GENERAL OPTICS DESIGN PART 2

Martin Traub – Fraunhofer ILT

martin.traub@ilt.fraunhofer.de - Tel: +49 241 8906-342



AGENDA

- Raytracing
 - Fundamentals
 - Aberration plots
 - Merit function
 - Correction of optical systems
- Optical materials
 - Bulk materials
 - Coatings
- Optimization of a simple optical system, evaluation of the system

Why raytracing?

- Real paths of rays in optical systems differ more or less to paraxial paths of rays
- These differences can be described by Seidel aberrations up to the 3rd order
- The calculation of the Seidel aberrations does not provide any information of the higher order aberrations
- Solution: numerical tracing of a large number of rays through the optical system
- High accuracy for the description of the optical system
- Fast calculation of the properties of the optical system
- Automatic, numerical optimization of the system

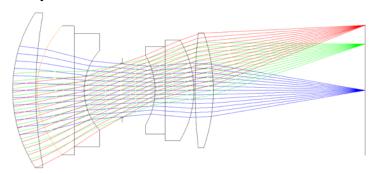
Principle of raytracing

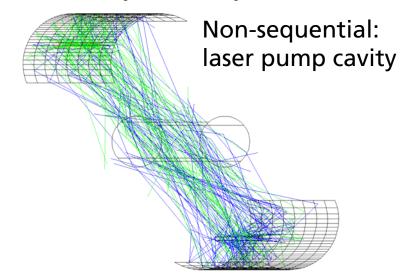
- The system is described by a number of surfaces arranged between the object and image plane (shape of the surfaces and index of refraction)
- 2. Definition of the aperture(s) and the ray bundle(s) launched from the object (field of view)
- 3. Calculation of the paths of all rays through the whole optical system (from object to image plane)
- 4. Calculation and analysis of diagrams suited to describe the properties of the optical system
- 5. Optimizing of the optical system and redesign if necessary (optics design is a highly iterative process)

Sequential vs. non sequential raytracing

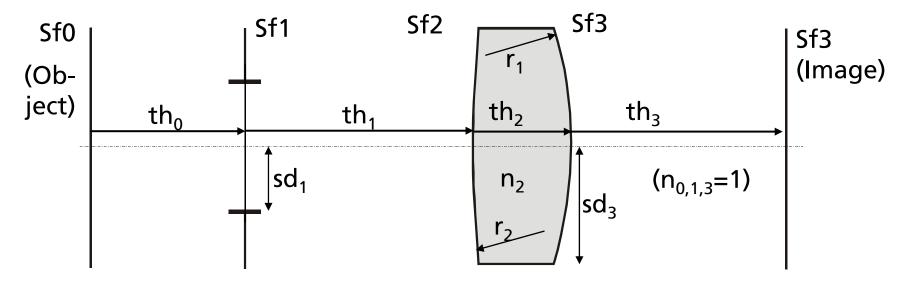
- In sequential raytracing, the order of surfaces is predefined
 → very fast and efficient calculations, but mainly limited to imaging optics, usually 100..1000 rays
- 2. In non-sequential raytracing, each ray may hit (or miss) each surface as often as necessary, and rays can be absorbed, new rays are generated at partially reflecting surfaces...
 - \rightarrow limited possibilities for optimization, usually > 10⁶ rays

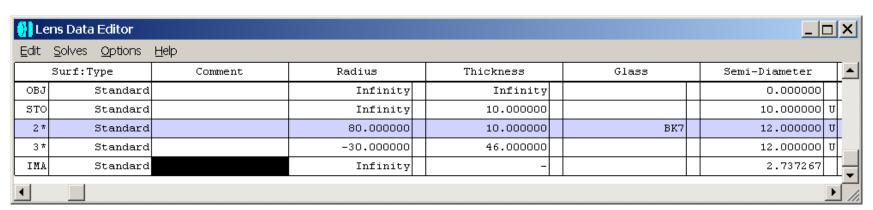
Sequential: Camera lens





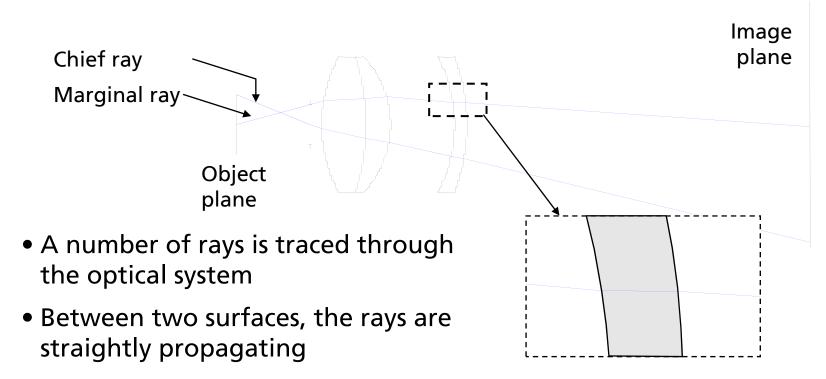
Description of an optical system





4) Index of refraction of the material following the surface

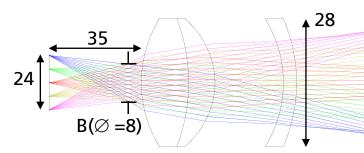
Example of traced rays



- At each interface with a change in index, refraction or reflection occurs
- The beam deflection is calculated applying the full law of refraction (the example shows the marginal and the chief ray)

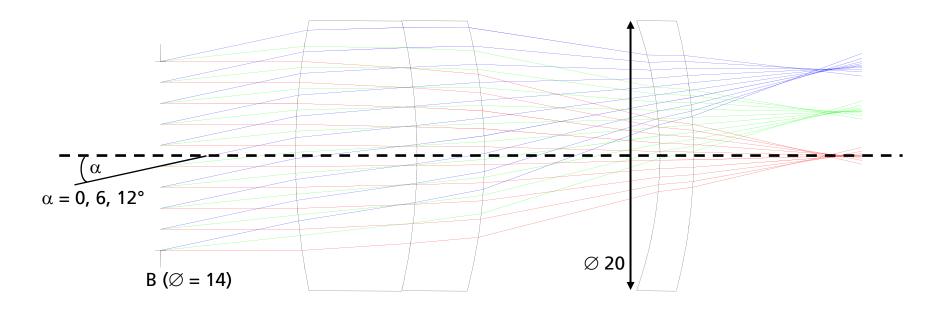
Definition of the field of view by the object size

 Image of an object formed by a three-lens system (not well corrected)



- Object size: 24 mm
- The ray bundle of each object point is limited by the aperture B
- To describe the system, ray bundles for different object points are calculated (here: +/-12, +/- 6, and 0 mm)
- As the system is symmetrical, rays below the optical axis are usually neglected

Definition of the field of view by field angle

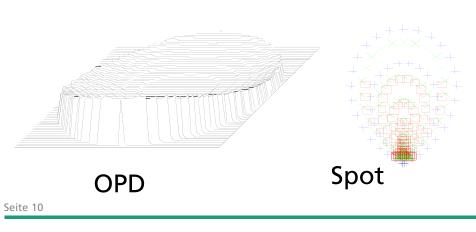


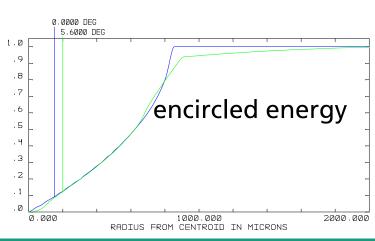
- If objects located far from the optical systems are imaged, the incoming rays are nearly parallel (e.g. stars)
- Therefore, the field of view is defined by the angle formed by the chief rays and the optical axis

Abberation plots

Raytracing software offer a number of diagrams and plots for analyzing the aberrations of an optical system:

