Massive black holes: gravitational waves and cosmology

Alberto Sesana

(Albert Einstein Institute, Golm)

EPTA

INTERNATIONAL PULSAR TIMING ARRAY

IPTA

Gravitational Wave Astronomy in Space

eLISA/NGO

OUTLINE

> Introduction to gravitational waves (Gws) from massive black hole binaries (MBHBs)

> Cosmology and fundamental physics with the (evolving) Laser Interferometer Space Antenna (eLISA)

> Probing gravity with Pulsar Timing Arrays (PTAs)

Gravitational waves: a short intro

Consider a small metric perturbation

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad h_{\mu\nu} \ll 1$$

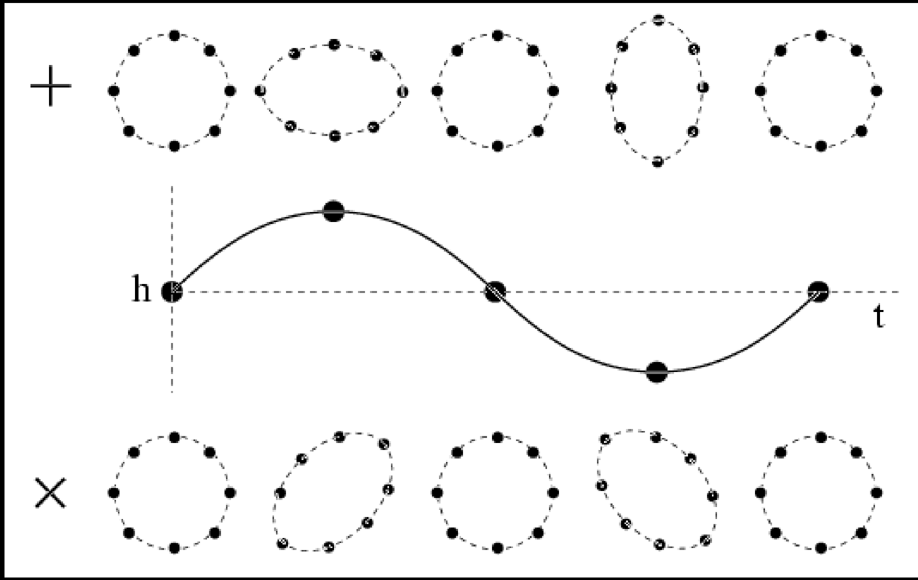
The linearization of the EEs results in a wave equation

$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 + h_+^{TT} & h_\times^{TT} \\ 0 & 0 & h_\times^{TT} & 1 - h_+^{TT} \end{pmatrix}$$

The solution is a wave travelling
At the speed of light:
GRAVITATIONAL WAVES

$$\bar{h}^{ij}(t, r) = \frac{2G}{c^4} \left[\frac{d^2}{dt^2} q^{ij} \left(t - \frac{r}{c} \right) \right]$$



They are proportional to the
Second derivative of the mass
quadrupol moment and they carry
an energy given by

$$L_{gw} = \frac{G}{5c^5} \left\langle \sum_{ij} \frac{d^3}{dt^3} Q_{ij} \left(t - \frac{x}{c} \right) \frac{d^3}{dt^3} Q^{ij} \left(t - \frac{x}{c} \right) \right\rangle$$

GWs are transversal and have two independent polarizations

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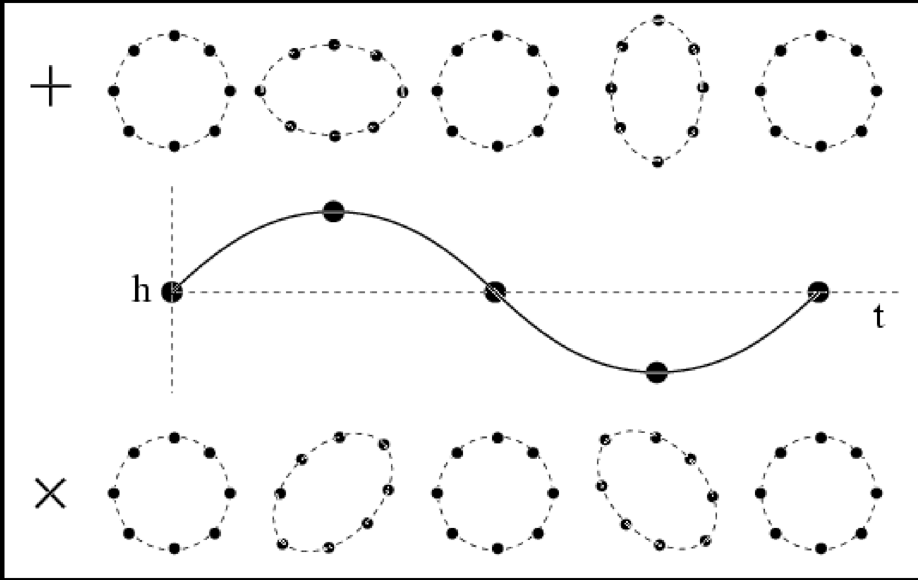

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The solu

At t

We need very massive systems with varying quadrupole moment: we need astrophysical binaries! possibly BH binaries!

They are proportio
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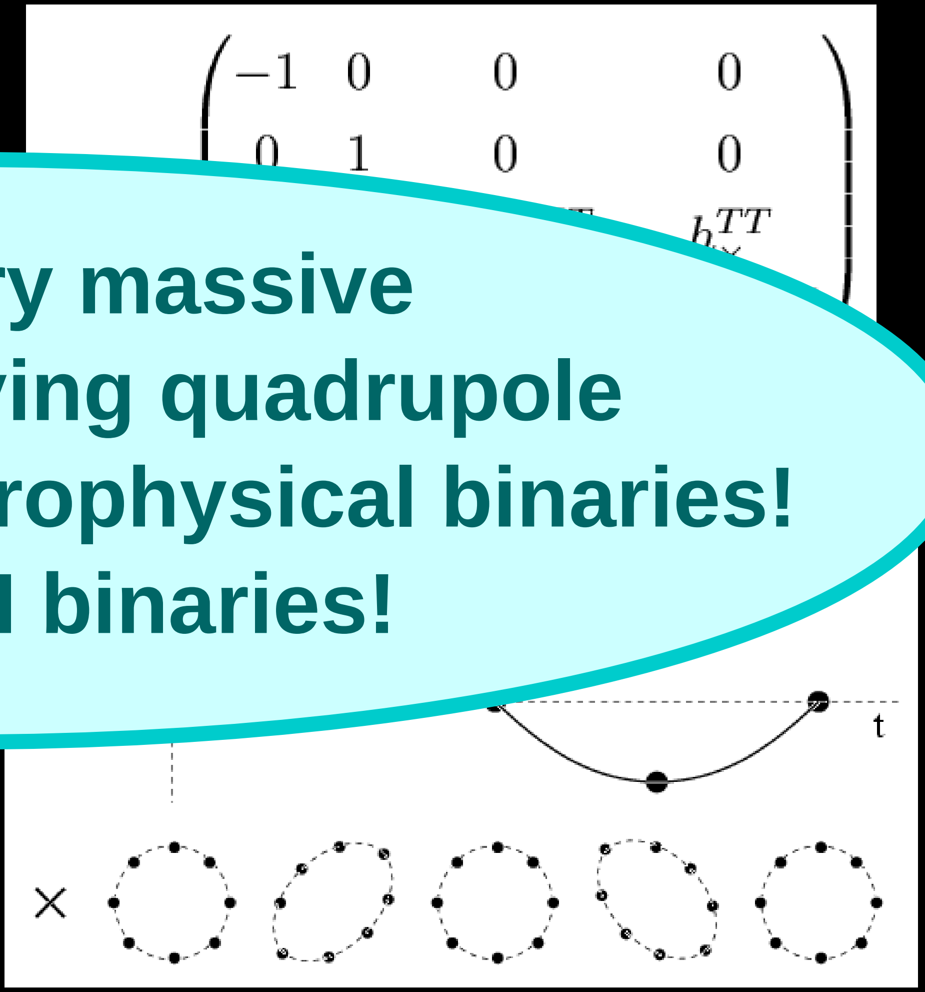

