QED PDFs in QCDNUM

Renat Sadykov

JINR, Dubna

07.08.2013, HERA Fitter User's Meeting

Outline

- Status and overview
- Comparison of `QCDNUM` and `partonevolution-1.1.3` in FFNS
- Comparison of `QCDNUM` and `MRST2004QED` in VFNS
- Next steps

Status

- The NLO QCD + LO QED evolution is implemented into `QCDNUM` beta version `17-01-0b` released by Michiel Botje on 26.07.2013. This version allows to solve n coupled evolution equations in FFNS and VFNS
- For $n_f = 5$ we have to solve 4 coupled equations for $\Delta, \Sigma, g, \gamma$ and 5 uncoupled equations for $d_v, u_v, \Delta_{ds}, \Delta_{uc}, \Delta_{sb}$ (assuming $s = \bar{s}$, $c = \bar{c}$, $b = \bar{b}$)
- It takes $\sim$ 5 s of CPU time (Intel Core i7-3630QM, 2.4 GHz) to fill weight tables of splitting functions and $\sim$ 0.5 s to evolve this distributions on 100×50 grid in x and μ^2
- The results in FFNS were cross-checked with `partonevolution-1.1.3` program of S. Weinzierl
- The comparison in VFNS with `MRST2004QED` pdf set is ongoing

PDF basis for QED-modified evolution

$$
\begin{aligned}
q_1 &= \Delta = u + \bar{u} + c + \bar{c} - d - \bar{d} - s - \bar{s} - b - \bar{b} \\
q_2 &= \Sigma = u + \bar{u} + c + \bar{c} + d + \bar{d} + s + \bar{s} + b + \bar{b} \\
q_3 &= g \\
q_4 &= \gamma \\
q_5 &= u_v \\
q_6 &= d_v \\
q_7 &= \Delta_{ds} = d + \bar{d} - s - \bar{s} \\
q_8 &= \Delta_{uc} = u + \bar{u} - c - \bar{c} \\
q_9 &= \Delta_{sb} = s + \bar{s} - b - \bar{b}
\end{aligned}
$$

Evolution equations

$$\frac{\partial}{\partial \ln \mu^2} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{pmatrix} = \begin{pmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44} \end{pmatrix} \otimes \begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{pmatrix}$$

$$\frac{\partial q_i}{\partial \ln \mu^2} = P_{ii} \otimes q_i, \quad i = 5, ..., 9$$

Splitting functions

$$P_{11} = a_s P_{qq}^{(0)} + a_s^2 P_+^{(1)} + \frac{e_u^2+e_d^2}{2} a\tilde{P}_{qq}^{(0)}$$
$$P_{12} = \frac{n_u-n_d}{n_f} a_s^2 (P_{qq}^{(1)} - P_+^{(1)}) + \frac{e_u^2-e_d^2}{2} a\tilde{P}_{qq}^{(0)}$$
$$P_{13} = \frac{n_u-n_d}{n_f} (a_s P_{qg}^{(0)} + a_s^2 P_{qg}^{(1)})$$
$$P_{14} = \frac{n_u e_u^2 - n_d e_d^2}{n_f} a P_{q\gamma}^{(0)}$$
$$P_{21} = \frac{e_u^2-e_d^2}{2} a\tilde{P}_{qq}^{(0)}$$
$$P_{22} = a_s P_{qq}^{(0)} + a_s^2 P_{qq}^{(1)} + \frac{e_u^2+e_d^2}{2} a\tilde{P}_{qq}^{(0)}$$
$$P_{23} = a_s P_{qg}^{(0)} + a_s^2 P_{qg}^{(1)}$$
$$P_{24} = \frac{n_u e_u^2 + n_d e_d^2}{n_f} a P_{q\gamma}^{(0)}$$
$$P_{31} = 0$$
$$P_{32} = a_s P_{gq}^{(0)} + a_s^2 P_{gq}^{(1)}$$
$$P_{33} = a_s P_{gg}^{(0)} + a_s^2 P_{gg}^{(1)}$$
$$P_{34} = 0$$
$$P_{41} = \frac{e_u^2-e_d^2}{2} a P_{\gamma q}^{(0)}$$
$$P_{42} = \frac{e_u^2+e_d^2}{2} a P_{\gamma q}^{(0)}$$
$$P_{43} = 0$$
$$P_{44} = a P_{\gamma\gamma}^{(0)}$$
$$P_{55} = a_s P_{qq}^{(0)} + a_s^2 P_-^{(1)} + a e_d^2 \tilde{P}_{qq}^{(0)}$$
$$P_{66} = a_s P_{qq}^{(0)} + a_s^2 P_-^{(1)} + a e_u^2 \tilde{P}_{qq}^{(0)}$$
$$P_{77} = P_{99} = a_s P_{qq}^{(0)} + a_s^2 P_+^{(1)} + a e_d^2 \tilde{P}_{qq}^{(0)}$$
$$P_{88} = a_s P_{qq}^{(0)} + a_s^2 P_+^{(1)} + a e_u^2 \tilde{P}_{qq}^{(0)}$$

$$a_s = \frac{\alpha_s}{2\pi}, \quad a = \frac{\alpha}{2\pi}, \quad e_u^2 = \frac{4}{9}, \quad e_d^2 = \frac{1}{9}, \quad n_u = \left\lfloor \frac{n_f}{2} \right\rfloor, \quad n_d = \left\lfloor \frac{n_f+1}{2} \right\rfloor$$

