Progress on Old and New Themes in Cosmology (PONT)
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Dark D-Brane Cosmology

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The disformal relation

- Generalisation of the conformal transformation

$$\bar{g}_{\mu\nu} = C(\phi) g_{\mu\nu} + D(\phi)\phi_{,\mu}\phi_{,\nu}$$

[Bekenstein, **Phys.Rev. D48 (1993)**]

- C: dilatation, D: distorts shapes

$$\dot{\bar{x}}^2 \equiv \bar{g}_{\mu\nu}\dot{x}^\mu \dot{x}^\nu = C\dot{x}^2 + D(\dot{x}\cdot\partial\phi)^2$$

$$\cos\theta = \frac{x\hat{\cdot}y - D(\partial\phi\cdot x)(\partial\phi\cdot y)}{|\,\hat{x}\,||\,\hat{y}\,|\sqrt{1-\frac{D}{\hat{x}^2}(\partial\phi\cdot x)^2}\sqrt{1-\frac{D}{\hat{y}^2}(\partial\phi\cdot y)^2}},$$

$$\ddot{x}^\mu = -\bar{\Gamma}^\mu_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta = -\Gamma^\mu_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta + \text{5th forces}$$

- 1) Phenomenology: automatic screening!

  2) Brane-worlds: generic prediction!

  3) Cosmology: unified dark sector!

Disformal screening

[TK, Mota & Zumalacarregui: PRL (2012)]

$$\nabla_\mu T_m^{\mu\nu} \equiv -Q\phi^{,\nu}\,, \quad Q = \frac{C'}{2C}T_m - \nabla_\lambda\left(\frac{D}{C}\phi_{,\mu}T_m^{\mu\lambda}\right) + \frac{D'}{2C}\phi_{,\mu}\phi_{,\nu}T_m^{\mu\nu}$$

- **Spherically symmetric, static NR configuration:**
  - Each term proportional to D *vanishes identically!*
- High density D ρ >>1 limit, allow time evolution:
  - The evolution is *independent of the density*

$$\ddot{\phi} \approx -\frac{D'}{2D}\dot{\phi}^2 + C'\left(\frac{\dot{\phi}^2}{C} - \frac{1}{2D}\right) = -\frac{\beta}{2M_p}\dot{\phi}^2$$

- Thus: disformal couplings even in the visible sector could be locally undetectable
  - Note: pretty much *regardless of the details of V and D*
  - Now what about nonspherical, dynamical or relativistic configurations…?
- **Coup**ling to photons is not screened
  - Constraints e.g. from distortion of the CMB
  - [Brax, Burrage, Davis & Gubitosi: JCAP (2013), van de Bruck, Morrice & Vu: PRL (2014)]

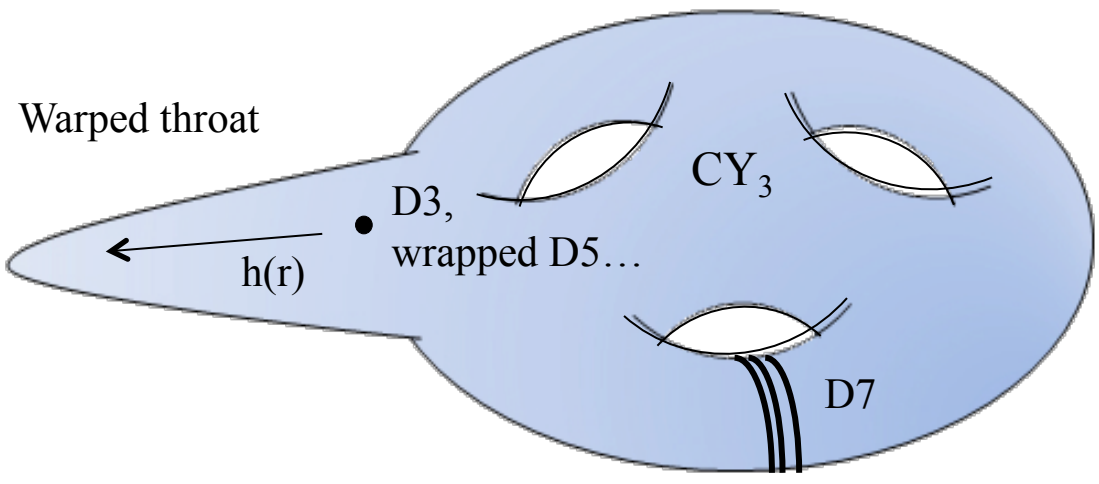
The extra-dimensional set-up

- In flux compactifications of Type IIB string theory, warping can arise from the backreaction of fluxes/objects onto the compact space → warped throats
- Single D*p*-branes can move as probes in this geometry described by the DBI action

$$S_{\rm DBI} = -\mu_p \int d^{p+1}\xi e^{\frac{(p-3)}{4}\varphi}\sqrt{-\det(\gamma_{ab} + e^{-\frac{\varphi}{2}}\mathcal{F}_{ab})}$$

$$\mathcal{F}_{ab} = \mathcal{B}_{ab} + 2\pi\alpha' F_{ab}$$

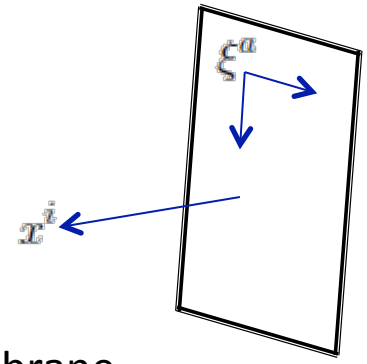
$$T_p = \mu_p e^{\frac{(p-3)}{4}\varphi}$$