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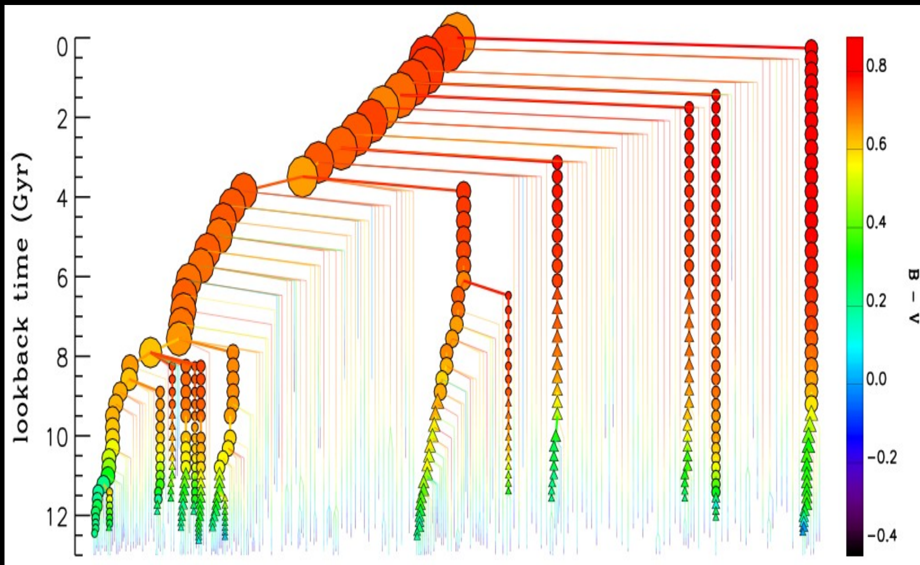
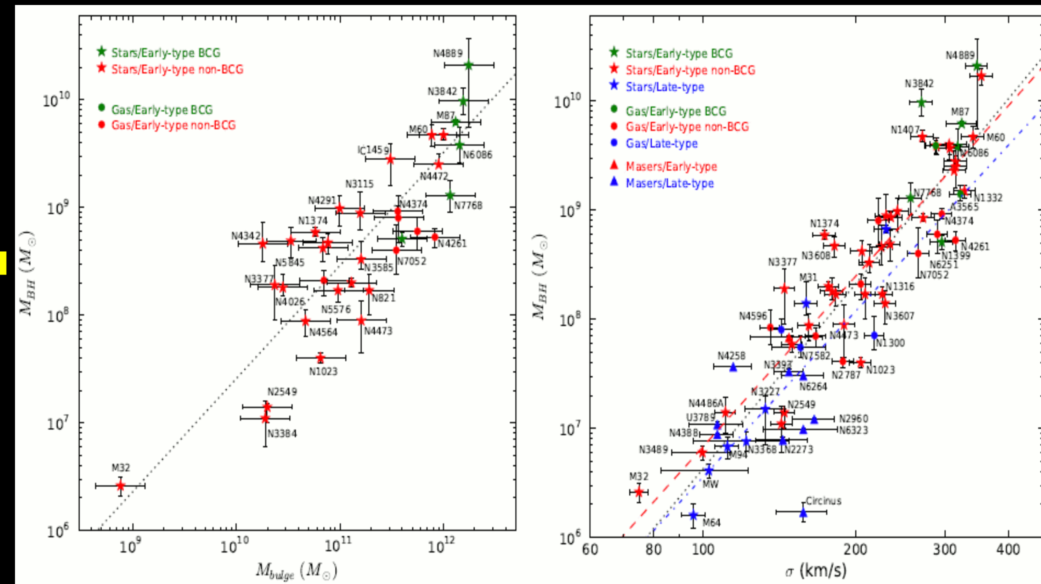
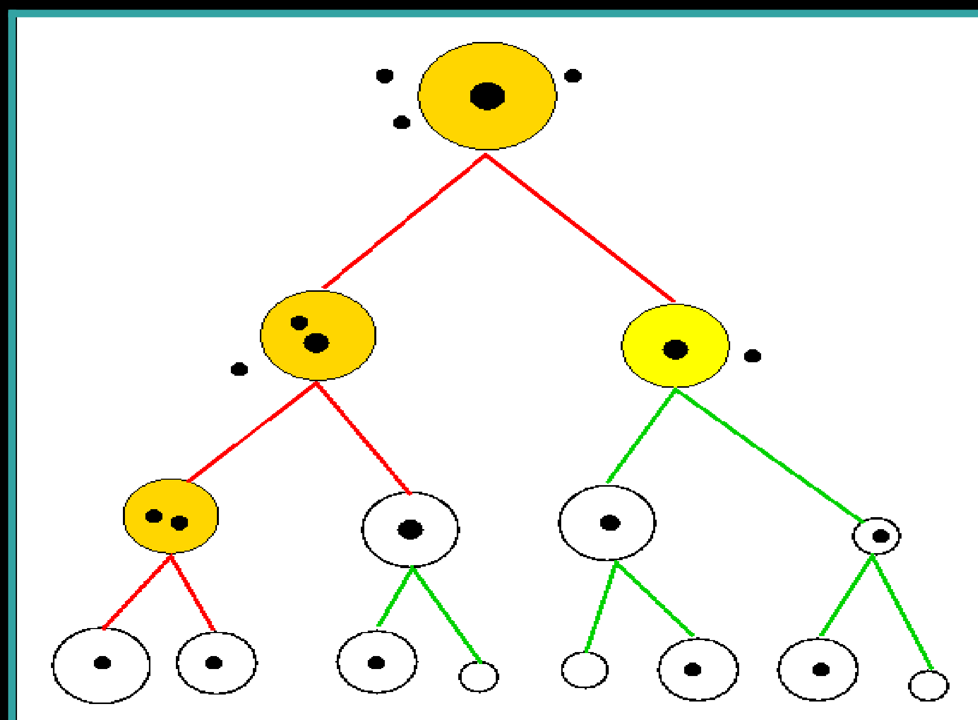
Structure formation in a nutshell