Splitting functions

$$P_{qq}^{(0)} = \frac{4}{3}\left(\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right)$$

$$P_{qg}^{(0)} = n_f\left(x^2 + (1-x)^2\right)$$

$$P_{gq}^{(0)} = \frac{4}{3}\left(\frac{1+(1-x)^2}{x}\right)$$

$$P_{gg}^{(0)} = 6\left(\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) + \left(\frac{11}{12} - \frac{n_f}{18}\right)\delta(1-x)\right)$$

$$\tilde{P}_{qq}^{(0)} = \frac{3}{4}P_{qq}^{(0)}$$

$$P_{q\gamma}^{(0)} = 2P_{qg}^{(0)}$$

$$P_{\gamma q}^{(0)} = \frac{3}{4}P_{gq}^{(0)}$$

$$P_{\gamma\gamma}^{(0)} = -\frac{2}{3}(n_u e_u^2 + n_d e_d^2)\delta(1-x)$$

Expressions for $P_{qq}^{(1)}$, $P_{qg}^{(1)}$, $P_{gq}^{(1)}$, $P_{gg}^{(1)}$ can be found in *Phys.Lett. B97 (1980) 437* and those for $P_{+}^{(1)}$, $P_{-}^{(1)}$ can be found in *Nucl.Phys. B175 (1980) 27*

Setup for comparison in FFNS

For numerical comparison of `QCDNUM` and `partonevolution-1.1.3` (`PE`) codes the toy model with $n_f = 4$ is adopted
(as in *arXiv:hep-ph/9609400* and *arXiv:hep-ph/0403200*)

PDFs at initial scale $\mu = 2$ GeV:

$$xu_v = \frac{35}{16} x^{0.5} (1-x)^3$$
$$xd_v = \frac{315}{256} x^{0.5} (1-x)^4$$
$$x\bar{d} = x\bar{u} = x\bar{s} = xs = \frac{0.673345}{6} x^{-0.2} (1-x)^7$$
$$x\bar{c} = xc = 0$$
$$xg = 1.90836 x^{-0.2} (1-x)^5$$
$$x\gamma = 0$$

Setup for comparison in FFNS

The running of the strong and the electromagnetic couplings:

$$a_s(\mu^2) = \frac{1}{\beta_0 L}\left(1 - \frac{\beta_1}{\beta_0^2}\frac{\ln L}{L}\right)$$

$$a(\mu^2) = \frac{a(m_\tau^2)}{1 - \frac{38}{9}a(m_\tau^2)\ln\frac{\mu^2}{m_\tau^2}}$$

Here

$$L = \ln\frac{\mu^2}{\Lambda_{QCD}^2}, \quad \Lambda_{QCD} = 0.25 \text{ GeV}, \quad \beta_0 = \frac{25}{6}, \quad \beta_1 = \frac{77}{6}$$

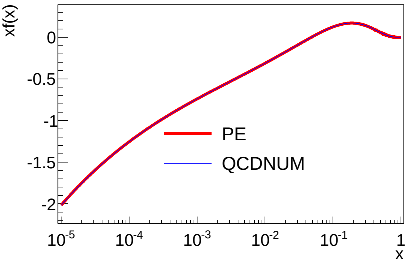
$$a(m_\tau^2) = \frac{1}{2\pi}\frac{1}{133.4}, \quad m_\tau = 1.777 \text{ GeV}$$

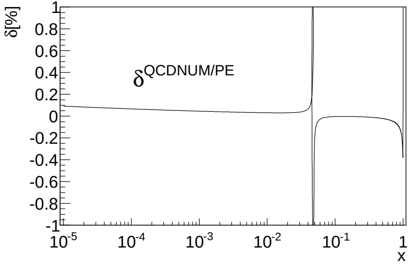
Delta definitions

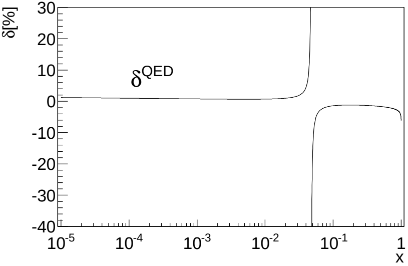
$$\delta^{\mathsf{QCDNUM/PE}} = \frac{xf(\mathsf{QCDNUM\ with\ QED}) - xf(\mathsf{PE})}{xf(\mathsf{PE})}$$

$$\delta^{\mathsf{QED}} = \frac{xf(\mathsf{QCDNUM\ with\ QED}) - xf(\mathsf{QCDNUM})}{xf(\mathsf{QCDNUM})}$$