- Wave front aberrations (Optical Path Difference: OPD)
- Transverse ray aberration (Transverse Ray Fan: TRF)
- Spot diagrams
- Encircled energy plots
- Modulation transfer function (MTF)
- Point Spread Function (PSF)







TRF

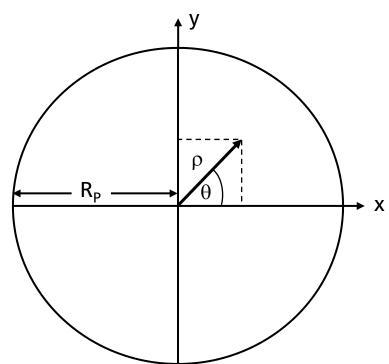
Normalized coordinates

Raytracing software uses normalized cartesian coordinates rather than polar coordinates (r, Θ) to define the position of a ray in the entrance pupil.

$$P_{x} = \frac{\rho \cdot \sin \theta}{R_{P}} = \frac{x}{R_{P}}$$

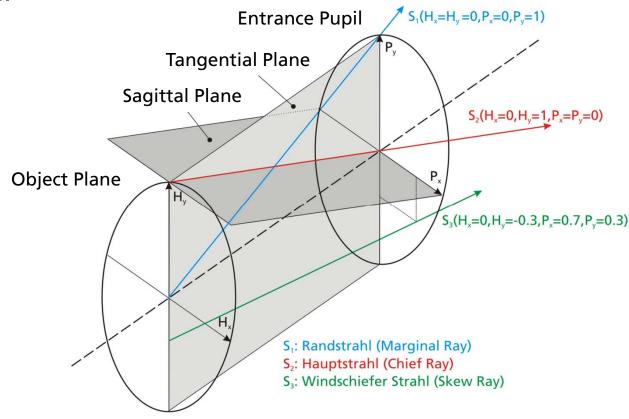
$$P_{y} = \frac{\rho \cdot \cos \theta}{R_{P}} = \frac{y}{R_{P}}$$

R_P – pupil radius



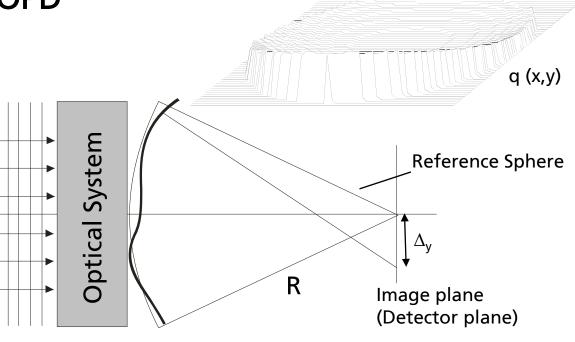
Ray definition

Raytracing software uses two points to define each ray: one is located in the object plane, and the other is located in the entrance pupil.



Calculation of the OPD

- Calculation of the optical path length for a number (i,j) of rays located in the entrance pupil: l_{i,i}
- Normalized to the wavelength and composed to a wavefront field W(x,y)

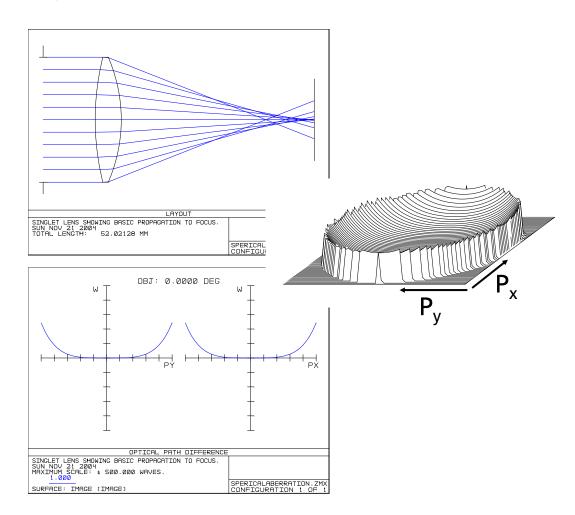


- Calculation of the spherical reference wave W_{sph,Ref}
- The OPD q(x,y) (wave front aberration) is calculated by $q(x,y) = W(x,y) W_{sph,Ref}(x,y)$
- The transverse ray aberrations can be calculated by:

$$\Delta x = -\frac{R}{n} \frac{\partial q}{\partial \rho_x} \qquad \Delta y = -\frac{R}{n} \frac{\partial q}{\partial \rho_y}$$

Example: OPD for a system with spherical aberrations

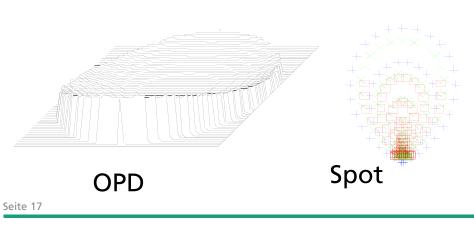
- Uncorrected singled with large spherical aberrations
- The complete 2D OPD (wavefront map) as a function of P_x and P_y is shown on the right hand side
- Usually only x and y cross sections are given (OPD fan)

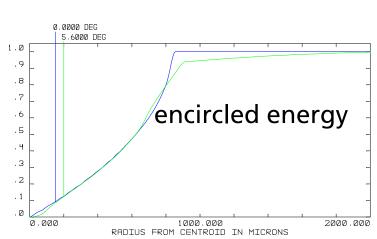


Abberation plots

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- Point Spread Function (PSF)

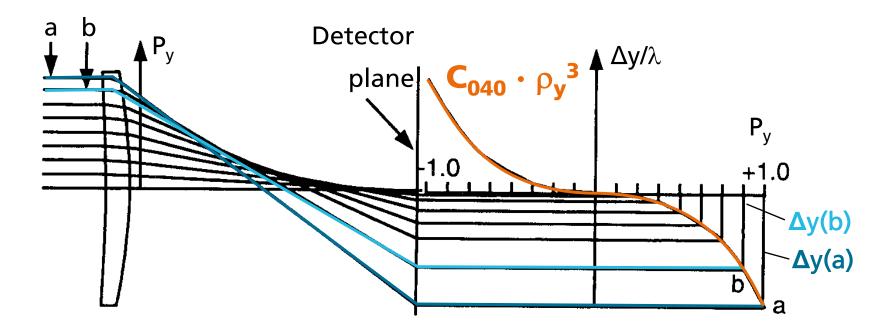






TRF

Transverse Ray Fan (TRF) Plot



P_{x,y} - Normalized pupil coordinates

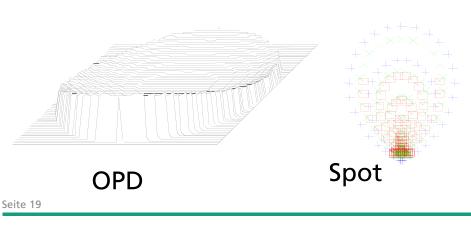
 Δx , Δy - Transverse distance from the axis or the chief ray if the ray bundle is tilted to the optical axis

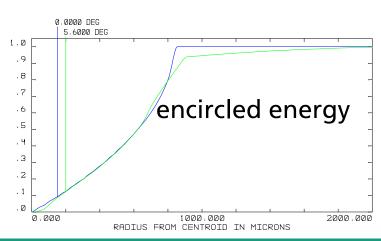
The aberration is measured in wavelengths

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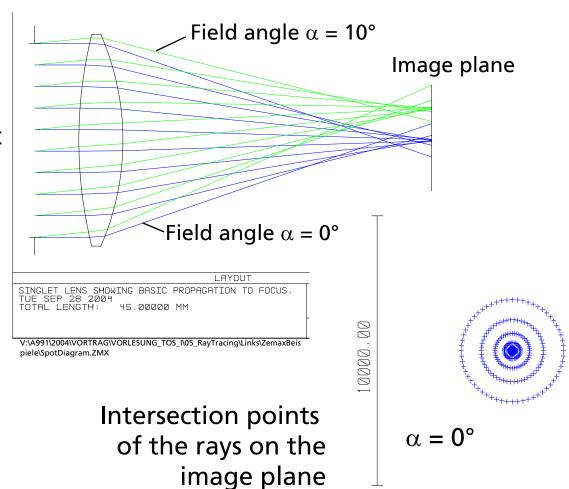


