Probing Magnetic Field Correlations with μ-distortion

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Our motivation: Learning about early magnetic fields?

Very brief abstract:

Magnetic fields damp during the early universe, generating μ-distortion. The μ-monopole constrains the field's strength. What else can we learn from μ-distortion?

We can learn about a primordial $\langle \mathbf{B} \cdot \mathbf{B}\, \zeta \rangle$ correlation.

Outline

The early universe Planck spectrum

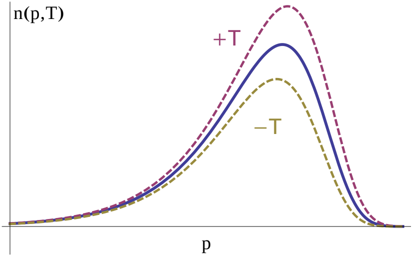
- At very early times in the universe, the photon spectrum has the pure Planckian form

$$n(p, T) \propto 1/(e^{p/T-1}) .$$

- Note that number density is uniquely related to temperature which is uniquely related to energy density.

- If energy is injected into the photons, the number density of the photons must change to maintain a Planck spectrum.

  This is accomplished in the very early universe ($z \gtrsim 10^6$) by double Compton scattering ($\gamma + e^- \rightarrow 2\gamma + e^-$) and bremsstrahlung.



But the photon spectrum gets distorted

- After $z_i \approx 2 \times 10^6$, double Compton scattering ($\gamma + e^- \rightarrow 2\gamma + e^-$) and bremsstrahlung become inefficient
  $\Rightarrow$ Photon number becomes essentially conserved.
- Meanwhile, the plasma is still in thermal equilibrium due to elastic Compton scattering, $\gamma + e^- \rightarrow \gamma + e^-$.
- The result: The photons settle into a Bose-Einstein distribution with a chemical potential μ:

$$n(P, T) \propto \frac{1}{e^{(p-\mu)/T} - 1} \,.$$

  We call this new feature *a μ-distortion*.
- After $z_f \approx 5 \times 10^4$, even elastic Compton scattering is inefficient and the photon spectrum is no longer thermalized after an energy injection.

Sunyaev & Zel'dovich 1970, Weynmann 1966

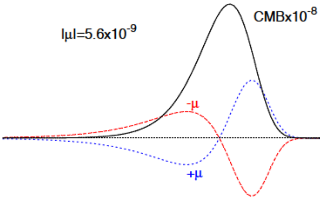
Current status of μ-distortion

- COBE/FIRAS, along with small improvements from TRIS, gives the current constraint

$$|\mu| < 6 \times 10^{-5}\,.$$

- μ-distortion is particularly promising because it is sensitive to earlier times ($2 \times 10^{6} > z > 5 \times 10^{4}$) and smaller scales ($1 \times 10^{4}\ \mathrm{Mpc}^{-1} < k < 50\ \mathrm{Mpc}^{-1}$) than most other observables.

- There have been many interesting recent papers on μ-distortion, e.g. papers by Khatri, Chluba, Sunyaev, Pajer, Zaldarriaga, Komatsu, Kunze, JG, Miyamoto, ...



From Khatri et. al. 2011

Anisotropic μ-distortion

- Until recently, investigations largely considered μ-distortion as a sky-averaged value, which would represent a correction to a Planck spectrum.
- Pajer and Zaldarriaga 2012 realized that the sources of μ-distortion, e.g. energy from density waves, magnetic fields, particle decay, could have correlations with other quantities. In their case, they correlated μ-distortion from density wave energy ($\propto \zeta^2$) with the CMB temperature anisotropy ($\propto \zeta$) to constrain $\langle \zeta^3 \rangle \propto f_{\rm NL}$.
- We will consider correlations between μ-distortion from magnetic wave energy ($\propto B^2$) and CMB temperature anisotropy to constrain $\langle \mathbf{B} \cdot \mathbf{B}\, \zeta \rangle$.

Damping of magnetic waves in the pre-CMB plasma

- The CMB is the remnant of a plasma of photons, electrons, and protons, which were ($z \gg 1000$) tightly coupled.
- Besides standard acoustic waves, this system supports waves where fluid motion is accompanied by magnetic fluid oscillations: magnetohydrodynamic (MHD) waves.
- The plasma is not an ideal fluid because the photons have a non-zero mean free path l_γ. This gives the fluid an effective viscosity $\propto \rho_\gamma l_\gamma$, damping the propagating waves.
- There is a characteristic damping scale:

$$k_D(a) \approx 7.44 \times 10^{-6}\ \text{Mpc}^{-1} \times a^{-3/2}$$

with $k_D^i = 2.1 \times 10^4\ \text{Mpc}^{-1}$, $k_D^f = 83\ \text{Mpc}^{-1}$
(cf. Silk damping scale of $k_{D\gamma} \approx 4.1 \times 10^{-6}\ \text{Mpc}^{-1} \times a^{3/2}$).