Geometry

– **The induced metric:** this arises from the pull-back of the 10 dimensional metric on the worldvolume of a moving D-brane

$$\gamma_{ab} = G_{MN}\partial_a x^M \partial_b x^N \qquad \begin{aligned} \xi^\mu &= x^\mu \\ x^i &= x^i(\xi^\mu) \end{aligned}$$



$$G_{MN}dx^M dx^N = h^{-1/2}(x^A)g_{\mu\nu}dx^\mu dx^\nu + h^{1/2}(x^A)g_{AB}dx^A dx^B$$

– **The scalar field:** the changing radial position coordinate of the brane

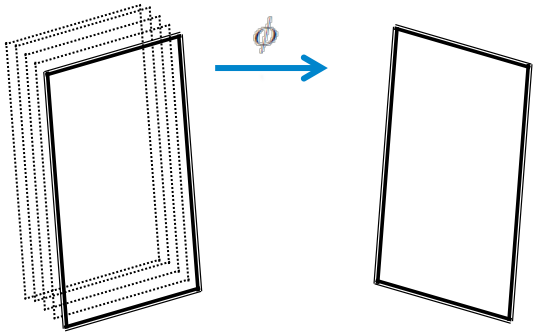
$$\phi \equiv \sqrt{T_3}r \quad \Rightarrow \quad \gamma_{\mu\nu} = h^{-1/2}(\phi)g_{\mu\nu} + h^{1/2}(\phi)\phi_{,\mu}\phi_{,\nu}$$

$$S_{\rm DBI} = -\mu_p \int d^{p+1}\xi e^{\frac{(p-3)}{4}\varphi}\sqrt{-\det(\gamma_{ab} + e^{-\frac{\varphi}{2}}\mathcal{F}_{ab})}$$

+ potential
+ charge

$$S_\phi = -\int d^4x\sqrt{-g}\left(h^{-1}\sqrt{1 + hg^{\mu\nu}\partial_\mu\phi\partial_\nu\phi} + V(\phi) - h^{-1}\right)$$

$$h(\phi) = \frac{\lambda}{\phi^4}$$

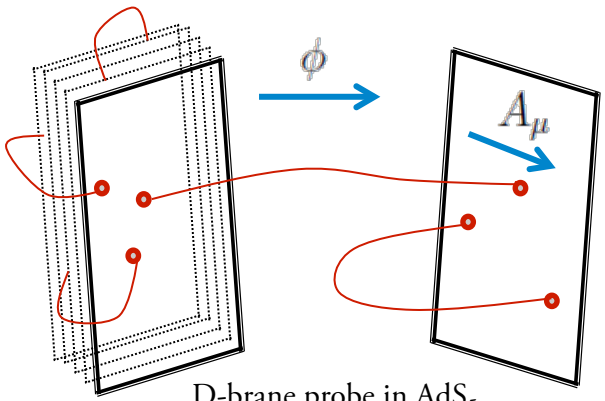


D-brane probe in AdS_5 geometry from stack of D3-branes

Matters on the brane

- Open string endpoints → U(1) vector fields on the world-volume
- These can acquire masses via Stückelberg couplings to bulk 2-forms

$$S_{\rm DBI} = -\mu_p \int d^{p+1}\xi e^{\frac{(p-3)}{4}\varphi} \underbrace{\sqrt{-\det(\gamma_{ab} + e^{-\frac{\varphi}{2}}\mathcal{F}_{ab})}}_{\sqrt{-\gamma}\sqrt{\det(\delta^c_b + e^{-\frac{\varphi}{2}}\gamma^{cd}\mathcal{F}_{db})}}$$



D-brane probe in AdS_5 geometry from stack of D3-branes

→ The massive vectors (or their decay products) are

- ***dark*** to our standard model
- ***disformally coupled*** to our gravity

$$\bar{g}_{\mu\nu} = C(\phi) g_{\mu\nu} + D(\phi)\phi_{,\mu}\phi_{,\nu}$$

$$h(\phi) = \frac{\lambda}{\phi^4}$$

$$C(\phi) \equiv (T_3\, h(\phi))^{-1/2} \text{ and } D(\phi) \equiv (h(\phi)/T_3)^{1/2}$$

$$S = \int d^4x \left[\sqrt{-g}\frac{R}{16\pi G} + \sqrt{-\bar{g}}\mathcal{L}\left(\text{matter}, \bar{g}_{\mu\nu}\right)\right]$$