TRF

Spot diagram

Spot diagram

- Start: field of rays in the entrance pupil with a specific pattern
- Calculation and plot of all intersection points of rays on the image plane
- Gives a good overview of the aberrations of the optical system



Spot diagram: Start patterns

Hexapolar:

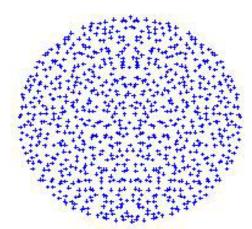
- Artifacts
- Well suited for 3rd order aberrations

Square:

- Artifacts
- Dithered:
- No artifacts
- Difficult to reproduce

Hexapolar distribution (rotationally symmetric)

Square distribution

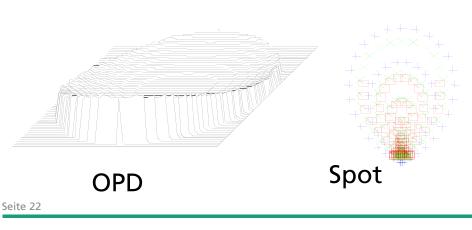


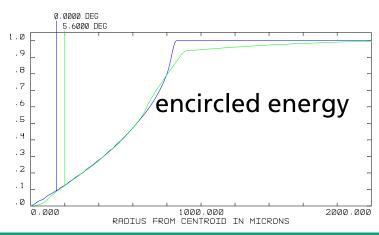
Dithered distribution

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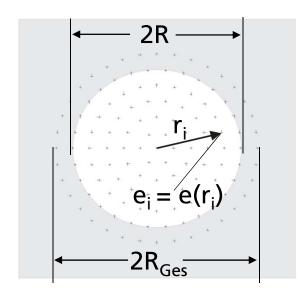






TRF

Calculation of the encircled energy P_{EP}



Spot diagram in the image plane

To every ray with index i in the entrance pupil a portion of the overall energy E_{tot} is assigned:

$$E_{Tot} = \sum_{r_i=0}^{R_{Tot}} e(r_i) = 1$$

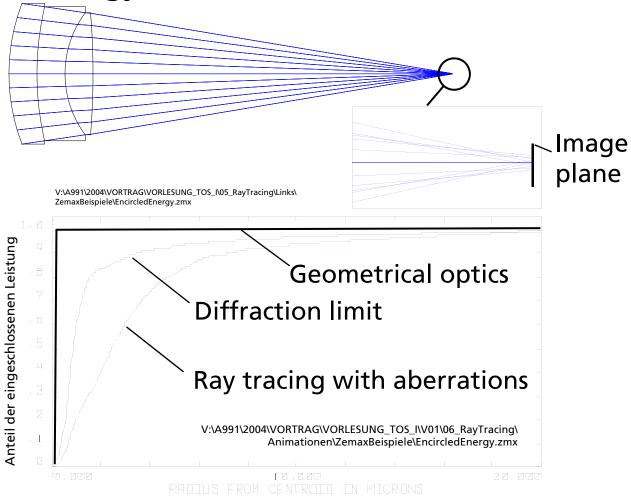
The energy $E_{EP}(R)$ encircled in the radius R is calculated by radially summarizing:

$$E_{EP}(R) = \sum_{r_i=0}^{R} e(r_i)$$

Example: encircled energy

 Focussing of a parallel ray bundle (with aberrations)

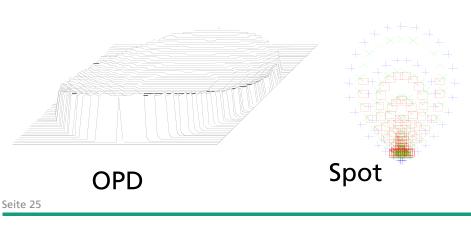
 Summation of the energy increments in the image plane

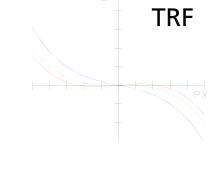


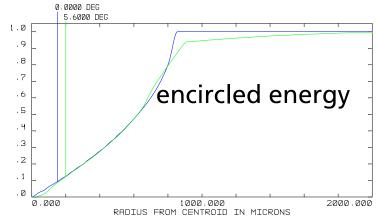
Abberation plots

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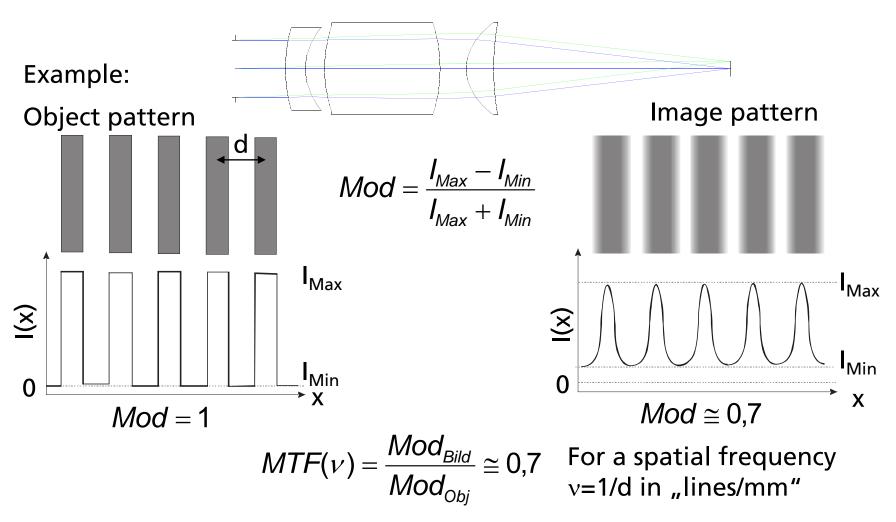
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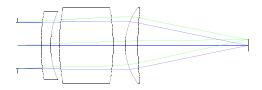




Modulation transfer function (MTF): definition



Modulation transfer funktion (MTF): Example



No aberrations, no diffraction:

$$\mathsf{MTF}(\vee)=1$$

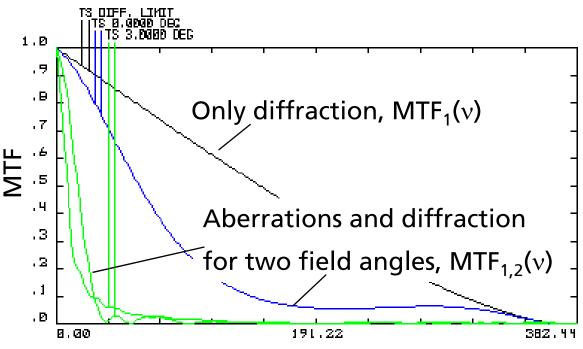
No aberrations, only diffraction:

$$MTF_1(v)$$

Aberrations and diffraction:

 $MTF_1(v)$, $MTF_2(v)$

Partly corrected focusing triplet with two field angles: 0° (blue) and 3°(green)

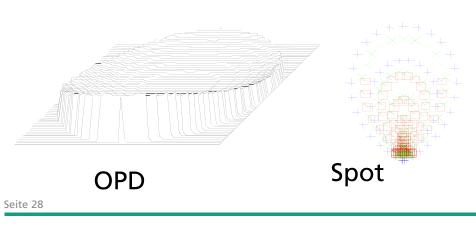


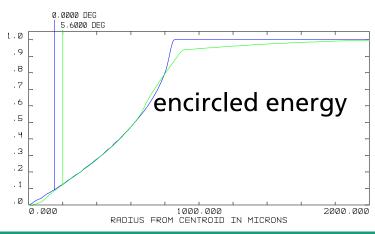
Spatial frequency v (Lines/mm)

