Probing non-standard gravity with the growth index of cosmological perturbations

Progress on Old and New Themes in Cosmology (P O N T)   2 0 1 4
april 16th

Heinrich Steigerwald

H. Steigerwald, J. Bel, C. Marinoni, (2014) [arXiv:1403:0898]   (accepted by JCAP)

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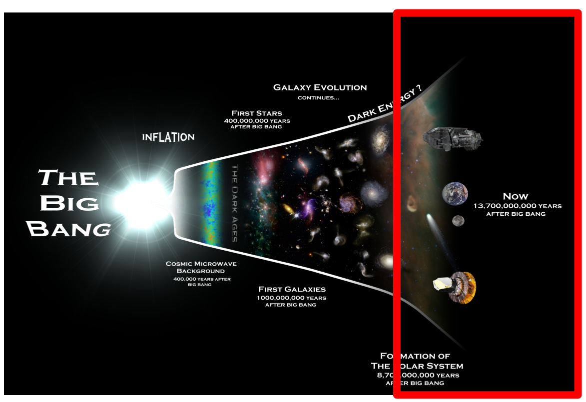

Λ
Modified Gravity
LTB
k-essence
DGP
$f(R)$
Quintessence
Cosmological constant
Dark Energy
Brane-word cosmology

Late periode acceleration :
SNIa, CMB, AP, ...

(measure background)

New physics :

… but which?

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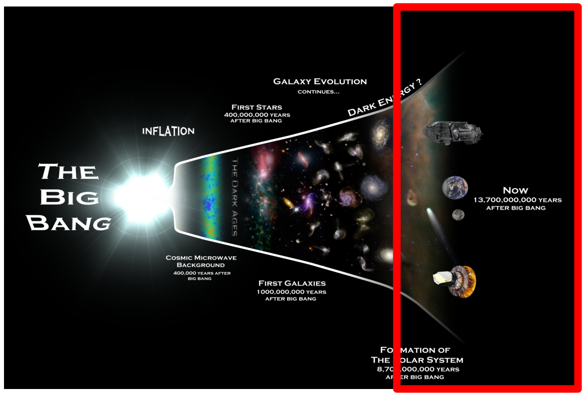
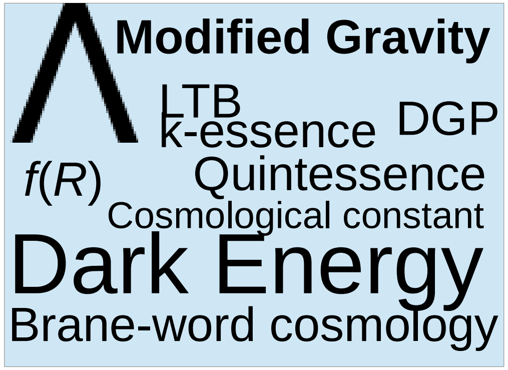
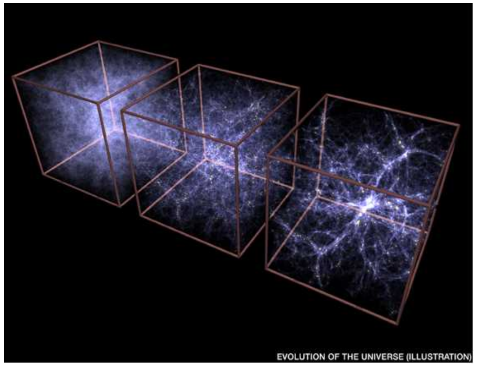






Late periode acceleration :
SNIa, CMB, AP, ...

(measure background)

New physics :

… but which?

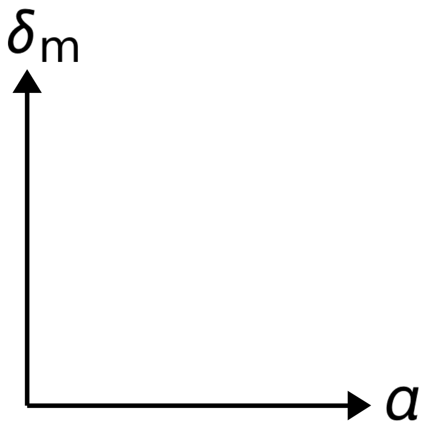
Decicive probes :
Large Scale Structure

(measure perturbations)

First order matter perturbations

$$\ddot{\delta}_{\mathrm{m}} + 2H\dot{\delta}_{\mathrm{m}} - 4\pi G\rho_{\mathrm{m}}\delta_{\mathrm{m}} = 0$$

$$\rho_{\mathrm{m}}(t,\mathbf{x}) = \bar{\rho}_{\mathrm{m}}(t)\big(1+\delta_{\mathrm{m}}(t,\mathbf{x})\big)$$



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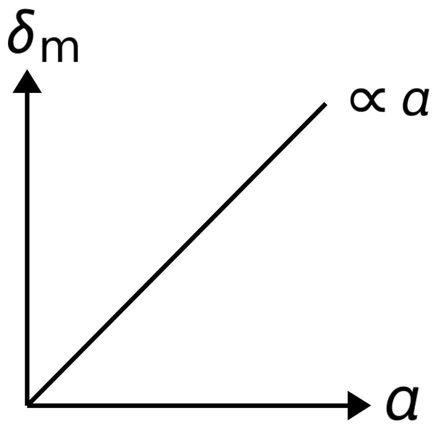

(matter only)

First order matter perturbations

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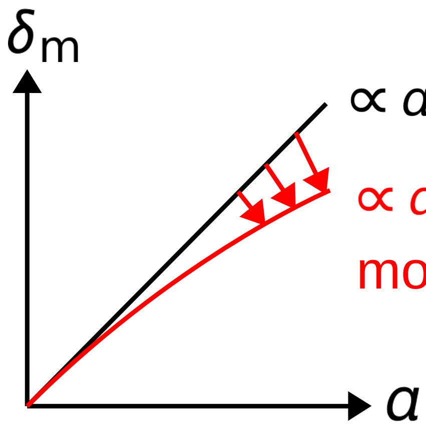


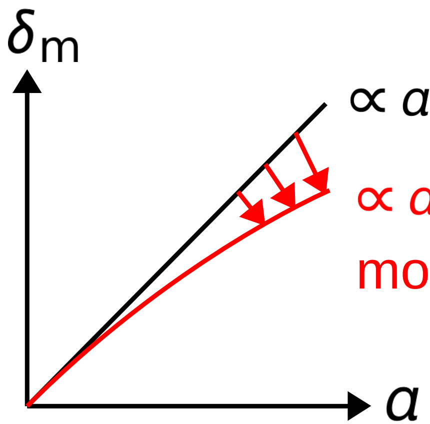
$\propto a$ (matter only)

$\propto a\,{}_2F_1\left(1, \frac{1}{3}; \frac{6}{11}; -\frac{\Omega_\Lambda}{\Omega_{\rm m}} a^3\right)$ (matter + Λ)

more damping because H is higher in the past!

First order matter perturbations

$$\ddot{\delta}_m + 2\nu H\dot{\delta}_m - 4\pi\mu G\rho_m\delta_m = 0 \qquad \rho_m(t,\mathbf{x}) = \bar{\rho}_m(t)(1+\delta_m(t,\mathbf{x}))$$



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In more sophisticated models, we have additional source μ and damping ν

$$f' + f^2 + (1+\nu+\frac{H'}{H})f - \frac{3}{2}\mu\Omega_m = 0 \qquad f = \frac{d\ln\delta_m}{d\ln a}$$

- Solution numerically in general
- μ and ν can depend on time (and Fourier scale)

How to parametrize the growth rate f?

$$f' + f^2 + \left(1 + \nu + \frac{H'}{H}\right) f - \frac{3}{2}\mu\,\Omega_m = 0$$

background

SNIa, CMB, ...

perturbations

LSS

How to parametrize the growth rate f?

$$f' + f^2 + (1 + \underbrace{\nu}_{} + \underbrace{\tfrac{H'}{H}}_{})f - \tfrac{3}{2}\underbrace{\mu}_{}\underbrace{\Omega_m}_{} = 0$$

growth index

$$f = f\left(\boldsymbol{P}_{(\boxed{\text{background}})}, \boldsymbol{P}_{(\boxed{\text{perturbations}})}\right) \sim \Omega_{\mathrm{m}}^{\gamma=0.6}$$

[Peebles 1980]

SNIa, CMB, ...

LSS

ΛCDM

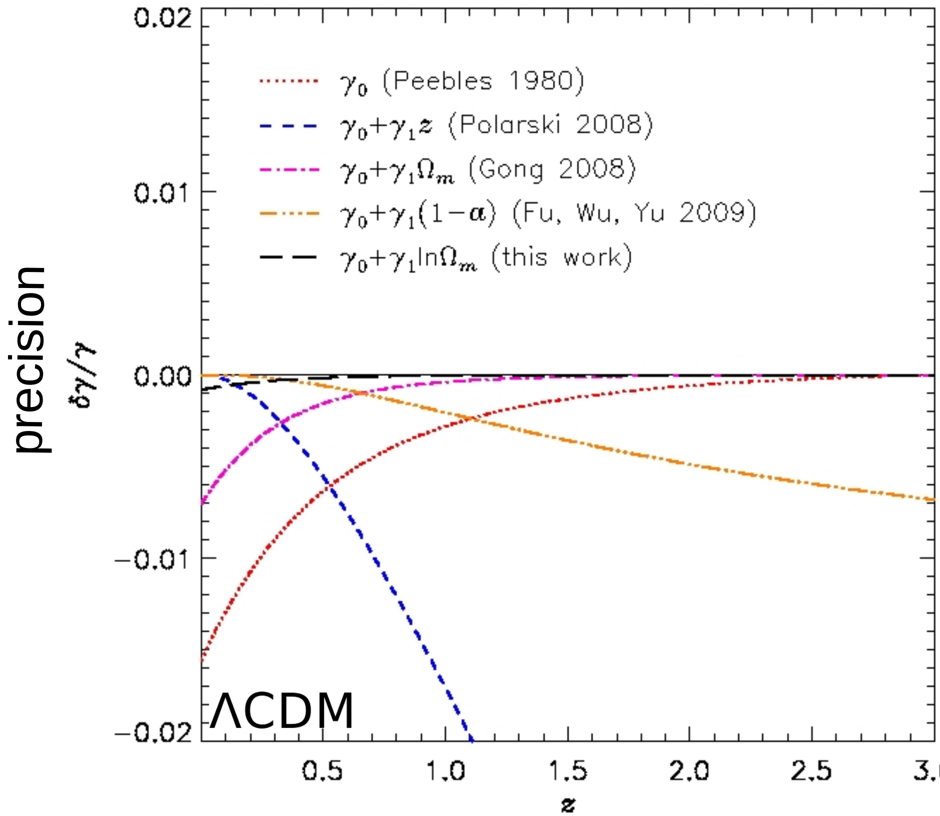
How to parametrize the growth rate f ?

$$f' + f^2 + (1 + \nu + \frac{H'}{H})f - \frac{3}{2}\mu\,\Omega_m = 0$$

growth index

(a function)

$$f = f\Big(\boldsymbol{P}(\boxed{\text{background}}), \boldsymbol{P}(\boxed{\text{perturbations}})\Big) = \Omega_\mathrm{m}^{\gamma}$$



Ansatz :

$$\boxed{\gamma = \sum_i \gamma_i \frac{(\ln \Omega_\mathrm{m})^i}{i!}}$$

Improvement factor: ~10

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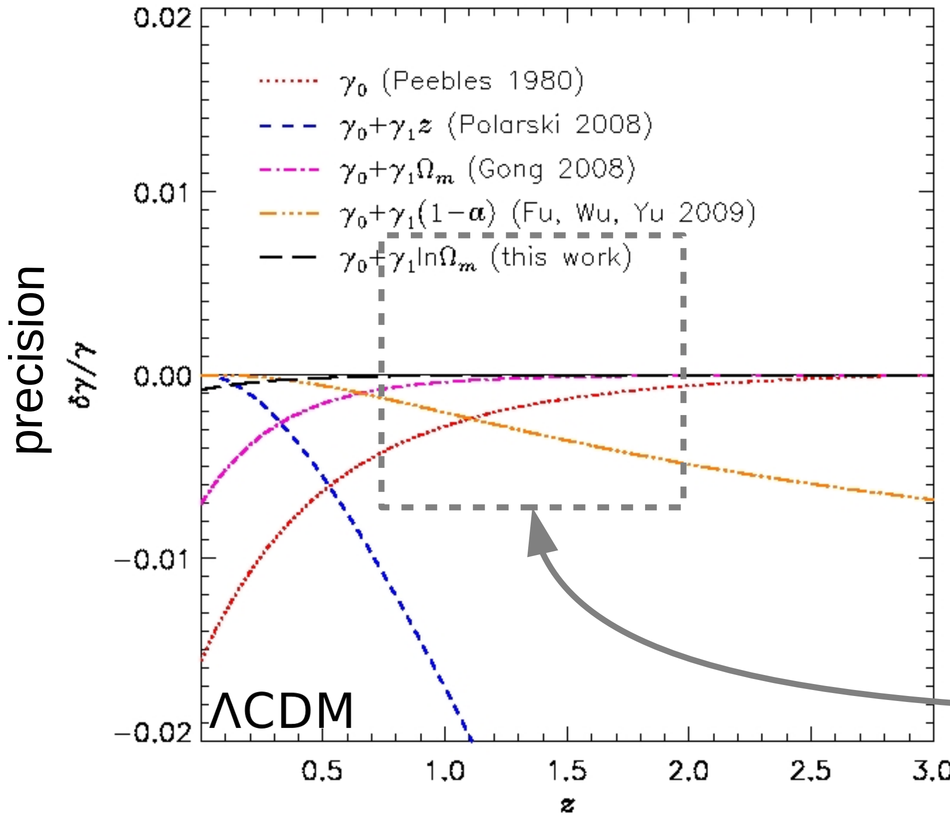

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$$\boxed{\gamma = \sum_i \gamma_i \frac{(\ln \Omega_{\rm m})^i}{i!}}$$

Improvement factor: ~10

Euclid + *Planck*
$\delta\gamma/\gamma \sim 0.7$

Current growth data (from redshift-space distortions)

Label	Reference	z	$f\sigma_8$
THF	Turnbull *et al.* (2012)	0.02	0.40 ± 0.07
DNM	Davis *et al.* (2011)	0.02	0.31 ± 0.05
6dFGS	Beutler *et al.* (2012)	0.07	0.42 ± 0.06
2dFGRS	Percival *et al.* (2004) , Song & Percival (2009)	0.17	0.51 ± 0.06
2SLAQ	Ross *et al.* (2007)	0.55	0.45 ± 0.05
SDSS	Cabré *et al.* (2009)	0.34	0.53 ± 0.07
SDSS II	Samushia *et al.* (2012)	0.25	0.35 ± 0.06
		0.37	0.46 ± 0.04
BOSS	Reid *et al.* (2012)	0.57	0.43 ± 0.07
WiggleZ	Contreras *et al.* (2013)	0.20	0.40 ± 0.13
		0.40	0.39 ± 0.08
		0.60	0.40 ± 0.07
		0.76	0.48 ± 0.09
VVDS	Guzzo *et al.* (2008) , Song & Percival (2009)	0.77	0.49 ± 0.18
VIPERS	De la Torre *et al.* (2013)	0.80	0.47 ± 0.08

Data analysis technique

Theory

$$f(\boldsymbol{\gamma},\mathbf{p},z) = \Omega_\mathrm{m}(\mathbf{p},z)^{\sum_i \gamma_i \left(\ln \Omega_\mathrm{m}(\mathbf{p},z)\right)^i / i!}$$

$$\sigma_8(\boldsymbol{\gamma},\mathbf{p},z) = \sigma_{8,0} D(\boldsymbol{\gamma},\mathbf{p},z) = \sigma_{8,0}\, e^{\int_0^z \frac{f(\boldsymbol{\gamma},\mathbf{p},z')}{1+z'} dz'}$$

Growth data

$$\chi^2(\underbrace{\boldsymbol{\gamma}},\underbrace{\mathbf{p}}) = \sum_{i=1}^{N} \left(\frac{\overbrace{(f\sigma_8)_\mathrm{obs}(z_i)}^{\text{Growth data}} - \overbrace{f(\boldsymbol{\gamma},\mathbf{p},z_i)\sigma_8(\boldsymbol{\gamma},\mathbf{p},z_i)}^{\text{Theory}}}{\sigma_i} \right)^2$$

$$\boxed{\mathbf{p} = (\sigma_{8,0}, \Omega_{\mathrm{m},0}, w_0, w_a)}$$

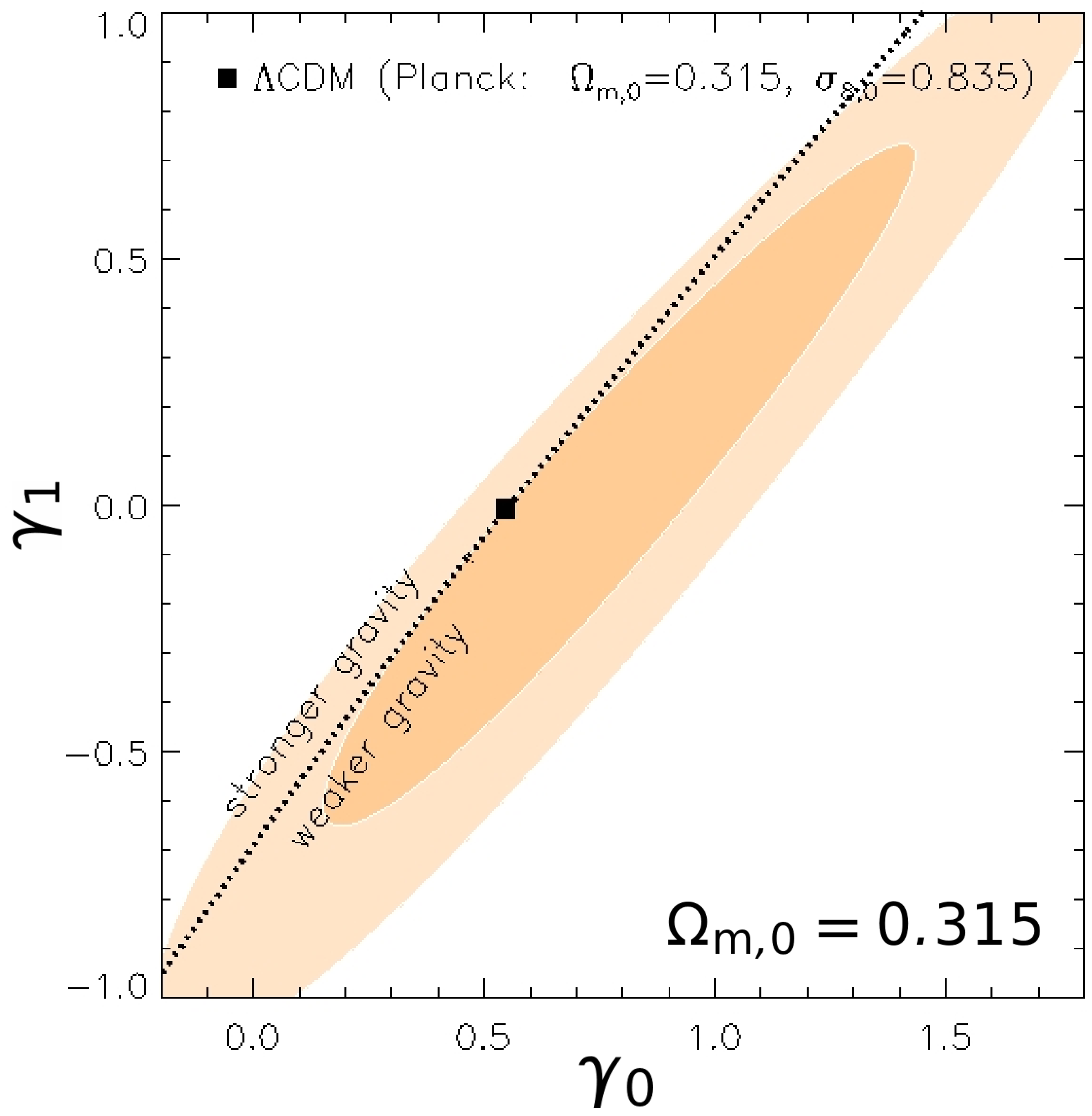
Background parameters
(except for σ_8)