Disformal geometry

$T_3 = (2\pi)^{-3}(\ell_s)^{-4}$ brane's tension

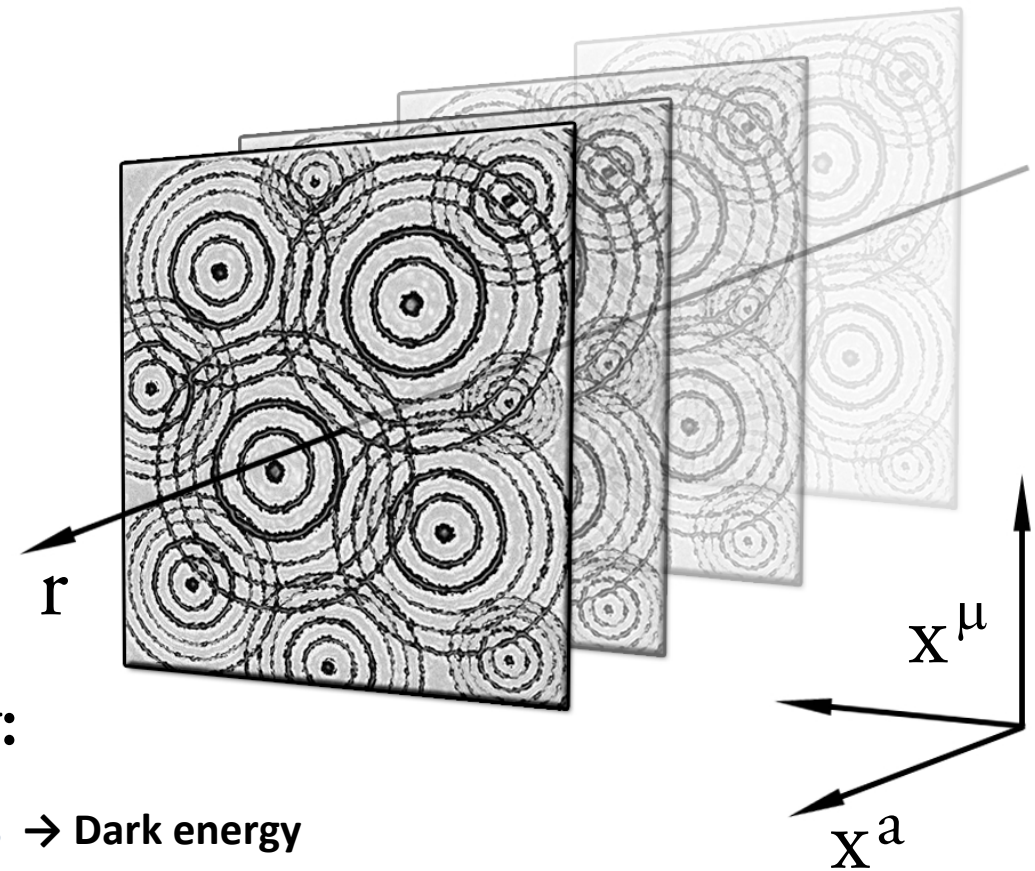
$\phi = \sqrt{T_3}\, r$ scalar associated to brane's position

$h(\phi) = h(r)/T_3$ warp factor in terms of the scalar

$C(\phi) = [T_3\, h(\phi)]^{-1/2}$ conformal piece

$D(\phi) = [h(\phi)/T_3]^{1/2}$ disformal piece

$$\tilde{g}_{\mu\nu} = h^{-1/2}(r) g_{\mu\nu} + h^{1/2}(r)\, \partial_\mu r \partial_\nu r$$



Dark D-brane cosmology:

- **Transverse open string oscillations → Dark energy**
- **Parallel massive oscillation modes → Dark matter**
- **Parallel massless modes → Dark radiation**
- **Closed string modes → Gravity disfomally coupled to the dark sector**

TK & Wills: IJMPD (2013)

Cosmology

The Friedmann and conservation equations become:

$$\gamma = \frac{1}{\sqrt{1 - h\dot{\phi}^2}}$$

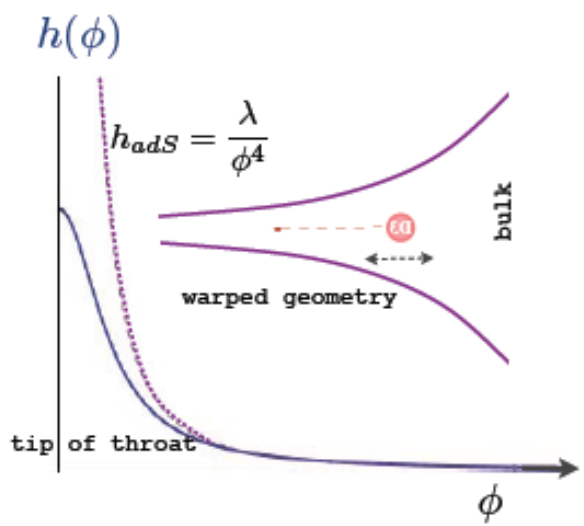
$$H^2 = \frac{\kappa^2}{3}\left[\rho_\phi + \rho\right],$$

$$\rho_\phi = \frac{\gamma - 1}{h} + V, \qquad P_\phi = \frac{1 - \gamma^{-1}}{h} - V$$

$$\dot{\rho}_\phi + 3H\left(\rho_\phi + P_\phi\right) = -Q_0\dot{\phi}, \qquad \dot{\rho} + 3H\rho = Q_0\dot{\phi}$$

$$Q_0 = -\left[\frac{h\left(V' + 3\gamma H\dot{\phi}\right) + \frac{h'}{h}\left(1 - \frac{3}{4}\gamma\right)}{\gamma + h\rho}\right]\rho$$