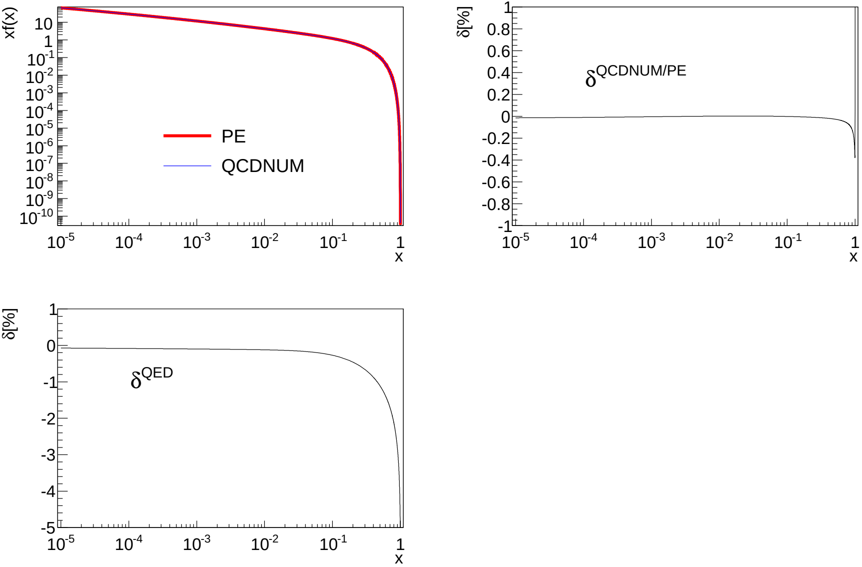
Δ distribution at $\mu = 100$ GeV



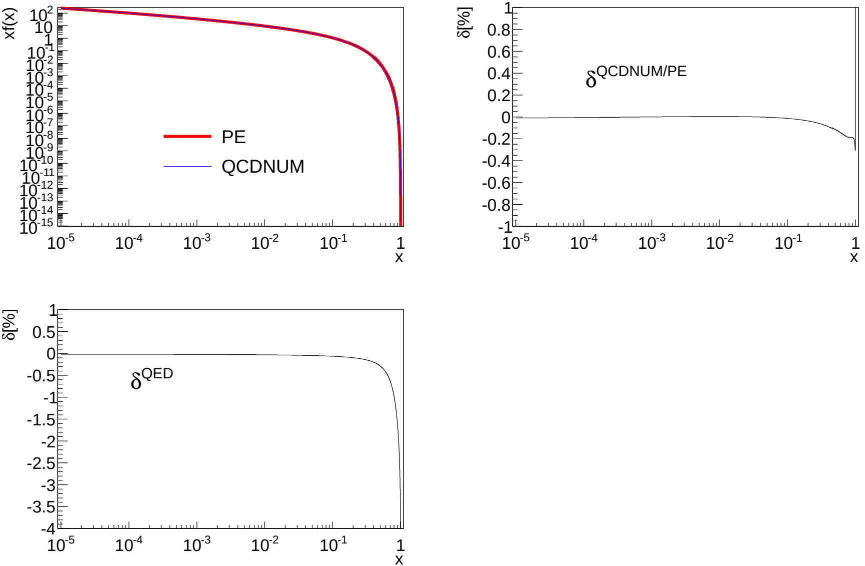




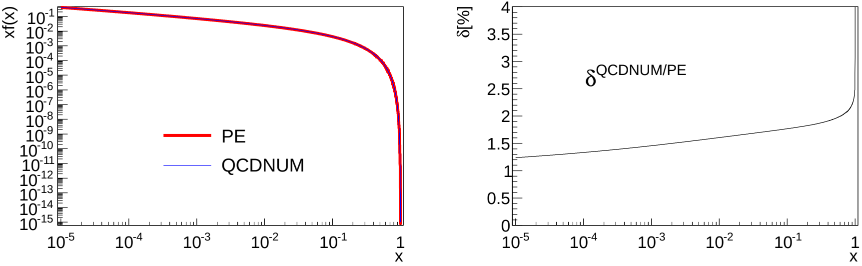
Σ distribution at $\mu = 100$ GeV



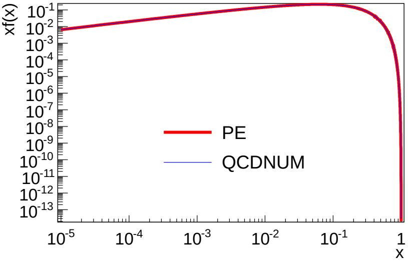
g distribution at $\mu = 100$ GeV

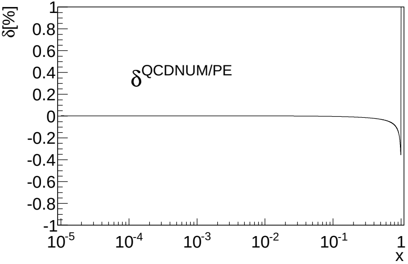


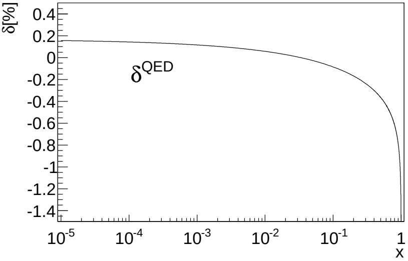
γ distribution at $\mu = 100$ GeV



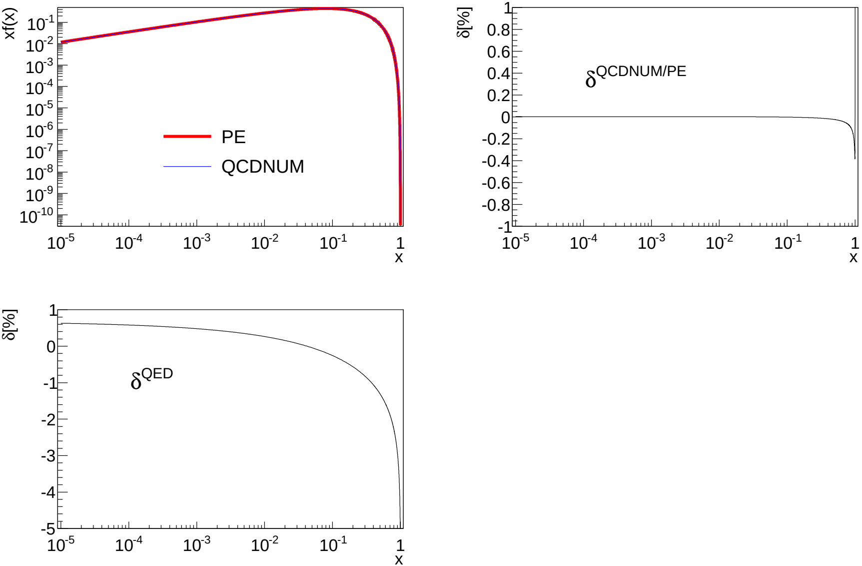
d_v distribution at $\mu = 100$ GeV



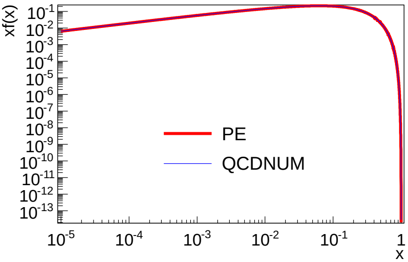


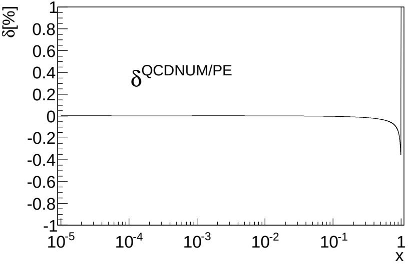


