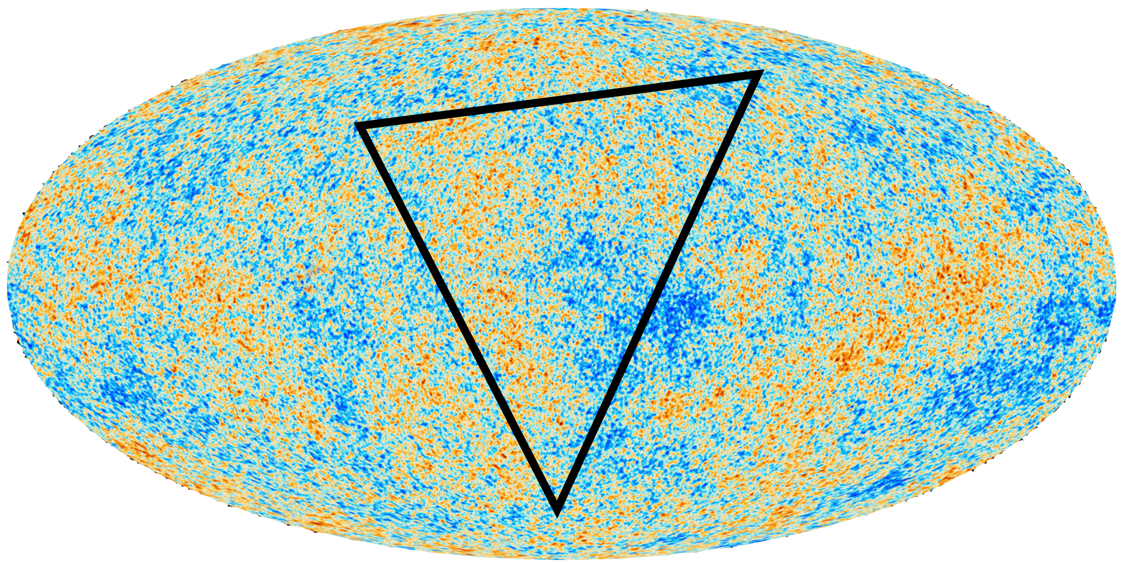
The full CMB bispectrum from single-field inflation



Filippo Vernizzi - IPhT, CEA Saclay

With Zhiqi Huang

PRL (1212.3573) & PRD (1311.6105)

April 14, 2014 - PONT, Avignon

Intrinsic nonlinear effects

Even in the absence of primordial non-Gaussianity, $\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = 0$, the CMB is non-Gaussian!

$$\zeta_{in} \qquad \Rightarrow \qquad \Theta = \frac{\delta T}{T}$$

$$\Theta_{\vec{k}} = T^{(1)}(t,k)\zeta_{\vec{k}} + T^{(2)}(t,k)(\zeta \star \zeta)_{\vec{k}}$$

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2nd-order effects induce non-Gaussianity:

- late time: ISW-lensing; Goldberg, Spergel, '99 $f_{\rm NL}^{\rm loc} = 7.1$ **Detected by Planck!**

(Planck '13)	**Independent** KSW	**ISW-lensing subtracted** KSW
SMICA Local	9.8 ± 5.8	$\mathbf{2.7} \pm \mathbf{5.8}$

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- at recombination: 2nd-order perturbations in the fluid + GR nonlinearities.

$$\delta = \delta^{(1)} + \delta^{(2)} \quad \Rightarrow \quad \begin{aligned} D[\delta^{(1)}] &= 0 \\ D[\delta^{(2)}] &= S[\delta^{(1)2}] \end{aligned} \quad \Rightarrow \quad f_{\rm NL} \sim \frac{\langle \delta^{(2)}\delta^{(1)}\delta^{(1)}\rangle}{\langle \delta^{(1)}\delta^{(1)}\rangle^2} \sim \text{few}$$

Why do we care?

- Reconstruct the 3-point function of the initial conditions

$$f_{\rm NL}^{\rm loc} = 2.7 \pm 5.8 \qquad \text{(Planck '13)} \qquad \Longrightarrow \qquad f_{\rm NL}^{\rm loc} \ll 1\ ? \qquad f_{\rm NL}^{\rm loc} \sim \text{few}\ ?$$

- Removing contamination is important to improve present constraints on primordial NG

- BICEP2 r=0.2 ⇒ Most probably single-field slow-roll inflation ⇒ $f_{\rm NL} \sim 0$

- Nonlinearities there <u>for sure</u>, if our picture of the universe is consistent

Numerical goals

- Boltzmann code:

Evolve cosmological perturbations up to second order by solving Boltzmann and Einstein equations

$$\frac{df_I}{d\eta} = C_I[f_I] \; , \quad I = \gamma, \nu, b, \mathrm{CDM}$$

$$\& \qquad G_{ij} = 8\pi G \sum_I T_{ij}^{(I)}$$

$$\Rightarrow \quad \Theta^{(2)}, \Phi^{(2)}, \Psi^{(2)}, \ldots$$

- Line-of-sight integral:

Compute CMB bispectrum from second order effects, by integrating the photon temperature along the line of sight

$$\Theta^{(2)}(\eta_0, \hat{n}) = \int_0^{\eta_0} d\eta S^{(2)}(\eta, \vec{x}(\eta), \hat{n})$$

$$\langle \Theta^{(2)}_{l_1 m_1} \Theta^{(1)}_{l_2 m_2} \Theta^{(1)}_{l_3 m_3} \rangle \propto \langle \zeta\zeta\zeta\zeta \rangle$$

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<u>Based on many contributions</u>

Bartolo, Matarrese, Riotto ’04, ’06; Bernardeau, Pitrou, Uzan ’08; Pitrou ’08 (CMBquick2); Bartolo, Riotto ’08; Khatri, Wandelt ’08; Senatore, Tassev, Zaldarriaga ’08; Nitta et al. ’09, Boubekeur, Creminelli, D’Amico, Norena, ’09, Beneke and Fidler ’10,...

<u>and previous codes</u>

- Bernardeau, Pitrou, Uzan ’08 (CMBquick2)
- Khatri, Wandelt ’08 (perturbed rec.)
- Senatore, Tassev, Zaldarriaga ’08 (perturbed recombination)

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• CosmoLib2nd - Huang, Vernizzi '12
• SONG - Pettinari, Fidler, Chrittenden, Koyama, Wands '13