We can integrate the matter equation: $\rho = (T_3/h)^{1/4}\,\gamma\rho_b\,, \quad \rho_b = \rho_0 a^{-3}$



- The Lorentz factor $\dot{\gamma} > 0$ $\quad (\dot{\gamma} \to 0)$
- The warp factor $h' < 0$
- The scalar $\begin{cases} \dot{\phi} < 0 & \text{down throat (UV)} \\ \dot{\phi} > 0 & \text{up throat (IR)} \end{cases}$

Phase space

- Define the dimensionless variables

$$x \equiv \frac{\kappa\gamma}{\sqrt{3\,(\gamma+1)}}\frac{\dot{\phi}}{H}\,, \quad z \equiv \frac{\kappa\sqrt{V}}{\sqrt{3}H}\,, \quad \Omega \equiv \frac{\kappa^2\rho}{3H^2}$$

$$\boxed{\Omega = 1 - x^2 - z^2}$$

- The system of equations turns out to depend only upon

$$\Gamma_0 = \lambda V_0$$

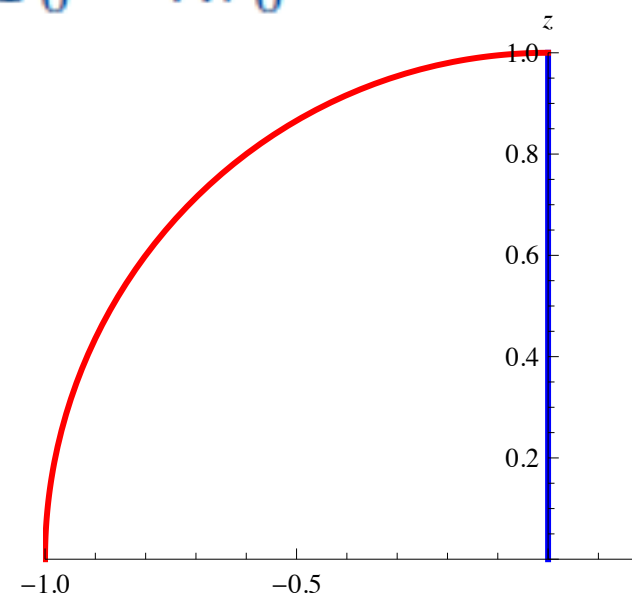
$$\boxed{h = \frac{\lambda}{\phi^4}\,, \qquad V = V_0\,\phi^2}$$

$$\frac{dx}{dN} = \frac{3x}{2}\left(\frac{(\tilde{\gamma}+1)(2\tilde{\gamma}-1)x^2}{\tilde{\gamma}\,(x^2+z^2-1)-z^2+1} + \tilde{\gamma}x^2 - z^2 + 1\right) + \frac{\sqrt{3\tilde{\gamma}(\tilde{\gamma}+1)}x^2\left[\mu\left((10\tilde{\gamma}-3)x^2 - 2\tilde{\gamma}+3\right) + z^2((4-8\tilde{\gamma})\lambda + (2\tilde{\gamma}-3)\mu)\right]}{8\,(\tilde{\gamma}\,(x^2+z^2-1)-z^2+1)}\,,$$

$$\frac{dz}{dN} = \frac{z}{2}\left(3 + 3\tilde{\gamma}x^2 - 3z^2 - \sqrt{3\tilde{\gamma}(\tilde{\gamma}+1)}\lambda\, x\right)\,,$$

$$\frac{d\tilde{\gamma}}{dN} = \frac{3\tilde{\gamma}\,(1-\tilde{\gamma}^2)\,x^2}{\tilde{\gamma}\,(x^2+z^2-1)-z^2+1} + \frac{\sqrt{3\tilde{\gamma}(\tilde{\gamma}+1)}(1-\tilde{\gamma})\tilde{\gamma}x\left(\mu + 3\mu x^2 - z^2(4\lambda+\mu)\right)}{4\,(\tilde{\gamma}\,(x^2+z^2-1)-z^2+1)}$$

$$\mu = m\left(\frac{(1-\tilde{\gamma})z^2}{\tilde{\gamma}\Gamma_0 x^2}\right)^{\frac{1}{m-n}}\,, \quad \lambda = -n\left(\frac{(1-\tilde{\gamma})z^2}{\tilde{\gamma}\Gamma_0 x^2}\right)^{\frac{1}{m-n}}$$