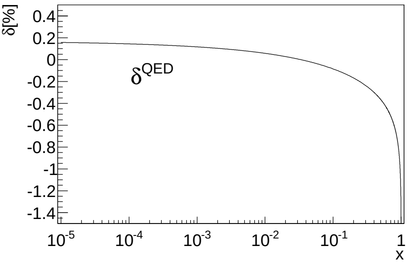
u_v distribution at $\mu = 100$ GeV



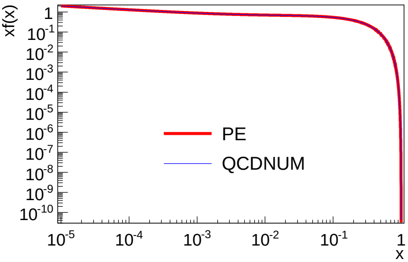
Δ_{ds} distribution at $\mu = 100$ GeV

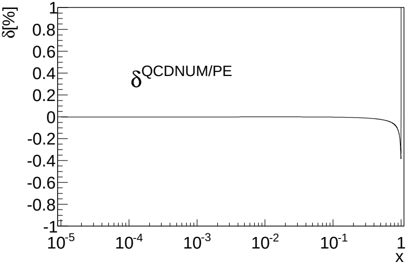


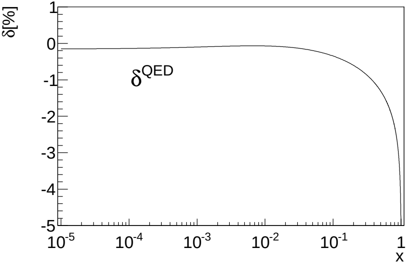




Δ_{uc} distribution at $\mu = 100$ GeV







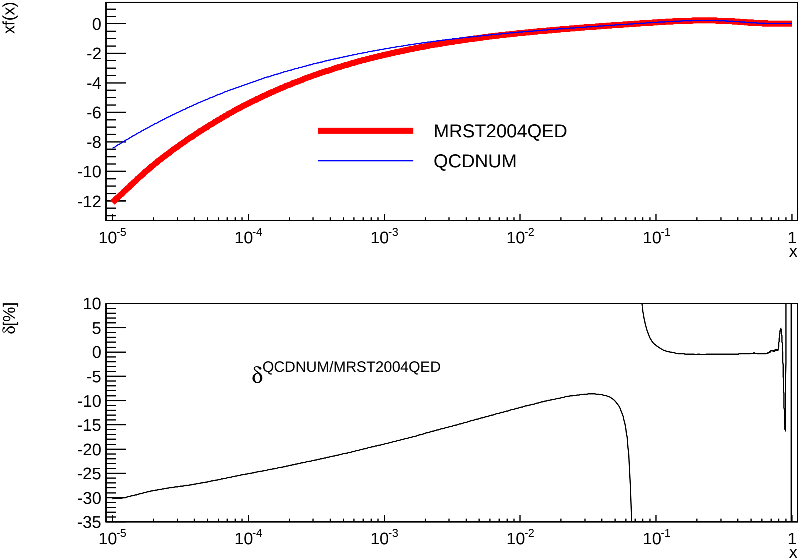
Comparison in VFNS

- For numerical comparison of `QCDNUM` and `MRST2004PDF` the distributions is evolved from initial scale $\mu_0^2 = 2$ GeV^2 to $\mu^2 = 10^4$ GeV^2 on 1000×1000 grid in x and μ^2