★ No license, parallelizable, full-sky

• Su, Lim, Shellard '12

★ Mathematica, flat-sky, …

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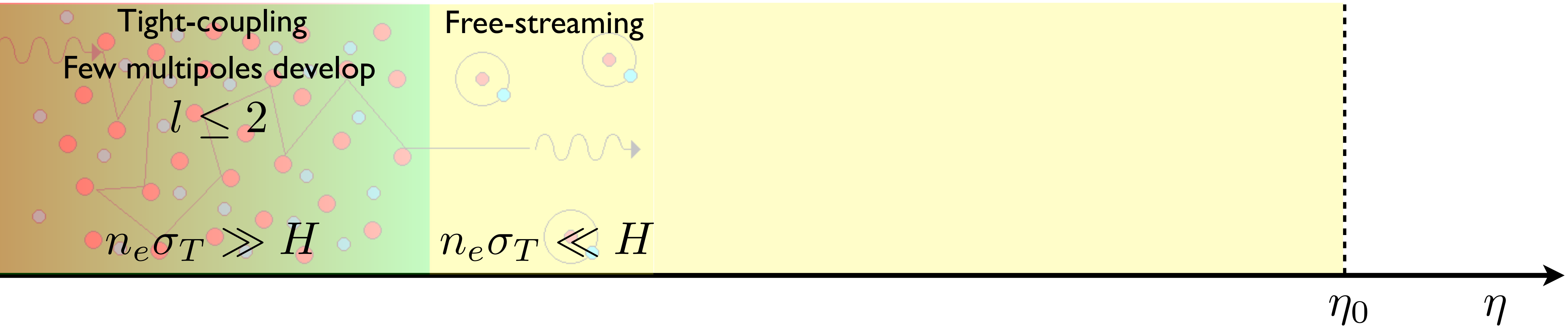
Need to include geodesic deviation: ensures that the final result is gauge invariant

Current numerical codes:

- CosmoLib2nd - Huang, Vernizzi '12 ★ Consistently includes lensing and time delay
- SONG - Pettinari, Fidler, Chrittenden, Koyama, Wands '13
- Su, Lim, Shellard '12

Line-of-sight treatment



- Photon temperature equation (first order):

$$\frac{d}{d\eta}(\Theta + \Phi) - E = -\dot{\tau} F$$

integrated effects

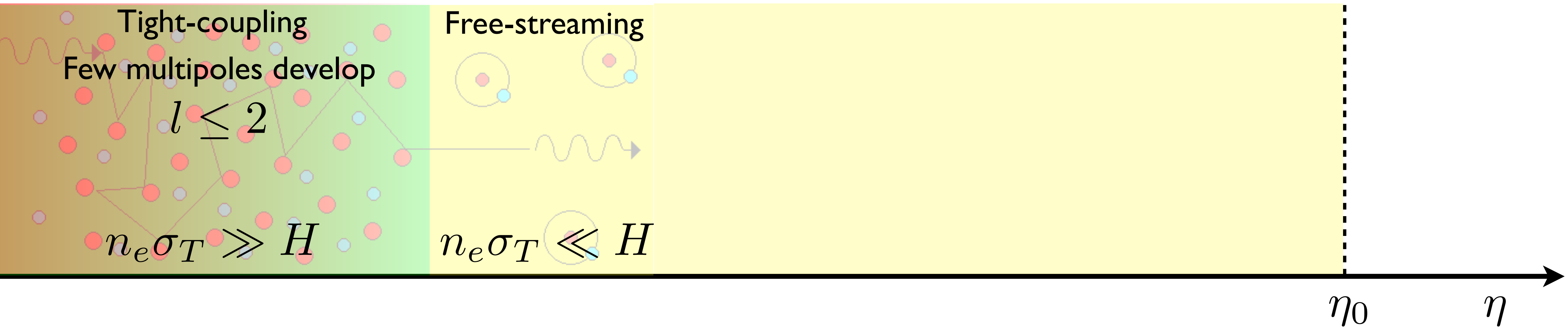
collision term

$$E \equiv (\dot{\Phi} + \dot{\Psi})$$

$$F \equiv \Theta_{00} - \Theta - \frac{1}{2}\sqrt{\frac{4\pi}{5^3}} \sum_m \Theta_{2m} Y_{2m}(\hat{n}) + \hat{n} \cdot \vec{v}$$

$$\dot{\tau} = -\bar{n}_e \sigma_T a$$

Line-of-sight treatment



- Photon temperature equation (second order):

$$\frac{d}{d\eta}(\Theta+\Phi) - \Theta(\dot{\Psi} - \Phi_{,i}n^i) - E + \left[(\Phi+\Psi)n^i\partial_i - \nabla^i_\perp(\Phi+\Psi)\partial_{\hat{n}^i}\right](\Theta+\Phi)$$

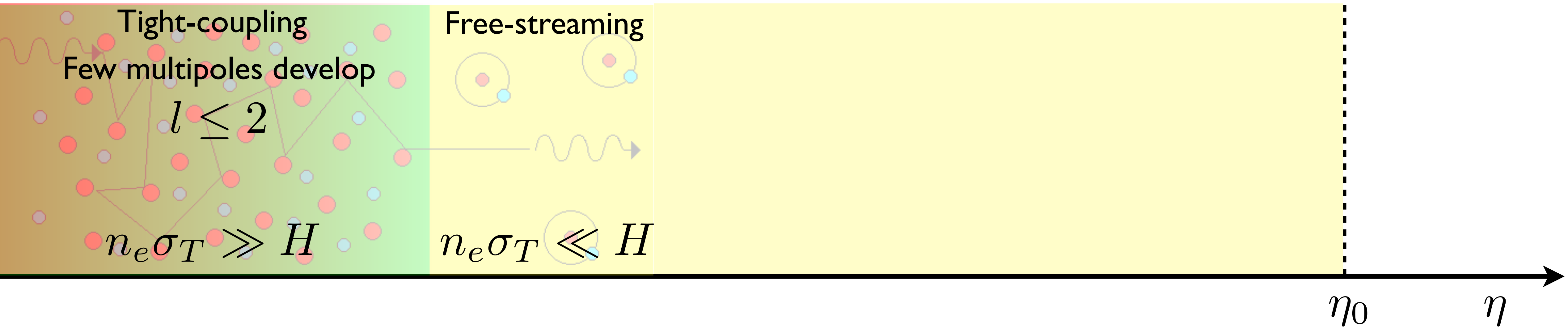
integrated effects

geodesic deviation

collision term

$$= -(\dot{\tau} + \delta\dot{\tau})F$$

Line-of-sight treatment



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$$= -(\dot\tau + \delta\dot\tau)F$$

$$E \equiv (\dot\Phi + \dot\Psi) - \dot\omega_i n^i - \dot\chi_{ij}n^i n^j/2$$

ISW (+ RS), vector and tensor contributions

$$F \equiv \Theta_{00} - \Theta - \frac{1}{2}\sqrt{\frac{4\pi}{5^3}}\sum_m \Theta_{2m}Y_{2m}(\hat n) + \hat n\cdot\vec v$$

$$+7(\hat n\cdot\vec v)^2/4 - v^2/4 + \hat n\cdot\vec v\left(\Theta + 3\Theta_{00} - \frac{1}{2}\sqrt{\frac{4\pi}{5^3}}\sum_m \Theta_{2m}Y_{2m}(\hat n) + i\sqrt{\frac{\pi}{3}}\sum_m \Theta_{1m}Y_{1m}(\hat n)\right)$$

$$+2\pi v\sqrt{\frac{2}{15}}\sum_{m,M}\begin{pmatrix}1 & 1 & 2\\ m & M & -m-M\end{pmatrix}\Theta_{2,m+M}Y_{1m}(\hat n)Y_{1M}(\hat v)(-1)^{m+M} + i\sqrt{\frac{\pi}{3}}v\sum_m \Theta_{1m}Y_{1m}(\hat v)$$