Aberration plots

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TRF

Point Spread Function (PSF)

The PSF calculates the twodimensional distribution of intensity in the image plane of the optics for a point source (e.g. a star)

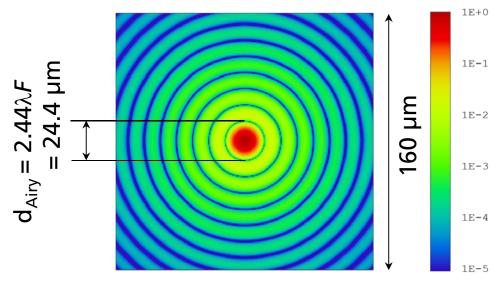
Both diffraction and aberrations are regarded

For a uniformly illuminated pupil and a system without aberrations, the PSF can be calculated by:

$$\frac{I(r)}{I_0} = \left(\frac{2J_1(x)}{x}\right)^2 \qquad x = \frac{\pi r}{\lambda F}$$

PSF (f=100 mm, EDP=10 mm, λ=1 μm):

Log scale plot:

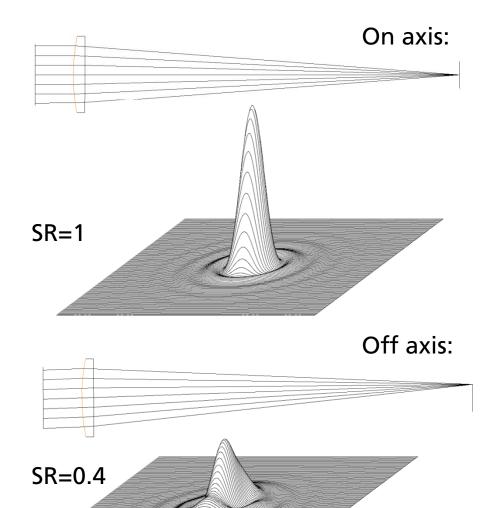


Point Spread Function: Aberrated systems and Strehl ratio

If aberrations decreases the optical performance of the system, the diameter of the PSF increases and the maximum intensity decreases:

$$P = \iint I(r, \varphi) \pi r dr d\varphi = const.$$

The Strehl ratio is the ratio of the peak irradiance of an aberrated system to the peak irradiance of the aberration free system.



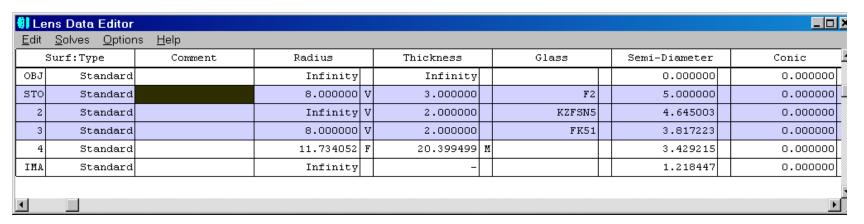
Optimization of optical systems

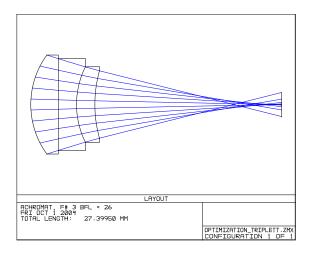
The main benefit of raytracing software is the ability to optimize the image quality of optical systems automatically (i.e. to reduce the aberrations).

The following steps are necessary for this:

- Declaration of variables V_i of the optical system (e.g. curvatures and distances of surfaces)
- 2. Definition of target values (e.g. focus diameter or max. OPD)
- 3. Definition of the merit function $\Phi(V_i)$. The merit function calculates a scalar based on the weighted deviation of the target values
- 4. Finding a local or global minimum of the merit function

Definition of variables of an optical system





Potential variables (indicated by "V"):

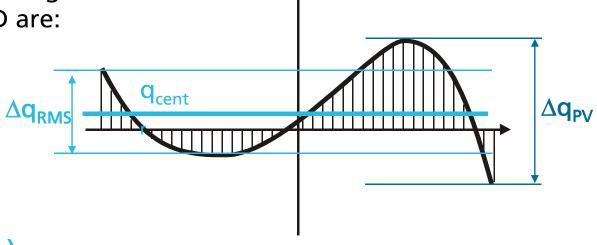
- Curvature of surfaces
- Distances of surfaces
- Optical materials
- Position and diameter of apertures
- Aspherical parameters of surfaces

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Modulation of the OPD function

Typical measures for the degree of modulation for the OPD are:

$$\Delta q_{PV} = |q_{i,Max}| - |q_{i,Min}|$$

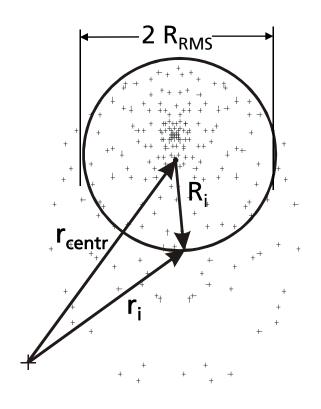


q_{RMS} (root mean square)

$$\Delta q_{centr} = \frac{1}{N} \sum_{i=1}^{N} q_i$$

$$\Delta q_{RMS} = \frac{1}{N} \sqrt{\sum_{i=1}^{N} (q_{centr} - q_i)^2}$$

Center of gravity and RMS of a spot distribution



Centroid calculated from N spots:

$$\vec{r}_{centr.} = \frac{1}{N} \sum_{i=1}^{N} \vec{r}_{i}$$

A typical radius of the spot distribution is calculated using the root-mean-square relative to the centroid:

$$R_{RMS} = \frac{1}{N} \sqrt{\sum_{i=1}^{N} (\vec{r}_i - \vec{r}_{centr})^2}$$

$$= \frac{1}{N} \sqrt{\sum_{i=1}^{N} (\vec{r}_i)^2} \qquad for \quad \vec{r}_{centr} = 0$$

RMS – root mean square

Definition of the Merit function Φ

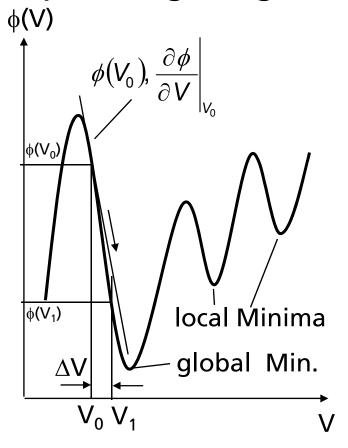
• Definition of parameters ϕ_i and target values $\phi_{i,target}$, e.g.:

$$\begin{array}{ll} \phi_{RMS} = R_{RMS} & \phi_{RMS,target} = 0 & RMS \ radius \ of the \ spot \ diagram \\ \phi_{PV} = \Delta q_{PV} & \phi_{PV,target} = 0 & Peak-to-valley \ modulation \ of \ OPD \\ \phi_{f/\#} = f/D & \phi_{f/\#,target} = 5 & F \ number \ in \ image \ space \end{array}$$

- The parameters ϕ_i depend on the variables V_i
- Calculation of the differences between parameter and target value: $\underline{\phi}_{i} = (\phi_{i} \phi_{i, \text{target}})$
- Squaring and weighting of the deviation: $w_i \cdot \underline{\phi_i}^2$
- The merit function is the square root of the sum of these values, normalized by the square root of the sum of the weights:

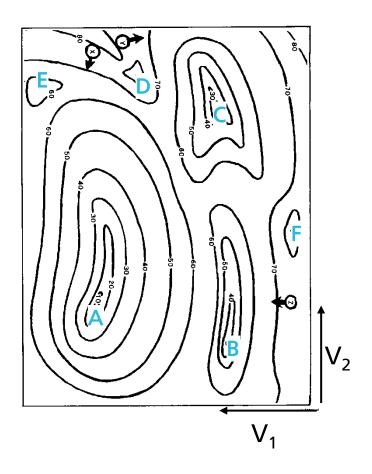
$$\Phi(V_i) = \sqrt{\frac{1}{\sum w_j} \sum_{j=1}^M w_j \cdot \underline{\varphi}_j^2(V_i)}$$

Optimizing using the merit function



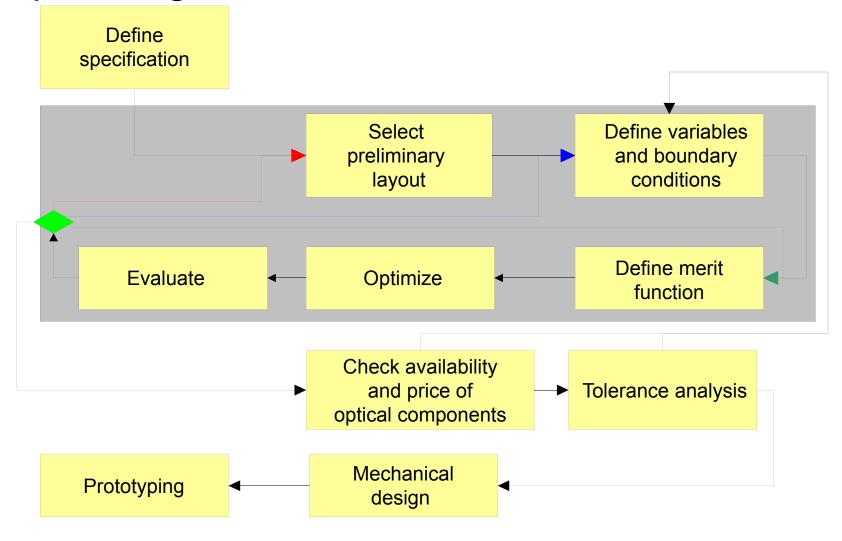
- Simple case: φ is only depending on one variable (V)
- Initial value V₀ is given
- Calculation of the derivative ∂φ/∂V at the initial value
- Based on the direction and the increment ∆V, the next value is calculated
- Recalculation of φ until the minimum is reached
- Mathematical method: (Damped) least squares: $grad(\Phi(\vec{V})) = 0$

Simple example: 2D merit function



- Given: merit function, depending on two variables
- The contour line plot shows 6 minima (A to F)
- Three weak minima (D to F; trivial solutions)
- Three strong minima (A to C; desired solutions)
- Depending on the chosen initial position, different solutions are found
- Evaluation of the solution, choosing a new initial solution if necessary

Optics design flow chart



Factors influencing the performance of an optical system

Optical design

 The correction of the aberrations is defined by the parameters chosen during the design process

Components and manufacturing

- The sensitivity to tolerances is also defined during the design process
- Mechanical tolerances of the optical components (curvatures, thicknesses, diameters, centering)
- Tolerances of the optical materials (index, dispersion, homogeneity)
- Mechanical tolerances of the lens mounts

Environment

- Temperature range
- Mechanical stress
- Dynamic loads (vibration and shock)

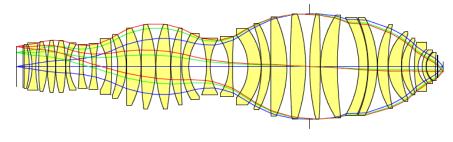
Design and optimization of optical ystems

Initial system for optimization (first layout):

- Catalogs of lens manufacturers (basic designs)
- Databases (patents, LensView, Zebase)
- Experience and intuition



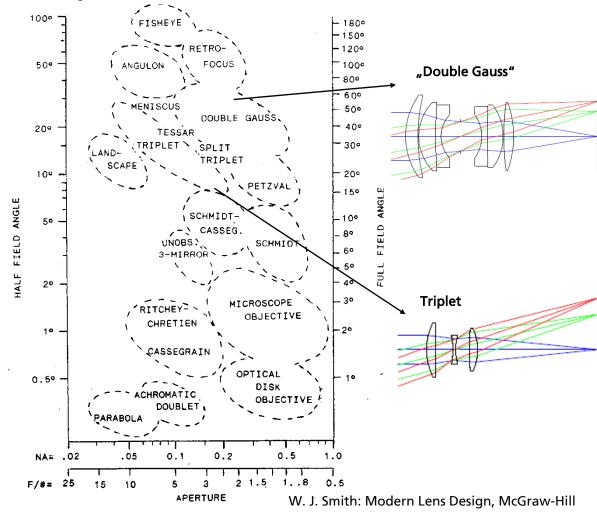
or



?

Basic designs based on aperture and field size

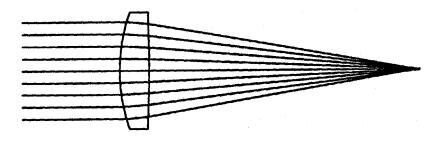
- The map shows well-known lens designs for different combinations of field and speed
- The performance and complexity increases from bottom to top (higher FoV) and from left to right (higher speed)



Basic designs

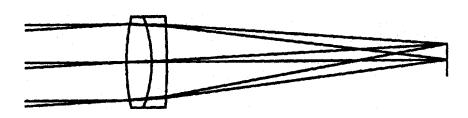
• Simplest design:

Singlet



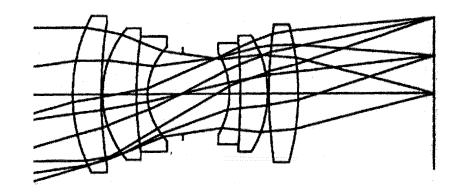
Doublet:

Correction of spherical aberrations and other 3rd order aberrations (limited in field size)



• "double gauss" lens:

Well corrected for large FoV and low F#

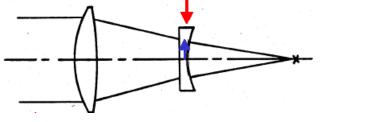


Basic designs: negative elements

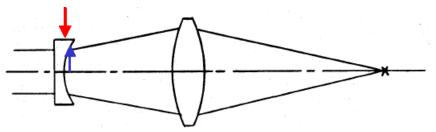
Field flattener

Cooke triplet

Telefocus lens

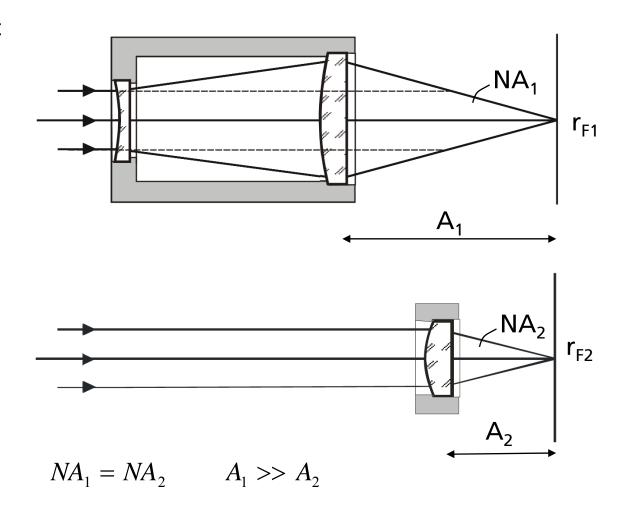


Retrofocus Iens



Basic designs: Retrofocus lens

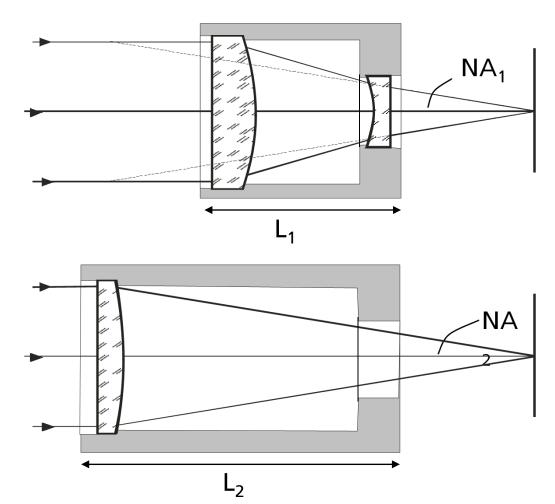
- The retrofocus lens at the top offers the same effective focal length (EFL) and NA compared to the singlet at the bottom
- For the retrofocus lens, the back focal length (BFL) is larger than the EFL