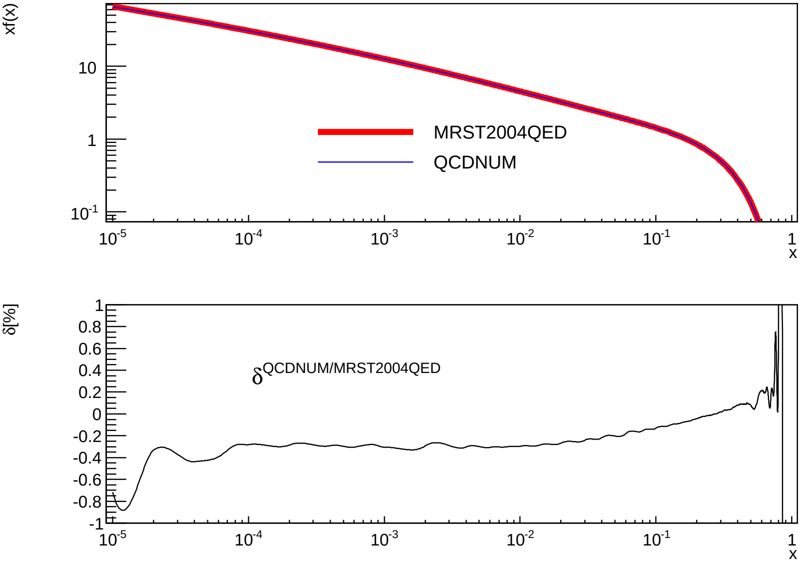
Delta definition

$$\delta^{\mathsf{QCDNUM/MRST2004QED}} = \frac{xf(\mathsf{QCDNUM\ with\ QED}) - xf(\mathsf{MRST2004QED})}{xf(\mathsf{MRST2004QED})}$$

