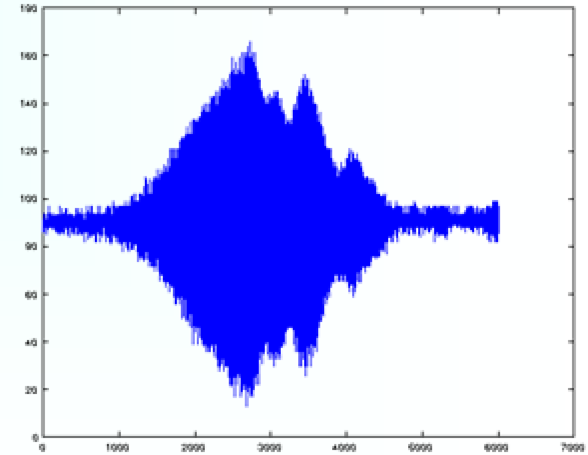
- A non-zero chromaticity corruptes the measurements by 2 ways: a) it modulates the decoherence and the beating of the beam envelope can be described analytically (ref.[4]); b) the chromatic contribution increases the decoherence
  - chromaticitiy $|\xi| \geq 0.5$ is sufficient to invalidate the measurement

    solution: check and eventually correct
    the chromaticity each time you change
    the working point by $\delta \geq 0.005$



- The bunch excitation is not continous (affecting the measure of the effective tune and modifying the phase space structure) and "saturates" at large amplitudes

  solution: move the working point to obtain the
  FM experimentally



- Beam position depends on current

  solution: re-check the kick/amplitude calibration
  for each current loss

Conclusions

- **FM simulations:**
  - a single particle/beam tracking code has been developed, extended to include a description of the nonlinear effects of variuos types of IDs
  - simulated asymmetry in the optics generates higher order (4^{th}, 5^{th}, 6^{th}), non-systematic, coupling resonances
  - unfolded frequency map (quite simple nonlinear physics; strong instabilities are avoided; 2^{nd} order perturbative terms dominate)

- **FM measurements:**
  - measurements of the diffusion rate, tune-shift with amplitude and phase space confirm the validity and the results of the frequency map simulation through the observation of predicted non-systematic high order resonances
  - measurements of the nonlinear coefficients confirm the general validity of the nonlinear model used for the machine simulations

- **Work in progress:**
  - measurements in the coupled (x,y) plane, with various IDs configurations.
  - comparison between the simulations and the whole measured map

References

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