Basic designs: Telephoto lens

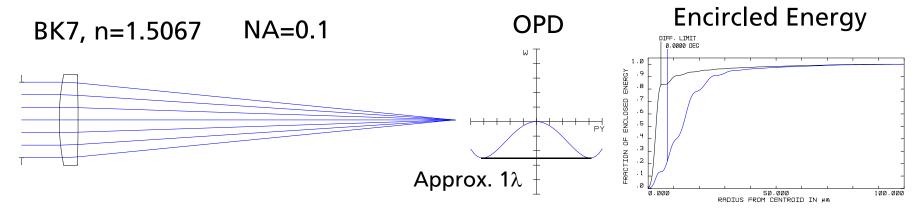
- Also, the telephoto lens has the same EFL than the singlet (bottom)
- The overall length of the telephoto lens is considerably smaller compared to the singlet

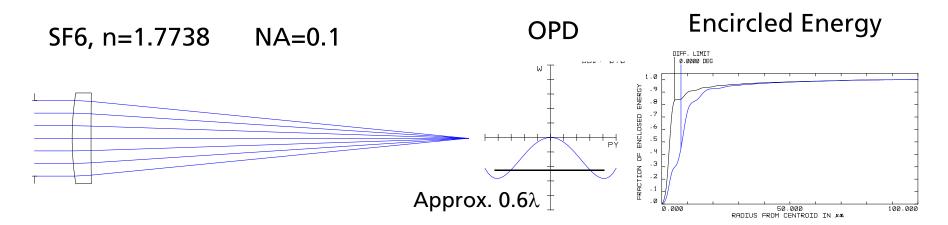
$$NA_1 = NA_2$$
 $L_1 \ll L_2$



Correcting optical systems: spherical aberrations

1) Increased index of refraction (choice of glass)



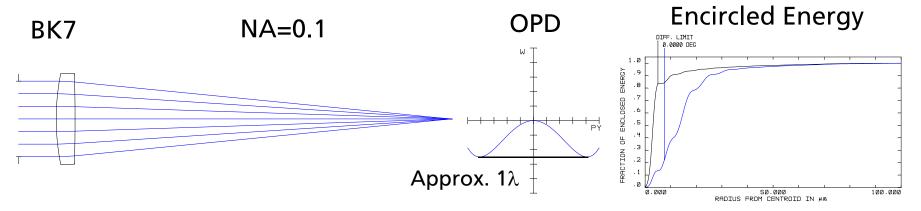


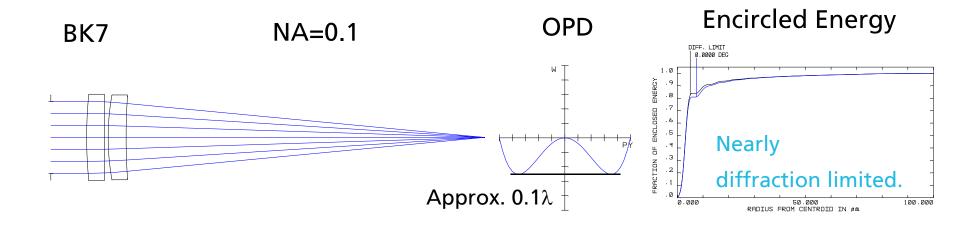
Radii are chosen to yield the same EFL.

Seite 46

Correcting optical systems: spherical aberrations

2) Split the power of the singlet to two lenses





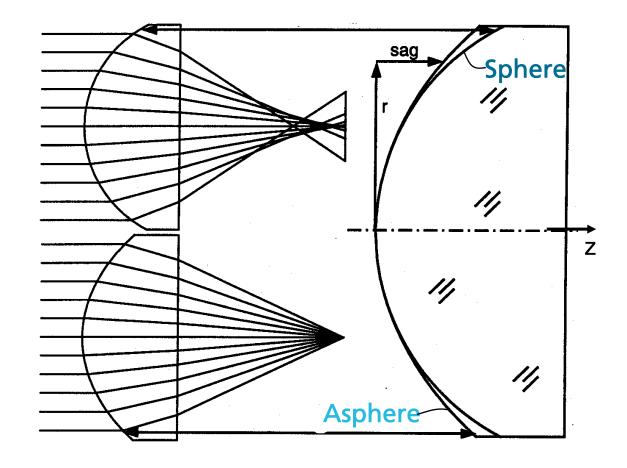
Correcting otical systems: Aspheric surfaces

Sphere:

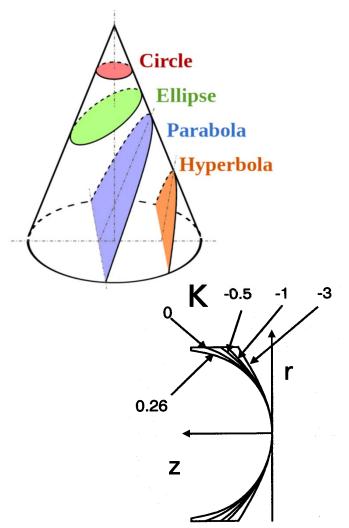
High bending of the peripheral rays (spherical aberration)

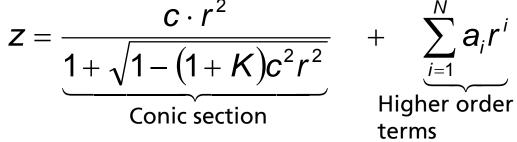
Asphere:

Decreased slope at the edge (weaker refraction) leads to a stigmatic on-axis image



Description of the asphere





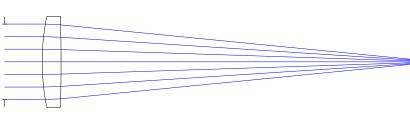
c – Curvature (=1/R)

K – conic constant

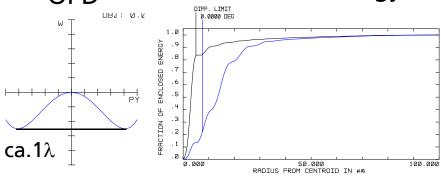
K	Conic section	Surface of revolution
0	Circle	Sphere
K < -1	Hyperbola	Hyperboloid
K = -1	Parabola	Paraboloid
-1 < K < 0	Ellipse	Ellipsoid
K > 0		"Oblate ellipsoid"

Correcting optical systems: spherical aberrations

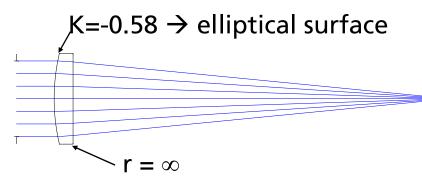
Spherical surface (BK7, NA=0.1):



OPD Encircled Energy

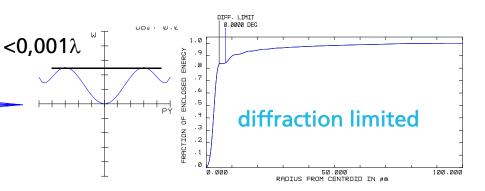


Aspherical surface (BK7, NA=0.1):