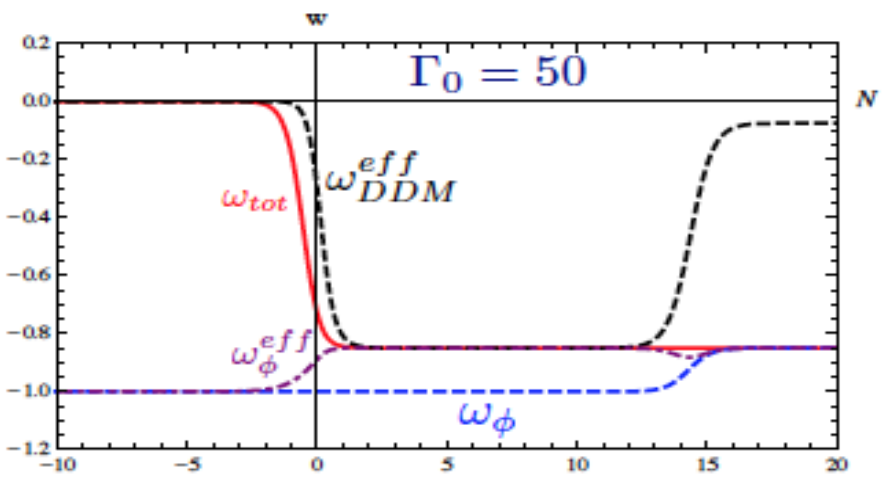
TK, Wills & Zavala : arXiv:1312.2597

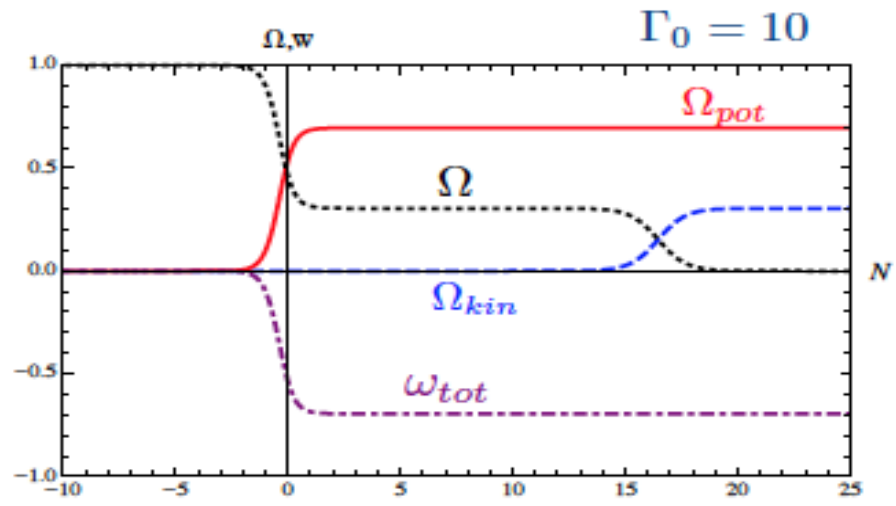
Dynamics

TK, Wills & Zavala : arXiv:1312.2597

	Ω_M	Ω_{kin}	Ω_{pot}	ω	γ	Stability
Matter Dominated	1	0	0	0	Any	Unstable
Potential Dominated	0	0	1	-1	1	Saddle
Matter Scaling	$\frac{2}{1+\sqrt{1+3\Gamma_0}}$	0	$\frac{-1+\sqrt{1+3\Gamma_0}}{\sqrt{3\Gamma_0}}$	$-\frac{(1-\sqrt{1+3\Gamma_0})^2}{3\Gamma_0}$	∞	$\Gamma_0 < 1$
Kinetic Scaling	0	$\sqrt{\frac{2}{1+\sqrt{1+3\Gamma_0}}}$	$\frac{-1+\sqrt{1+3\Gamma_0}}{\sqrt{3\Gamma_0}}$	$-\frac{(1-\sqrt{1+3\Gamma_0})^2}{3\Gamma_0}$	∞	$\Gamma_0 > 1$

Acceleration requires $\omega < -1/3 \qquad \Gamma_0 > 1$





Numbers

- Recall technical unnaturalities of canonical quintessence

$$\ddot{\phi} + 3H\dot{\phi} + V_\phi \simeq 3H\dot{\phi} + V_\phi \qquad \phi \sim \frac{V}{V_\phi} \sim \frac{V}{H\dot{\phi}} \gtrsim M_P$$

$$3M_P^2 H^2 = (\dot{\phi}^2/2 + V) \simeq V \qquad m^2 \sim V_{\phi\phi} \sim \frac{V}{\phi^2} \lesssim \frac{V}{M_P^2} \sim 10^{-120} M_P^2 \sim (10^{-33}\,\mathrm{eV})^2$$

$$H \sim 10^{-33}\ \mathrm{eV} \longrightarrow V_{vac} \sim (10^{-3}\mathrm{eV})^4 \longrightarrow (\kappa m)(\kappa\phi_0) \sim 10^{-60}$$

- *In our case the mass needs not to be small:*

$$(\kappa m)^2 \gtrsim 1/\lambda_{adS} \qquad \lambda_{adS} \sim 10^{5-10} \qquad \kappa\phi_0 \gtrsim 10^{-60}$$

- The coincidence problem

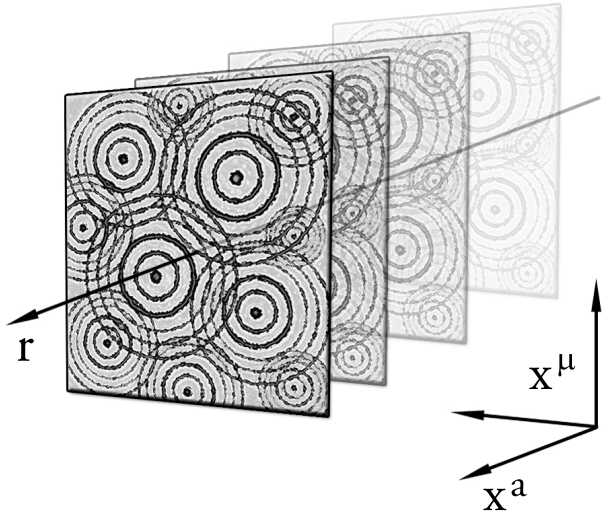
  $\gamma \to \infty$ as $\phi \to 0$: the tiny value of the field is not fine-tuned