$$\boxed{\boldsymbol{\gamma} = (\gamma_0, \gamma_1, \ldots)}$$

Perturbation parameters

Current observations

Planck



Current observations

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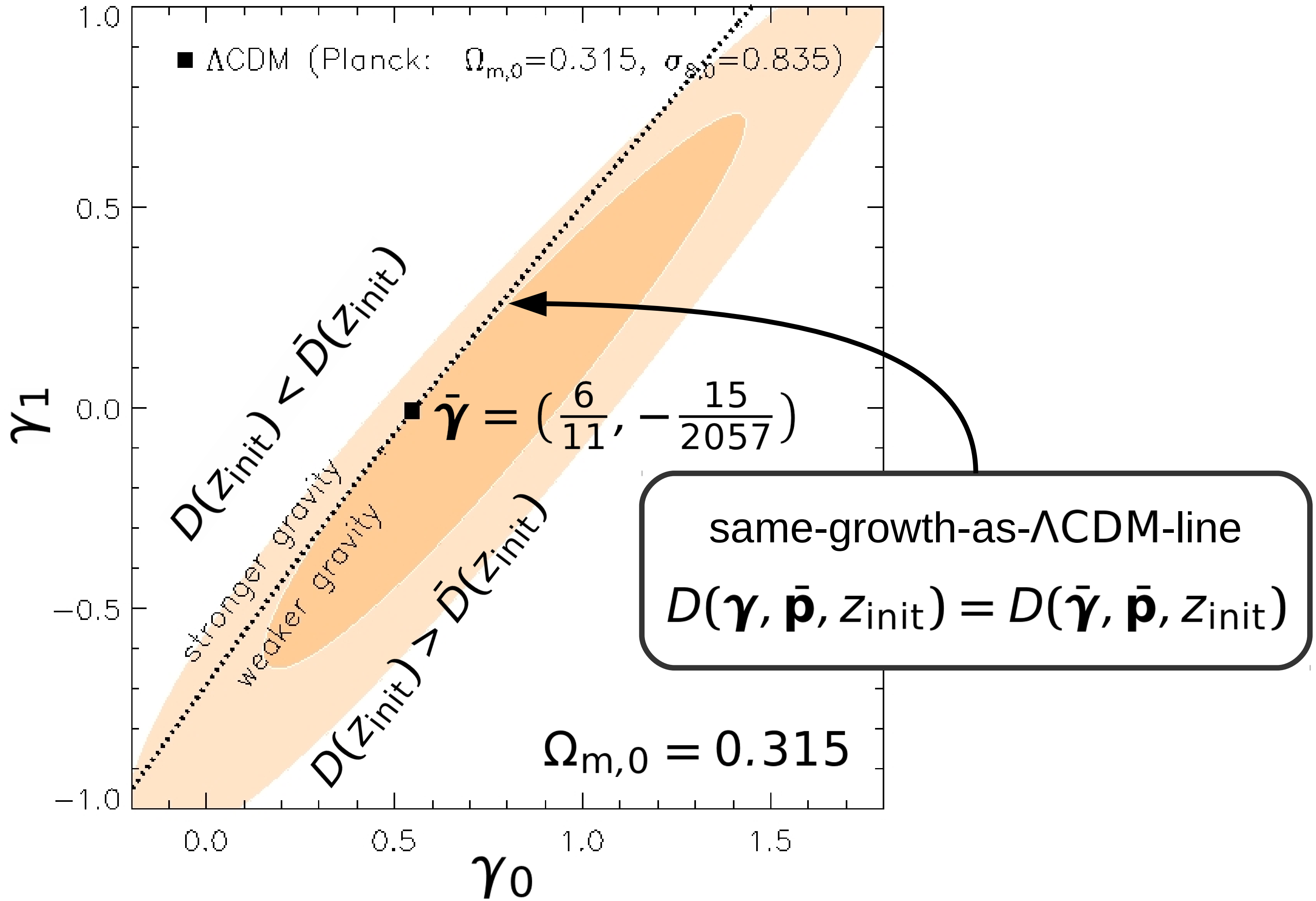
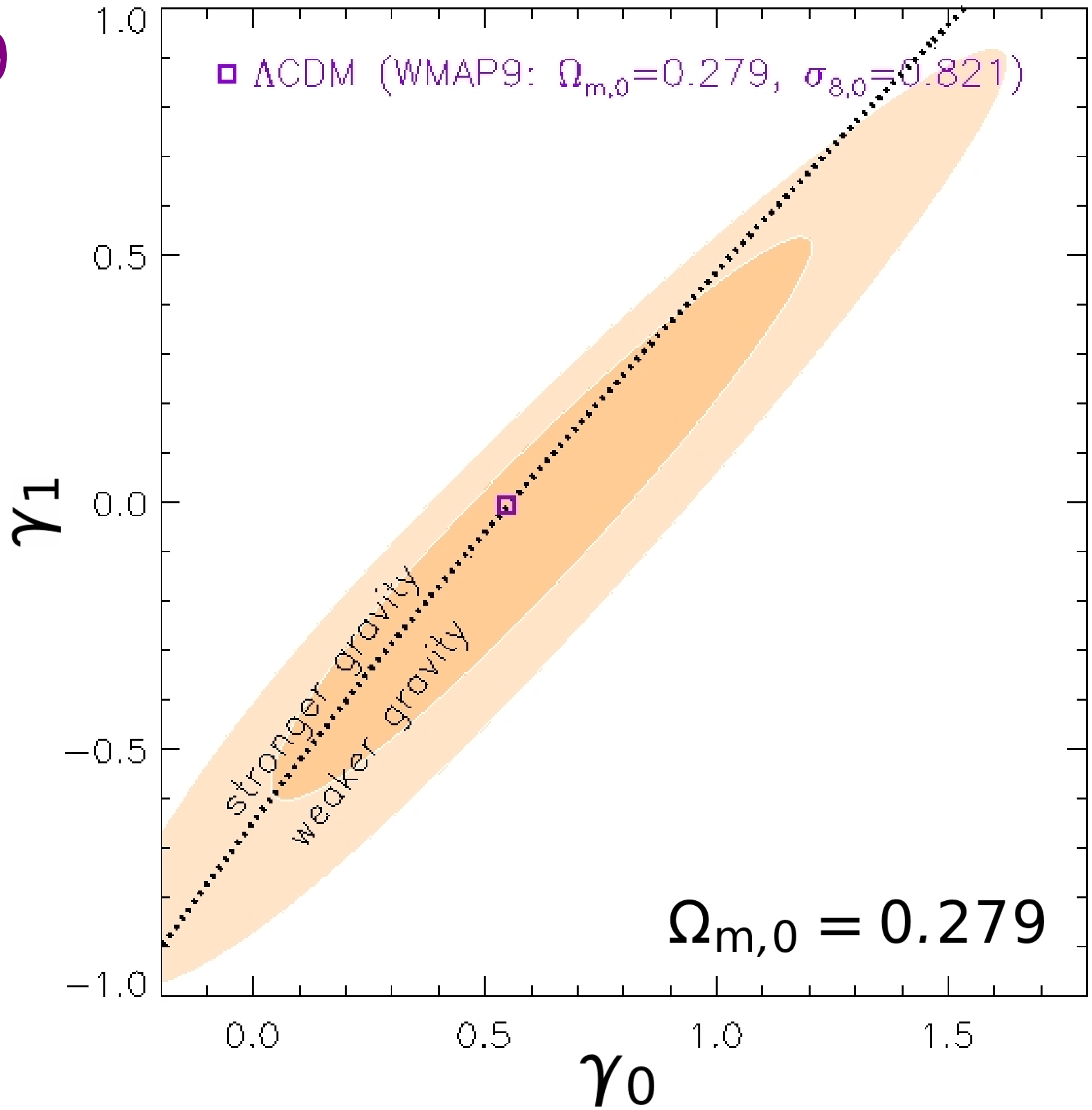


Current observations

WMAP9



Current observations

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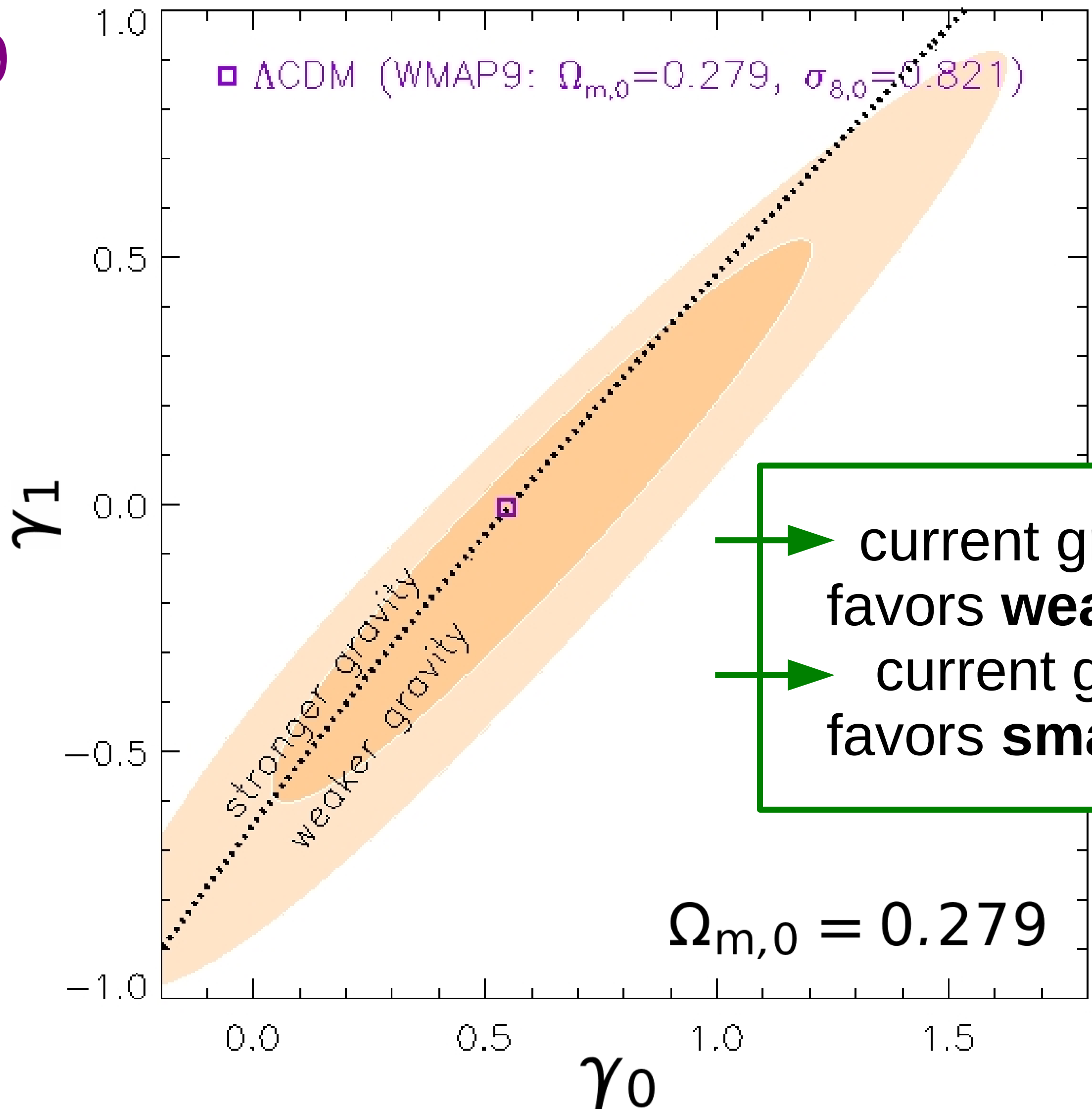


- current growth data mildly favors **weaker gravity**
- current growth data mildly favors **smaller $\boldsymbol{\Omega}_{m,0}$**

Remapping growth indexes into the fiducial background

- Example: wCDM model:

Growth indexes:

$$\gamma_0 = \frac{3(1-w)}{5-6w}$$

$$\gamma_1 = -\frac{3(1-w)(2-3w)}{2(5-12w)(5-6w)^2}$$

Model parameters

$$\mathbf{p} = (0.835, 0.315, w, 0)$$

$$f(\boldsymbol{\gamma}^*, \bar{\mathbf{p}}, z) = f(\boldsymbol{\gamma}, \mathbf{p}, z)$$

z in [0,2]

Fiducial model parameters

$$\bar{\mathbf{p}} = (0.835, 0.315, -1, 0)$$

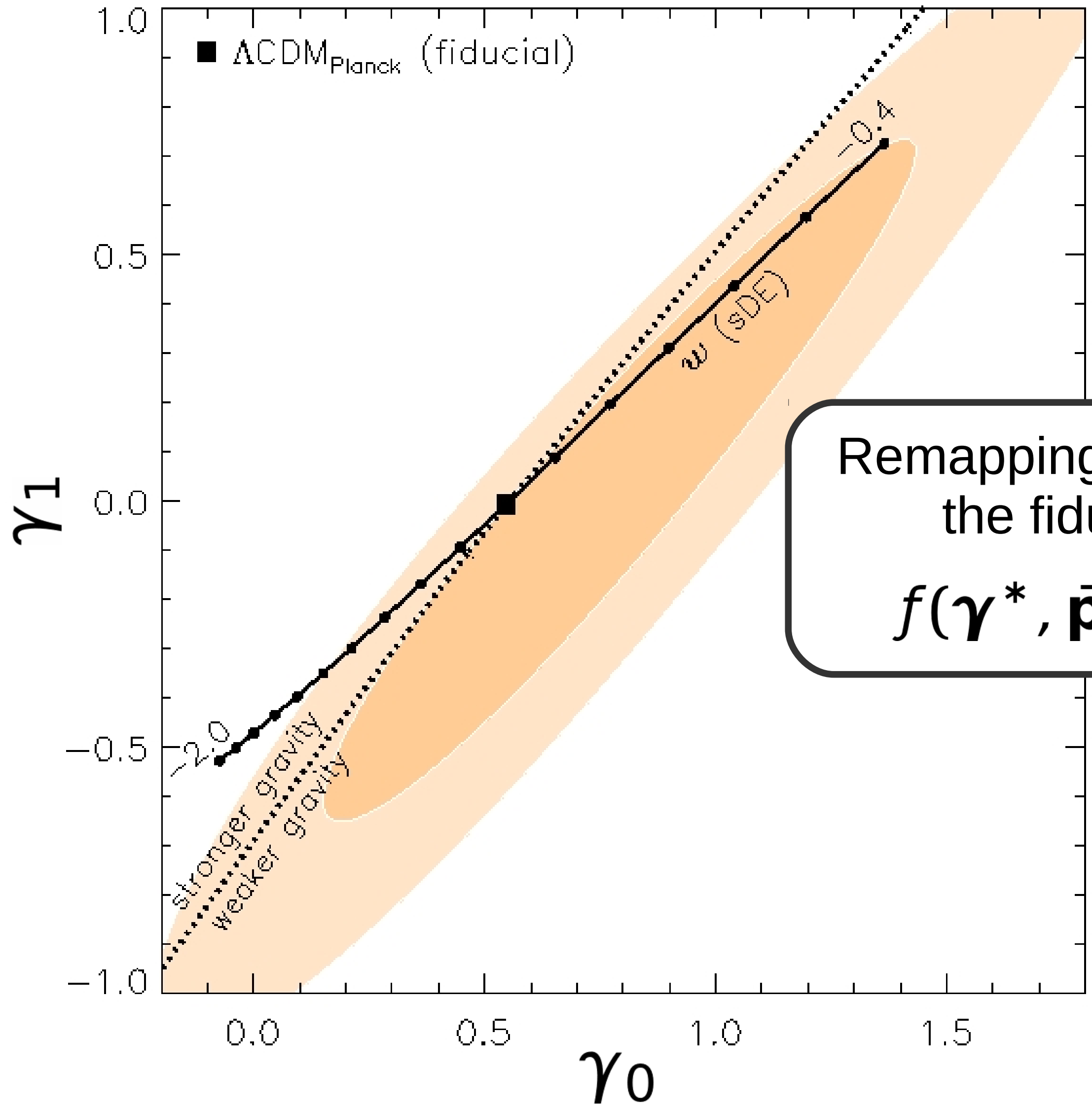
$$\boldsymbol{\gamma}^* = (\gamma_0^*, \gamma_1^*)$$

Effective growth indexes

- Errors are of order 0.3% for Euclid and 3% for current data (and the most extreme models)

Current observations

Planck



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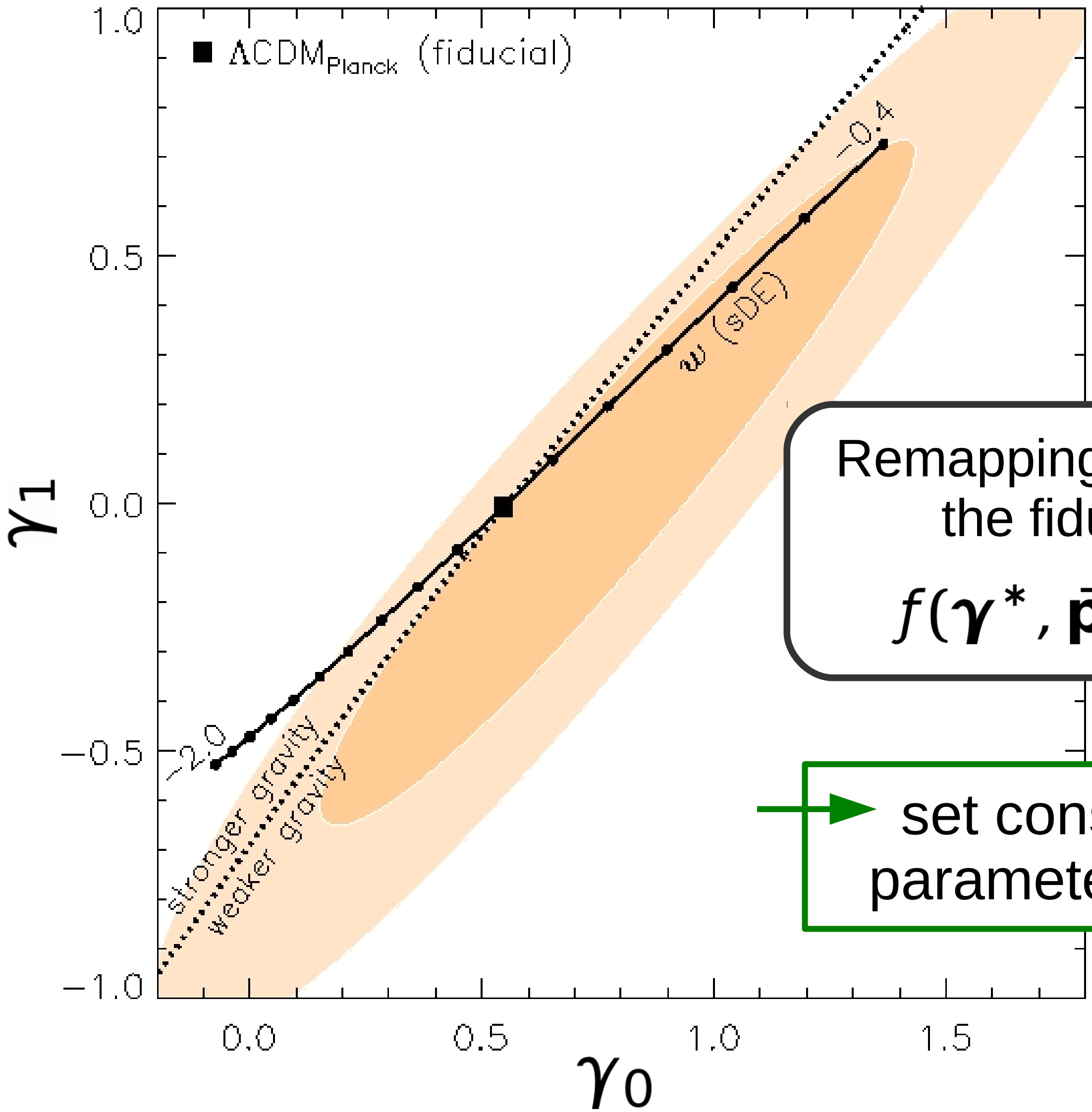


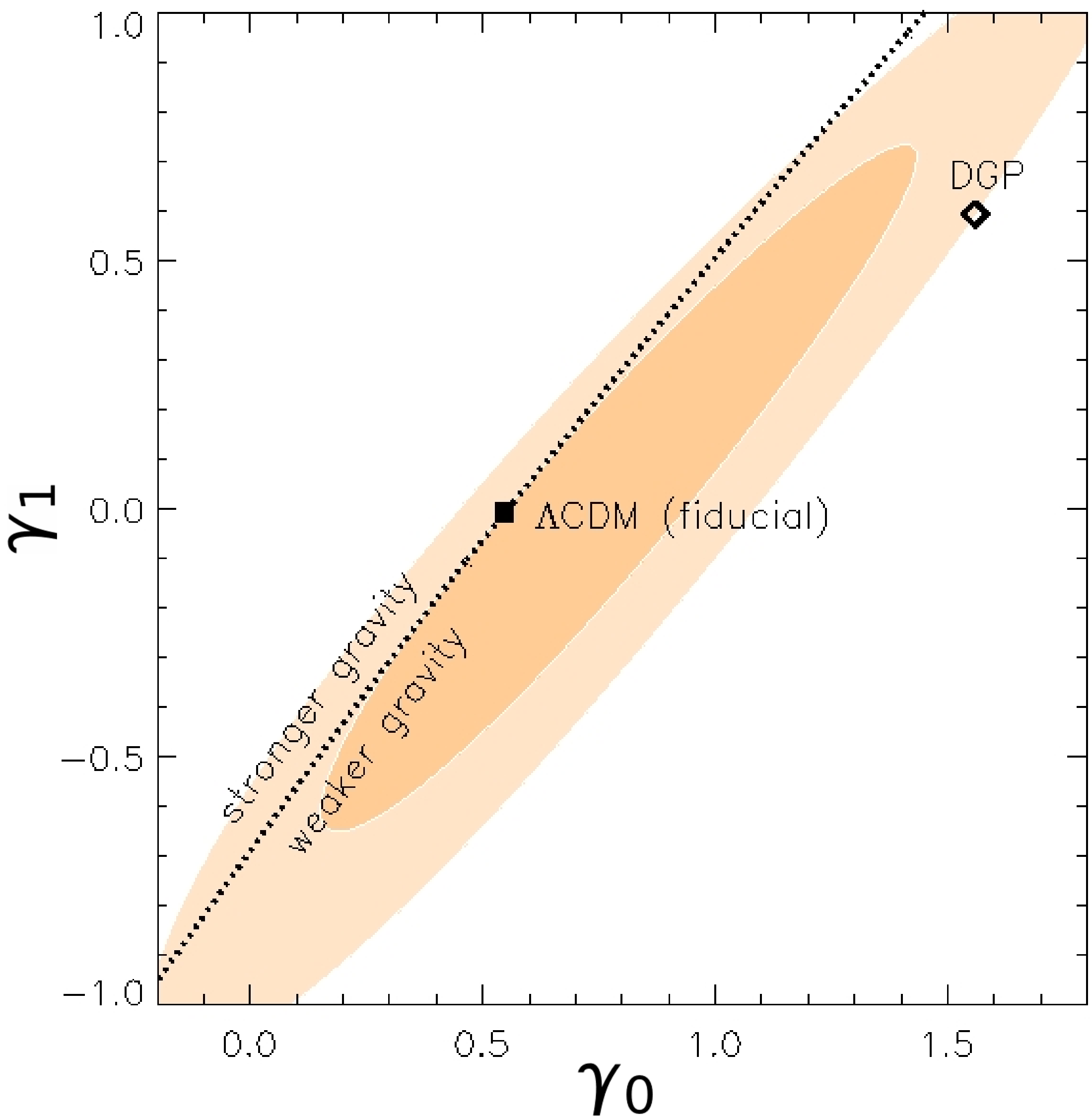
Remapping growth indexes into the fiducial background

$$f(\boldsymbol{\gamma}^*, \bar{\mathbf{p}}, z) = f(\boldsymbol{\gamma}, \mathbf{p}, z).$$

→ set constraints on model parameters: w >-1.5 95% c.l.