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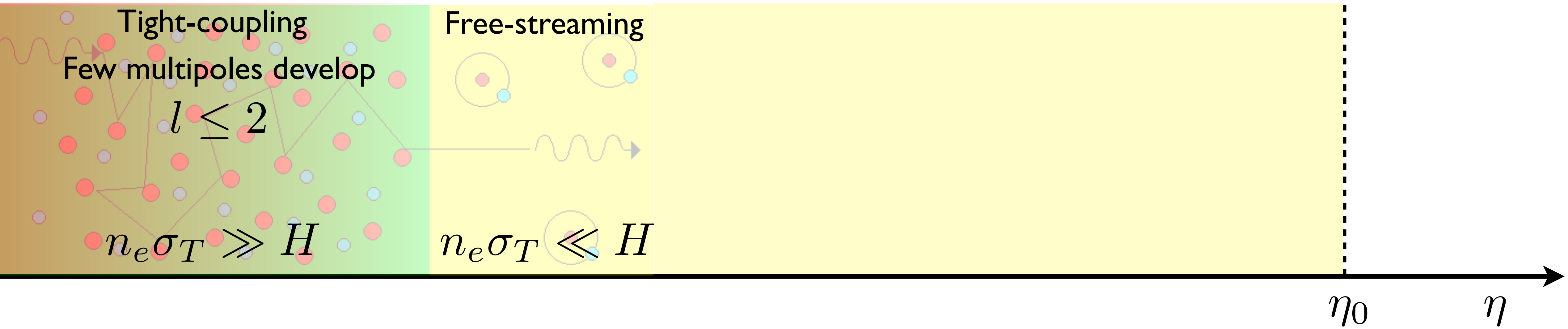

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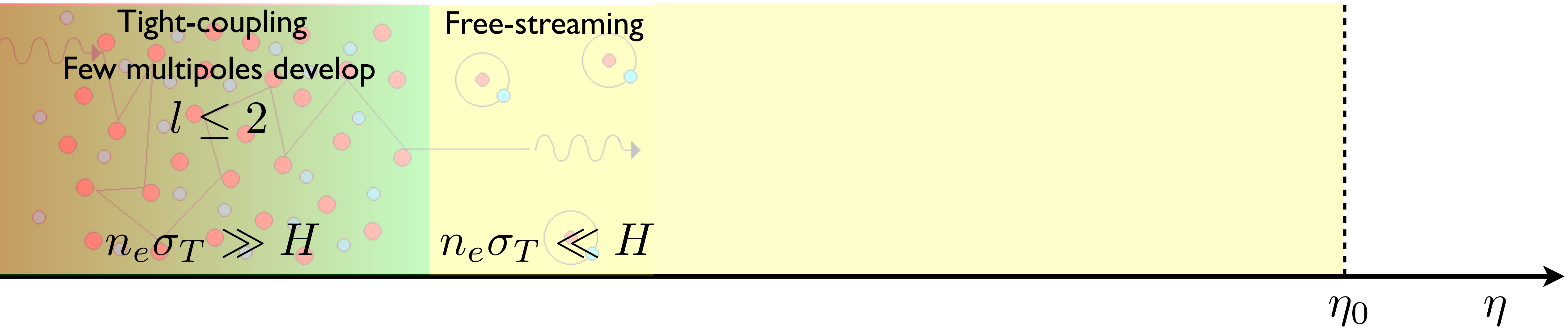
$$= -(\dot{\tau} + \delta\dot{\tau}) F$$

Long wavelength temperature mode ⇔ Rescaling of the background

$$T \propto \frac{1}{a} \quad \Rightarrow \quad T \equiv \bar{T}(t) e^{\tilde{\Theta}}$$

$$\Theta \equiv \frac{\delta T}{\bar{T}} = \tilde{\Theta} + \frac{1}{2} \tilde{\Theta}^2$$

Line-of-sight treatment



- Photon temperature equation (second order):

$$\frac{d}{d\eta}(\tilde{\Theta} + \Phi) - E \; + \left[(\Phi + \Psi) n^i \partial_i - \nabla_\perp^i (\Phi + \Psi) \partial_{\hat{n}^i} \right] (\Theta + \Phi) \; = -(\dot{\tau} + \delta\dot{\tau}) F$$

integrated effects

geodesic deviation

collision term

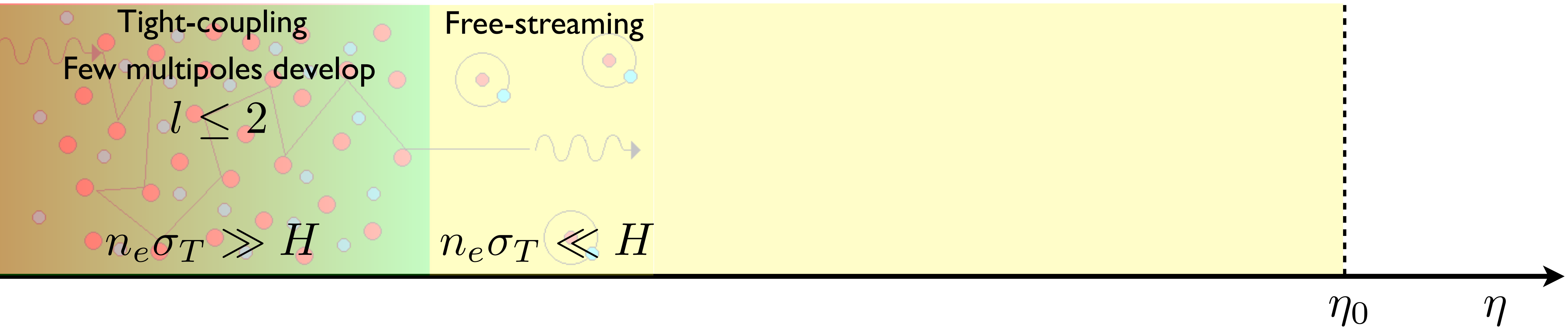
- Change of variable - identify boundary term:

$$\Theta^{(2)}_{\rm obs}(\hat{n}) = \tilde{\Theta}^{(2)}_{\rm obs}(\hat{n}) + \frac{1}{2} [\Theta_{\rm obs}(\hat{n})]^2$$

local redefinition

$$b_{l_1 l_2 l_3} = (C_{l_1} C_{l_2} + \text{perms}) + \tilde{b}_{l_1 l_2 l_3}$$

Line-of-sight treatment



- Photon temperature equation (second order):

$$\frac{d}{d\eta}(\tilde{\Theta} + \Phi) - (1 + \mathcal{D} n^i \partial_i + \partial_{n^i} \psi \frac{d}{dn^i}) E = (\dot{\tau} + \delta\dot{\tau})(1 + \mathcal{D} n^i \partial_i + \partial_{n^i} \psi \frac{d}{dn^i}) F$$

integrated effects — collision term

- Change of variable - identify boundary term:

$$\Theta^{(2)}_{\mathrm{obs}}(\hat{n}) = \tilde{\Theta}^{(2)}_{\mathrm{obs}}(\hat{n}) + \frac{1}{2}[\Theta_{\mathrm{obs}}(\hat{n})]^2 + \mathcal{D}(\eta_0, \hat{n}) n^i \partial_i \Theta_{\mathrm{obs}}(\hat{n}) + \partial_{n^i} \psi(\eta_0, \hat{n}) \partial_{n^i} \Theta_{\mathrm{obs}}(\hat{n})$$

local redefinition — time delay — lensing

- Neglecting time delay (suppressed by η_* / η_0):