From De Lucia et al 2006

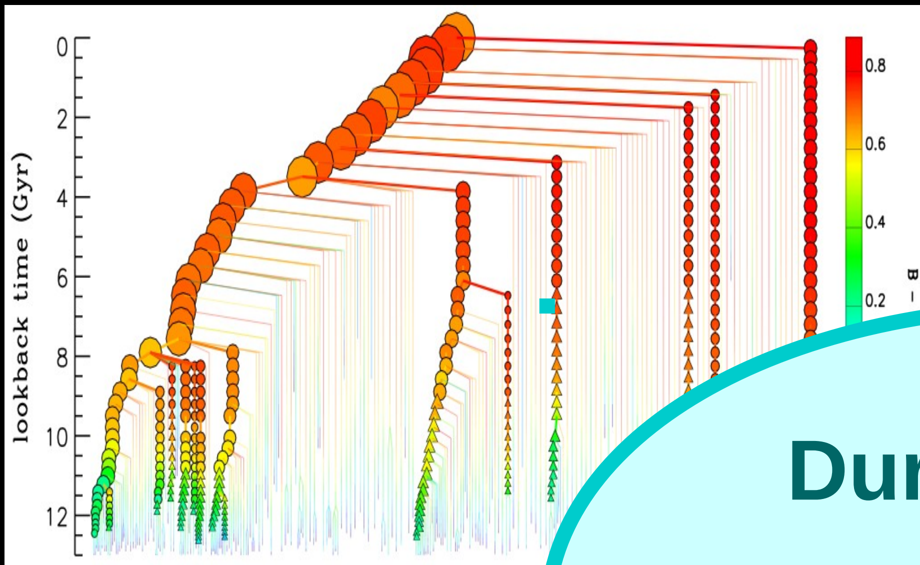


Ferrarese & Merritt 2000, Gebhardt et al. 2000



Volonteri Haardt & Madau 2003

Structure formation in a nutshell



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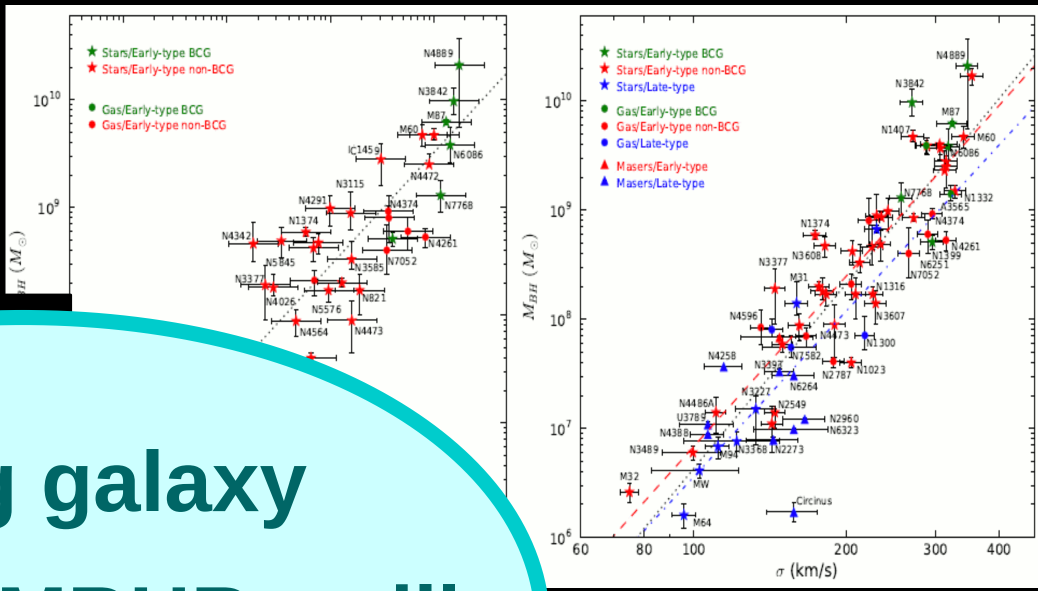
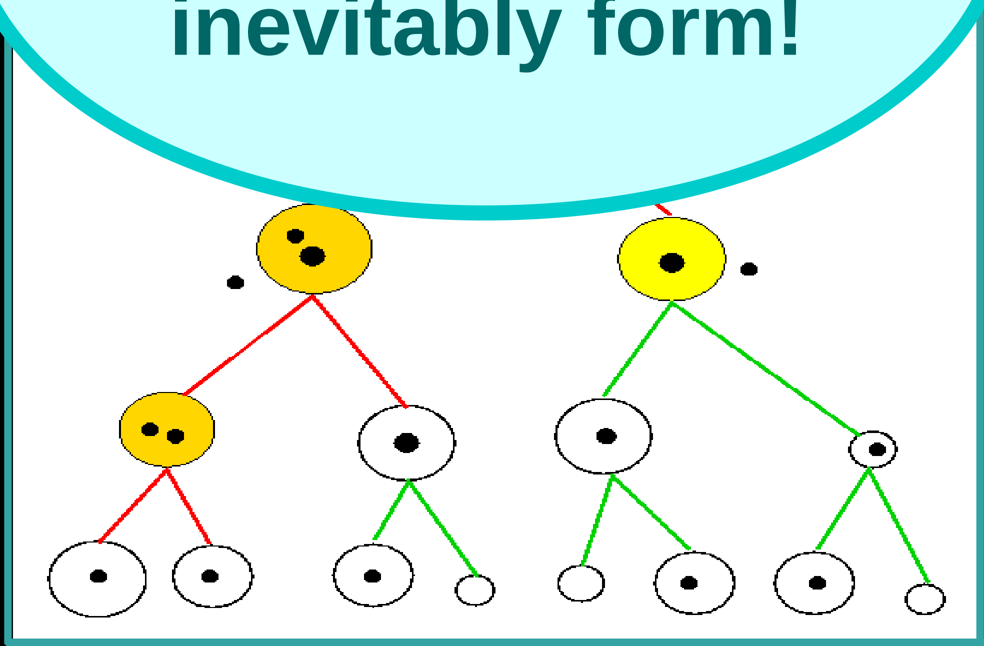


, Gebhardt et al. 2000

During galaxy mergers, MBHBs will inevitably form!



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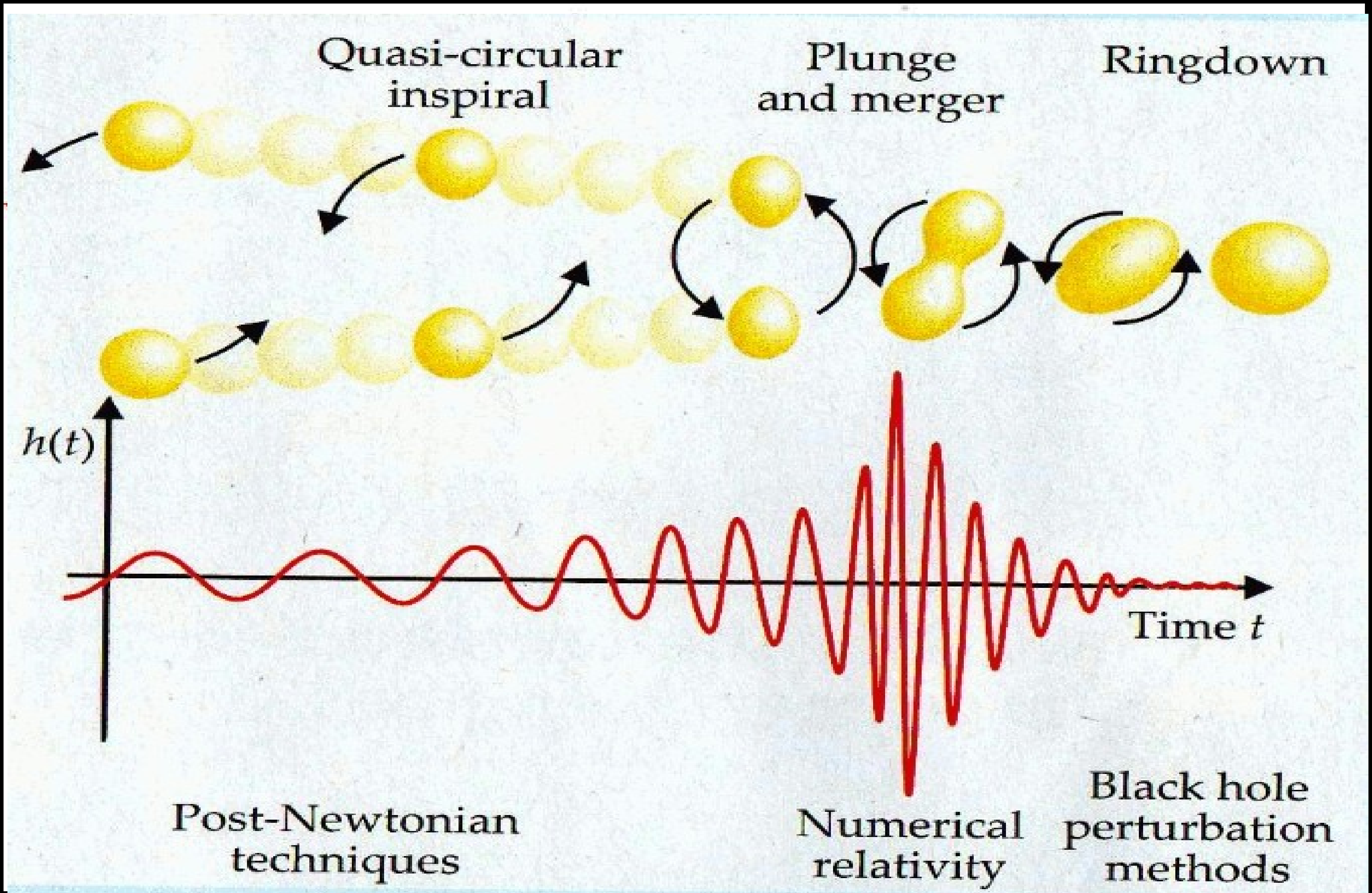
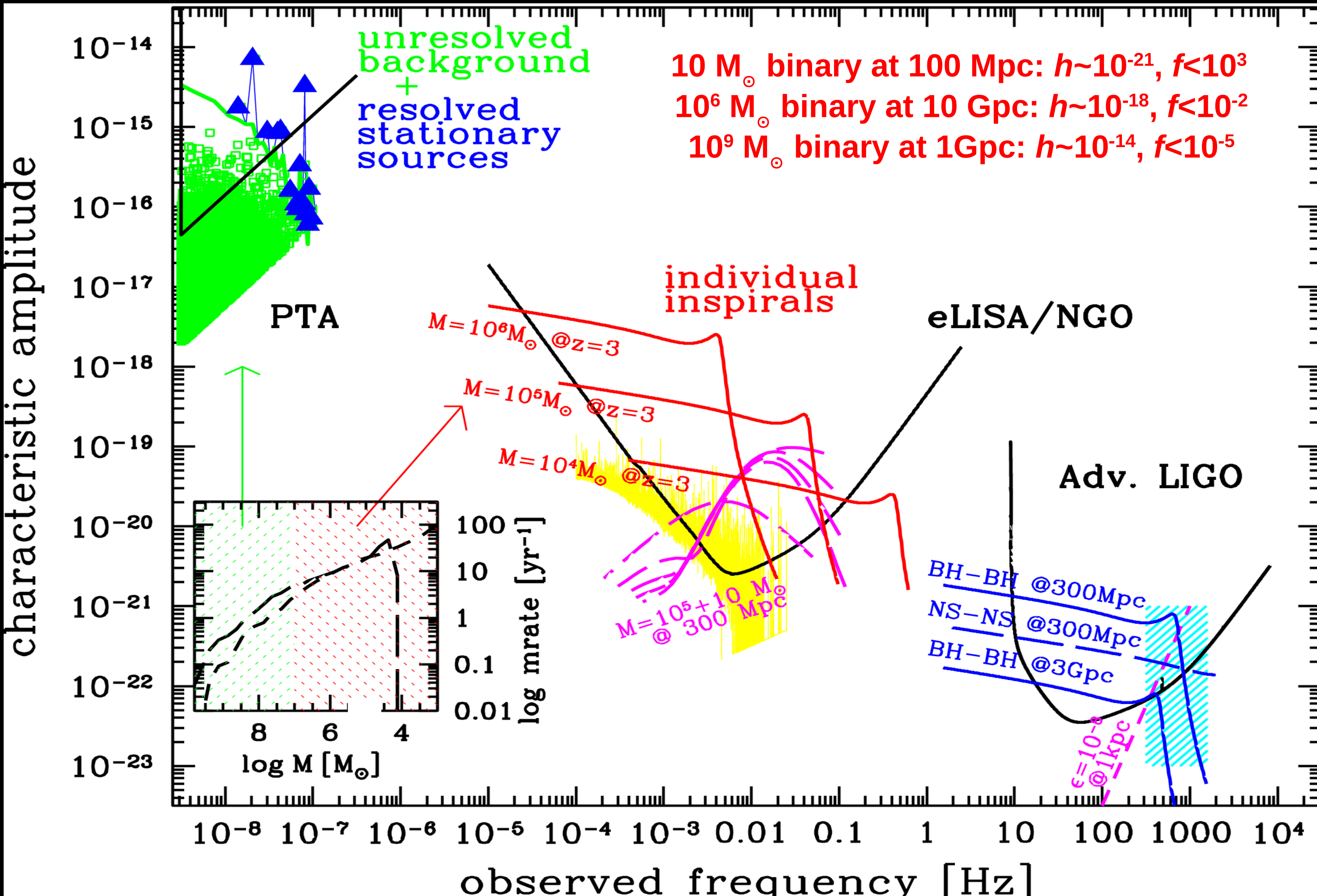
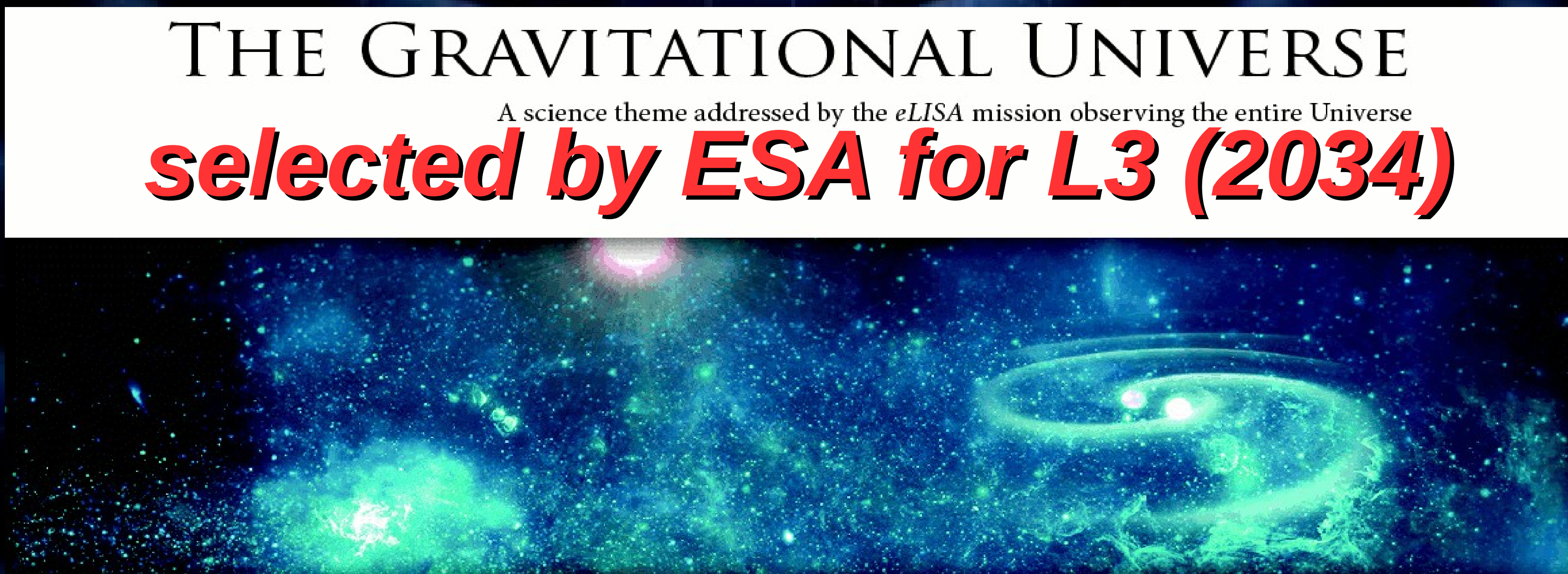