Current observations

Planck



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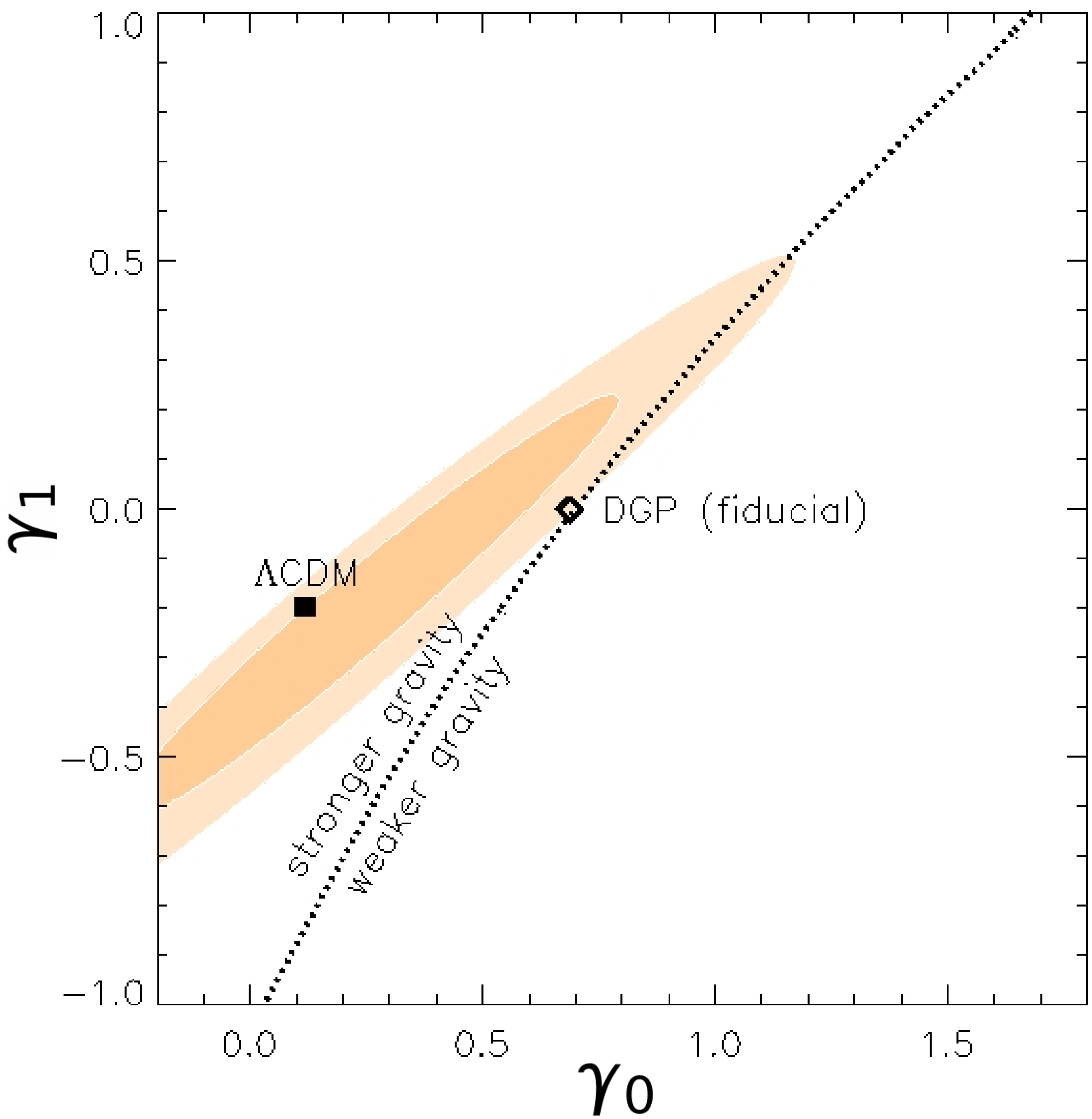

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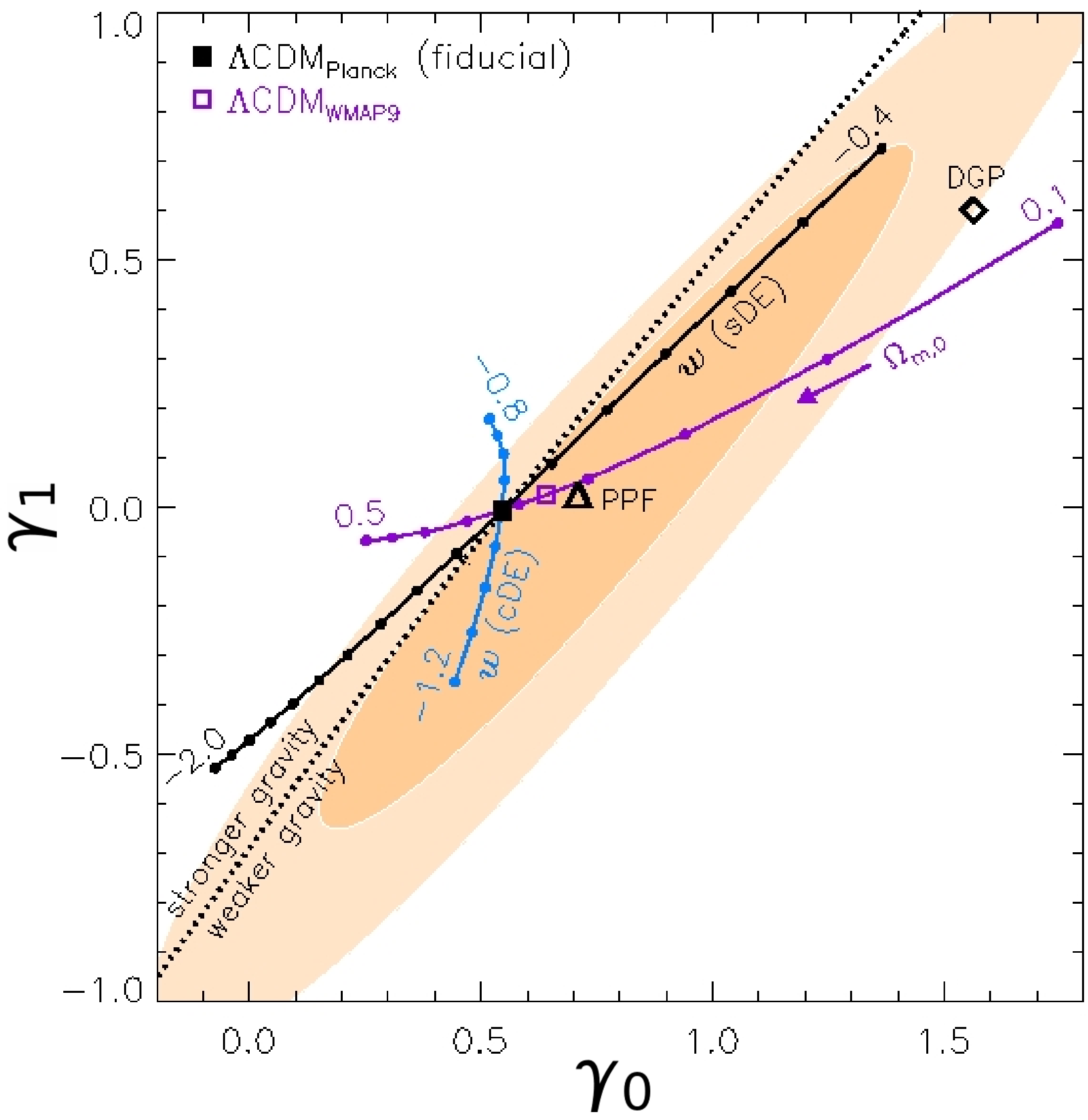

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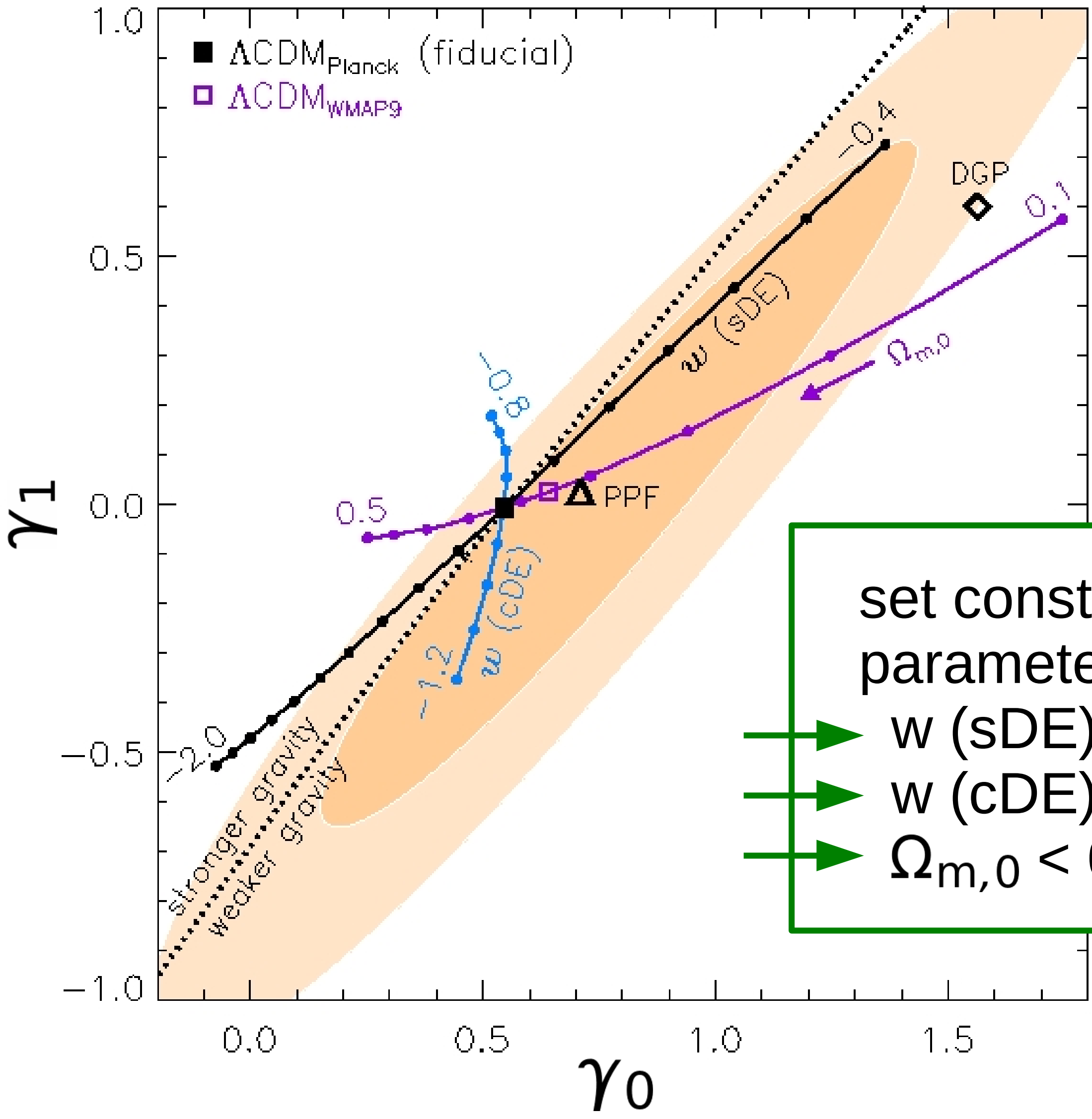
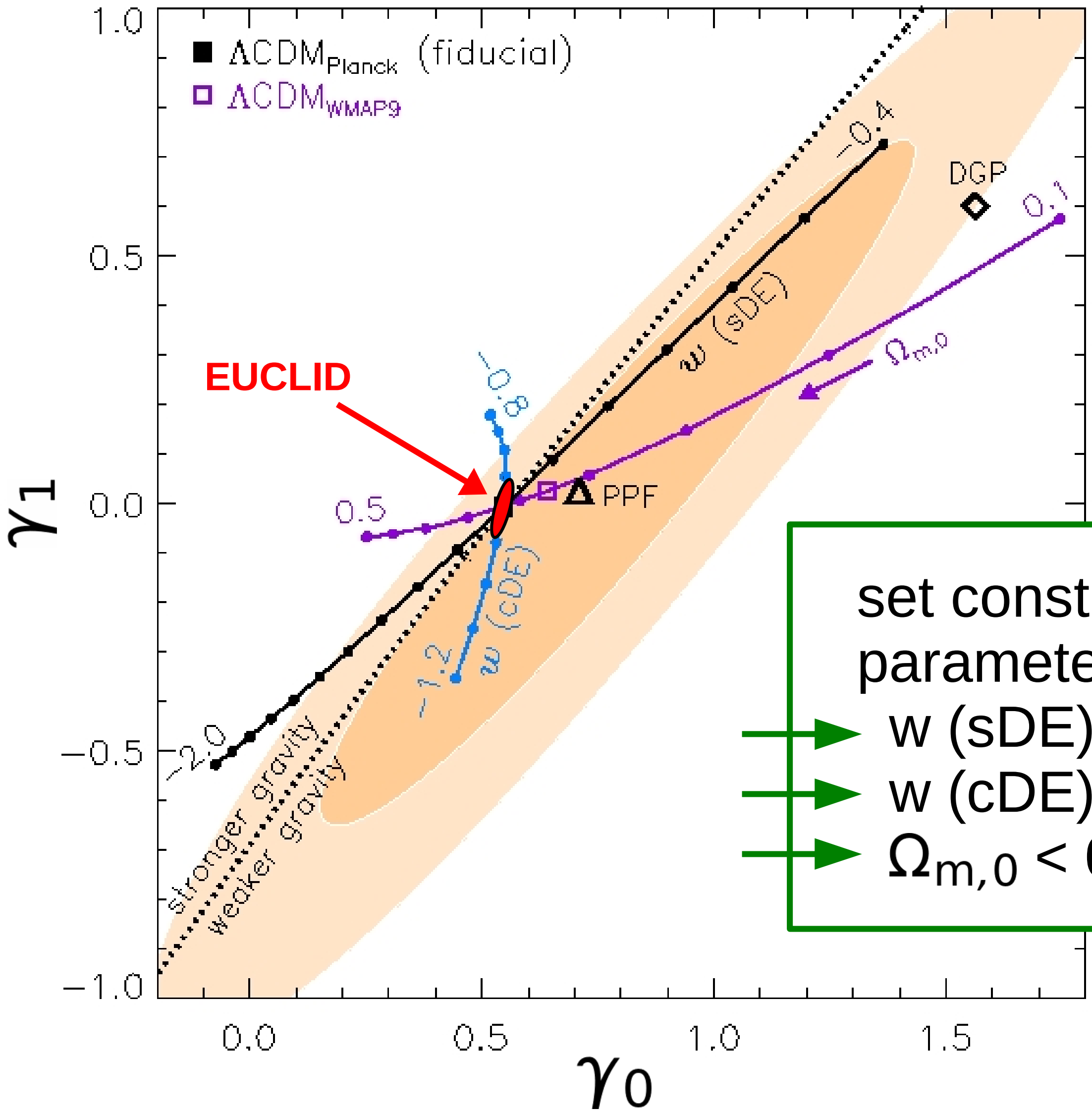


set constraints on model parameters:

- w (sDE) > - 1.5 at 95% c.l.
- w (cDE) < - 0.9 at 95% c.l.
- $\Omega_{m,0}$ < 0.38 at 95% c.l.