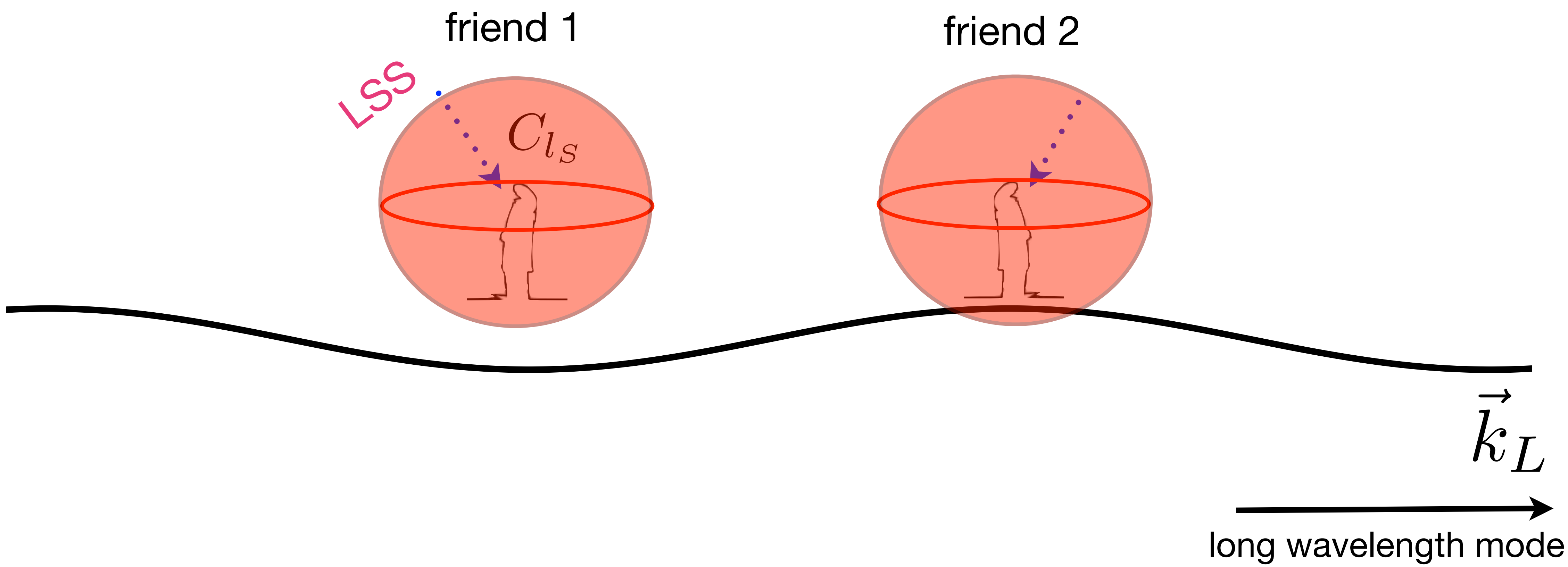
$$b_{l_1 l_2 l_3} = \left(C_{l_1} C_{l_2} + L_{l_1 l_2 l_3} C^{T\psi}_{l_1} C_{l_2} + \mathrm{perms} \right) + \tilde{b}_{l_1 l_2 l_3}$$

$$L_{l_1 l_2 l_3} \equiv \left[l_1(l_1+1) + l_2(l_2+1) - l_3(l_3+1) \right] / 2$$

Consistency relation

Creminelli, Zaldarriaga ’04
with Creminelli, Pitrou ’11

Single-field inflation: 1 clock, e.g. everything is determined by T.



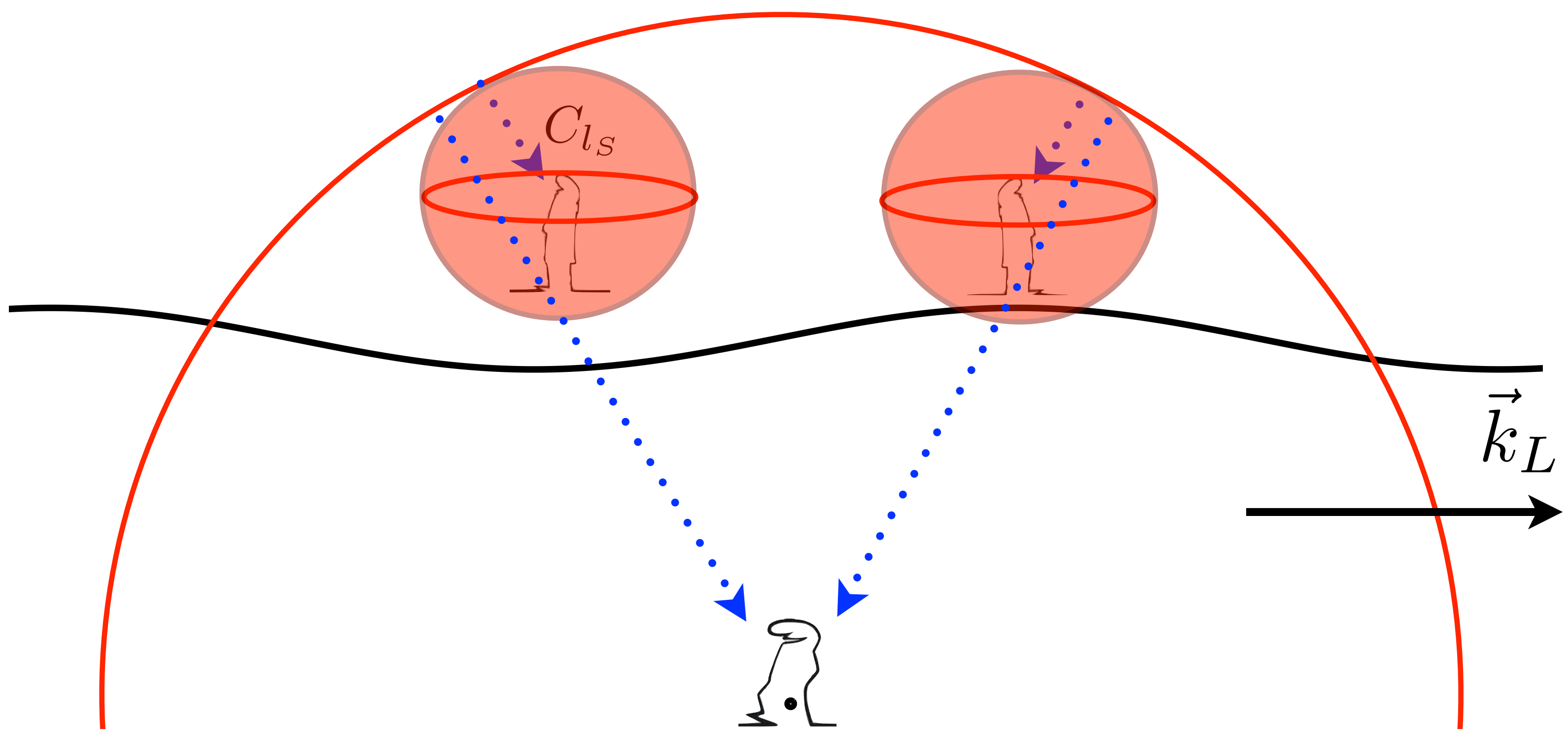
Local physics is identical in Hubble patches that differ only by super-horizon modes: two observers in different places on LSS will see exactly the same CMB anisotropies (at given T).