Quasi-circular inspiral
Plunge and merger
Ringdown
h(t)
Time t
Post-Newtonian techniques
Numerical relativity
Black hole perturbation methods

Coverage of the GW spectrum



AS 2013

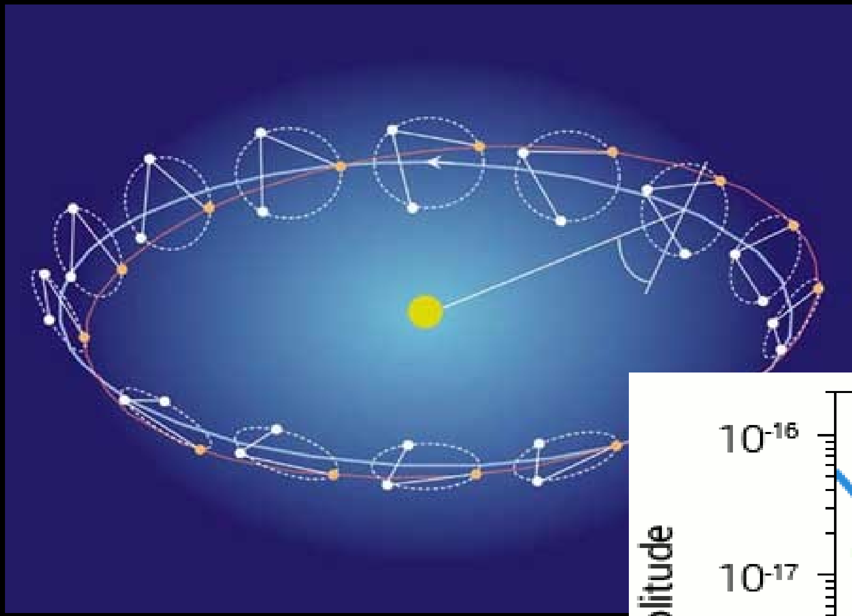
eLISA science



selected by ESA for L3 (2034)

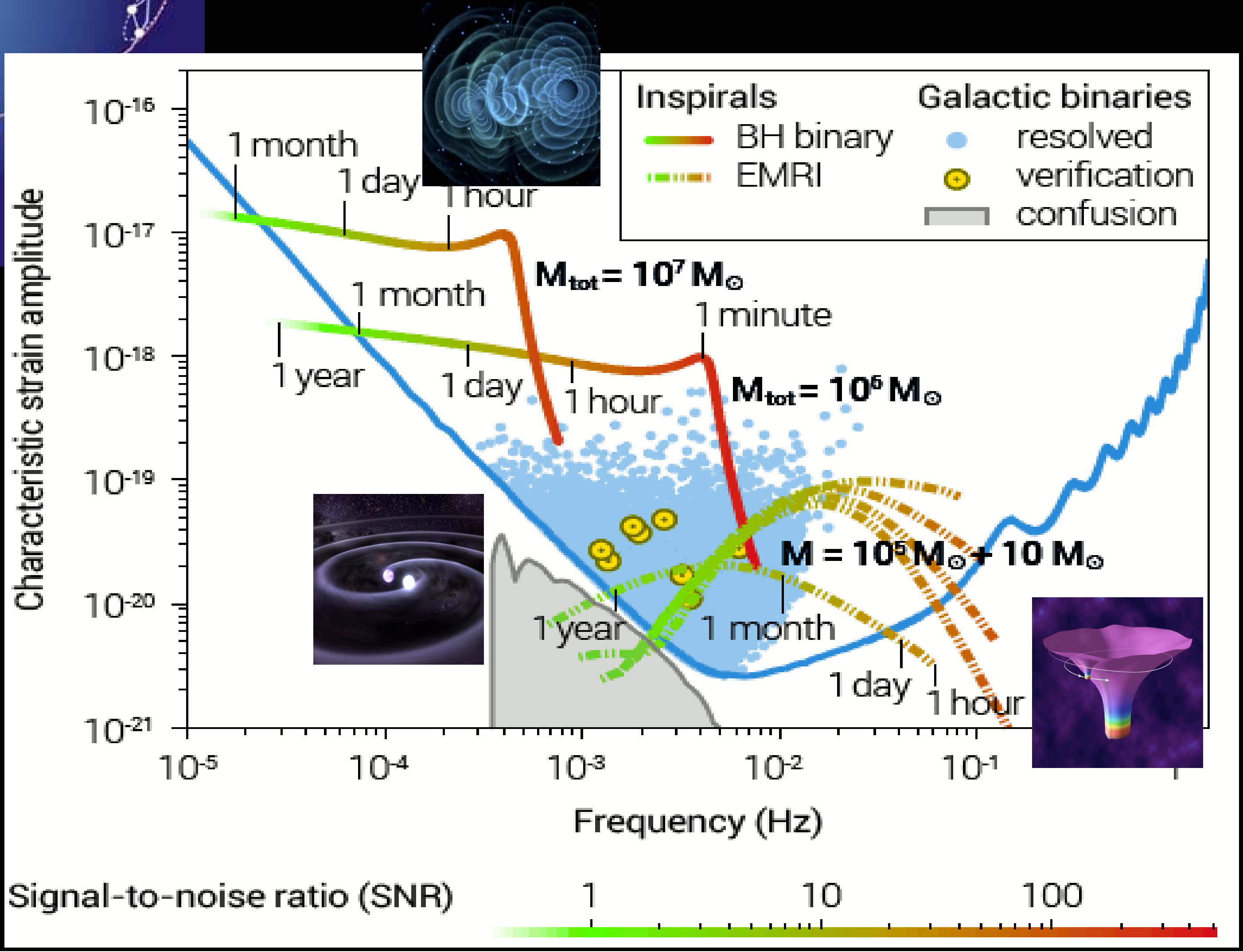
The eLISA Consortium, arXiv:1305.5720

Interferometry in space:
***e**volving **L**aser **I**nterferometer **S**pace **A**ntenna*