Current observations

Planck



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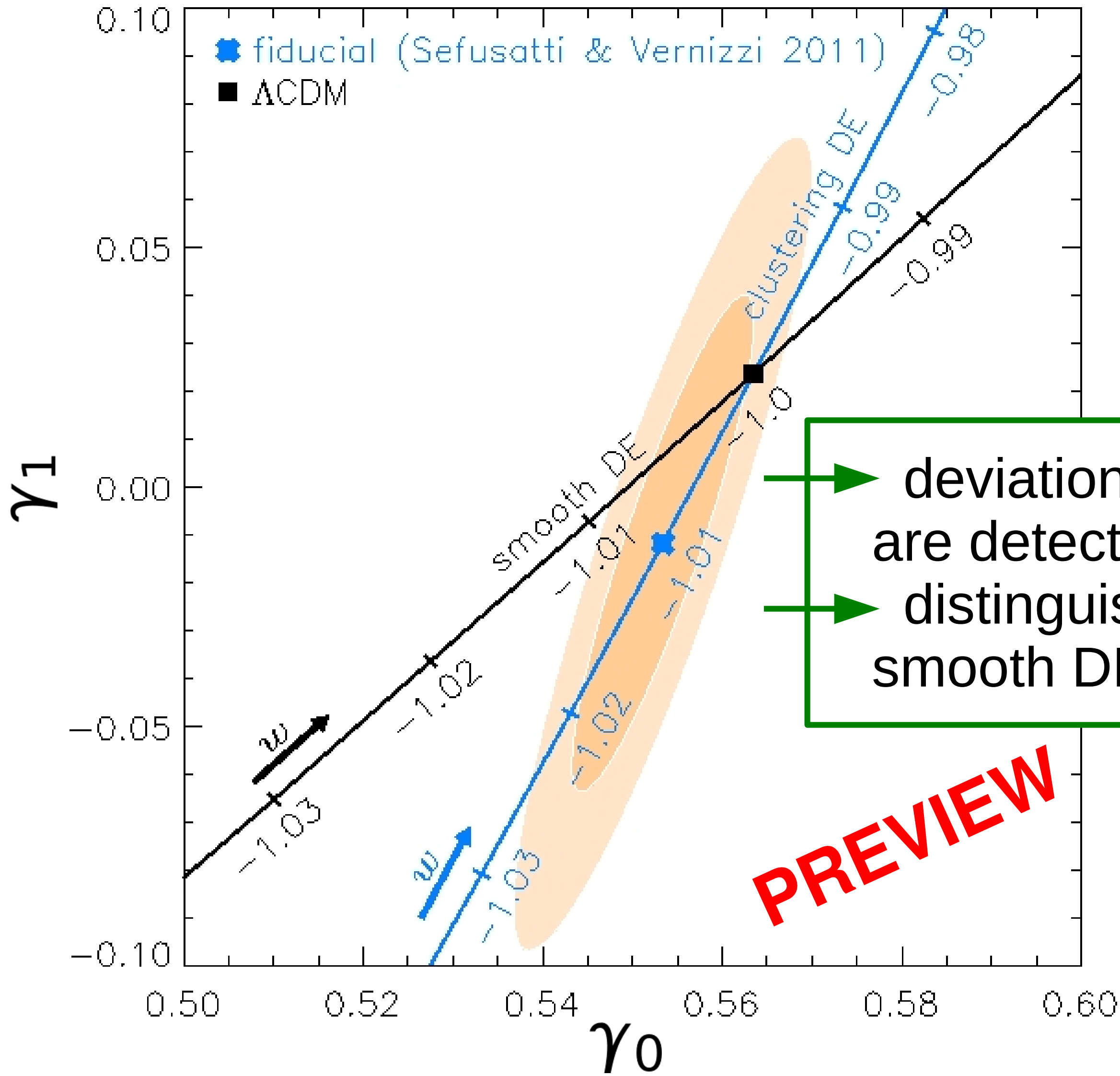
EUCLID

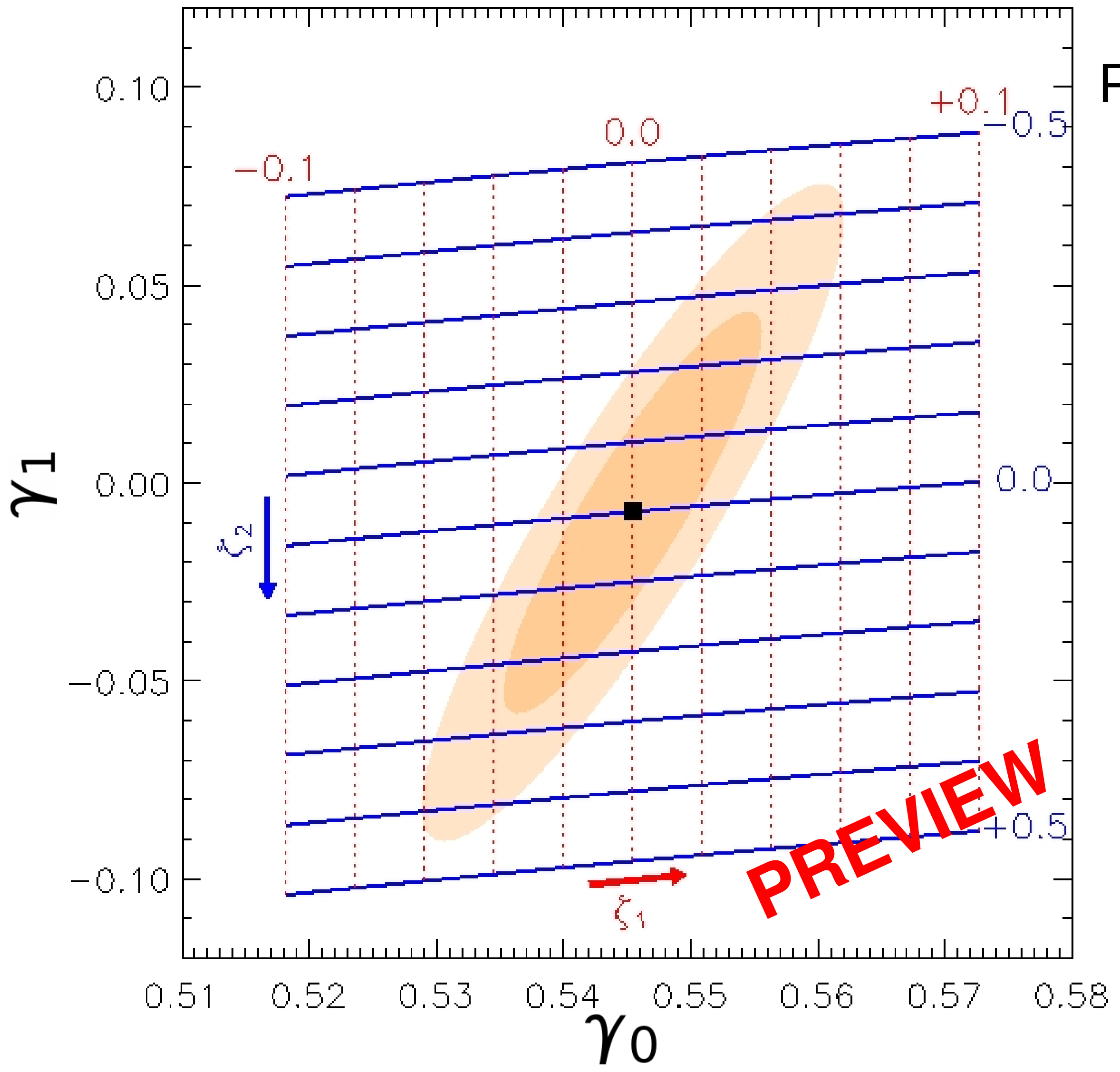
[Euclid Red Book]

Assume
w=-1.01 ...



- deviations of 1% from w=-1 are detectable at 95% c.l.
- distinguish clustering from smooth DE.

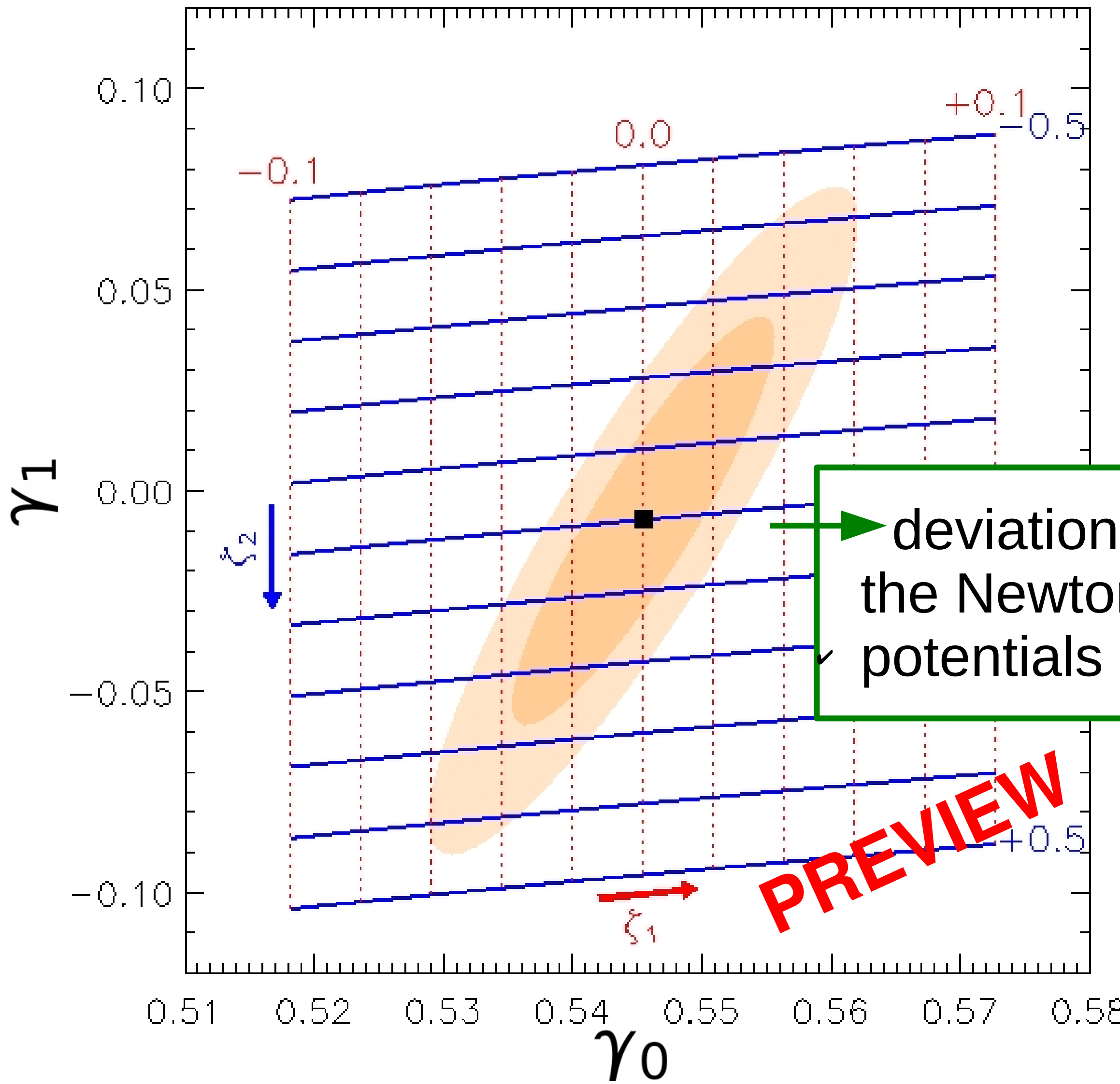
EUCLID



Parametrized Post-Friedmann formalism

Slip parameter
$\zeta = 1 - \Phi/\Psi$

EUCLID



Parametrized Post-Friedmann formalism

Slip parameter
$\zeta = 1 - \Phi/\Psi$

✓ deviations of 2.5% between the Newton and curvature potentials are seen at 95% c.l.

Conclusions

- Mild tension between *Planck* and LSS measurements in SM.
- We present an analytical parametrization for the growth index that is

  a) observer friendly (few coefficients needed)

  b) theorist friendly (links observational results to specific models)

  c) precise (<1%)
- Shortcomings :

  - scale dependent MG models are difficult to integrate → future work
- We introduce a background model independent way to analyse growth data and many models at the same time.
- It would be good to have k-binned LSS data

Details : H. Steigerwald, J. Bel, C. Marinoni, (2014) [arXiv:1403:0898] (accepted by JCAP)

How to parametrize the growth rate f?

$$f' + f^2 + \left(1 + \nu + \frac{H'}{H}\right)f - \frac{3}{2}\mu\,\Omega_m = 0$$

$$f = f\Big(\boldsymbol{P}(\boxed{\text{background}}),\ \boldsymbol{P}(\boxed{\text{perturbations}})\Big)$$

- For ΛCDM $f \sim \Omega^{\gamma=0.6}$ [Peebles 1980]

growth index

- We need a parametrization for the growth rate satisfying
  - high precision (<1% of error)
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Connection theory-observation

Model	γ_0	γ_1	γ_2	$\ldots$
ΛCDM	$\frac{6}{11}$	$-\frac{15}{2057}$	$\frac{410}{520\,421}$	$\ldots$
wCDM ($w=-0.9$)	0.5481	-0.0078	0.0009	$\ldots$
wCDM ($w=-1.1$)	0.5431	-0.0068	0.0007	$\ldots$
w(a)CDM ($w_o=-0.9, w_a=0.5$)	0.5538	-0.0072	-0.0010	$\ldots$
w(a)CDM ($w_o=-1.1, w_a=0.5$)	0.5467	-0.0064	0.0004	$\ldots$
Clust. Quint. ($w=-0.9$)	$\frac{243}{520}$	$\frac{1\,356\,669}{42\,723\,200}$	$\frac{6\,260\,762\,727}{294\,362\,848\,000}$	$\ldots$
Clust. Quint. ($w=-1.1$)	$\frac{363}{580}$	$-\frac{3\,467\,739}{61\,224\,800}$	$\frac{733\,481\,309}{17\,200\,342\,250}$	$\ldots$
flat DGP	$\frac{11}{16}$	$-\frac{7}{5632}$	$-\frac{1051}{22\,528}$	$\ldots$

- All coefficients are analytically predicted
- Higher orders decrease rapidly

Outline

- $\boxed{\text{The problem: how to parametrize the growth index?}}$
- Test of our formalism
- Prospects with Euclid-like surveys
- Application to EFT

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The universe is currently accelerating [Perlmutter et al.,Planck13]

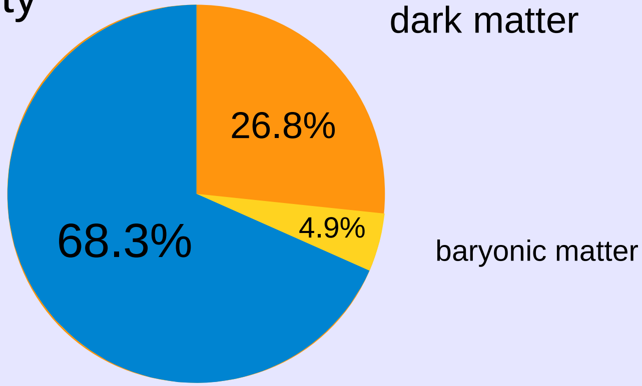
$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$$

$$S_{\mathrm{EH}} = \int d^4x \sqrt{-g} R$$

Standard Gravity

$T_{\mu\nu} =$



dark energy

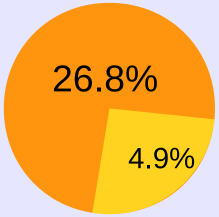
- = Lambda ?
- = quintessence ?
- = quintessence without soundspeed ?

Non-Standard Gravity

$S_{\text{Modified Gravity}}$

- = brane world ?
- = f(R), … ?
- = EFT ?

$T_{\mu\nu} =$



The universe is currently accelerating [Perlmutter et al.,Planck13]

- The universe is dominated today by a hidden form of energy, dubbed Dark Energy. $w = p/\rho < -1/3$
- **OR . . .** Einsteins fields equations break down at very large cosmological scales modified gravity
- Measurements of background not enough to determine the source of acceleration $\longrightarrow$ **perturbations**

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Simulation of Dark Matter clustering

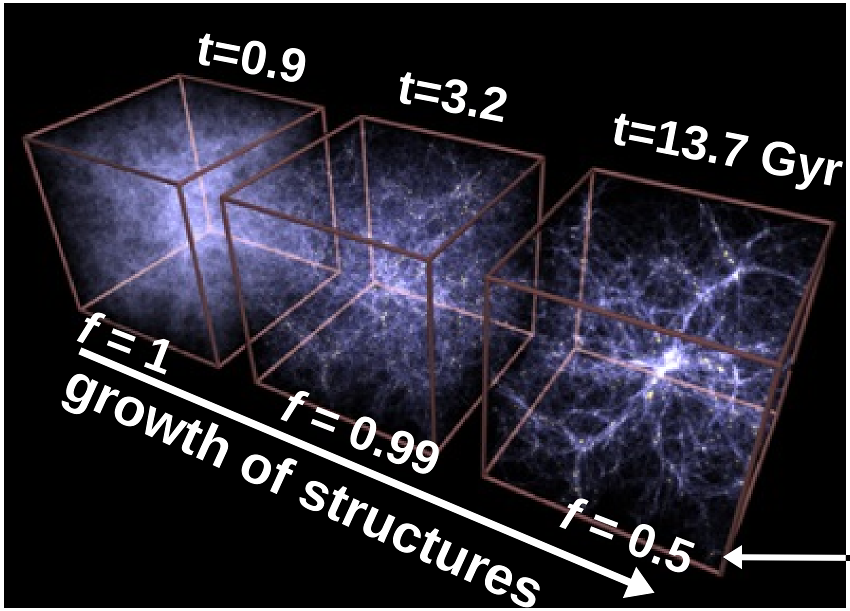


Image: MPE/V.Springel.

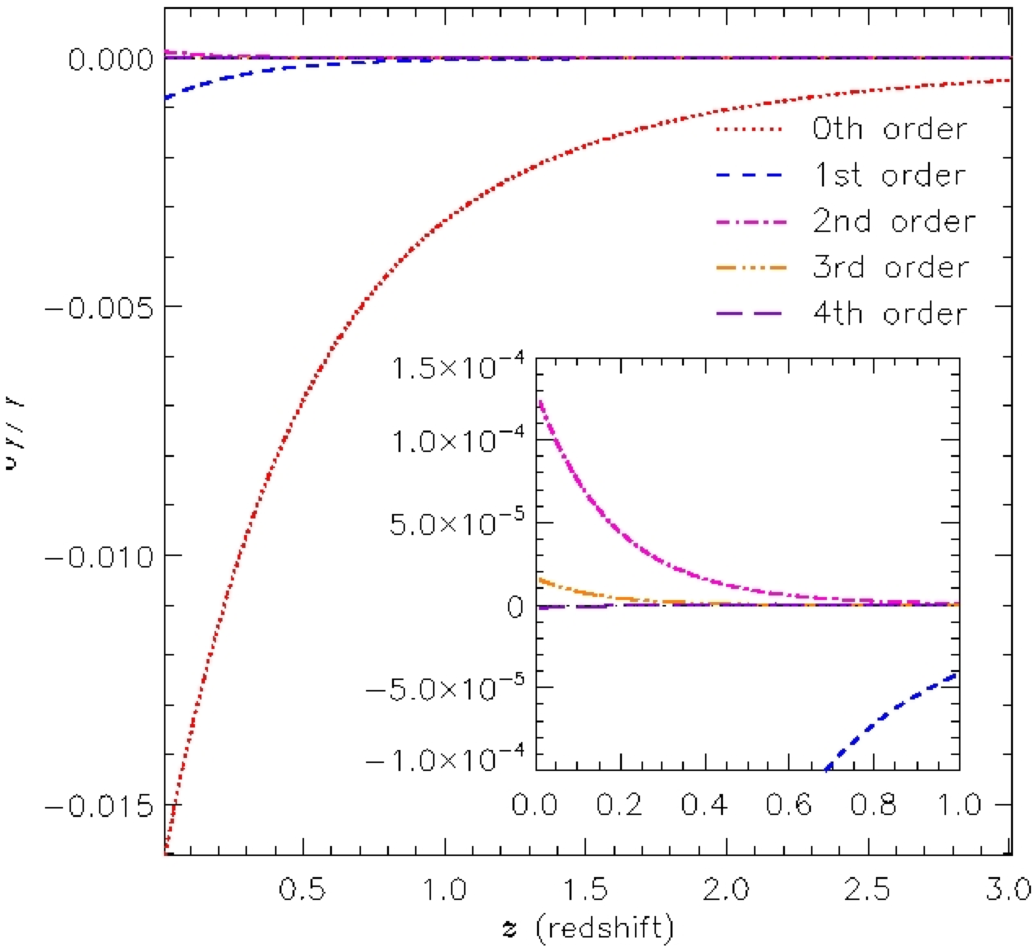
Can the linear growth of structures disentangle these hypotheses?