Coordinate trasformation:

$$\langle \zeta_{\vec{k}_L} \zeta_{\vec{k}_S} \zeta_{-\vec{k}_L} \rangle = -(n_s - 1) P_\zeta(k_L) P_\zeta(k_S) \quad \rightarrow \quad \langle \zeta_{\vec{k}_L} \zeta_{\vec{k}_S} \zeta_{-\vec{k}_L} \rangle = 0$$

Maldacena ’02

Projection effects



The long mode is inside the horizon and I can compare different patches. Will see a modulation of the 2-point function due to large scale T:

Transverse rescaling of spatial coords ⇒ rescaling of angles: $C_l \to C_l + \zeta(\hat{n} \cdot \nabla_{\hat{n}} C_l)$

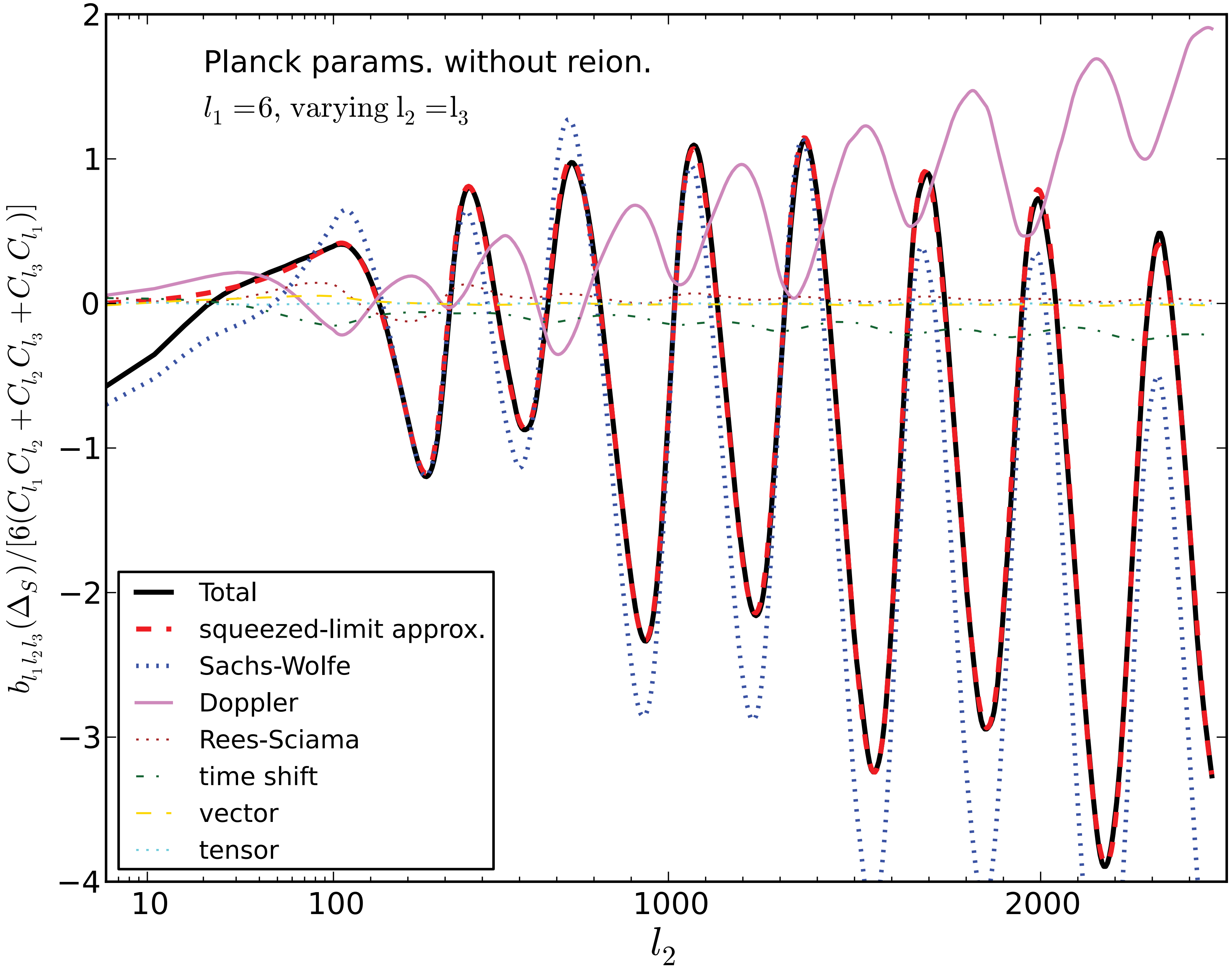
• Squeezed limit consistency relation:

$$\Rightarrow \qquad \tilde{b}_{l_1 l_2 l_3} = -\frac{1}{2} C_{l_1}^{T\zeta} \left(C_{l_2} \frac{d\ln(l_2^2 C_{l_2})}{d\ln l_2} + C_{l_3} \frac{d\ln(l_3^2 C_{l_3})}{d\ln l_3} \right) \qquad \begin{array}{c} l_1 \ll l_2, l_3 \\ l_1 \ll l_H \simeq 110 \end{array}$$

with Creminelli, Pitrou ’11; Bartolo, Matarrese, Riotto; ’11, Lewis ’12, Pajer, Schmidt, Zaldarriaga ‘13

This relation can be used as consistency check of Boltzmann codes based on a physical limit

The squeezed limit



- Comparison with the analytic formula:

$$\tilde{b}_{l_1 l_2 l_3} = -\frac{1}{2} C_{l_1}^{T\zeta} \Big(C_{l_2} \frac{d \ln(l_2^2 C_{l_2})}{d \ln l_2} + C_{l_3} \frac{d \ln(l_3^2 C_{l_3})}{d \ln l_3} \Big) \qquad l_1 \ll l_2, l_3$$