Jedamzik, Katalinic, & Olinto 1998

Correlations between a B and ζ

In analogy with f_{NL}, it is natural to parameterize a magnetic field B^{cor}'s correlations by supposing they are produced through local processing of an initially uncorrelated B^{uncor} field:

$$\mathbf{B}^{\mathrm{corr}} \simeq \mathbf{B}^{\mathrm{uncor}} + b_{\mathrm{NL}} \mathbf{B}^{\mathrm{uncor}} \zeta + \dots ,$$

In the squeezed limit $k_1 \approx k_2 \ll k$, this gives

$$\left\langle \mathbf{B}^*_{\mathbf{k}_1} \cdot \mathbf{B}_{\mathbf{k}_2} \, \zeta_{\mathbf{k}_3} \right\rangle'_{k_1 \approx k_2 \ll k_3} = b_{\mathrm{NL}} P_B(k_1) P_\zeta(k_3) \, .$$

This will be our definition of b_{NL}.

Jain & Sloth 2012

Outline

Calculating μ-distortion

The μ-distortion from a change $\Delta\rho_\gamma$ in the photon energy ρ_γ is given by (Sunyaev & Zel'dovich 1970, Hu & Silk 1993, Pajer & Zaldarriaga 2013)

$$\mu = 1.4 \frac{\Delta\rho_\gamma}{\rho_\gamma} \,.$$

Thus, the μ-distortion from magnetic field damping is

$$\mu \approx 1.4 \frac{\left[\rho_B\right]_f^i}{\rho_\gamma} \propto \int d^3k_1 d^3k_2 \ldots \underbrace{\mathbf{B}_{\mathbf{k}_1} \cdot \mathbf{B}^*_{\mathbf{k}_2}}_{\substack{\text{magnetic}\\ \text{fields}}} \underbrace{\left[e^{-(k_1^2+k_2^2)/k_D^2(a)}\right]_f^i}_{\text{damping}} e^{i\mathbf{k}_-\cdot\mathbf{x}} \,,$$

where $\mathbf{k}_- \equiv \mathbf{k}_1 - \mathbf{k}_2$.

Finding $C_l^{\mu T}$

For the temperature perturbation[1]

$$\frac{\Delta T}{\bar{T}}(\hat{\mathbf{n}}) \approx -\frac{1}{5}\zeta(r_L \hat{\mathbf{n}}),$$

where r_L is the distance to surface of last scattering.
To project onto the 2D CMB (for $X = \mu, T$), we need to define

$$a_{lm}^X \equiv \int d^2\hat{\mathbf{n}}\, X(\hat{\mathbf{n}}) Y_{lm}^*(\hat{\mathbf{n}}), \qquad \text{and} \qquad \langle a_{lm}^{*\mu} a_{l'm'}^T \rangle \equiv \delta_{ll'}\delta_{mm'} C_l .$$

Then we find for the correlation of μ with temperature (note that $\mathbf{k} \equiv \mathbf{k}_1 - \mathbf{k}_2$),

$$C_l^{\mu T} \propto \int dk\, dk_1\, du \,\ldots\, \langle \mathbf{B}_{\mathbf{k}_1}^* \cdot \mathbf{B}_{\mathbf{k}_2} \zeta(\mathbf{k}) \rangle' \left[e^{-(k_1^2 + k_2^2)/k_D^2} \right]_f^i j_l(k r_L).$$

[1]Note that we actually use the full radiation transfer function from CAMB.

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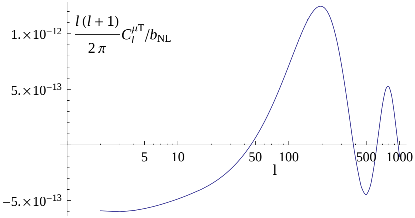
The angular power spectrum from $b_{\rm NL}$

- Plugging the definition for $\langle \mathbf{B} \cdot \mathbf{B}\, \zeta \rangle$ into the formula for $C_l^{\mu T}$, we find

$$C^{\mu T}_{l, b_{\rm NL}} \approx \ldots b_{\rm NL} \tilde{B}^2_\mu \int dk \ldots P_\zeta(k) j_l(k r_L)\,.$$

- $\tilde{B}_\mu$ is the magnetic field strength on μ-distortion scale modes only (multiplied by a^2, to remove the effect of Hubble expansion).