  The coupling provides connection to the DDM dominated era

Conclusion:

$$\bar{g}_{\mu\nu} = C(\phi) g_{\mu\nu} + D(\phi)\phi_{,\mu}\phi_{,\nu}$$

- **Screening mechanism**
  - The 5th force disappears in spherical, pressureless & static case
- **Generic for matters on a brane**
  - An avenue to test XD phenomenology
- **Unified dark sector in cosmology**
  - New coupled quintessence approach

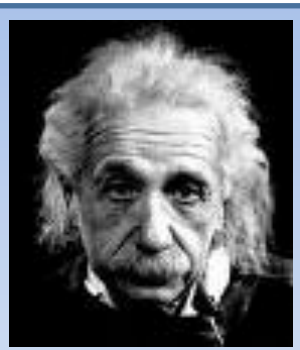


Appendix I:Frames of gravity

Brans-Dicke theory, e.g. f(R):

$$L = \phi R + V(\phi)$$

$$\tilde{g}_{\mu\nu} = \phi g_{\mu\nu}$$



$$\tilde{L} = \tilde{R} - \frac{1}{2}(\partial\varphi)^2 + U(\varphi) + L_m(\varphi)$$

- The generalisation of conformal mapping $\bar{g}_{\mu\nu} = C(\phi)g_{\mu\nu} + D(\phi)\phi_{,\mu}\phi_{,\nu}$
- Is contained in modified gravities beyond f(R) (e.g. Ricci-squared)
- and in scalar-tensor theories beyond Brans-Dicke (e.g. galileon)

Appendix 2A: Perturbations for canonic field

- We'll use the longitudinal gauge: $ds^2 = a^2(\eta)[-(1+2\Psi)d\eta^2 + (1-2\Phi)\delta_{ij}dx^i dx^j].$
- The field equations look like usual
- The coupling term Q affects only the conservation equations

  - **Continuity eq.** for the matter density contrast δ and **Euler eq.** for the matter velocity perturbation θ :

$$\dot{\delta}_i = -(1+w_i)(\theta_i - 3\dot{\Phi}) - 3\mathcal{H}\left(\frac{\delta P_i}{\delta\rho_i} - w_i\right)\delta_i + \frac{Q_0}{\rho_i}\dot{\phi}\,\delta_i - \frac{Q_0}{\rho_i}\delta\dot{\phi} - \frac{\dot{\phi}}{\rho_i}\delta Q,$$

$$\dot{\theta}_i = -\mathcal{H}(1-3w_i)\theta_i - \frac{\dot{w}_i}{1+w_i}\theta_i + k^2\Psi + \frac{\delta P_i/\delta\rho_i}{1+w_i}k^2\delta_i - k^2\sigma_i + \frac{Q_0}{\rho_i}\dot{\phi}\theta_i - \frac{Q_0}{(1+w_i)\rho_i}k^2\delta\phi,$$

  - **Klein-Gordon eq.** for the field $\delta\,\Phi$:

$$\delta\ddot{\phi} + 2\mathcal{H}\delta\dot{\phi} + (k^2 + a^2V'')\delta\phi = \dot{\phi}(\dot{\Psi} + 3\dot{\Phi}) - 2a^2(V' - Q_0)\Psi + a^2\delta Q.$$

- In general it is a mess:

$$\delta Q = -\frac{\rho_i}{a^2C + D(a^2\rho_i - \dot{\phi}^2)}[\mathcal{B}_1\delta_i + \mathcal{B}_2\dot{\Phi} + \mathcal{B}_3\Psi + \mathcal{B}_4\delta\dot{\phi} + \mathcal{B}_5\delta\phi]$$

$$\mathcal{B}_1 = \frac{a^2C'}{2}\left(1 - 3\frac{\delta P_i}{\delta\rho_i}\right) - 3D\mathcal{H}\dot{\phi}\left(1 + \frac{\delta P_i}{\delta\rho_i}\right) - Da^2(V' - Q_0) - D\dot{\phi}^2\left(\frac{C'}{C} - \frac{D'}{2D}\right),$$

$$\mathcal{B}_2 = 3D\dot{\phi}(1+w_i),$$

$$\mathcal{B}_3 = 6D\mathcal{H}\dot{\phi}(1+w_i) + 2D\dot{\phi}^2\left(\frac{C'}{C} - \frac{D'}{2D} + \frac{Q_0}{\rho_i}\right),$$

$$\mathcal{B}_4 = -3D\mathcal{H}\dot{\phi}(1+w_i) - 2D\dot{\phi}\left(\frac{C'}{C} - \frac{D'}{2D} + \frac{Q_0}{\rho_i}\right),$$

$$\mathcal{B}_5 = \frac{a^2C''(1-3w_i)}{2} - Dk^2(1+w_i) - Da^2V'' - D'a^2V' - 3D'\mathcal{H}\dot{\phi}(1+w_i) - D\dot{\phi}^2\left(\frac{C''}{C} - \left(\frac{C'}{C}\right)^2 + \frac{C'D'}{CD} - \frac{D''}{2D}\right) + (a^2C' + D'a^2\rho_i - D'\dot{\phi}^2)\frac{Q_0}{\rho_i},$$