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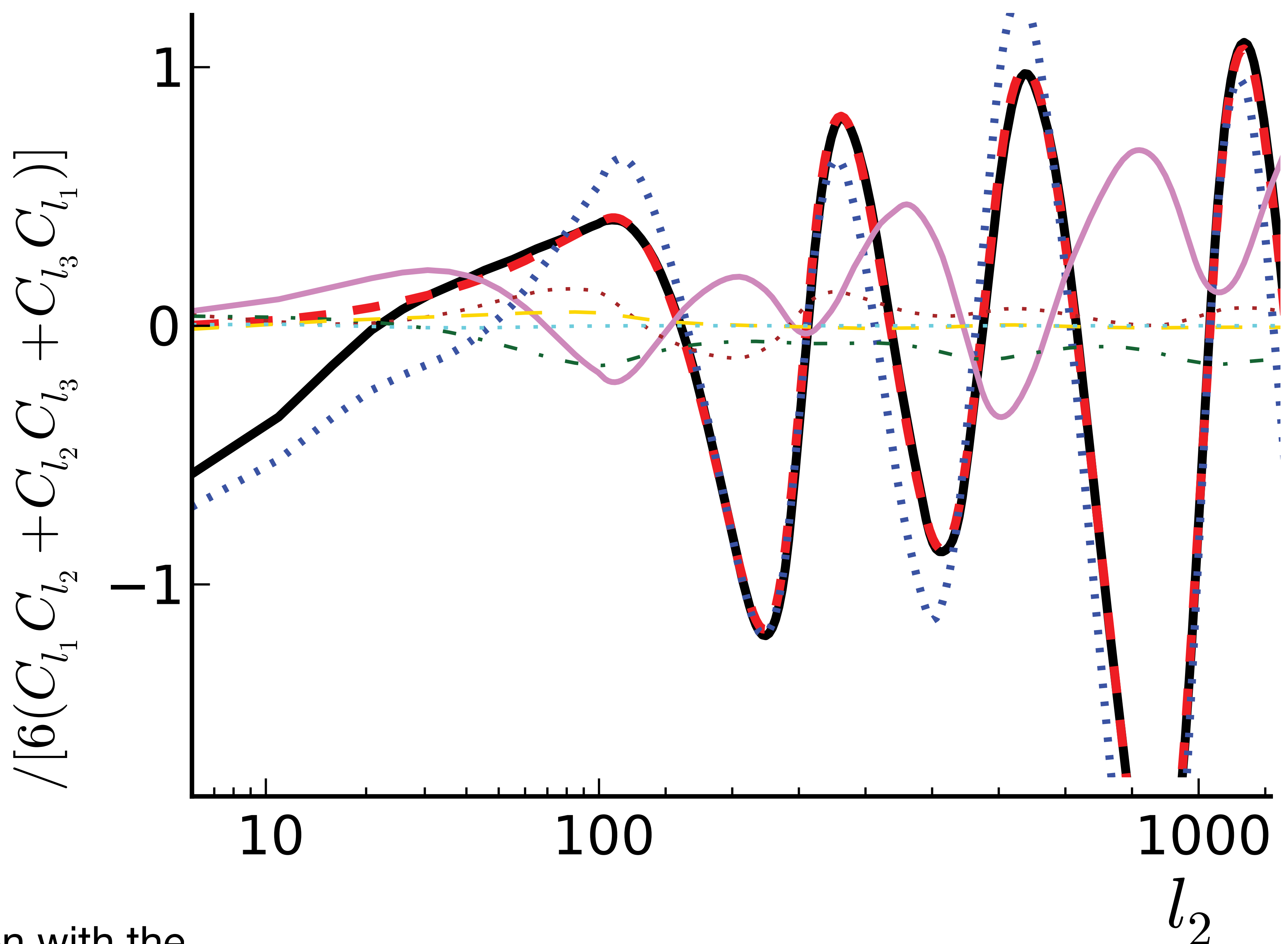

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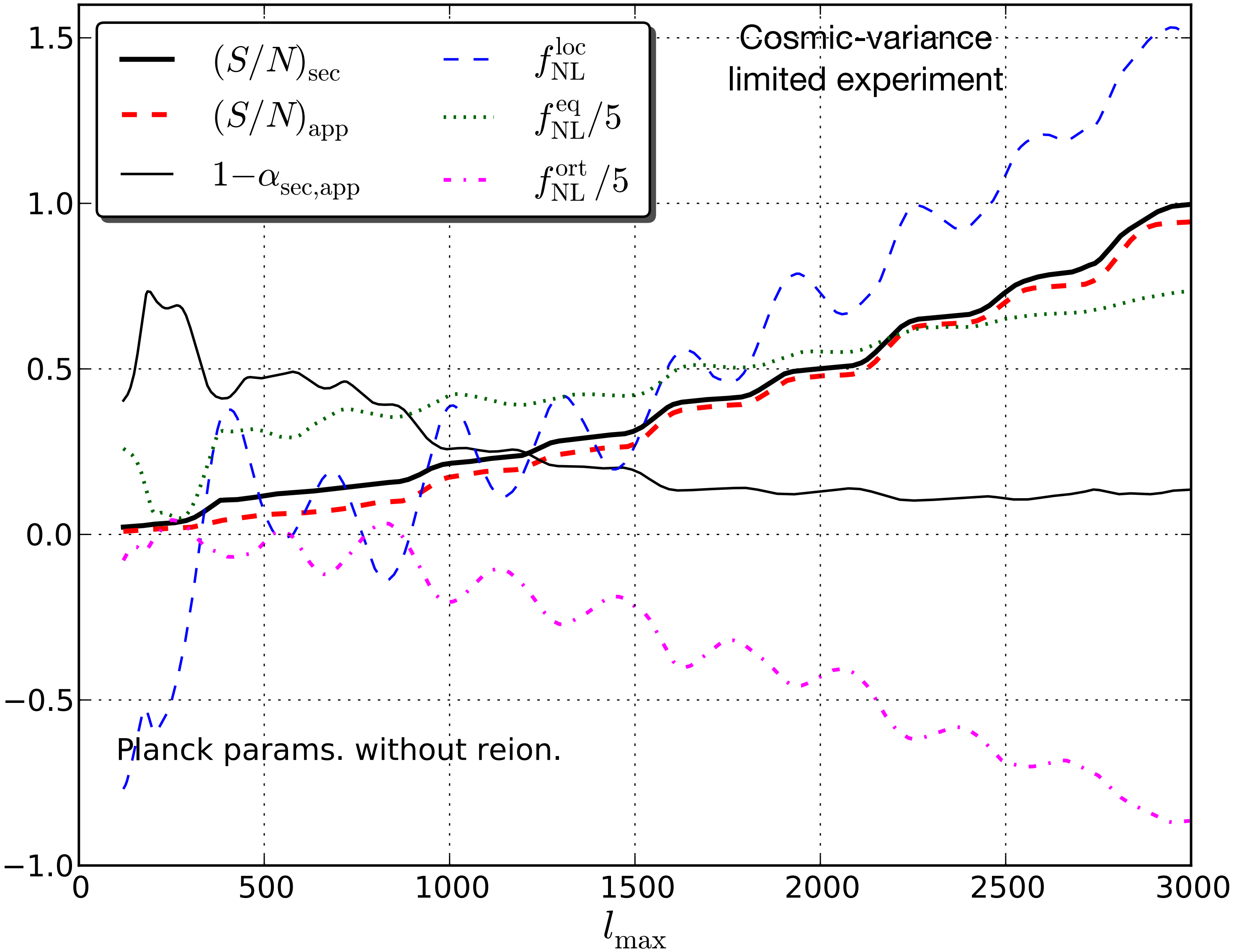
Observability and contamination