- We can plot the spectrum:



with $b_{NL} = 1$, $\tilde{B}_\mu = 1\ nG$

- Note that, for $b_{\rm NL}$ scale invariant, the shape of $C_l^{\mu T}$ is not a function of the magnetic field's details.

Detectability of a $\langle \mu T \rangle$ signal

- The signal-to-noise for $\langle \mu T \rangle$ is given by (Pajer & Zaldarriaga 2012)

$$\left(\frac{S}{N}\right)^2 = \sum_l (2l+1) \frac{e^{-l^2/l_{\max}^2}}{N_\mu} \frac{\left(C_l^{\mu T}\right)^2}{C_l^{TT}} ,$$

  where the sensitivity N_μ^{-1} and beam size $l_{\max}$ are parameters of the detector.
- We find

$$\boxed{\frac{S}{N} = b_{\rm NL} \left(\frac{\tilde{B}_\mu}{10\ {\rm nG}}\right)^2 \times \begin{cases} 8.4 \times 10^{-2} & \text{PIXIE} \\ 2.4 & \text{CMBPol} \end{cases}} .$$

  (PIXIE and CMBPol are proposed experiments that could measure the $\langle \mu T \rangle$ correlation).
- For $\tilde{B}_\mu = \mathcal{O}(10)$ nG, CMBPol could constrain $b_{\rm NL} \lesssim 1$.

Outline

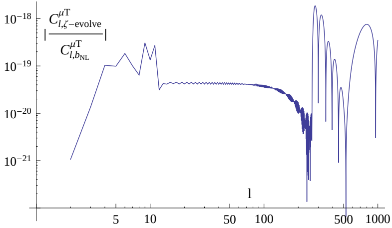
Magnetic fields affect ζ. Can this produce $\langle \mathbf{B} \cdot \mathbf{B}\, \zeta \rangle$?

- Treated in Miyamoto et. al. 2013, though with slight differences in approach to magnetic damping.
- The anisotropic stress of inhomogeneous magnetic fields causes evolution of the curvature perturbation (Shaw & Lewis 2009), giving a correlation (with an angular dependence $g(k_1, k, u)$):

$$\left\langle \mathbf{B}^*_{\mathbf{k}_1} \cdot \mathbf{B}_{\mathbf{k}_2}\, \zeta_{\mathbf{k}_3} \right\rangle' \propto P_B(k_1) P_B(k_2) g(k_1, k, u)\,.$$

- But...

$C^{\mu T}_{l,\zeta\text{-evolve}} / C^{\mu T}_{l,b_{NL}}$



with $b_{NL} = 1,\ n_B = -1,\ B_\mu = 10\ nG$

- For PIXIE:

$$\left(\frac{S}{N}\right)_{\zeta\text{-evolve}} \approx 2 \times 10^{-19} \left(\frac{B_0}{1\ \text{nG}}\right)^4 .$$

The signal is negligible.

Magnetic fields evolve in the presence of moving charges

- One of Maxwell's equations in curved space tells us that magnetic fields evolve in the present of moving ions: $\mathbf{B}' = \nabla \times [\mathbf{v}_b \times \mathbf{B}]$, where $\mathbf{v}_b$ is the baryon velocity.
- In the tightly coupled early universe (e.g. Doran et. al. 2003), we can relate the change in density perturbation with the baryon velocity $\delta'_{\mathbf{k}} + \frac{4}{3} k v_{\mathbf{k}} = 0$, giving us that (a slightly different result is derived in Kunze 2012)

$$\Delta B(\mathbf{k}, t) \approx \int d^3 q \ldots \frac{q^2 - (\mathbf{k} \cdot \mathbf{q})}{q} B_{\mathbf{k}-\mathbf{q}} \Delta \delta_{\mathbf{q}} \,.$$

- But δ is constant in Newtonian gauge for superhorizon modes during radiation domination, so that $\Delta \delta = 0$.
- Thus, there is no contribution from the evolution of B.

Outline

Silk (or diffusion) damping of density perturbations

- In the pre-recombination plasma, hot regions emit more photons than cold regions, evening out the temperature and damping anisotropies. This is called Silk (or diffusion) damping.
- If we look at this effect as the damping of density waves, then the energy dissipated goes into μ-distortion (as well as raising the photon temperature). Thus, $\mu \propto \Delta\rho \sim \rho\zeta^2$
- This produces a contribution to $\langle \mu T \rangle \propto \langle \zeta^3 \rangle \propto f_{\rm NL}$, i.e. $\langle \mu T \rangle$ has a contribution from the bispectrum.