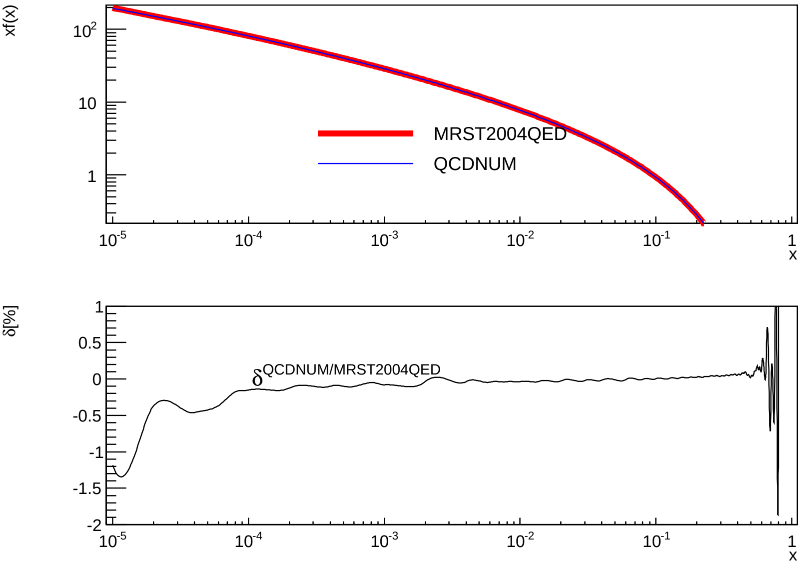
Δ distribution at $\mu = 100$ GeV



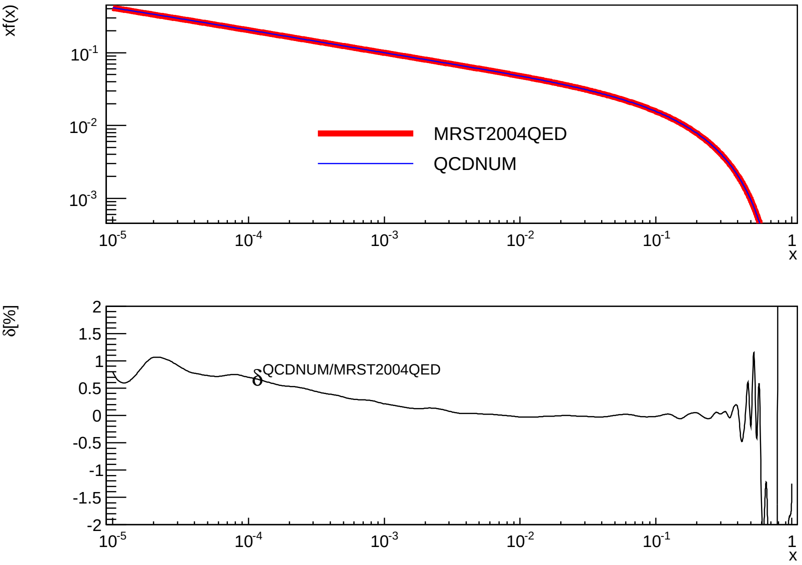
Σ distribution at $\mu = 100$ GeV



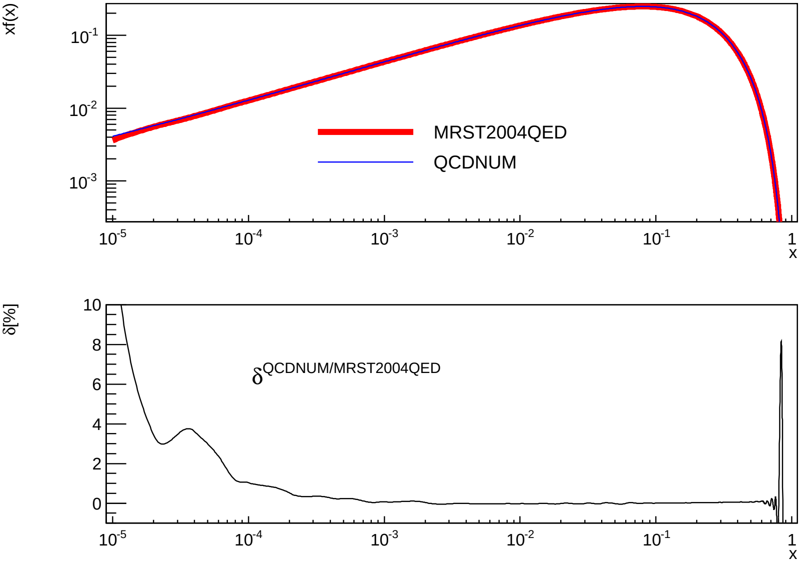
g distribution at $\mu = 100$ GeV



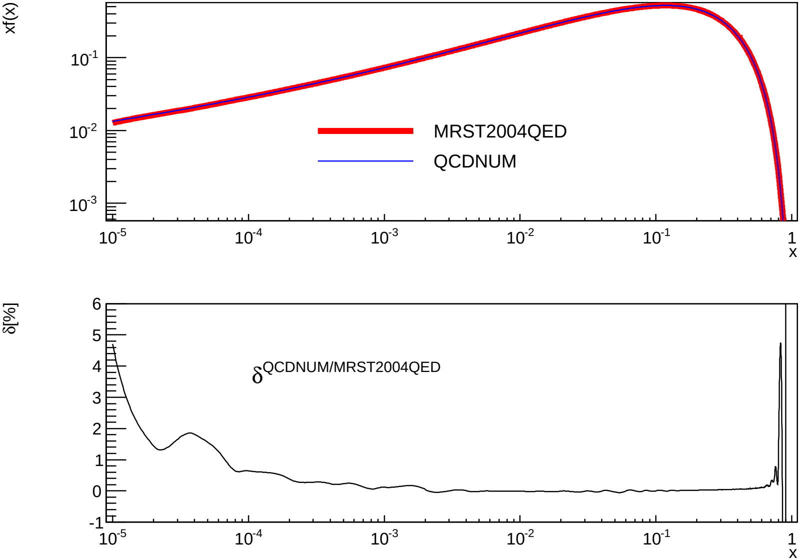
γ distribution at $\mu = 100$ GeV



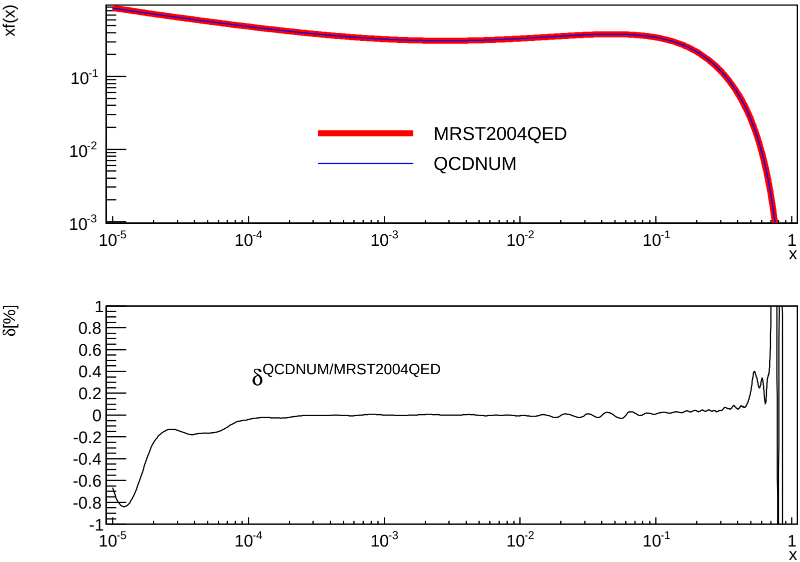
d_v distribution at $\mu = 100$ GeV



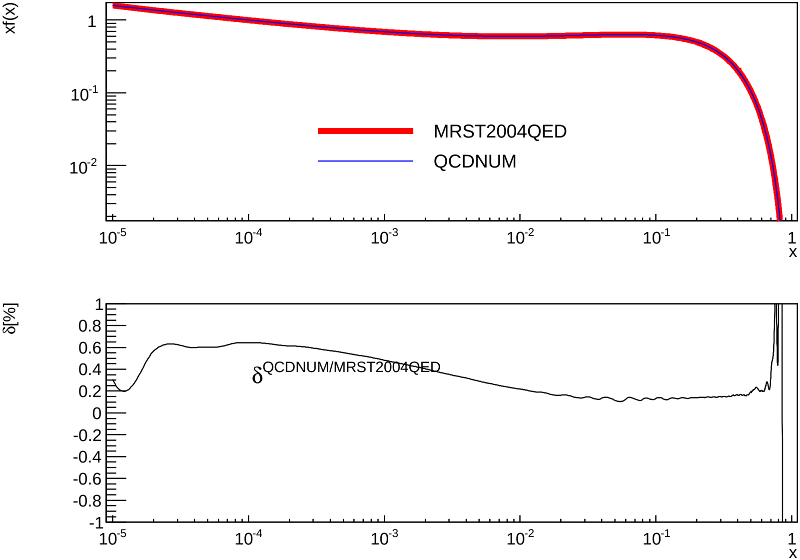