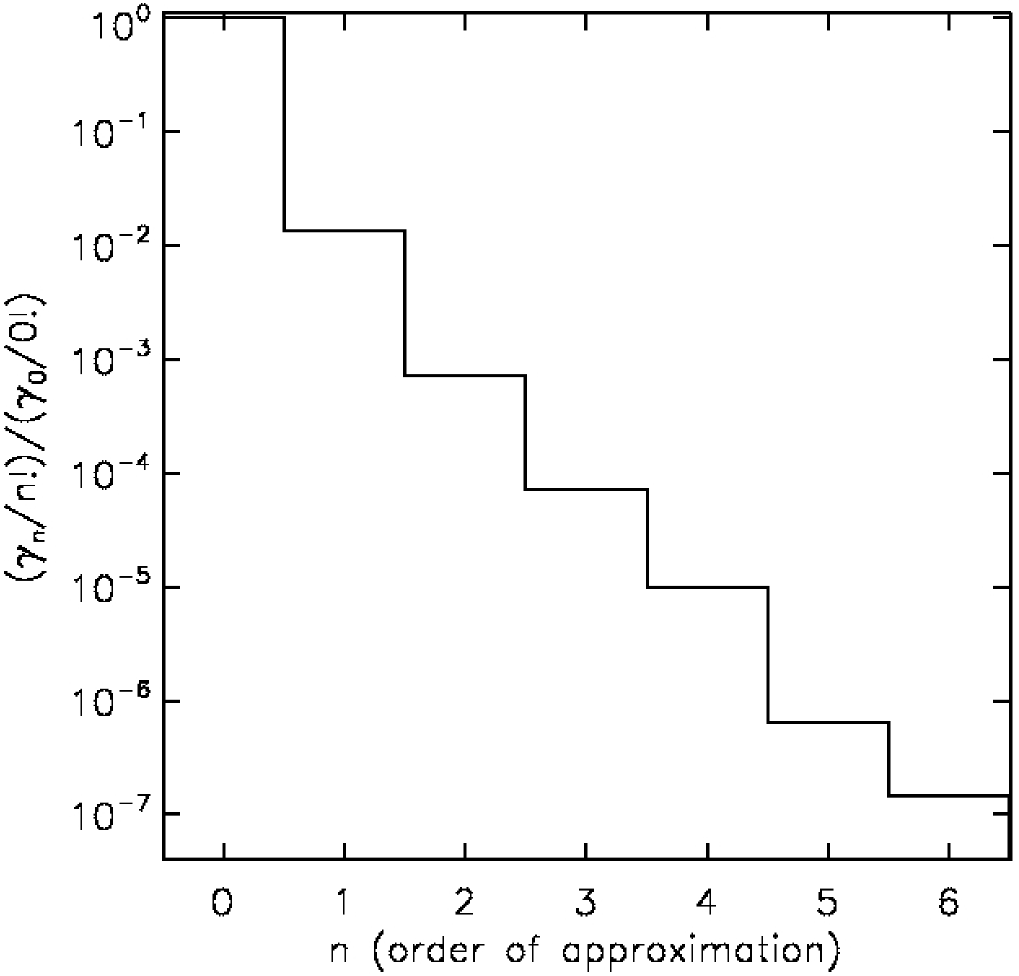
$$f' + f^2 + (1 + \nu + \frac{H'}{H})f - \frac{3}{2}\mu\Omega_m = 0$$

growth rate

Precision

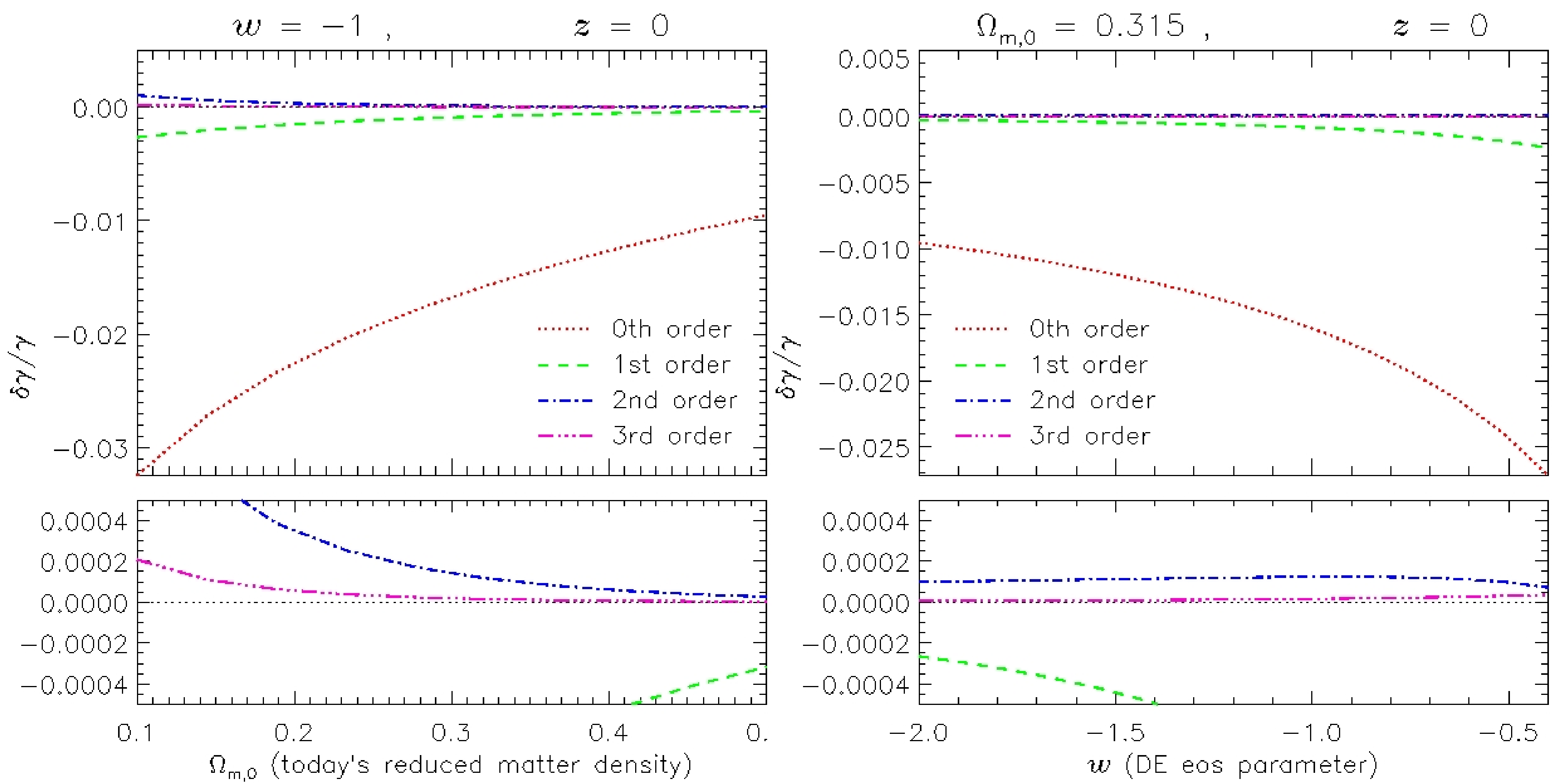
- What is the precision of the approximation? ΛCDM





Precision

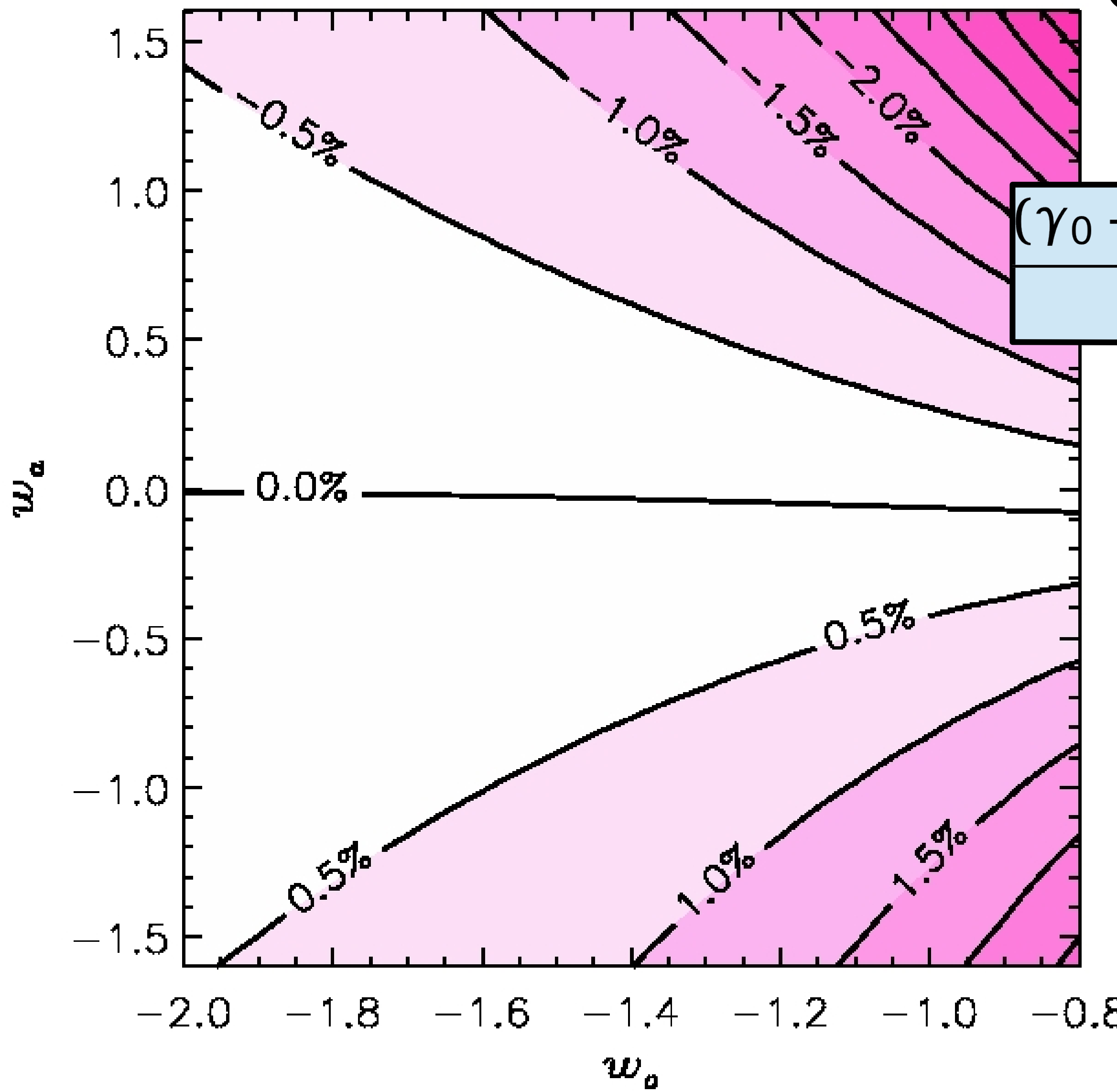
- What is the precision of the approximation? wCDM



Precision

- What is the precision of the approximation?

Smooth Quintessence

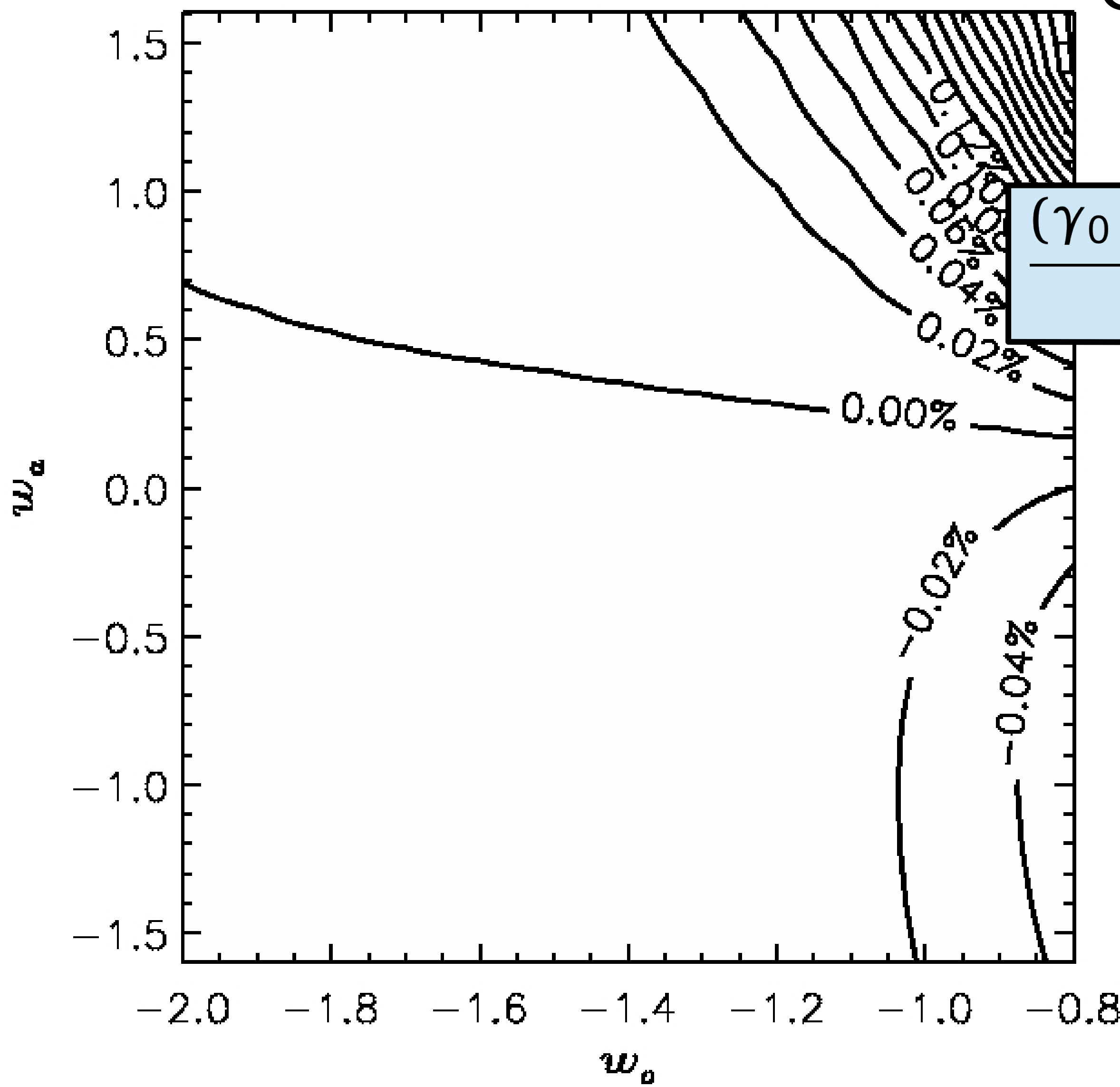


$$\frac{(\gamma_0 + \gamma_1(1 - \Omega_\mathrm{m})) - \gamma_\mathrm{num}}{\gamma_\mathrm{num}}$$

Precision

- What is the precision of the approximation?

Smooth
Quintessence

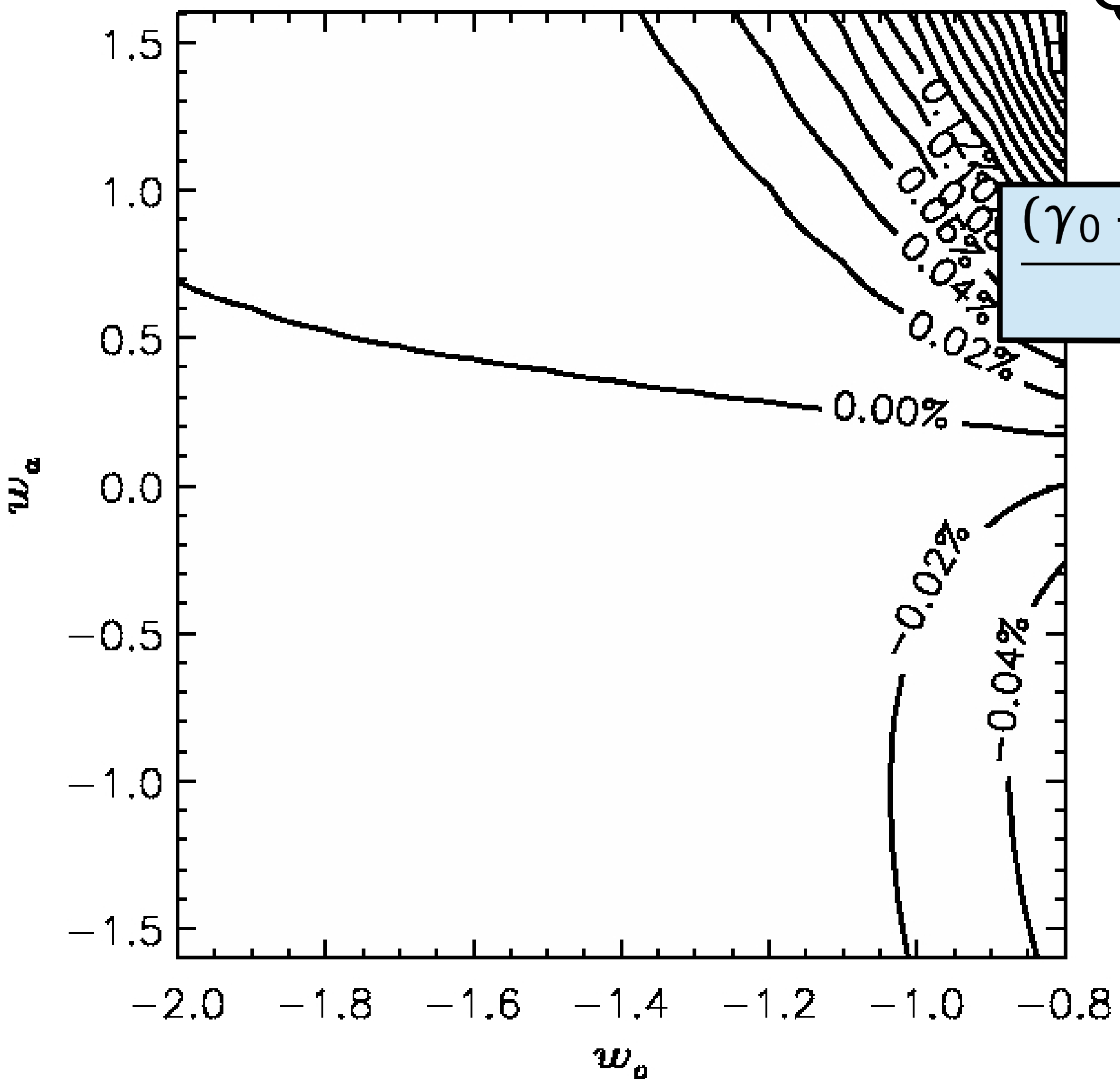


$$\frac{(\gamma_0 + \gamma_1 \ln \Omega_{\mathrm{m}}) - \gamma_{\mathrm{num}}}{\gamma_{\mathrm{num}}}$$

Precision

- What is the precision of the approximation?

Smooth Quintessence



$$\frac{(\gamma_0 + \gamma_1 \ln \Omega_m) - \gamma_{num}}{\gamma_{num}}$$

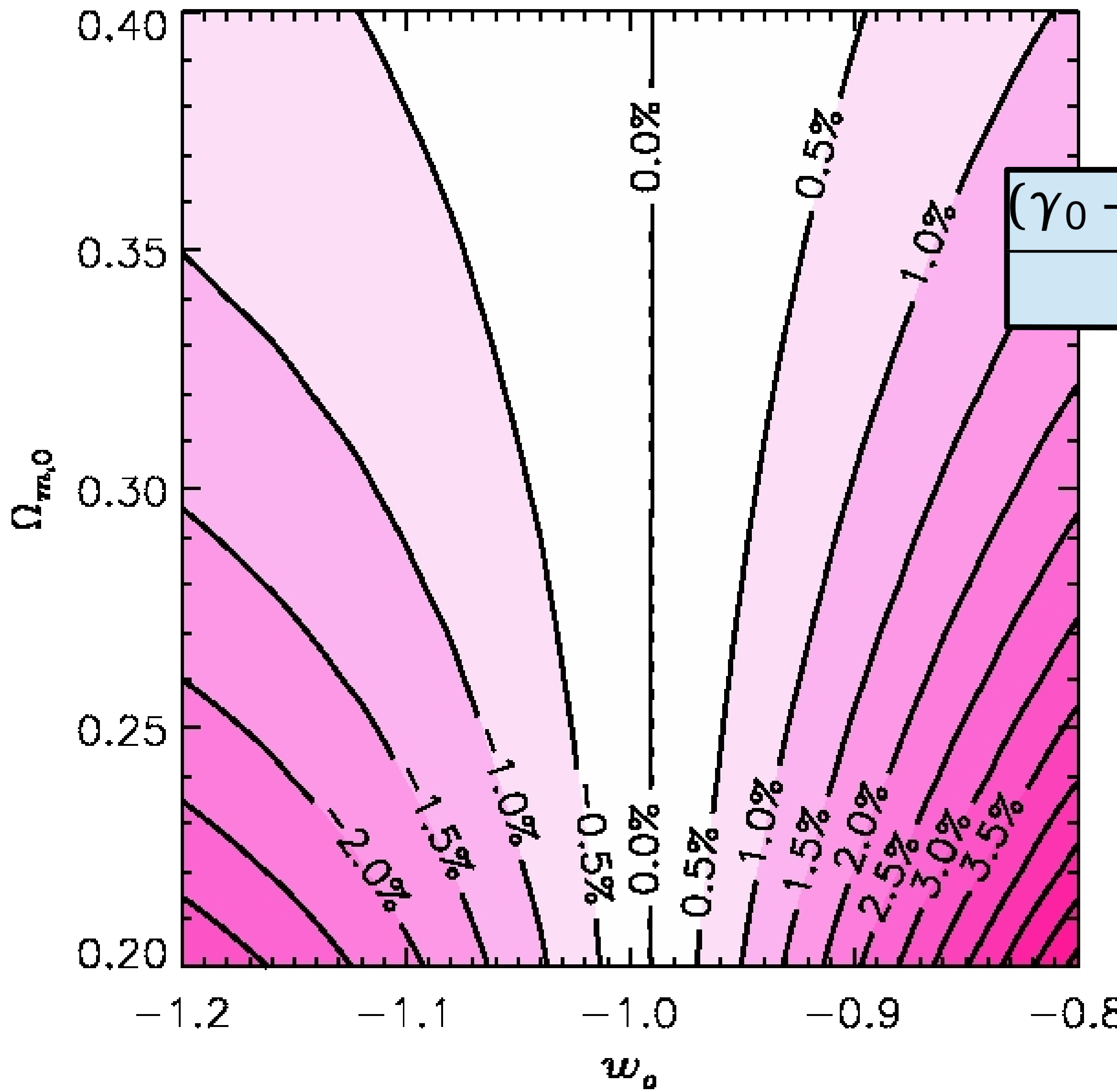
Improvement factor: ~10

Precision

- What is the precision of the approximation?