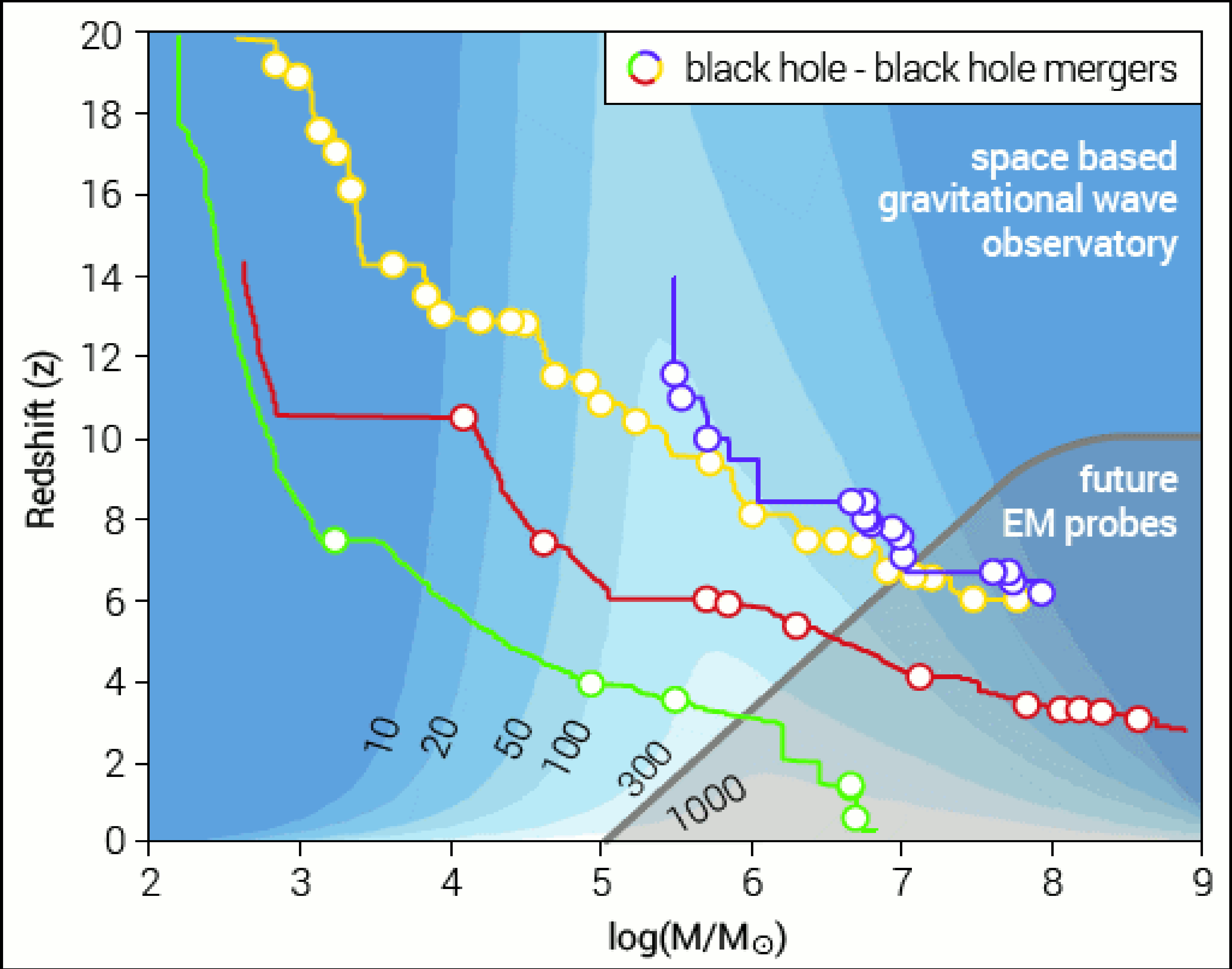


eLISA **is sensitive at mHz frequency, where the evolution of MBH binaries is fast.**
eLISA **will detect MBH binary inspirals and mergers.**

-same orbit as *LISA*
-1Gm armlength
-four laser links
-max 6 year lifetime



eLISA coverage of the Universe



Source parameter extraction

$$h^{(\iota)}(t) = \frac{\sqrt{3}}{2}\left[F_{+}^{(\iota)}(t)h_{+}(t) + F_{\times}^{(\iota)}(t)h_{\times}(t)\right]$$

Detected signal: combination of the two wave *polarization amplitude* and the *antenna beam pattern*

$$h_{+} = 2\frac{\mathcal{M}^{5/3}}{D_L}\left[1+\left(\hat{\mathbf{L}}\cdot\hat{\mathbf{N}}\right)^2\right](\pi f)^{2/3}\cos\phi(t),$$

$$h_{\times} = -4\frac{\mathcal{M}^{5/3}}{D_L}\left(\hat{\mathbf{L}}\cdot\hat{\mathbf{N}}\right)(\pi f)^{2/3}\sin\phi(t),$$

polarization amplitude:

function of the source intrinsic parameters (*M, f, ϕ*), of the source distance *D_L*, and of the source inclination *i=L•N*

$$F_{+}(\theta'_N,\phi'_N\,\psi'_N) = \frac{1}{2}(1+\cos\theta'^2_N)\cos 2\phi'_N\cos 2\psi'_N - \cos\theta'_N\sin 2\phi'_N\sin 2\psi'_N$$

$$F_{\times}(\theta'_N,\phi'_N\,\psi'_N) = \frac{1}{2}(1+\cos\theta'^2_N)\cos 2\phi'_N\sin 2\psi'_N + \cos\theta'_N\sin 2\phi'_N\cos 2\psi'_N$$

Antenna pattern:

function of the relative source-detector orientation. Depends on: souce sky location and polarization (θ,φ,ψ)

$$\phi(f) = \phi_c - \frac{1}{16}(\pi f\mathcal{M})^{-5/3}\left[1+\frac{5}{3}\left(\frac{743}{336}+\frac{11}{4}\eta\right)(\pi M f)^{2/3} - \frac{5}{2}(4\pi-\beta)(\pi M f)\right.$$
$$\left.+5\left(\frac{3058673}{1016064}+\frac{5429}{1008}\eta+\frac{617}{144}\eta^2-\sigma\right)(\pi M f)^{4/3}\right];$$

Phase evolution:

depends on the system masses and spins and eccentricity (*M_1,M_2,a_1,a_2,e*)

The full waveform for an eccentric spinning binary depends on *17 parameters*. Each of them leave a peculiar imprint in the waveform amplitude and phase.