u_v distribution at $\mu = 100$ GeV



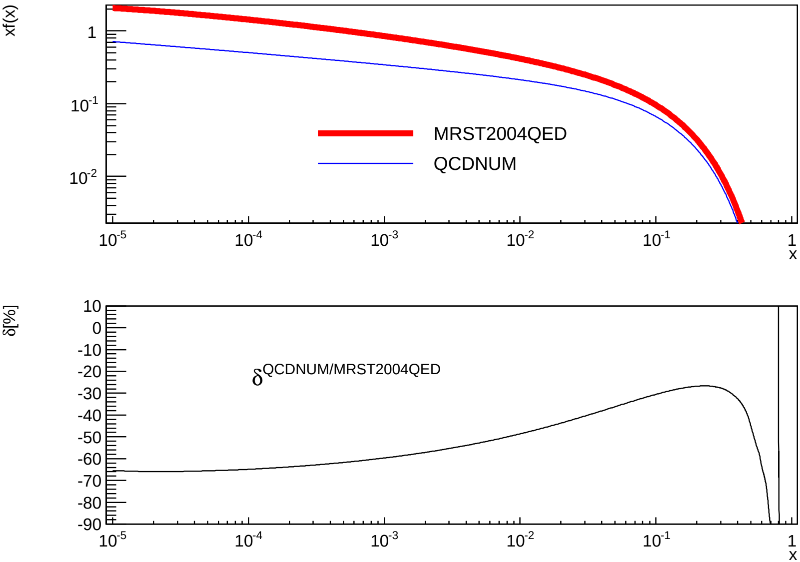
Δ_{ds} distribution at $\mu = 100$ GeV



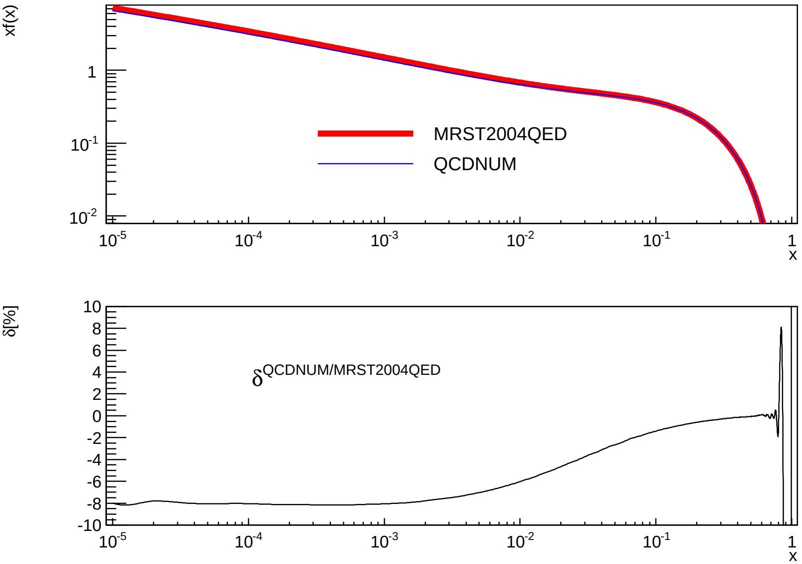
Δ_{uc} distribution at $\mu = 100$ GeV



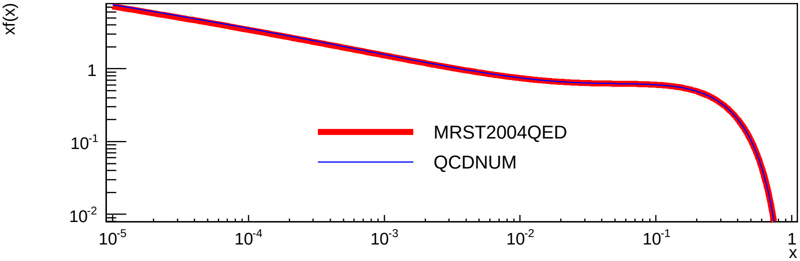
Δ_{sb} distribution at $\mu = 100$ GeV

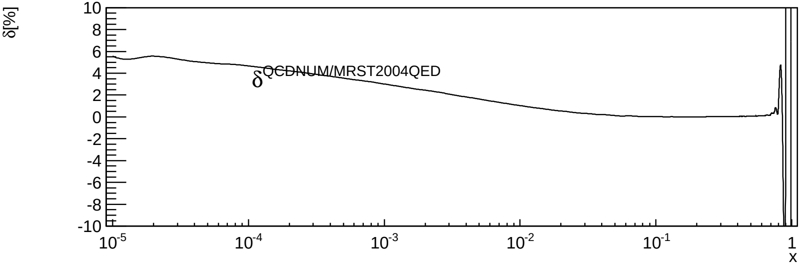


$\bar{d}$ distribution at $\mu = 100$ GeV

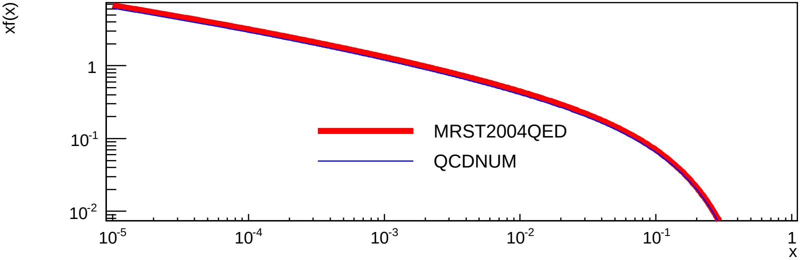