Clustering Quintessence

[Sefusatti & Vernizzi 11]



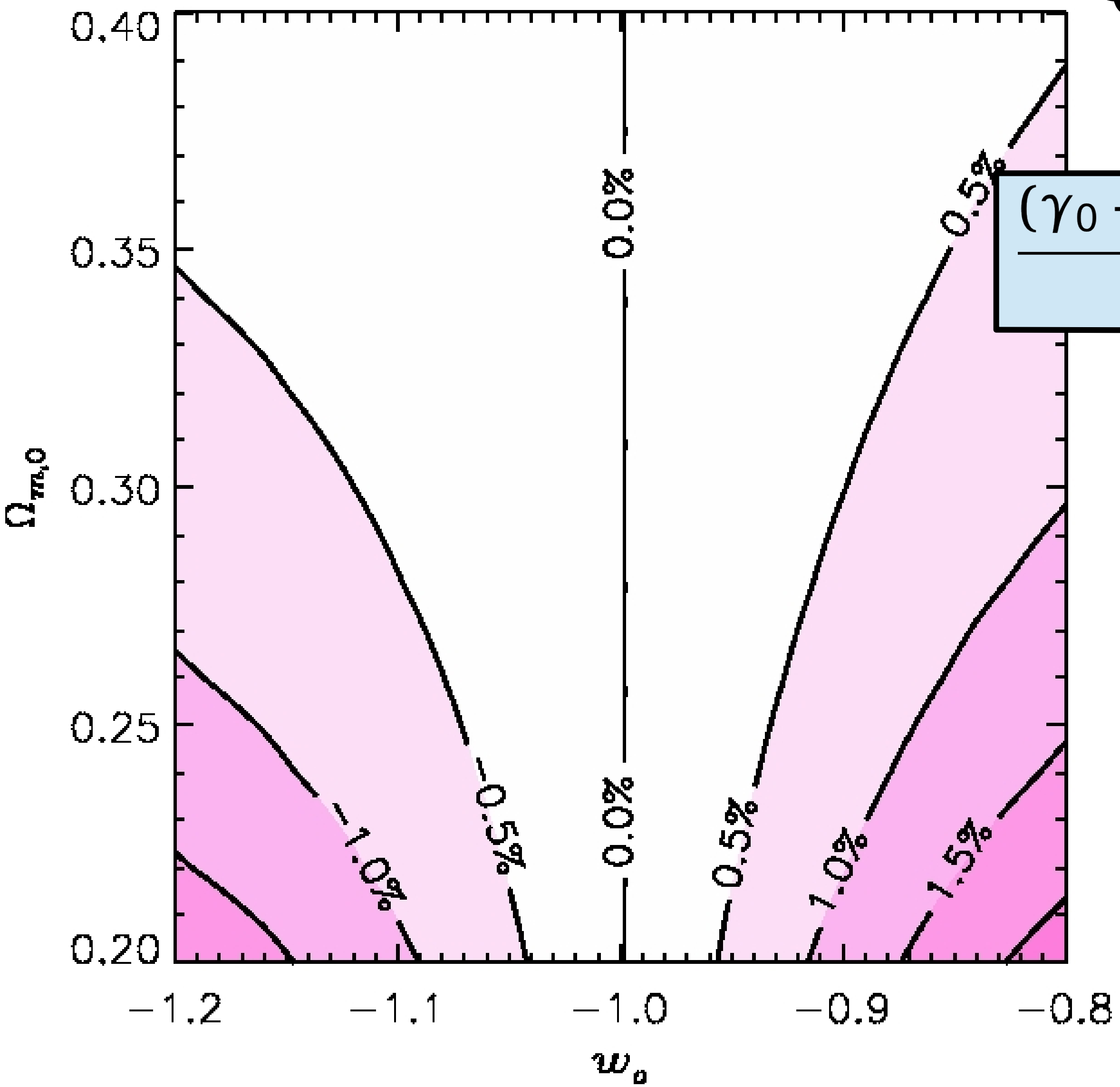
$$\frac{(\gamma_0 + \gamma_1(1 - \Omega_m)) - \gamma_{num}}{\gamma_{num}}$$

Precision

- What is the precision of the approximation?

Clustering Quintessence

[Sefusatti & Vernizzi 11]



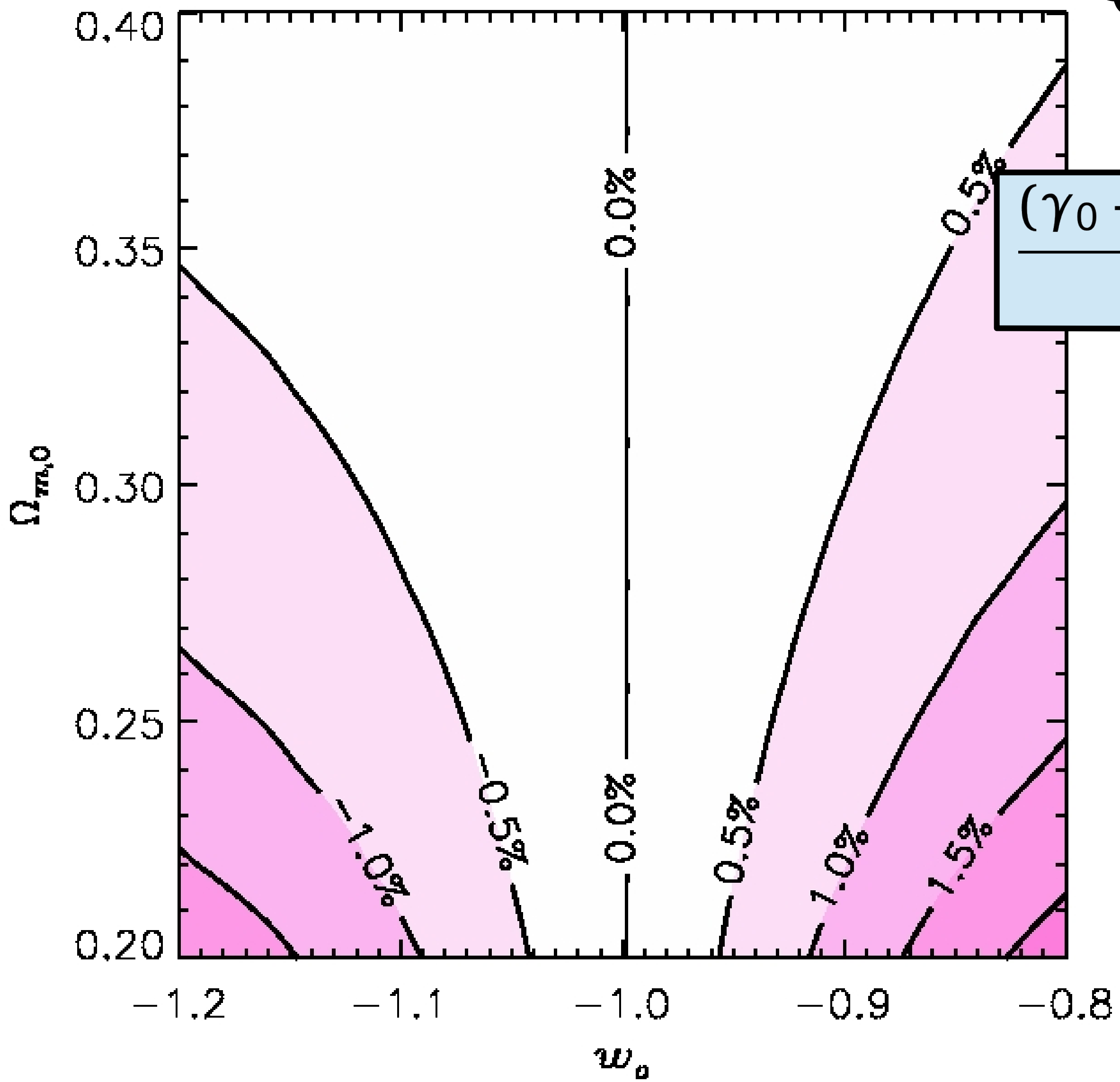
$$\frac{(\gamma_0 + \gamma_1 \ln \Omega_m) - \gamma_{num}}{\gamma_{num}}$$

Precision

- What is the precision of the approximation?

Clustering Quintessence

[Sefusatti & Vernizzi 11]



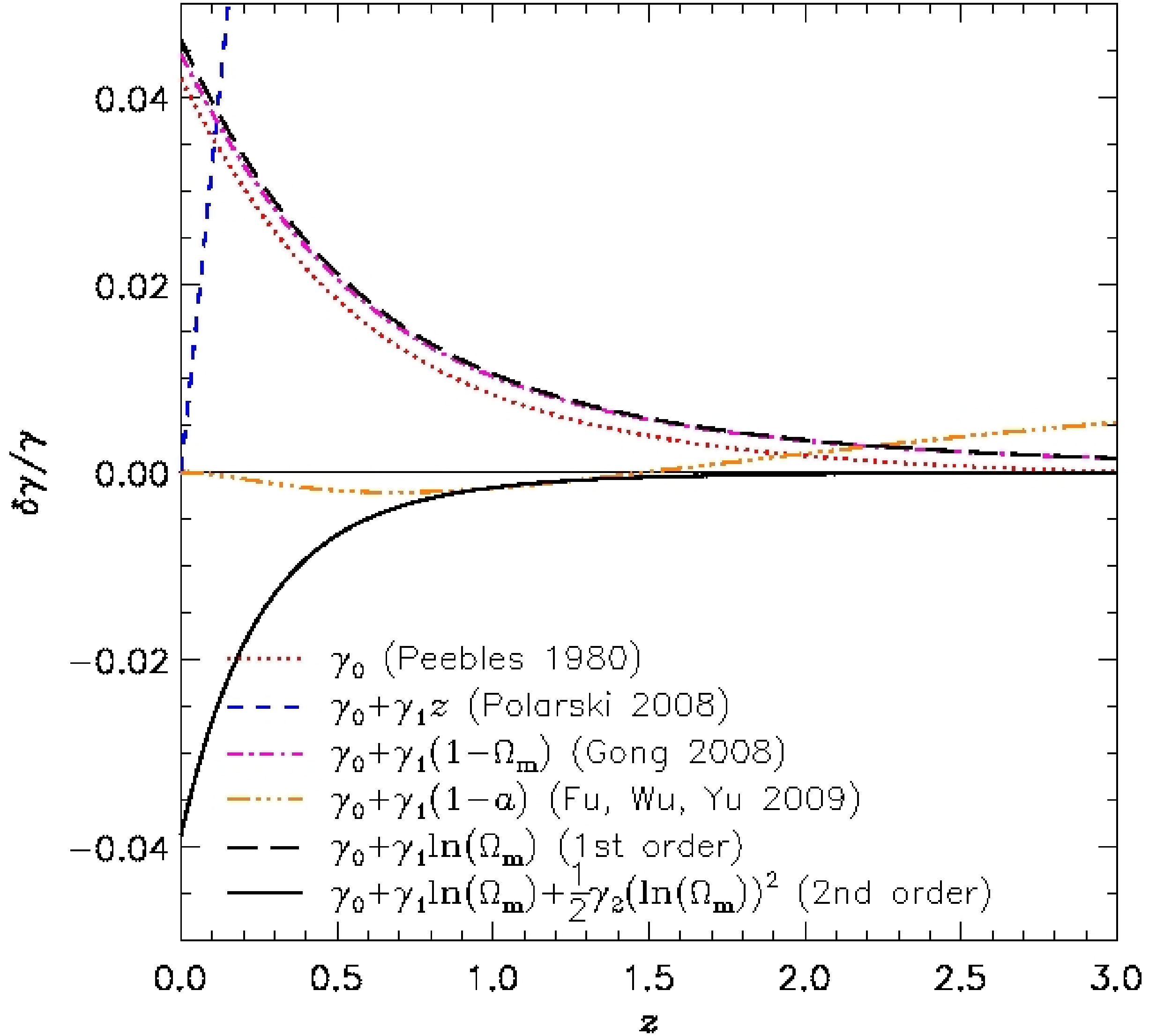
$$\frac{(\gamma_0 + \gamma_1 \ln \Omega_m) - \gamma_{num}}{\gamma_{num}}$$

Improvement factor: ~2

Precision

- What is the precision of the approximation?

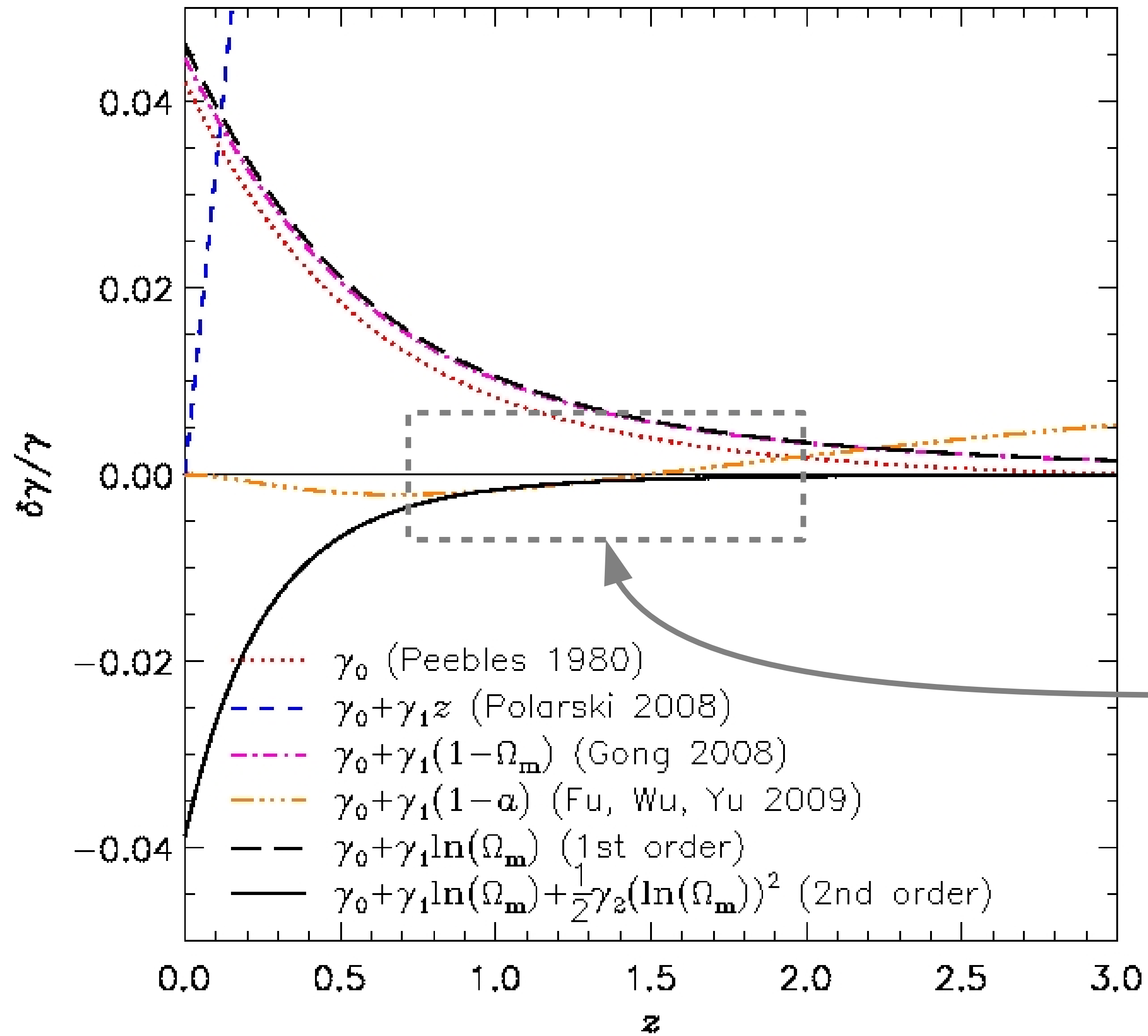
DGP



Precision

- What is the precision of the approximation?

DGP



No improvement

Euclid + *Planck*
$\delta\gamma/\gamma \sim 0.7$

But... current constraints are enough!

Outline

- The problem: how to parametrize the growth index?
- Test of our formalism
- Prospects with Euclid-like surveys
- Application to Effective Field Theory

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EUCLID

Theory

$$f' + f^2 + \left(1 + \nu + \tfrac{H'}{H}\right)f - \tfrac{3}{2}\,\mu\,\Omega_\mathrm{m} = 0$$

Simulated data

$$\gamma(\boldsymbol{\gamma}, \mathbf{p}, z) = \sum_i \gamma_i \left(\ln \Omega_\mathrm{m}(\mathbf{p}, z)\right)^i / i!$$

$$\chi^2(\boldsymbol{\gamma}, \mathbf{p}) = \sum_{i=1}^{N} \left(\frac{\gamma_\mathrm{sim}(z_i) - \gamma(\boldsymbol{\gamma}, \mathbf{p}, z_i)}{\sigma_i} \right)^2$$

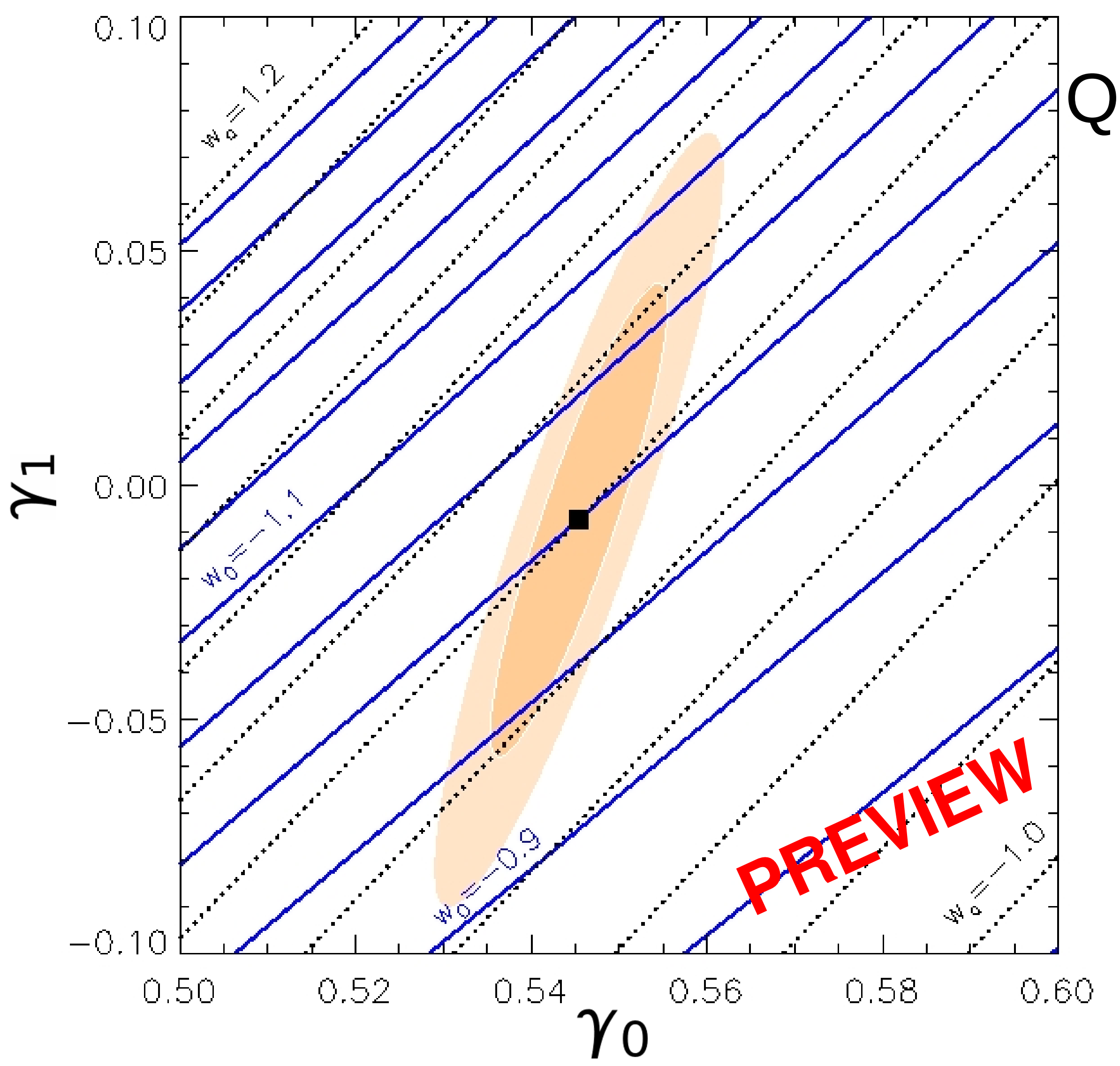
$$\mathbf{p} = (-, \Omega_{\mathrm{m},0}, w_0, w_a)$$