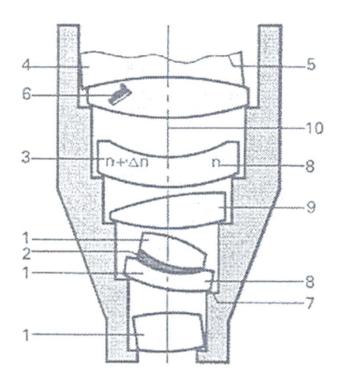


OPD

Encircled Energy



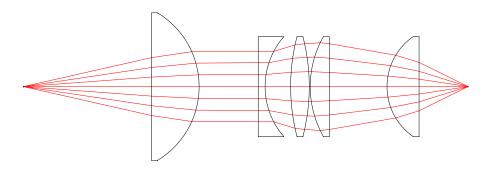
Tolerances

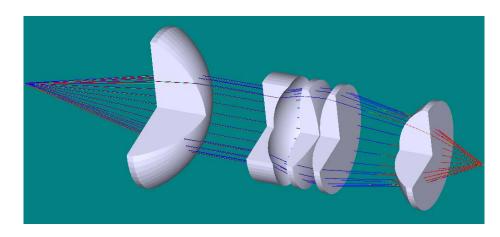


- 1. Tilt of lens
- 2. Wedge error caused by cementing
- 3. Variation of index
- 4. Decentered doublet
- 5. Surface irragulations
- 6. Striae, Bubbles
- 7. Lens mount tolerances (tilt)
- 8. Center thickness deviations
- 9. Decentered lens
- 10. Air space tolerances
- 11. ...

Mechanical design of a complex lens mount

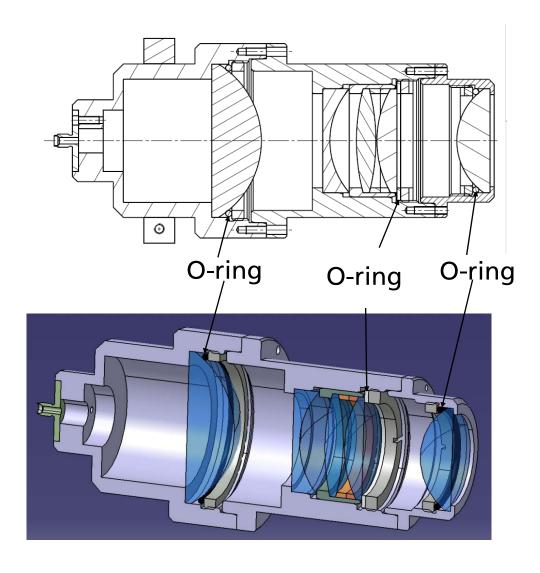
- Example: optical system for imaging an optical fiber exit on a workpiece (demagnified, M1:2)
- Boundary condition:
 Working distance > 25 mm,
 only stock lenses (lead time,
 costs)
- Result: 5-lens-system, retrofocus
- Interface Raytracing/CAD:IGES





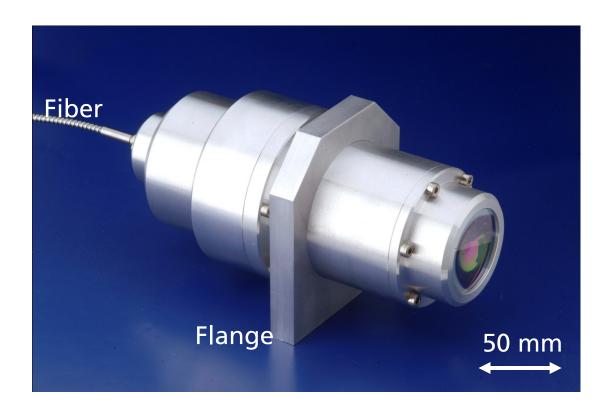
Mechanical design

- All lenses are mounted in a metal cylinder
- The distances are defined by spacers
- The lenses are centered by the cylinder
- Tilt errors are cumulated
- To compensate CTE mismatch, elastic rings are foreseen

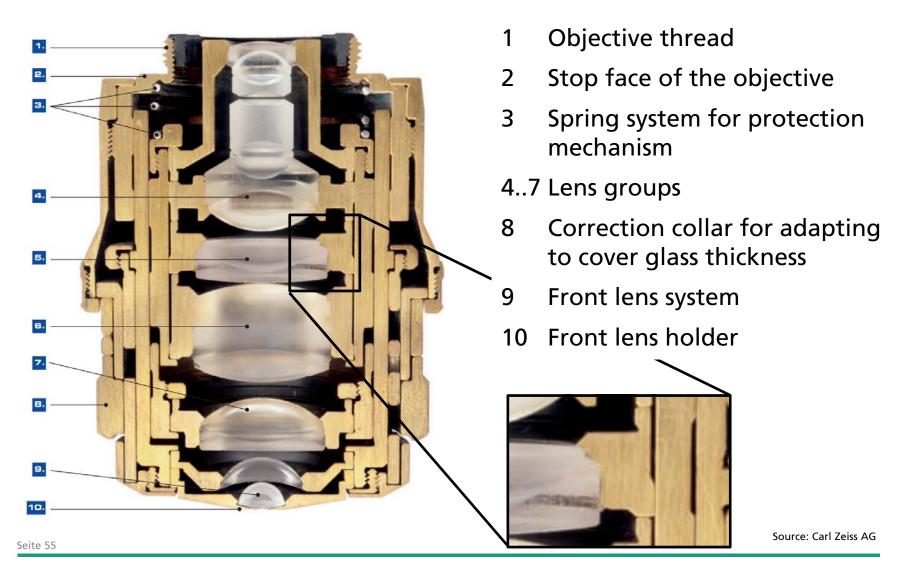


Lens mount

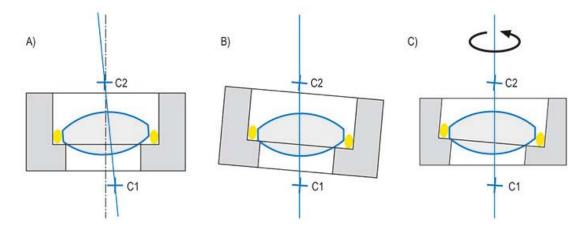
- Aluminum housing
- Three subassemblies
- Flange for mounting the optics to the robot



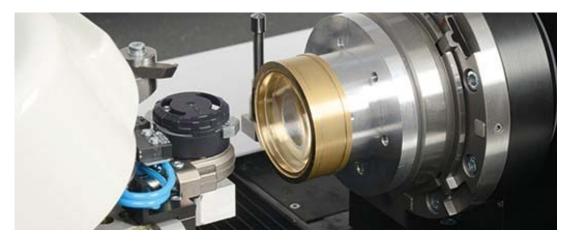
Complex lens mount: Microscope objective



Lathe centering



Source: Trioptics GmbH



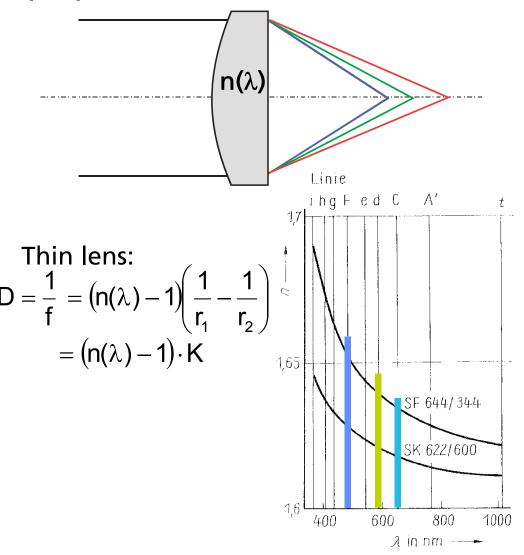
Source: Fraunhofer IPT

Seite 56

- A) The lens is decentered to the outer cylinder of lens mount
- B) The mount is tilted and shifted (C1 and C2 coincide with the axis of revolution of the lathe)
- C) The outer cylinder and the plane surface of the mount are machined