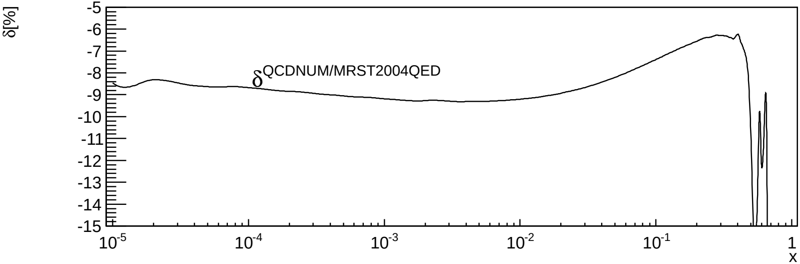
$\bar{u}$ distribution at $\mu = 100$ GeV



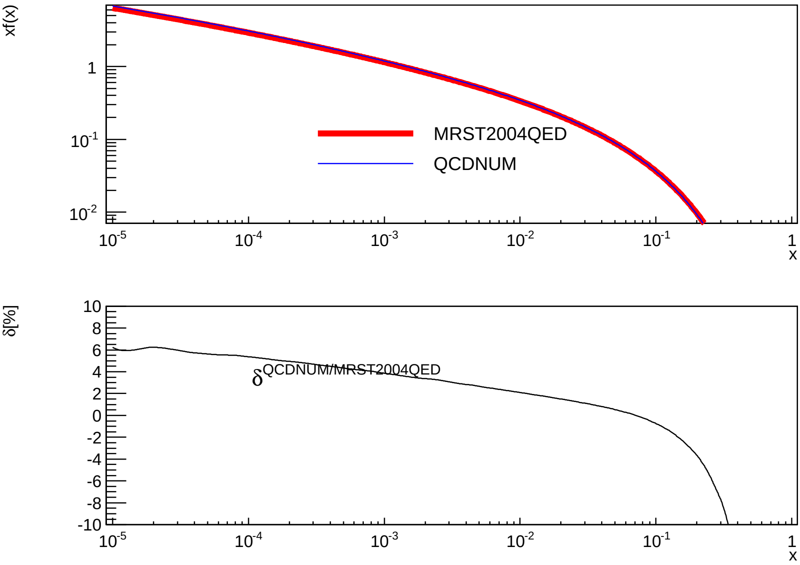


s distribution at $\mu = 100$ GeV

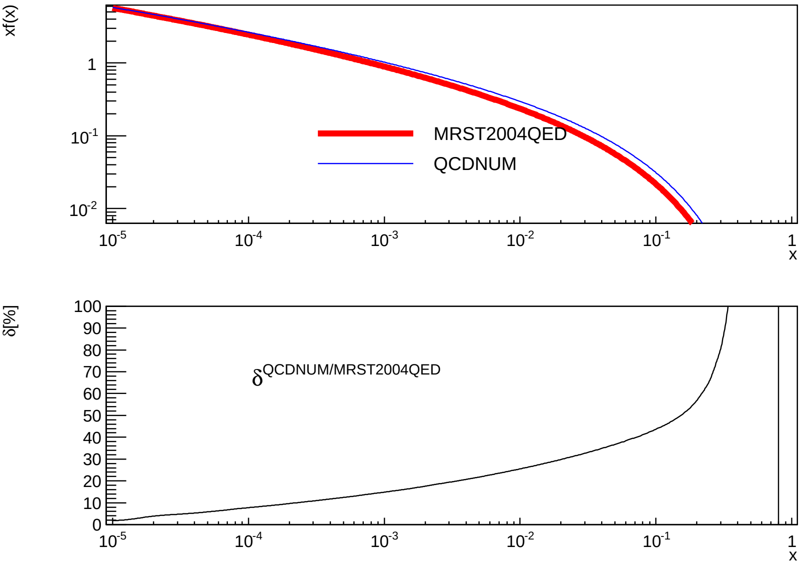




c distribution at $\mu = 100$ GeV



b distribution at $\mu = 100$ GeV



Next steps

- Investigate the reason of differencies between `QCDNUM` and MRST2004PDF set for quark distributions
- Implement into HERAfitter