(Planck '13)	**ISW-lensing subtracted** KSW
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Local	**2.7** ± **5.8**
Equilateral	**−42** ± **75**
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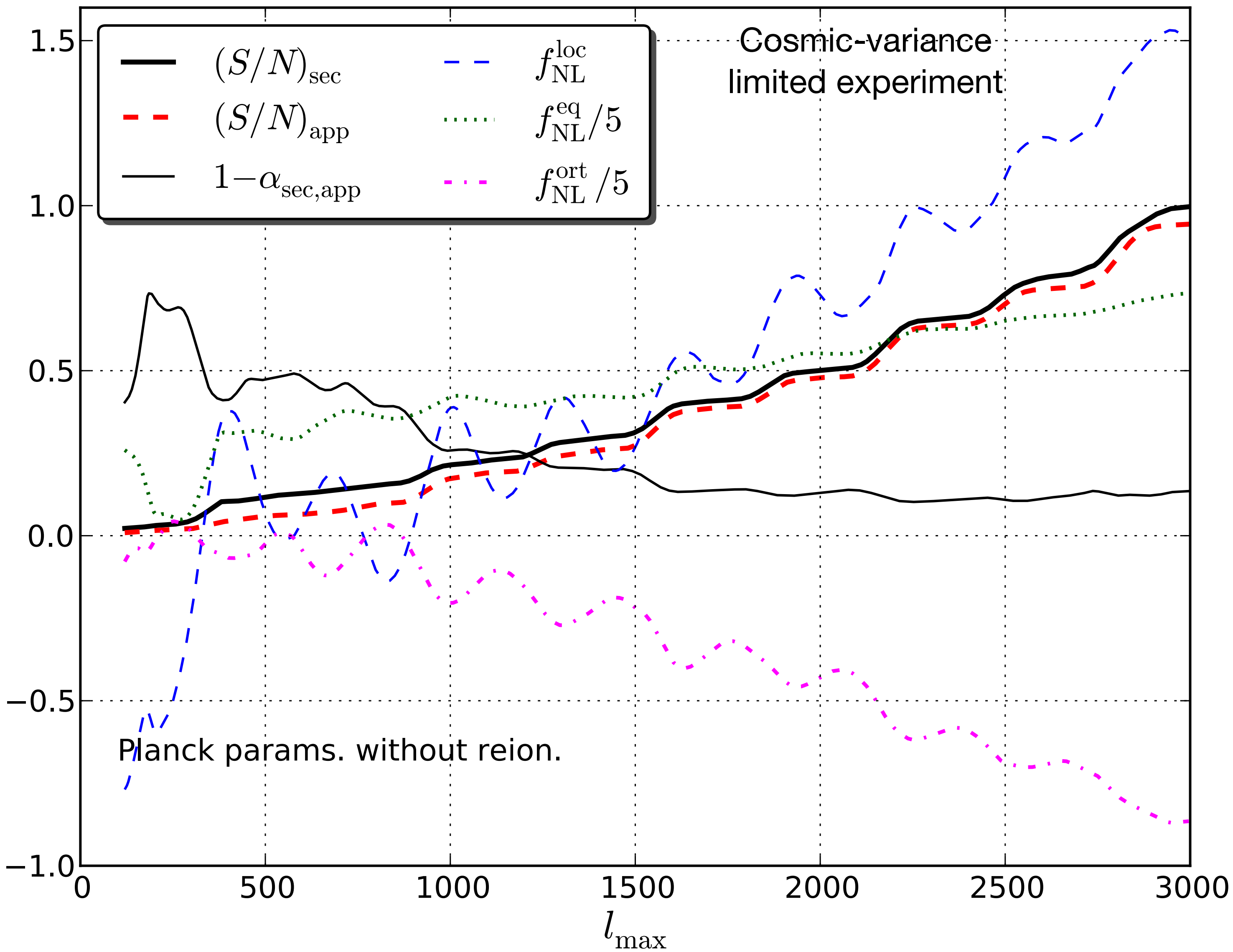

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- Comparison with other references for $l_{\rm max} = 2000$:

agrees with Senatore, Tassev, Zaldarriaga '08 for $l_{\rm min}$=100:

Su, Lim, Shellard '12: $S/N = 0.69; \quad f_{\rm NL}^{\rm loc} = 0.88;$

Pettinari, Fidler, Chrittenden, Koyama, Wands '13: $S/N = 0.47; \quad f_{\rm NL}^{\rm loc} = 0.57$

Conclusion

- **Second order effects are finally under control!**
- CosmoLib2nd: full calculation, on all scales, of bispectrum from nonlinear effects at recombination
- Consistent separation between second-order sources at recombination from better known ISW-lensing correlation
- Perfect agreement with squeezed limit formula and previous literature. Squeezed limit formula can be practically employed to compute the S/N.
- Small S/N but signal likely to be detectable with l_{max}~*3000* and including polarization
- Small contamination to local primordial non-Gaussianity for Planck but sizeable on local primordial signal:

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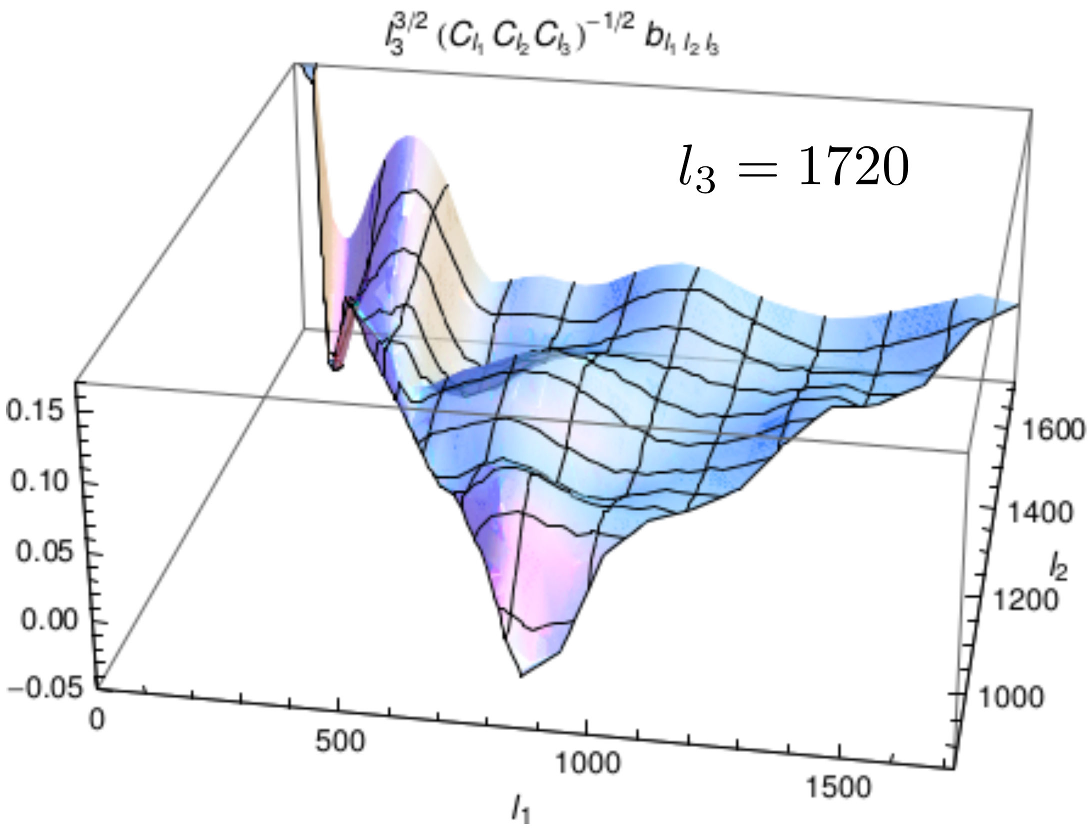
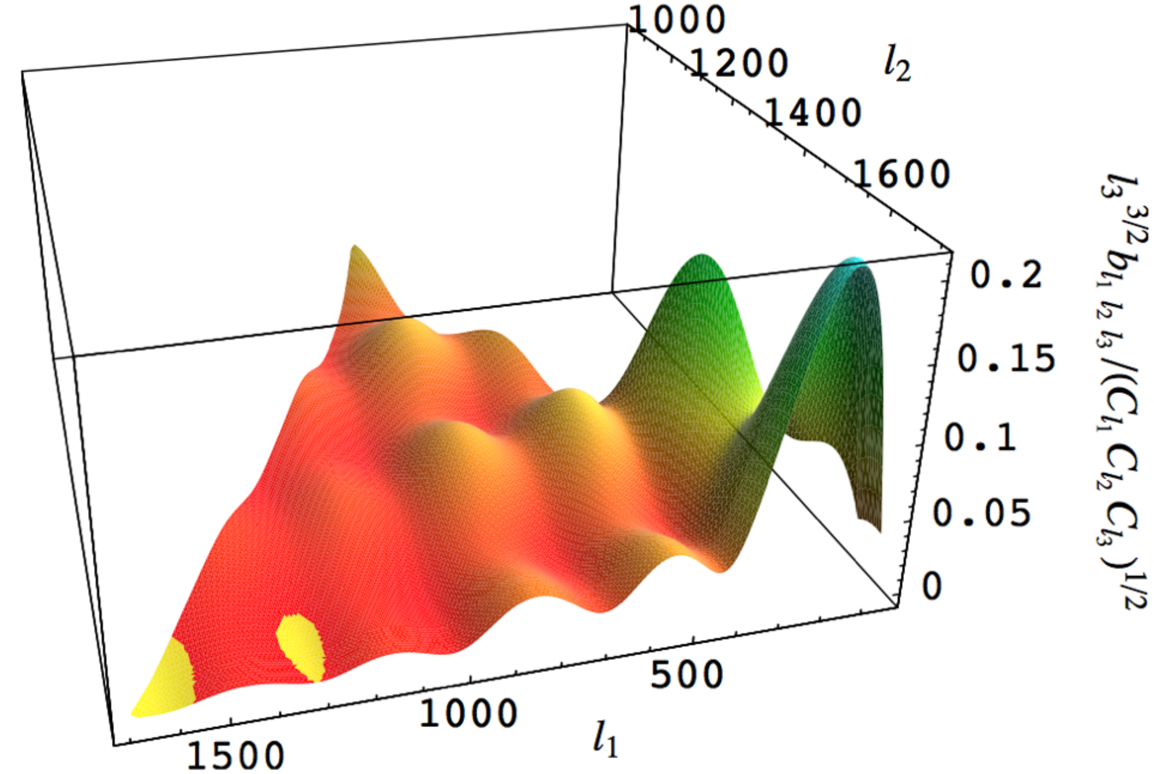
Let's include them in the next analysis!

The shape of the bispectrum



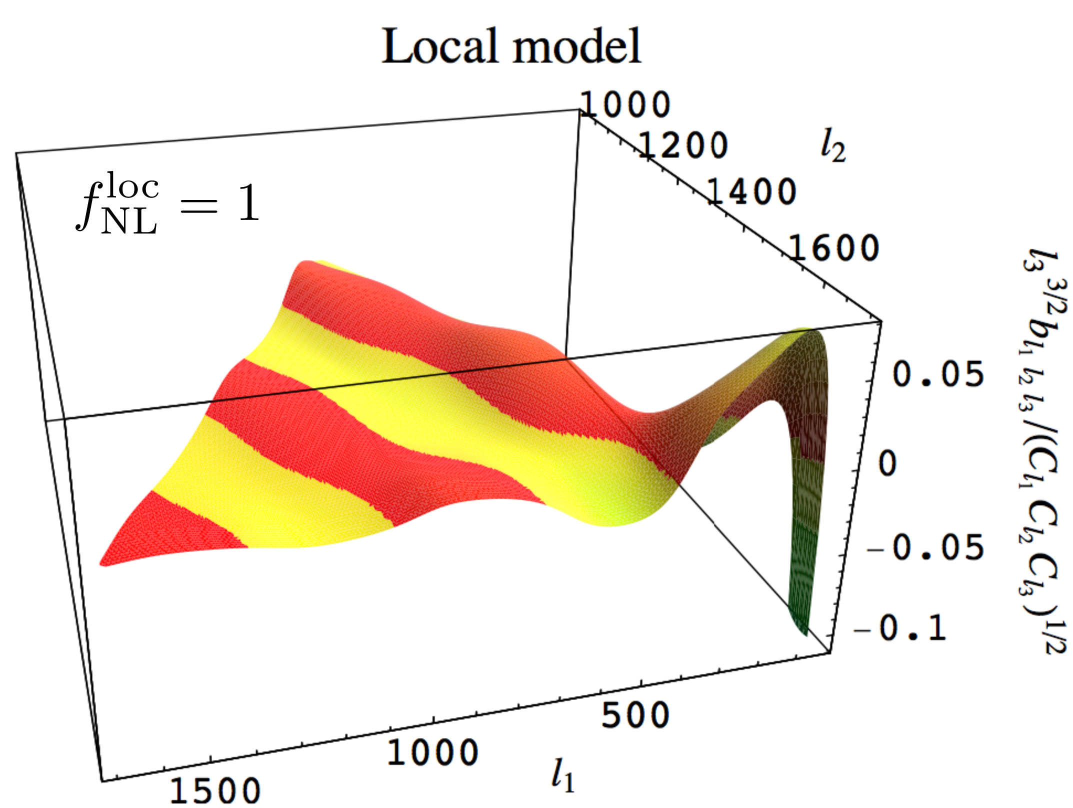
with Z. Huang, '12



Senatore, Tassev, Zaldarriaga, '08 (l>100)

- Signal-to-noise density:

$$l_3^{2/3}b_{l_1l_2l_3}/(C_{l_1}C_{l_2}C_{l_3})^{1/2}$$
$$\text{with} \quad l_1 \leq l_2 \leq l_3$$



Comparison with SONG