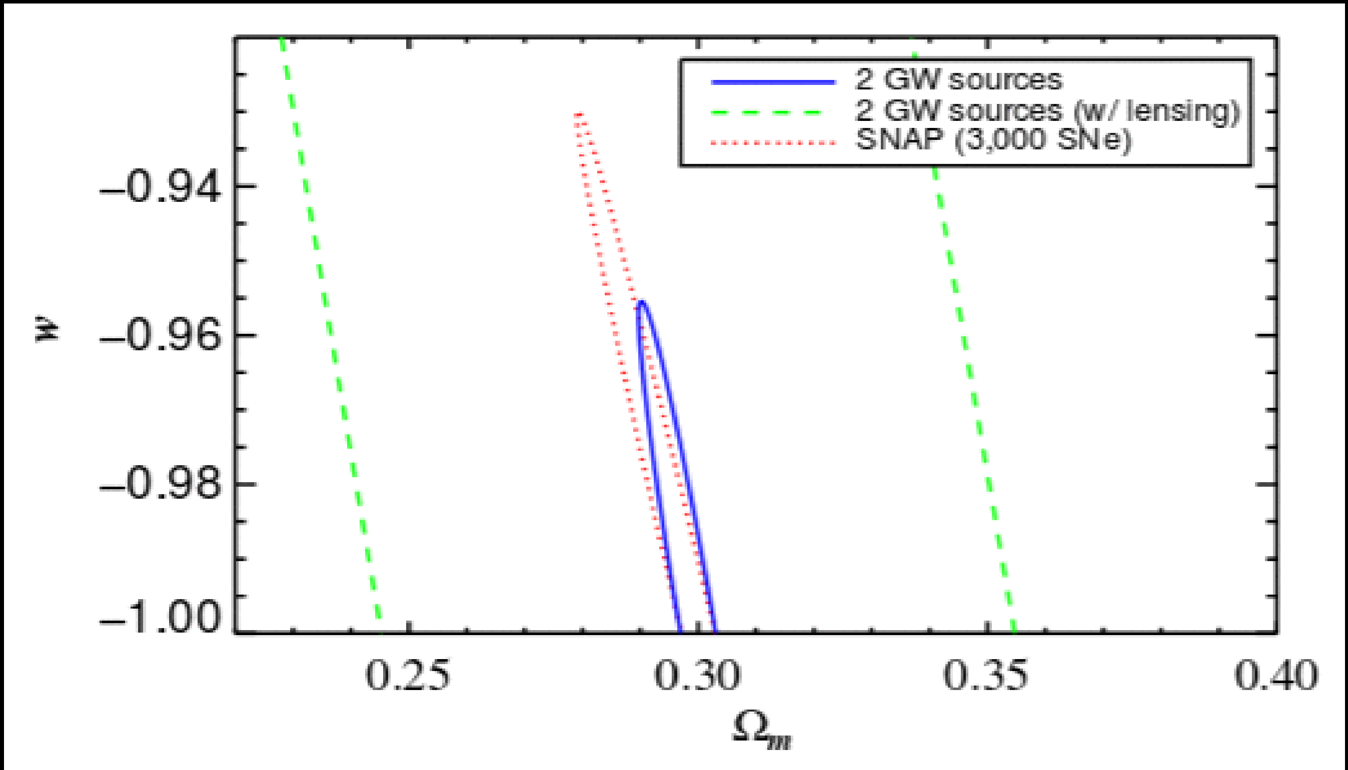
Cosmology with standard sirens

The gravitational wave signal gives a direct measurement of the luminosity distance (Schutz 1986)

$$h_{+\times}(t) \propto \frac{[(1+z)M_c]^{5/3}\, f^{2/3}}{D_L}$$

$$\dot{f}(t) \propto [(1+z)M_c]^{5/3}\, f^{11/3}$$

Identification of a counterpart will place a point on the D-z diagram, allowing measurements of the euqation of state of the Universe (but weak lensing)

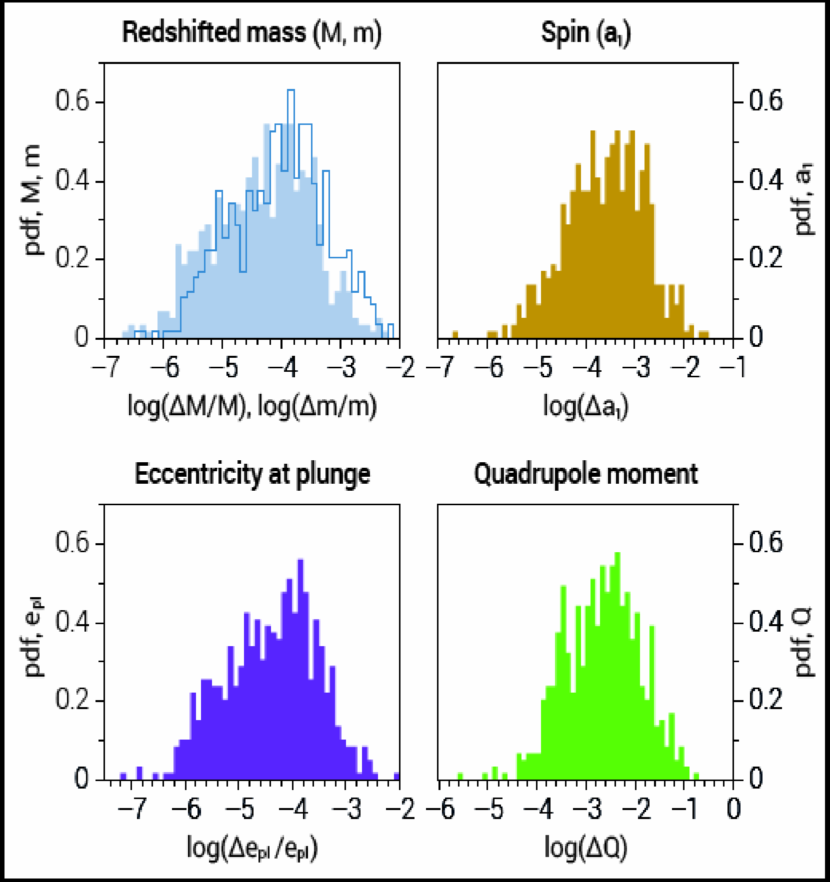


Holz & Huges 2005

eLISA potential: EMRIs

Credits: J. Gair

Configuration	Two Michelson Streams			One Michelson Stream		
	Black hole spin			Black hole spin		
	0	0.5	0.9	0	0.5	0.9
LISA5	1000	1100	1200	550	600	700
LISA25	300	350	500	135	150	235
LISA1	70	80	130	30	35	60
Config 1	40	45	75	15	20	30
Config 2	90	110	175	45	50	90
Config 3	60	65	105	25	30	50
Config 4	185	210	320	80	90	145
Config 5	310	335	465	140	155	235



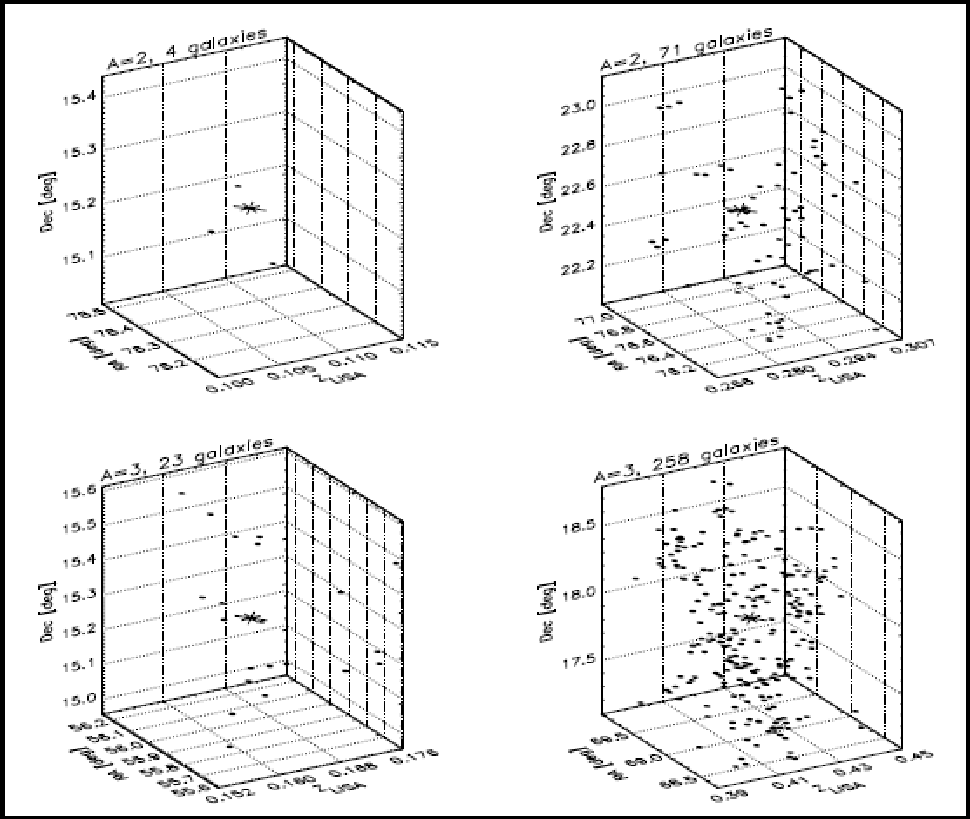
eLISA will give us:

- MBH mass to <0.1% relative accuracy
- spin of the primary hole to <0.01
- sky location to few deg^2
- luminosity distance to few%

(Barack & Cutler 2004, eLISA science team, Amaro-Seoane et al. 2012)

Measuring H_0 with EMRIs

No electromagnetic counterpart expected: cannot measure *z*!
Need to consider all galaxy distribution in the measurement errorbox