Chromatic aberrations (CA)

- The index of refraction of transparent materials depends on the wavelength (dispersion, n=n(λ))
- Therefore the axial focal point depends on the wavelength
- For normal dispersion glasses, $dn/d\lambda < 0$
- To correct CAs, positive and negative lenses of different index and dispersion are combined ("Achromat")



Spectral lines as wavelength reference

The index n is depending on the wavelength n is usually given for specific spectral lines, and a corresponding suffix is added to the index (e.g. n_d)

Symbol	Wavelength (nm)	Element	Spectral region
i	365.0	Hg	Ultraviolet
g	435.83	Hg	Blue
F	486.1	H ₂	Blue
е	546.07	Hg	Green
d	587.56	He	Green-Yellow
С	656.3	H ₂	Red
t	1013.9	Hg	Infrared

Important data on transparent optical materials

- Mean index of refraction n_d (@ d = 587.56 nm)
- Abbe number (@ d line)

$$v_d = \frac{n_d - 1}{n_F - n_C} > 0$$
 (for normal dispersion)

Large Abbe number: low dispersion

Low Abbe number: large dispersion

- Additional data:
 - Price
 - Hardness, Machinability
 - Resistance (Acids, base, water)
 - Absorption, homogeneity
 - Thermal expansion and dn/dT

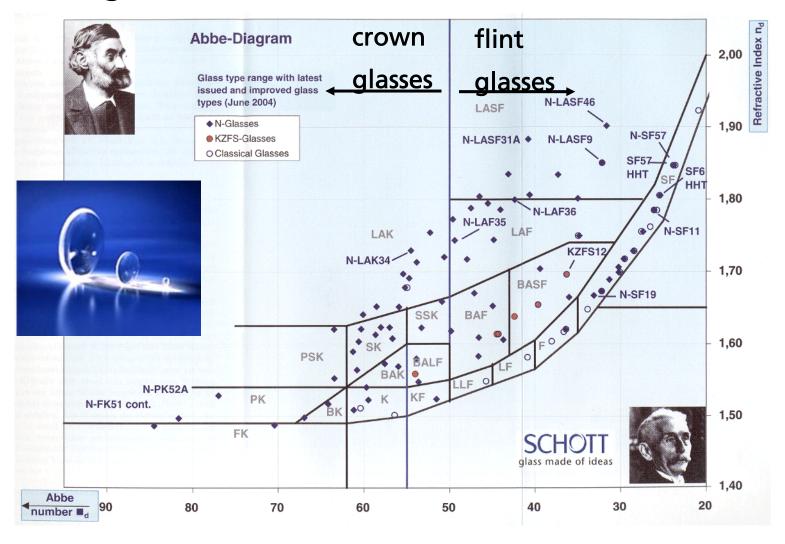
Comparison of different optical materials

- Optical glass is a inorganic melting product Solidifies as amorphous material Highly transparent, homogeneous and isotropic
- Glass consists of network formers (e.g. SiO₂), modifiers (e.g. Ca, Pb) and intermediates
- The modifiers change the physical properties of the glass

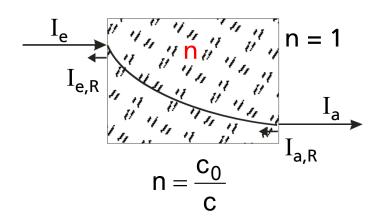


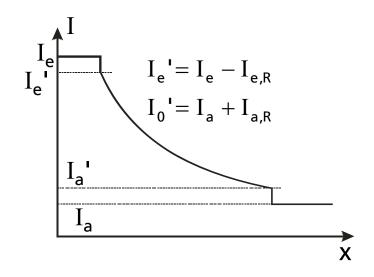
- Crystals are grown from the liquid phase (very slow, energy-consuming and thus expensive process)
- Crystals offer unique features (laser active media, nonlinear properties, anisotropic properties, high thermal conductivity, hardness)
- Metals are non-transparent, but offer a high reflectivity and high damage threshold

Glass diagram (Schott)



Transmission





Transmission:

$$T_e = \frac{I_a}{I_e}$$

Internal Transmission:

$$T_i = \frac{\mathbf{I}_a}{\mathbf{I}_e}'$$

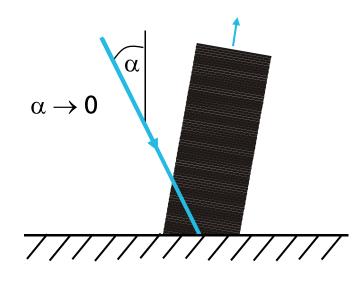
$$\frac{I_{a,R}}{I_a} = \frac{I_{e,R}}{I_e} = r^2 = R = \frac{(n-1)^2}{(n+1)^2} = \frac{(c_0 - c)^2}{(c_0 + c)^2}$$

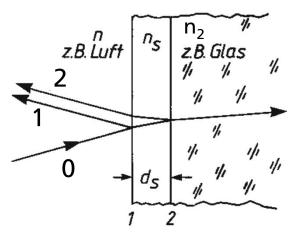
Reflectivity R: typically <10%, weakly depending on λ (due to dispersion)

$$I_a' = I_e' \cdot \exp(-\kappa \cdot x)$$

Coefficient of absorption κ (1/cm) strongly depending on λ

Principle of antireflection coatings





Destructive interference of partial waves which are reflected at the coating-air-interface (1) and at the coating-substrate-interface (2)

Simple case: single layer, 0° A.o.I.

Phase condition:

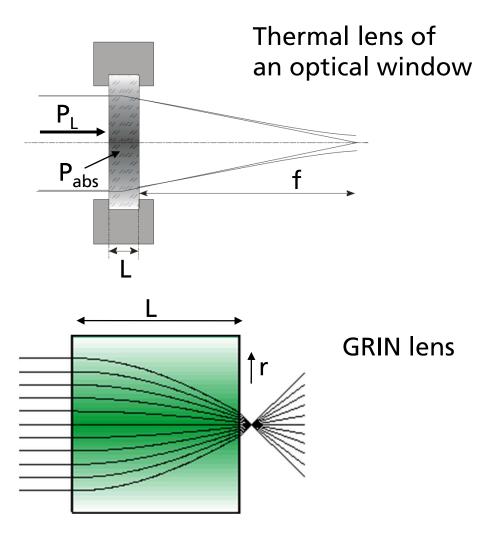
$$2n \cdot d_s = \frac{\lambda_0}{2}$$
 \Rightarrow $d_s = \frac{\lambda_0}{4n_s}$

Equal amplitude condition:

$$r_{01} = \frac{n_s - n_1}{n_s + n_1} = r_{12} = \frac{n_2 - n_s}{n_2 + n_s} \implies \frac{n_s = \sqrt{n_1 n_2}}{n_2 + n_s}$$

Gradient index materials, thermal lenses

- Even optical flat components may focus light
- The beam deviation is caused by a radial index profile
- First example: Thermal lens of a high power laser window
- Second example: Gradient index materials



Metal optics

Folding mirrors



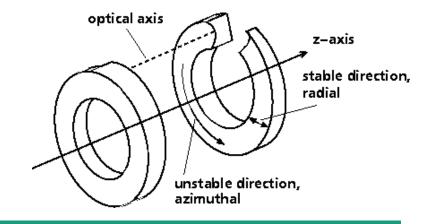
Off-axis paraboloid



Seite 66

- Very high thermal conductivity ($k_{cu} \approx 300^{\circ} k_{Silica}$), efficient cooling
- High reflectivity in the NIR spectral range
- Well machinable (UP turning and milling yields to optical surfaces)
- High damage threshold mirrors for high power CO₂ lasers

Mirrors of a coaxial ring resonator



Reflectivity of metals

- Copper, silver and gold reaches a reflectivity >99% in the FIR region (e.g. 10.6µm)
- Copper is commonly used
- Typical absorption:0.5-2%