- For conformally coupled DM:

$$\delta Q^{(c)} = \tfrac{1}{2}\log(C)'\rho\delta + \tfrac{1}{2}\log(C)''\rho\delta\phi\ [$$

Appendix 2B: The Newtonian Limit for canonic field

- Consider subhorizon scales, k/H>>1
- The coupling $\delta Q^{(\mathrm{N})} = Q_0 \delta_{\mathrm{dc}}$ given by

$$Q_0 = \frac{C' - 2D(3H\dot{\phi} + V' + \frac{C'}{C}\dot{\phi}^2) + D'\dot{\phi}^2}{2\left(C + D(\rho - \dot{\phi}^2)\right)}\rho$$

- We obtain the evolution equation

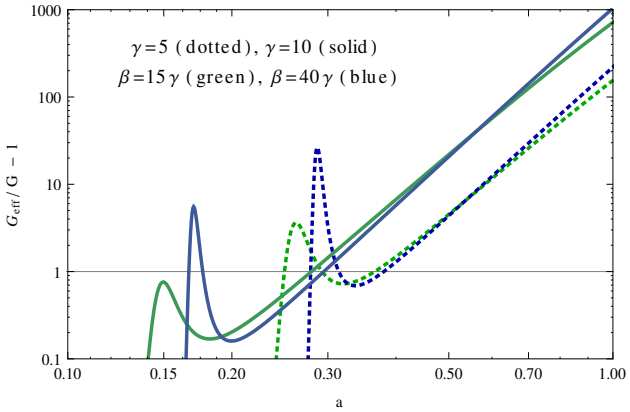
$$\ddot{\delta}_{\mathrm{dc}} + \left[2H + \frac{Q_0}{\rho}\dot{\phi}\right]\dot{\delta}_{\mathrm{dc}} = 4\pi G_{\mathrm{eff}}\rho\delta_{\mathrm{dc}}\,.$$

$$\frac{G_{\mathrm{eff}}}{G} - 1 = \frac{Q_0^2}{4\pi G\rho^2}$$

- Time-dependent effective gravitational constant
- An extra friction term
- In the exponential toy model:

  - scalar force >> gravity

  ➔ ruled out by LSS!



Appendix 3: DBI inflation

$$\ddot{\phi} + \frac{h'}{2h^2}(1 - 3\gamma^{-2} + 2\gamma^{-3}) + \gamma^{-3}(V' + Q_0) + 3H\gamma^{-2}\dot{\phi} = 0$$

- As the field rolls down its potential, the boost factor γ becomes large

$$\gamma = \frac{1}{\sqrt{1 - h\,\dot{\phi}^2}}$$

- For the adS warp the late-time solution is $\ddot{\phi} + \frac{2}{\lambda}\phi^3 \approx 0 \quad \Rightarrow \quad \phi \to \sqrt{\lambda}/t$
- This gives rise to power-law inflation $a(t) = a^{t/\epsilon} \qquad \epsilon \approx \sqrt{\frac{\lambda}{3}}m\,, \quad \gamma \to 2\epsilon\frac{t^2}{\lambda}$
- Fast-roll inflation! However needs some fine-tuning to get 60 e-folds
- Due to $c_s^2 = \gamma^{-2}$ there are non-gaussianities $f_{NL} \approx 0.32\gamma^2$
- The model is in tension with the Planck data