One can then write the likelihood of H_0 combining all galaxy distributions in the error boxes of several events

$$\ln \mathcal{L}(H_0) = \sum_i \sum_j N_j^{-1} \ln \mathcal{L}_j(D_j = cz_i/H_0).$$

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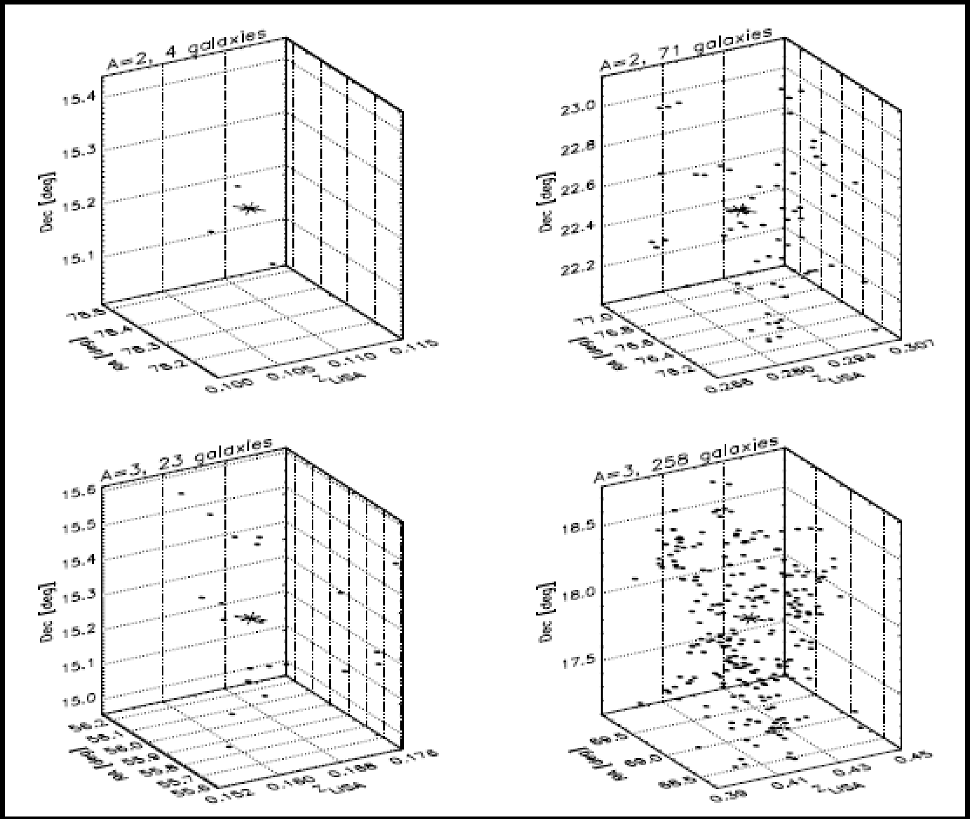

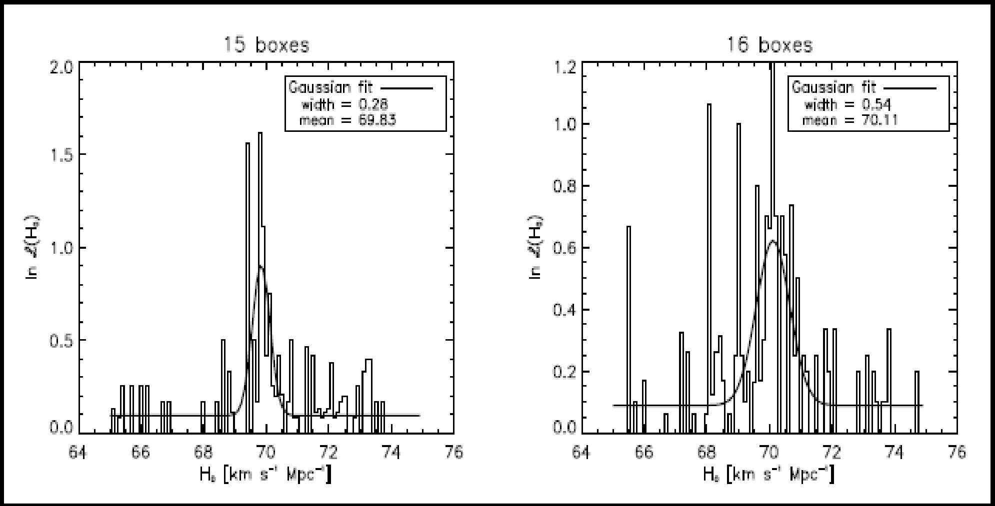

MacLeod & Hogan 2008

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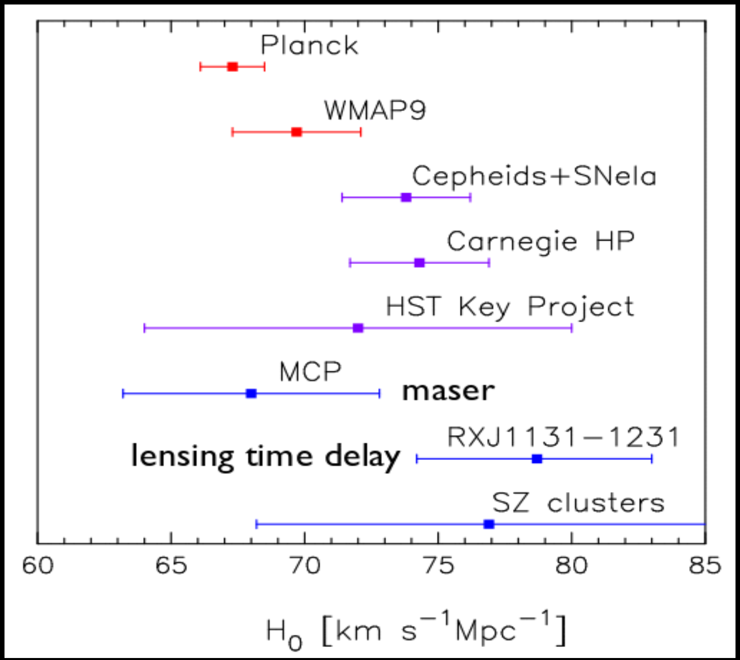
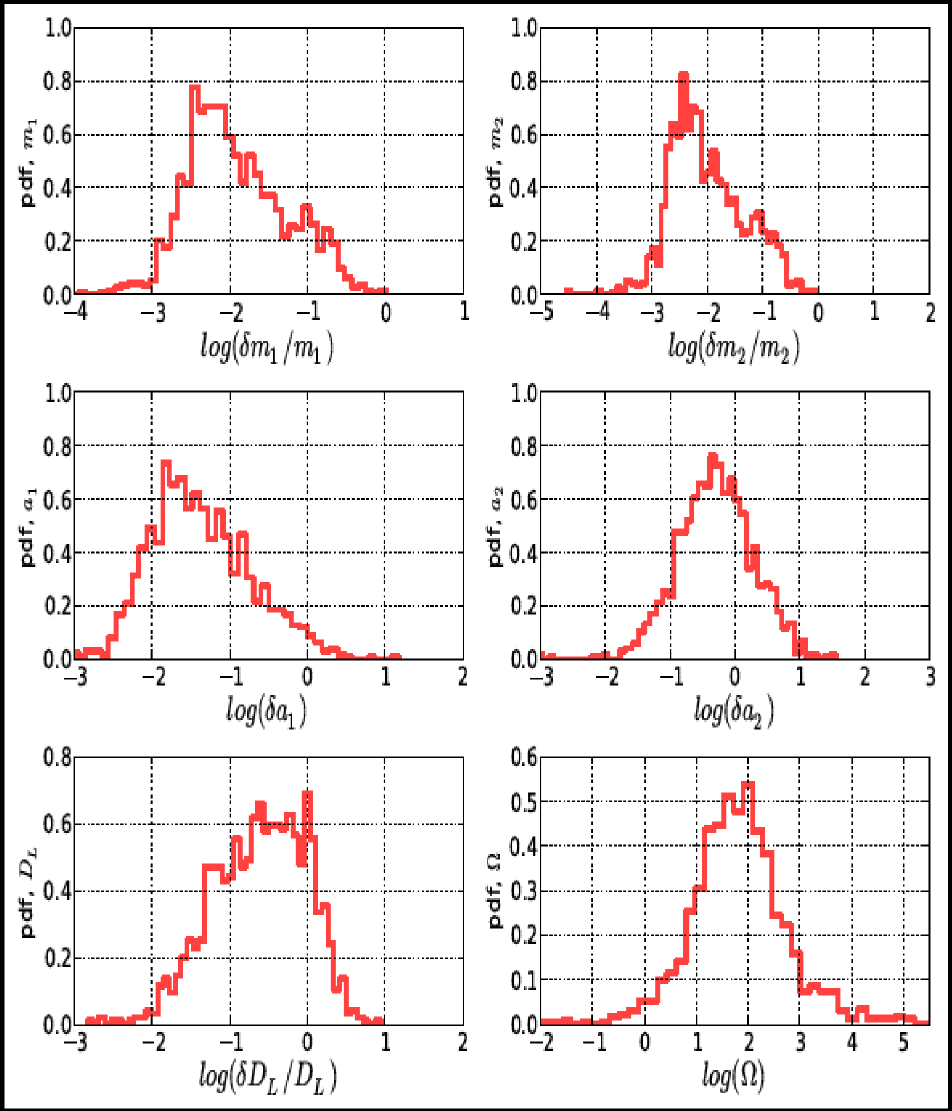


eLISA potential: MBHBs



(Results by N. Cornish,
using spinning full IMR waveforms)