Sunyaev & Zel'dovich 1969, Khatri et al 2011, Chluba et. al. 2012, Pajer & Zaldarriaga 2013

Characteristics of $\langle \mu T \rangle$ from diffusion damping

Calculating the $\langle \mu T \rangle$ correlation (Pajer & Zaldarriaga 2012) gives:

$$C_{l,f_{\rm NL}}^{\mu T} \propto f_{\rm NL} \int dk \ldots P_\zeta(k) j_l(kr_L) \int_{k^f_{D\rho}}^{k^i_{D\rho}} dk_1 \ldots P_\zeta(k_1)\,.$$

For a PIXIE-like experiment:

$$\boxed{\left(\frac{S}{N}\right)_{f_{\rm NL}} = |f_{\rm NL}| \left(4 \times 10^{-4}\right)}$$

Some comments on this result:

- Though the power spectrum on μ-scales is largely unconstrained, we assume here that it simply has the same tilt and amplitude as measured today.
- Note that there is no detectable signal from most single-field models (where $f_{\rm NL} \approx 0.01$), though see, e.g., JG 2012.
- $C_{l,f_{\rm NL}}^{\mu T}$ has the same l-dependence as $C_l^{\mu T}$ from $b_{\rm NL}$, as long as $b_{\rm NL}$ and $f_{\rm NL}$ are scale invariant. One needs theoretical priors to distinguish the signals.

Outline

Implications of some recent results

- Recent results (Ferreira et. al. 2013, 2014, Fujita & Yokoyama 2014) have indicated that magnetic fields' strength and temperature are inversely related for inflationary magnetogenesis. We find a bound for inflationary magnetogenesis that respects EM gauge invariance and does not have strong coupling:

$$B < \left(5 \times 10^{-14}\ \mathrm{nG}\right) \left(\frac{\rho_{\rm inf}^{1/4}}{10^{14}\ \mathrm{GeV}}\right)^{-1} \left(\frac{k_B}{k_D^i}\right)^{\frac{5}{4}} ,$$

  where k_B is the wavenumber where B's power is concentrated.
- BICEP2 (Ade et. al. 2014), taken at face value, gives us that $\rho_{\rm inf} \simeq 10^{14}$ GeV. If true, the $\langle \mu T \rangle$ correlation from $b_{\rm NL}$ would be $\frac{S}{N} < b_{\rm NL} 2 \times 10^{-30}$, i.e. undetectable.
- Possible workarounds: break gauge invariance, accept strong coupling, or find a different method for magnetogenesis.

Magnetic fields during inflation induce $f_{\rm NL}$

If magnetic fields are present during inflation, their non-adiabatic pressure adds an additional component $\zeta_B \propto B^2$ to the curvature perturbation (Fujita & Yokoyama 2013, Nurmi & Sloth 2014), which can contribute to $f_{\rm NL}$.

- There is a contribution from $\langle \zeta_B^3 \rangle \propto \tilde{B}^2 \tilde{B}_\mu^4$, which can be large if the magnetic spectrum is scale-invariant.
- There is a contribution from $\langle \zeta \zeta_B^2 \rangle \propto b_{\rm NL} \tilde{B}_\mu^4$.

These effects must also be considered when interpreting the results of a $\langle \mu T \rangle$ measurement and may give the stronger constraints in the near future.

- Note that these effects scale as at least B^4 while the direct contribution from $\langle \mathbf{B} \cdot \mathbf{B}\, \zeta \rangle$ scales as B^2. Thus, eventually the $\langle \mathbf{B} \cdot \mathbf{B}\, \zeta \rangle$ contribution will be larger.

Outline

Conclusion

- The $\langle \mu T \rangle$ signal in the CMB offers a potential way to constrain a primordial $\langle \mathbf{B} \cdot \mathbf{B}\, \zeta \rangle$ correlation; the proposed PIXIE experiment could constrain $b_{\mathrm{NL}} = 1$ for fields with $\tilde{B}_\mu \gtrsim 100$ nG.
- There is no contamination in the $\langle \mathbf{B} \cdot \mathbf{B}\, \zeta \rangle$ correlator from post-magnetogenesis effects.
- A primordial bispectrum produces a degenerate effect (particularly if both b_{NL} and f_{NL} are scale invariant).

Caveats:

- At face value, BICEP2 means that we will not observe a $\langle \mathbf{B} \cdot \mathbf{B}\, \zeta \rangle$ correlation unless i) gauge invariance is broken during inflation, ii) the fields are produced during strong coupling, or iii) they have a non-inflationary origin.
- Magnetic fields during inflation induce a bispectrum, which may produce a stronger constraint for the near future.