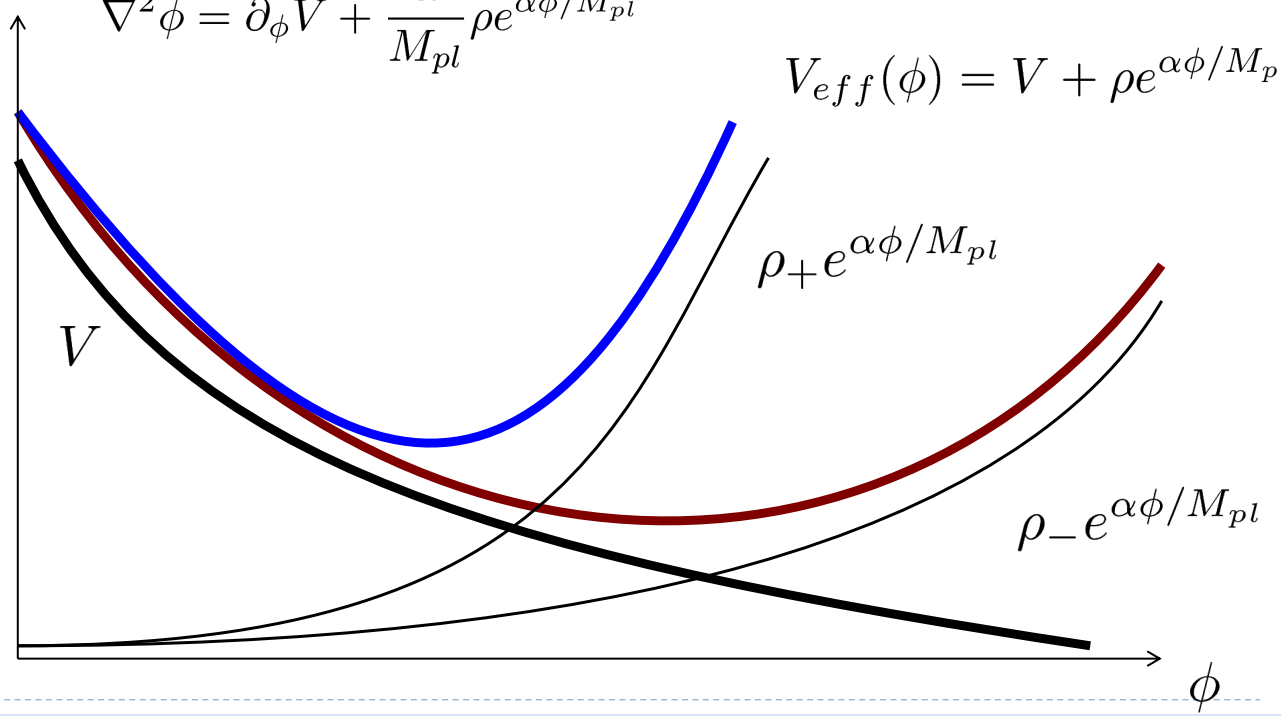
Appendix 4: Chameleonic screening

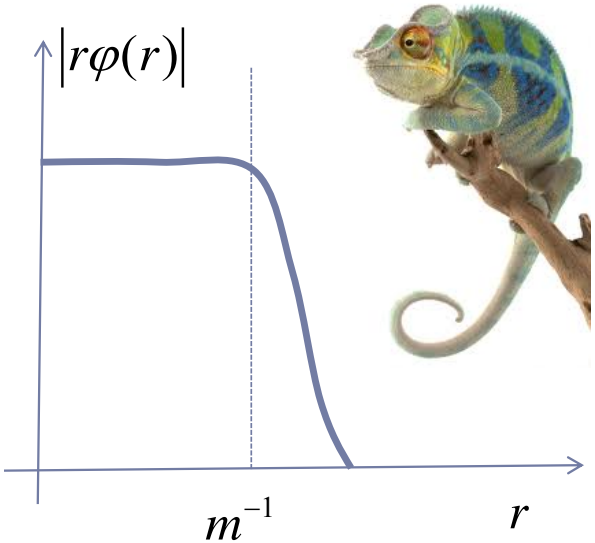
[Khoury and Weltman, **PRL (2004)**]

- Spherically symmetric NR point matter source
- The scalar potential decays beyond the Compton wavelenght 1/m
- High effective mass in high density regions

$$\nabla^2\phi = \partial_\phi V + \frac{\alpha}{M_{pl}}\rho e^{\alpha\phi/M_{pl}}$$

$$V_{eff}(\phi) = V + \rho e^{\alpha\phi/M_{pl}}$$





$$\varphi(r) = -\frac{GM}{r}\exp(-mr)$$

$$G_{eff} = \left(1 + \frac{\alpha^2}{2}e^{-rm_\phi}\right)G$$

Appendix 5: Vector on the brane

- The action written in terms of the physical metric:

$$-4S = \int d^4x\sqrt{-g}F^2 \quad \rightarrow$$

$$\int d^4x\sqrt{-\bar{g}}\bar{g}^{\alpha\beta}\bar{g}^{\gamma\delta}F_{\alpha\gamma}F_{\beta\gamma} = \int d^4x\sqrt{-g}\left[\gamma^{-2}F^2 - 2\frac{D}{C}\phi^{,\alpha}\phi^{,\beta}F_{\gamma\alpha}F^{\gamma}{}_{\beta}\right]$$

- The Maxwell equations generalise as:

$$\nabla_\alpha F^{\alpha\nu} = 0 \quad \rightarrow$$

$$\gamma^{-2}\nabla_\alpha F^{\alpha\nu} + \frac{D}{C}\phi^{,\alpha}\left[\nabla_\alpha\phi_{,\mu}F^{\mu\nu} - \nabla_\mu\phi_{,\nu}F^{\mu}{}_{\alpha} + \Box\phi F^{\nu}{}_{\alpha} + \phi^{,\mu}\nabla_\mu F^{\nu}{}_{\alpha} - \phi^{,\nu}\nabla_\mu F^{\mu}{}_{\alpha}\right] = 0$$

- In a homogenenous universe this becomes just

$$\boxed{\ddot{A}^T + k^2A^T = 0 \quad \rightarrow \quad \ddot{A}^T + \gamma^{-2}k^2A^T = 0}$$

- Relevant for inflationary magnetogenesis?
- Laboratory tests of low-energy photons, distortion of CMB

DARK ENERGY INTERACTIONS
THE NORDIC COSMOLOGY WORKSHOP 1-3. OCTOBER 2014.
HTTP://WWW.NORDITA.ORG/DE2014/