Background parameters

$$\boldsymbol{\gamma} = (\gamma_0, \gamma_1, \dots)$$

Perturbation parameters

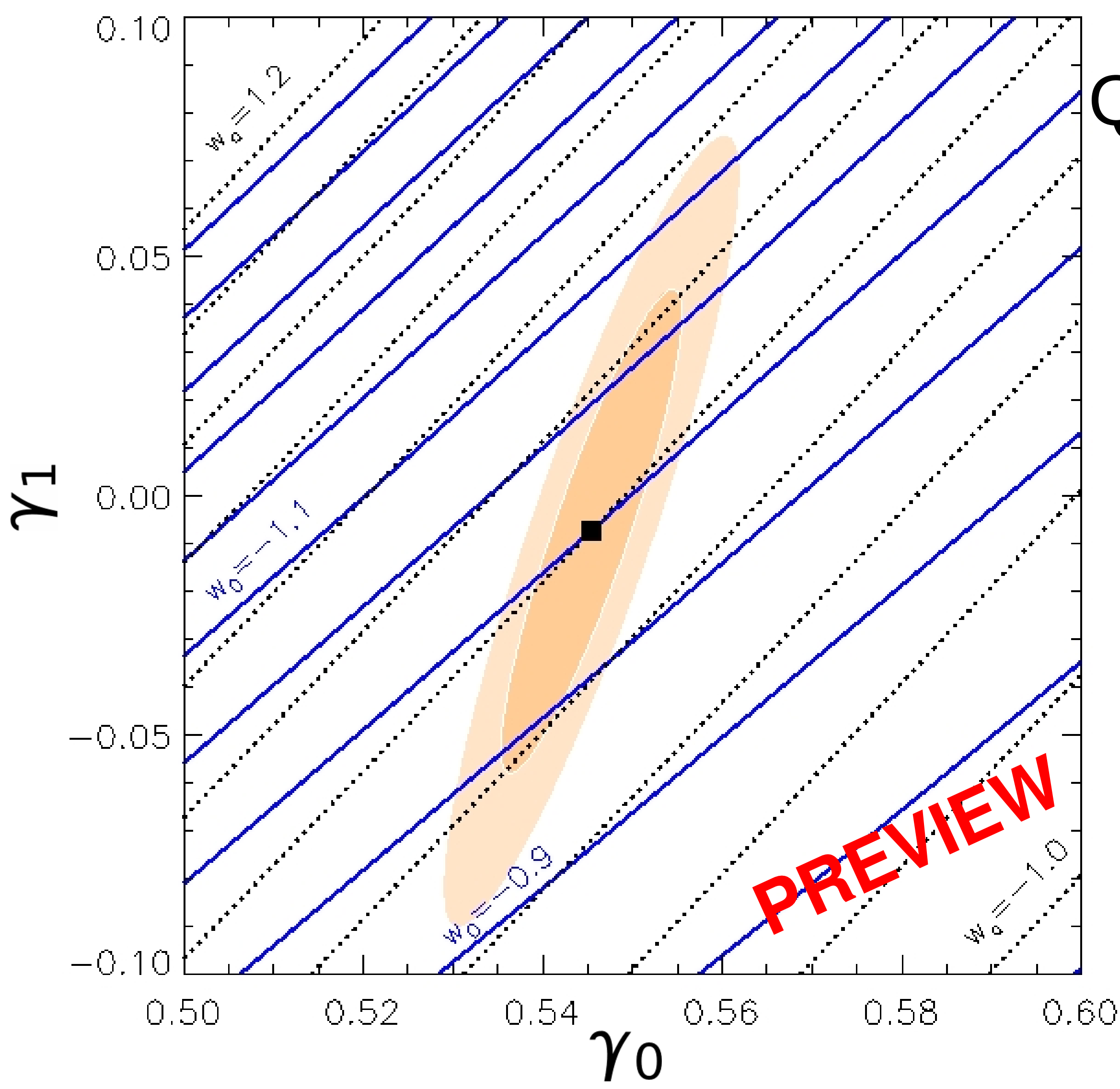
EUCLID



Smooth Quintessence

Fiducial model: ΛCDM

EUCLID



Smooth Quintessence

Fiducial model: ΛCDM

FoM increase

~ 550

A solution? Growth index as Taylor developpement

Ansatz : $$\gamma = \sum_{n=0}^{N} \gamma_n \frac{(\ln \Omega)^n}{n!}$$

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$$\boxed{\gamma_0 = \frac{3(\mathcal{M}_0 + \mathcal{M}_1) - 2(\mathcal{H}_1 + \mathcal{N}_1)}{2 + 2\mathcal{X}_1 + 3\mathcal{M}_0}}$$

$$\mathcal{X}_n = \left[\frac{d^n}{d(\ln\Omega)^n}\left(\frac{d\ln\Omega}{d\ln a}\right)\right]_{\ln\Omega=0}$$

$$\mathcal{H}_n = \left[\frac{d^n}{d(\ln\Omega)^n}\left(\frac{d\ln H}{d\ln a}\right)\right]_{\ln\Omega=0}$$

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Exemple: wCDM

$$\mathcal{H}_1 = \frac{3}{2}w,\ \mathcal{X}_1 = -3w,\ \mathcal{N}_0 = 1,\ \mathcal{M}_0 = 1,\ \mathcal{M}_1 = 0 \longrightarrow \gamma_0 = \frac{3(1-w)}{5-6w}$$

A solution? Growth index as Taylor developpement

Ansatz : $$\gamma = \sum_{n=0}^{N} \gamma_n \frac{(\ln \Omega)^n}{n!}$$

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A general recursion formula for higher order coefficients (n>0):

$$\gamma_n = 3\frac{\mathcal{M}_{n+1} + \sum_{k=1}^{n+1}\binom{n+1}{k}\mathcal{M}_{n+1-k}B_k(1-y_1, -y_2, -y_3, \ldots, -y_k)}{(n+1)(2+2(n+1)\mathcal{X}_1 + 3\mathcal{M}_0)}$$
$$-2\frac{B_{n+1}(y_1, y_2, \ldots, y_{n+1}) + \sum_{k=2}^{n+1}\binom{n+1}{k}\mathcal{X}_k(n+2-k)\gamma_{n+1-k} + \mathcal{H}_{n+1} + \mathcal{N}_{n+1}}{(n+1)(2+2(n+1)\mathcal{X}_1 + 3\mathcal{M}_0)}$$

Outline

- The problem: how to parametrize the growth index?
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The Effective Field Theory of dark energy [Arkani-Hamed et. al. 04]

The action (in unitary gauge):

$$S \;=\; S_{\mathrm{m}}[g_{\mu\nu},\Psi_i] \;+\; \int d^4x\,\sqrt{-g}\,\frac{M^2(t)}{2}\Bigg[R \,-\, 2\lambda(t) \,-\, 2\mathcal{C}(t)g^{00}$$

$$+\mu_2^2(t)(\delta g^{00})^2 \,-\, \mu_3(t)\,\delta K\delta g^{00} + \epsilon_4(t)\left(\underset{\cdot}{K}^{\mu}{}_{\nu}\,\underset{\cdot}{K}^{\nu}{}_{\mu} - \delta K^2 + \frac{{}^{(3)}R\,\delta g^{00}}{2}\right)\Bigg]$$

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- time coordinate t is proportional to the scalar field π (unitary gauge)
- can reproduce Hordeski / generalized Galileon theories

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- time coordinate t is proportional to the scalar field π (unitary gauge)
- can reproduce Hordeski / generalized Galileon theories
- matter fields are minimally coupled to the metric $g_{\mu\nu}$ (Jordan frame)
- neat separation between the background evolution ($M^2, \lambda, \mathcal{C}$) and the perturbation terms ($\mu_2^2, \mu_3, \epsilon_4$)

The Effective Field Theory of dark energy [Arkani-Hamed et. al. 04]

The action :

$$S \;=\; S_{\rm m}[g_{\mu\nu},\Psi_i] \;+\; \int d^4x\,\sqrt{-g}\,\frac{M_{\rm Pl}^2}{2}\left[R - 2\Lambda\right]$$

	$\mu = \frac{d\ln M^2(t)}{dt}$	λ	$\mathcal{C}$	μ_2^2	μ_3	ϵ_4
ΛCDM	0	const.	0	0	0	0

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Quintessence	0	✓	✓	0	0	0

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ΛCDM	0	const.	0	0	0	0
Quintessence	0	✓	✓	0	0	0
k-essence	0	✓	✓	✓	0	0
Brans-Dicke	✓	✓	✓	0	0	0
$f(R)$	✓	✓	0	0	0	0
Kinetic braiding	0	✓	✓	✓	✓	0
DGP	✓	✓	✓	✓	✓	0
Galileon Cosmology	✓	✓	✓	✓	✓	0
$f(G)$ -Gauss-Bonnet	✓	✓	✓	✓	✓	✓
Galileons	✓	✓	✓	✓	✓	✓
Horndeski	✓	✓	✓	✓	✓	✓

The background equations

Define a Dark Energy ρ_D with eos $w(t)$

$$H^2 = \frac{1}{3M(t)^2}(\rho_m + \rho_D)\,,$$

$$p_D(t) = w(t)\rho_D(t)$$

$$\dot{H} = -\frac{1}{2M(t)^2}(\rho_m + \rho_D + p_D)\,.$$

Modified Friedmann equations

$$\mathcal{C} = \frac{1}{2}(H\mu - \dot{\mu} - \mu^2) + \frac{1}{2M(t)^2}(\rho_D + p_D)\,,$$

$$\lambda = \frac{1}{2}(5H\mu + \dot{\mu} + \mu^2) + \frac{1}{2M(t)^2}(\rho_D - p_D)\,.$$

Matter is scaling as $\rho_m \propto a^{-3}$

Conservation equation for the dark energy:

$$\dot{\rho}_D + 3H(\rho_D + p_D) = 3\mu M(t)^2 H^2$$

The perturbation equations

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{\mathrm{eff}}\rho_m \delta = 0$$

with effective Newton constant

$$G_{\mathrm{eff}} = \frac{1}{8\pi M(t)^2(1+\epsilon_4)^2} \frac{2\mathcal{C} + \mathring{\mu}_3 - 2\dot{H}\epsilon_4 + 2H\mathring{\epsilon}_4 + 2(\mu + \mathring{\epsilon}_4)^2 + Y_{\mathrm{IR}}}{2\mathcal{C} + \mathring{\mu}_3 - 2\dot{H}\epsilon_4 + 2H\mathring{\epsilon}_4 + 2\frac{(\mu+\mathring{\epsilon}_4)(\mu-\mu_3)}{1+\epsilon_4} - \frac{(\mu-\mu_3)^2}{2(1+\epsilon_4)^2} + Y_{\mathrm{IR}}}$$

and

$$\mathring{\mu}_3 \equiv \dot{\mu}_3 + \mu\mu_3 + H\mu_3$$

$$\mathring{\epsilon}_4 \equiv \dot{\epsilon}_4 + \mu\epsilon_4 + H\epsilon_4$$

$$Y_{\mathrm{IR}} \equiv 3\left(\frac{a}{k}\right)^2 \left[2\dot{H} - \dot{H}\mathring{\mu}_3 + \ddot{H}(\mu-\mu_3) - 2H\dot{H}\mu_3 - 2H^2(\mu^2 + \dot{\mu})\right]$$

solar system tests:

$$G_N \simeq \frac{1}{8\pi M(t)^2(t_0)}$$

BBN constraints:

$$\frac{|M^2(z>10) - M_{\mathrm{Pl}}^2|}{M_{\mathrm{Pl}}^2} \lesssim \frac{1}{10}.$$

Fiducial background model

Define an effective dark energy component (minimally coupled)

$$\bar{p}_D(t) = \bar{w}(t)\,\bar{\rho}_D(t) \qquad\qquad H^2 = \frac{1}{3M_{\rm Pl}^2}(\bar{\rho}_m + \bar{\rho}_D)$$

then, matter density is scaling as in a minimally couple DE model

$$\bar{\Omega}_{\rm m}(y) = \frac{\bar{\Omega}_{\rm m,0}}{\bar{\Omega}_{\rm m,0} + (1-\bar{\Omega}_{\rm m,0})\,e^{-3\int_0^y \bar{w}(y')dy'}}$$

and is related to the physical background evolution by

$$\bar{w}(t)\left(1-\bar{\Omega}_{\rm m}(t)\right) = w(t)\left(1-\Omega_{\rm m}(t)\right)$$

Dimensionless couplings and parametrization

- In the fiducial background

$$\mu = \frac{H(1-\bar{\Omega}_{\rm m})}{w-\bar{w}+\bar{w}\,\bar{\Omega}_{\rm m}}\left[3\bar{w}\,(w-\bar{w})+\frac{d\bar{w}}{dy}-\frac{\bar{w}}{w}\,\frac{dw}{dy}\right].$$

- We factor out $H(1-\bar{\Omega}_{\rm m})$ in the couplings

$$\begin{aligned}
\mu &= \eta\, H\,(1-\bar{\Omega}_{\rm m})\\
\mu_2^2 &= \eta_2\, H^2\,(1-\bar{\Omega}_{\rm m})\\
\mu_3 &= \eta_3\, H\,(1-\bar{\Omega}_{\rm m})\\
\epsilon_4 &= \eta_4\,(1-\bar{\Omega}_{\rm m})
\end{aligned}$$

- Parametrize dimensionless functions:

$$\bar{w}(\bar{\Omega}_{\rm m}) = \bar{w} = constant$$

$$\eta(\bar{\Omega}_{\rm m}) = (\beta-\alpha)\frac{\Omega_{\rm m\,0}}{\bar{\Omega}_{\rm m}} + [\alpha-\beta(2+\bar{\Omega}_{\rm m\,0})]\,\bar{\Omega}_{\rm m} + 2\beta\,\bar{\Omega}_{\rm m}^2$$

and η_2, η_3, η_4 constants.

Viable theories...

Reintroduce the scalar field's degree of freedom π in the action

$$S_\pi = \int a^3 M(t)^2 \left[\boxed{A\left(\mu, \mu_2^2, \mu_3, \epsilon_4\right)}\, \dot{\pi}^2 - \boxed{B\left(\mu, \mu_3, \epsilon_4\right)} \frac{(\vec{\nabla}\pi)^2}{a^2} \right]$$

$$A = (\mathcal{C} + 2\mu_2^2)(1+\epsilon_4) + \frac{3}{4}(\mu - \mu_3)^2 \geq 0$$ (no-ghost condition)

$$B = (\mathcal{C} + \frac{\mathring{\mu}_3}{2} - \dot{H}\epsilon_4 + H\mathring{\epsilon}_4)(1+\epsilon_4) - (\mu - \mu_3)\left(\frac{\mu - \mu_3}{4(1+\epsilon_4)} - \mu - \mathring{\epsilon}_4\right)$$
$$\geq 0$$ (gradient-stability condition)

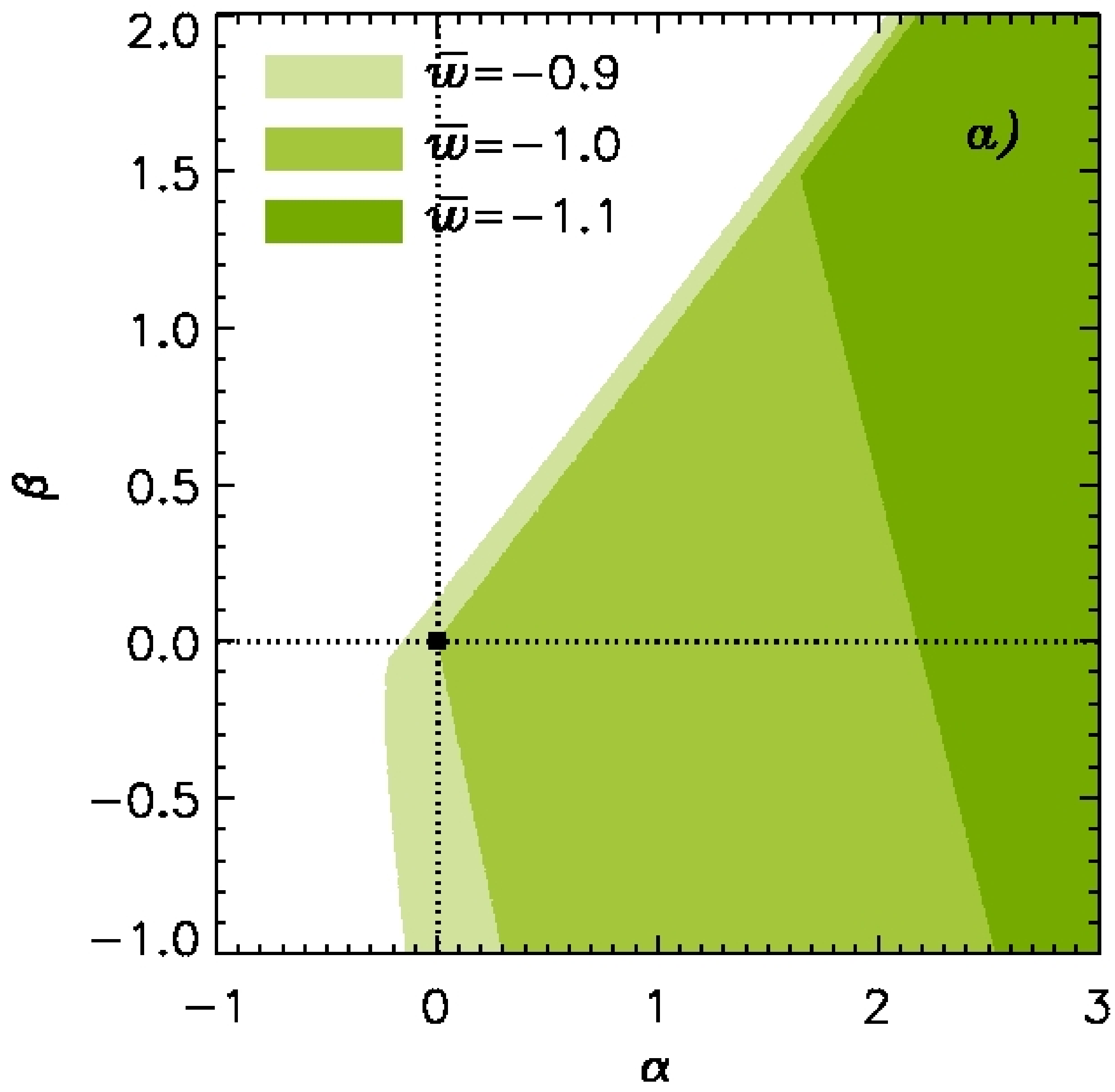
Brans-Dicke sector

Reintroduce the scalar field's degree of freedom π in the action

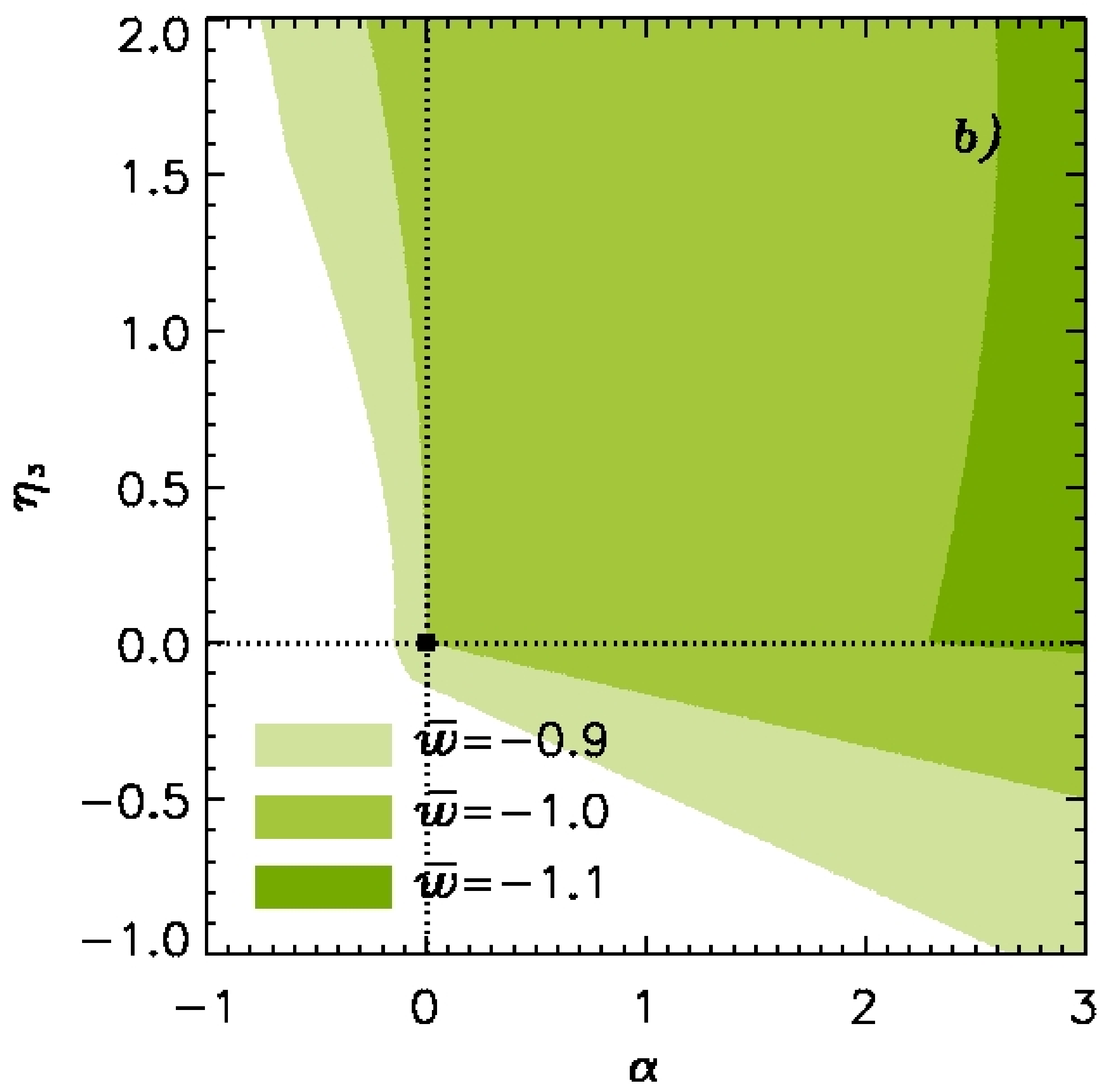
$$S_\pi = \int a^3 M(t)^2 \boxed{\left(\mathcal{C} + \tfrac{3}{4}\mu^2\right)} \left[\dot{\pi}^2 - \frac{(\vec{\nabla}\pi)^2}{a^2} \right] + \ldots$$

$$\boxed{3\bar{w}\,\frac{1+w}{w} + (5 + 3\bar{w} + 3\bar{w}\bar{\Omega}_{\mathrm{m}})\frac{\eta}{2} + \frac{1}{2}(1-\bar{\Omega}_{\mathrm{m}})\eta^2 - 3\bar{w}\bar{\Omega}_{\mathrm{m}}(1-\bar{\Omega}_{\mathrm{m}})\eta' \geq 0}$$

Brans-Dicke sector



Brans-Dicke sector



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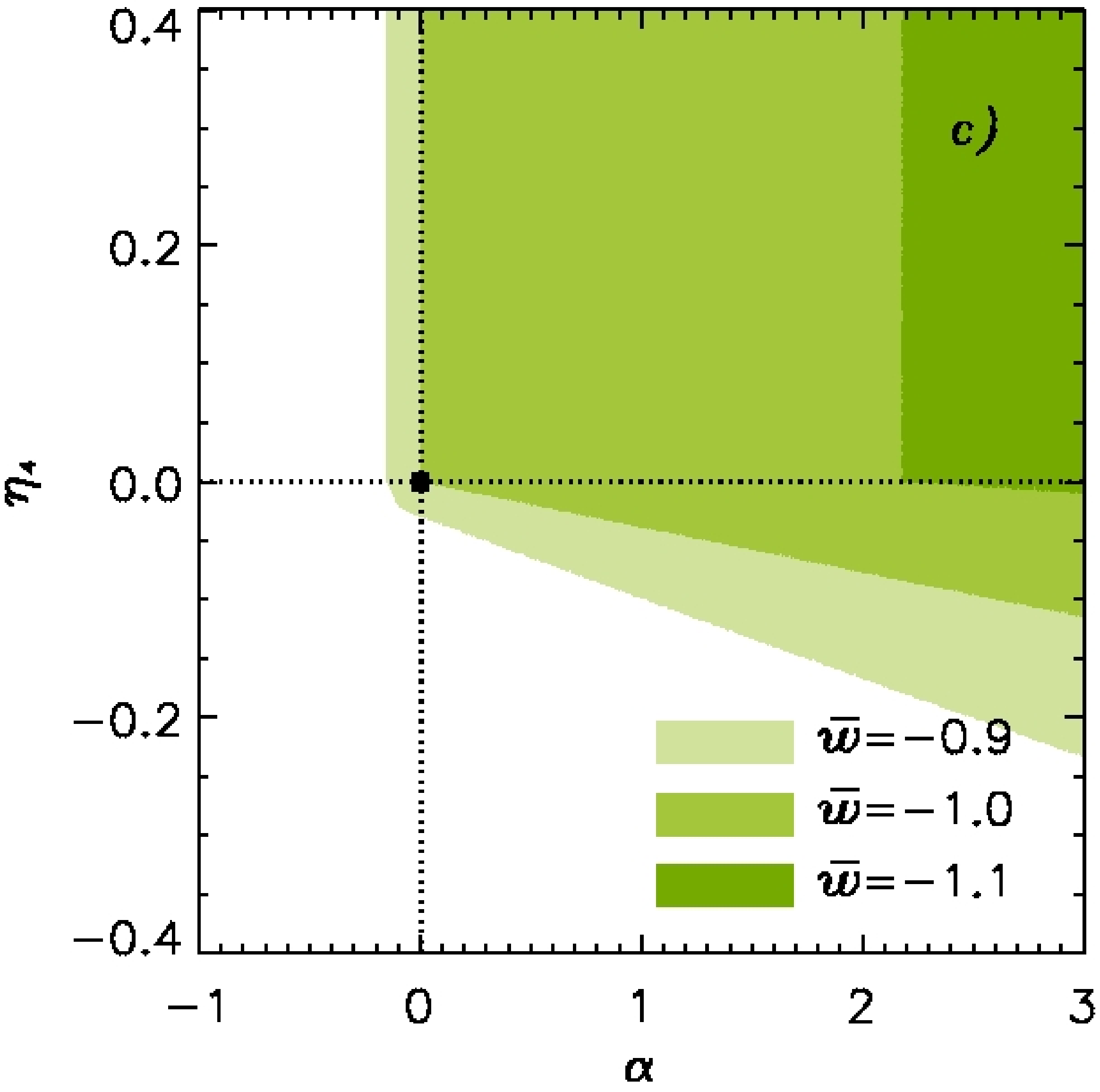

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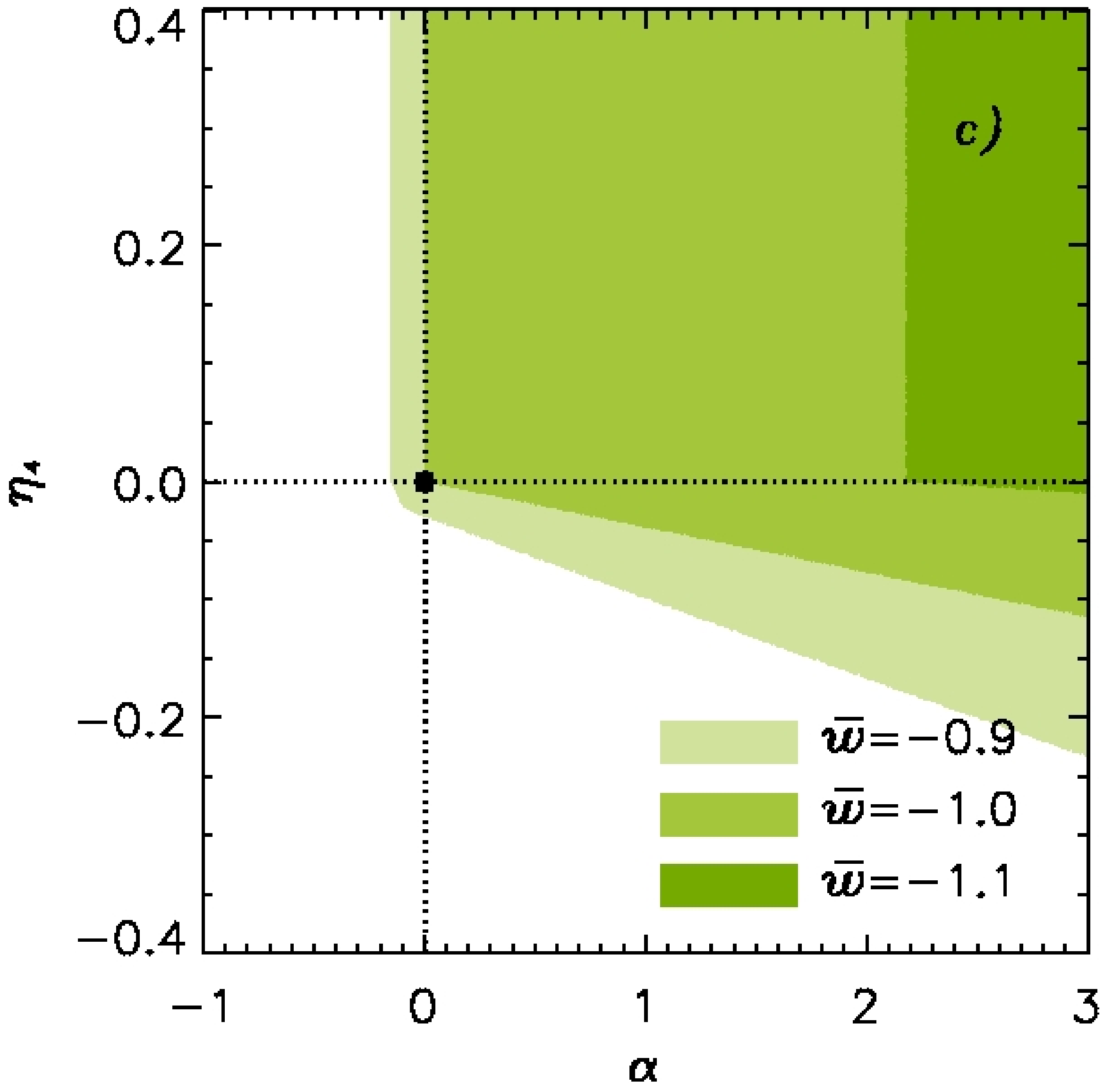

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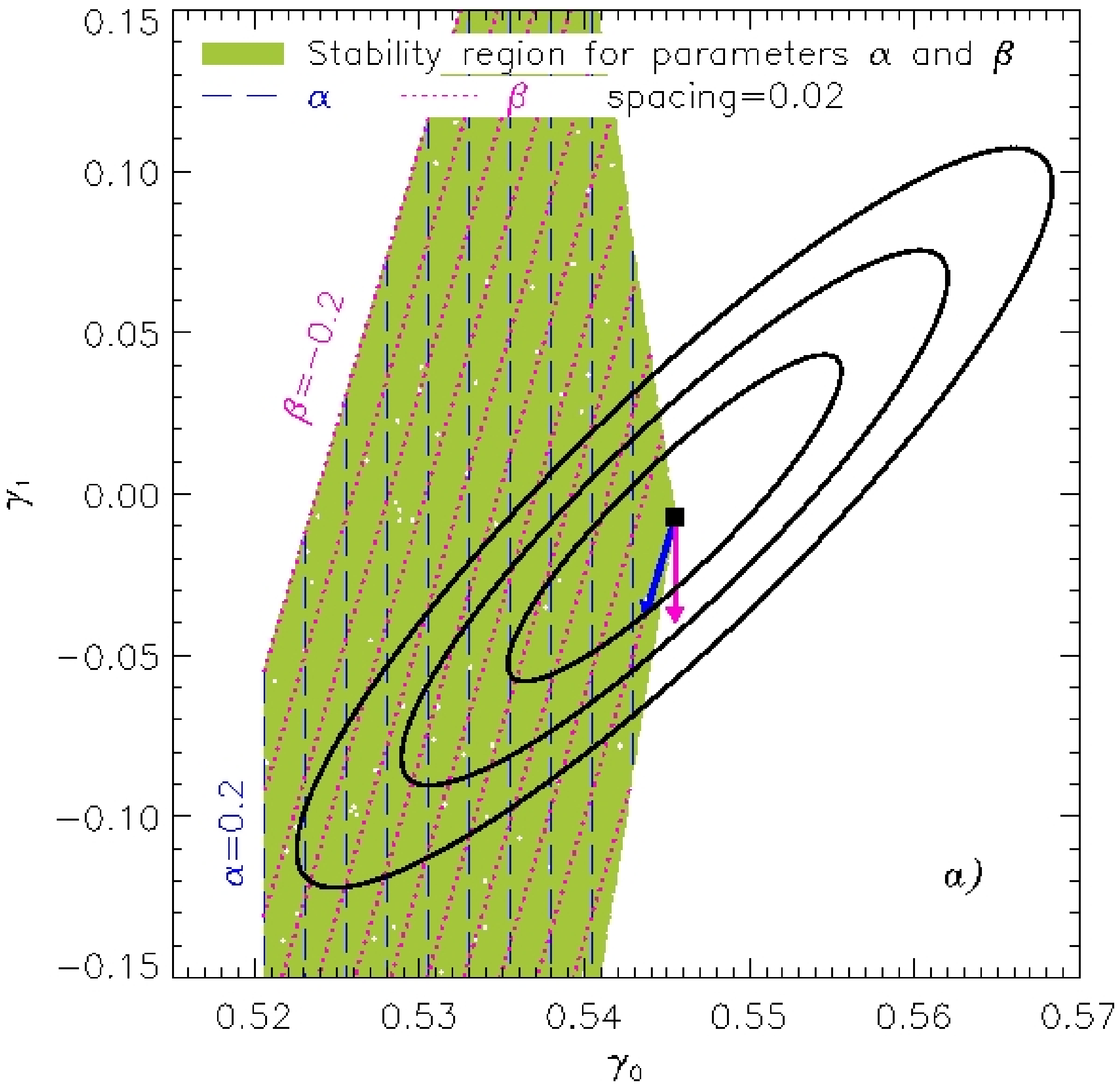


Conclusions

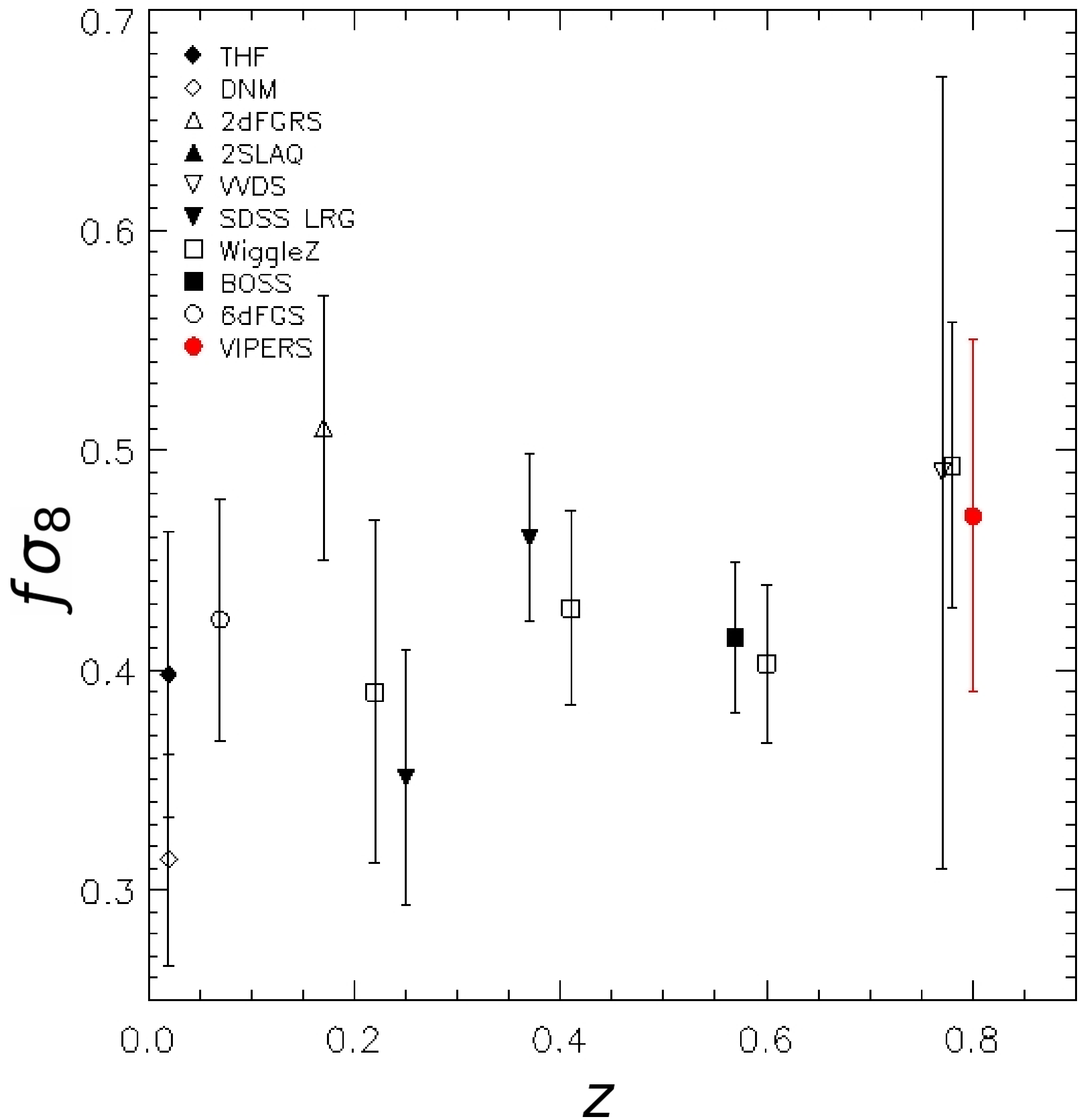
- We present an analytical parametrization for the growth index that is

  a) observer friendly (few coefficients needed)

  b) theorist friendly (links observational results to specific models)

  c) precise

Current measurements



Current measurements

Label	z	$f\sigma_8$	γ	Ref
THF	0.02	0.398 ± 0.065	$0.56^{+0.11}_{-0.09}$	1
DNM	0.02	0.314 ± 0.048	$0.71^{+0.10}_{-0.09}$	2
6dF	0.07	0.423 ± 0.055	$0.54^{+0.09}_{-0.08}$	3
2dF	0.17	0.510 ± 0.060	$0.43^{+0.10}_{-0.08}$	4
LRG1	0.25	0.351 ± 0.058	$0.77^{+0.16}_{-0.14}$	5
LRG2	0.37	0.460 ± 0.038	$0.55^{+0.09}_{-0.08}$	5
WZ1	0.22	0.390 ± 0.078	$0.67^{+0.19}_{-0.15}$	6
WZ2	0.41	0.428 ± 0.044	$0.64^{+0.12}_{-0.11}$	6
WZ3	0.6	0.403 ± 0.036	$0.76^{+0.14}_{-0.13}$	6
WZ4	0.78	0.493 ± 0.065	$0.38^{+0.28}_{-0.24}$	6
BOSS	0.57	0.415 ± 0.034	$0.71^{+0.12}_{-0.11}$	7
VVDS	0.77	0.490 ± 0.180	$0.40^{+0.89}_{-0.59}$	8
Euclid	$0.7 - 2.0$		$\delta\gamma/\gamma \sim 0.01$	2018

The coefficients mu and nu

$$f' + f^2 + (1 + \nu + \frac{H'}{H})f - \frac{3}{2}\mu\,\Omega_m = 0$$

Model	μ	ν
Smooth Quintessence	1	1
Clustering Quintessence	$1 + (1 + w)\frac{1-\Omega_m}{\Omega_m}$	$-\frac{3(1+w)w(1-\Omega_m)}{1+w(1-\Omega_m)}$
DGP	$1 - \frac{1-\Omega_m^2}{3(1+\Omega_m^2)}$	1
(Modified Gravity)	$\frac{G_{\mathrm{eff}}}{G}$	1

Current growth data (from redshift-space distortions)

$$z_{\mathrm{obs}} = z_{\mathrm{cosm}} + \frac{u}{c}(1 + z_{\mathrm{cosm}})$$

$f\sigma_8$ model independant combination

σ_8 rms density contrast in 8 h^{-1} Mpc

Label	Reference	z	$f\sigma_8$
THF	Turnbull *et al.* (2012)	0.02	0.40 ± 0.07
DNM	Davis *et al.* (2011)	0.02	0.31 ± 0.05
6dFGS	Beutler *et al.* (2012)	0.07	0.42 ± 0.06
2dFGRS	Percival *et al.* (2004) , Song & Percival (2009)	0.17	0.51 ± 0.06
2SLAQ	Ross *et al.* (2007)	0.55	0.45 ± 0.05
SDSS	Cabré *et al.* (2009)	0.34	0.53 ± 0.07
SDSS II	Samushia *et al.* (2012)	0.25	0.35 ± 0.06
		0.37	0.46 ± 0.04
BOSS	Reid *et al.* (2012)	0.57	0.43 ± 0.07
WiggleZ	Contreras *et al.* (2013)	0.20	0.40 ± 0.13
		0.40	0.39 ± 0.08
		0.60	0.40 ± 0.07
		0.76	0.48 ± 0.09
VVDS	Guzzo *et al.* (2008) , Song & Percival (2009)	0.77	0.49 ± 0.18
VIPERS	De la Torre *et al.* (2013)	0.80	0.47 ± 0.08