Results of the eLISA science case:

>Individual sources:

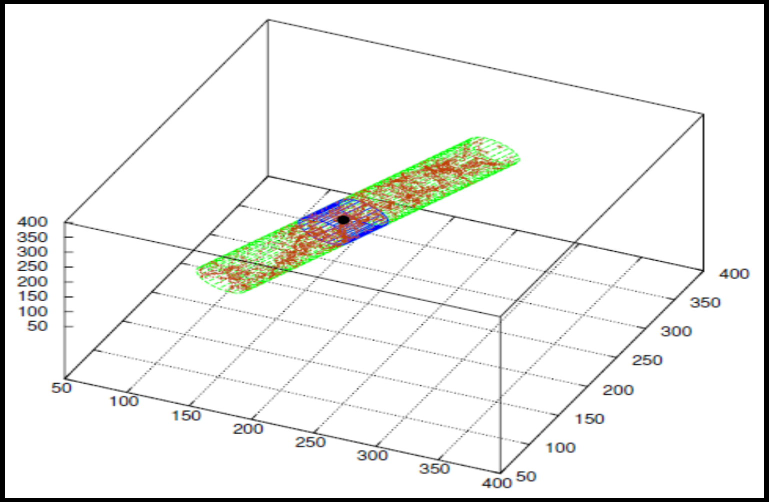
- -Individual (redshifted) masses to <1% relative accuracy
- -spin of the primary hole to <0.1 (in many cases to <0.01)
- -sky location to 1-100 deg
- -luminosity distance to <10% in most cases (better at low z)
- -no emphasis on multimessenger astronomy

>Population studies:

- -few detection will enable sensible astrophysical statements about MBH seeds and cosmic growth
- -test made mainly on a discrete set of models

Measuring w without counterparts

Electromagnetic counterpart difficult to identify: cannot measure *z*!
Play the same game as before: need to consider all galaxy distribution in the measurement errorbox



One can then write the posterior for w considering all distributions in the error boxes of several events

$$P(w) = \frac{p_0(w)\prod_{j=1}^{N_{ev}} P_j(s|w)}{\int p_0(w)\prod_{j=1}^{N_{ev}} P_j(s|w)dw}$$

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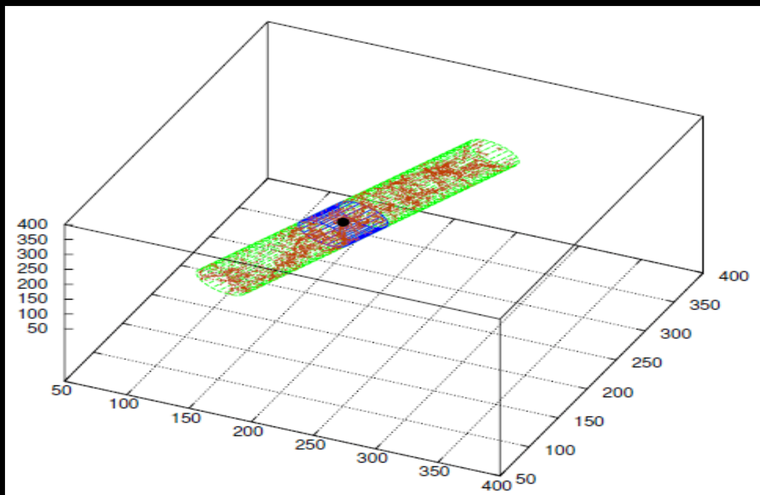

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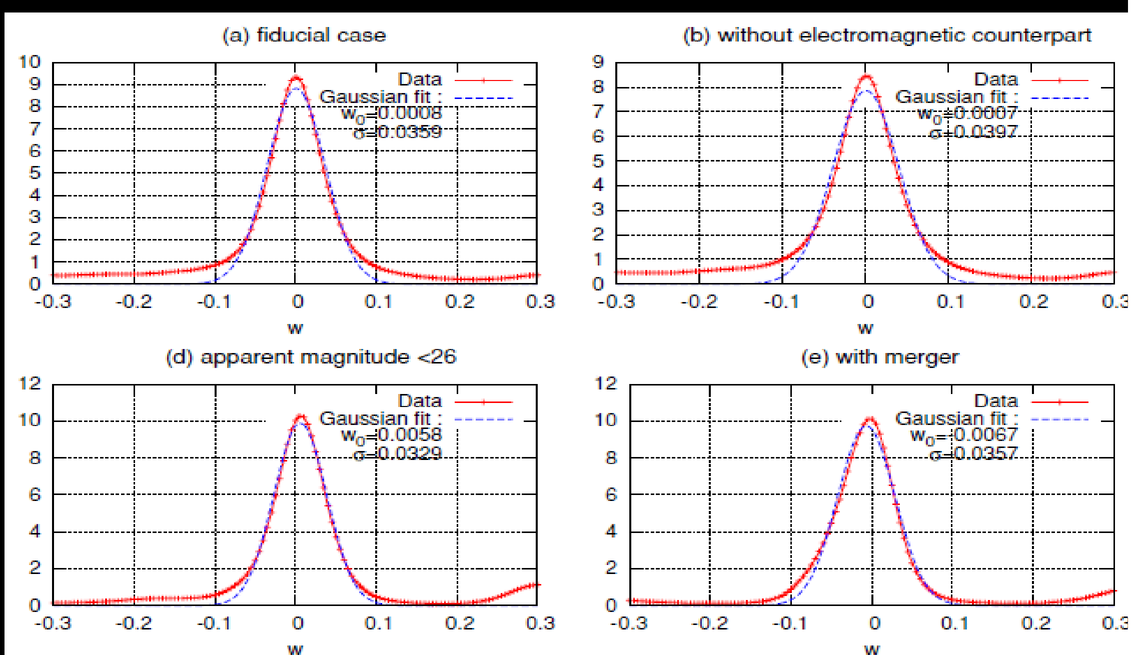
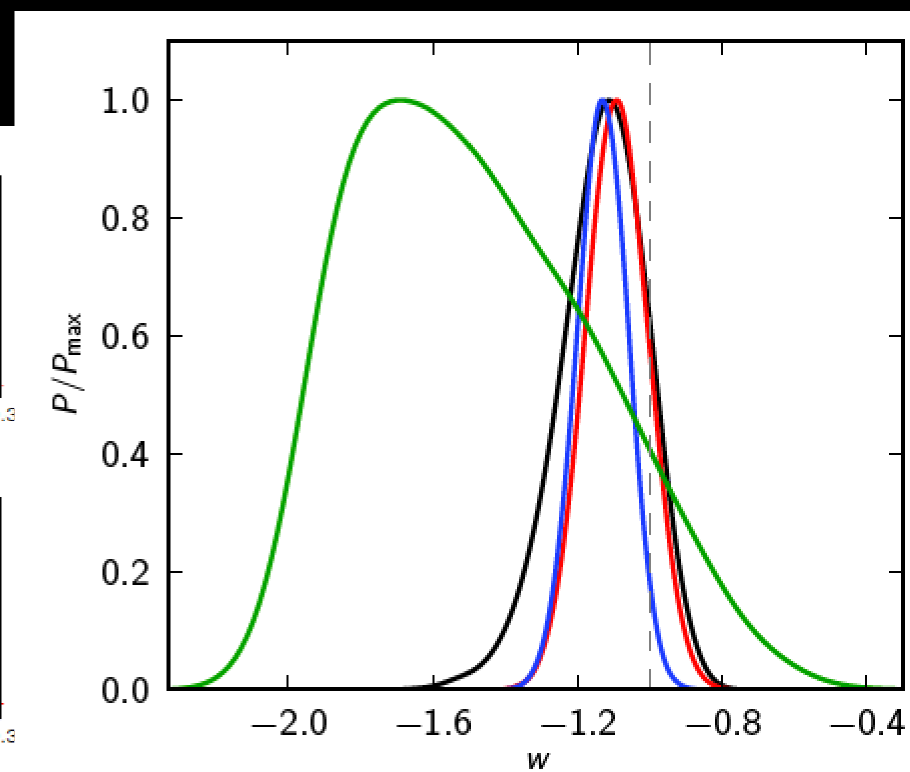
Petiteau Babak AS 2011



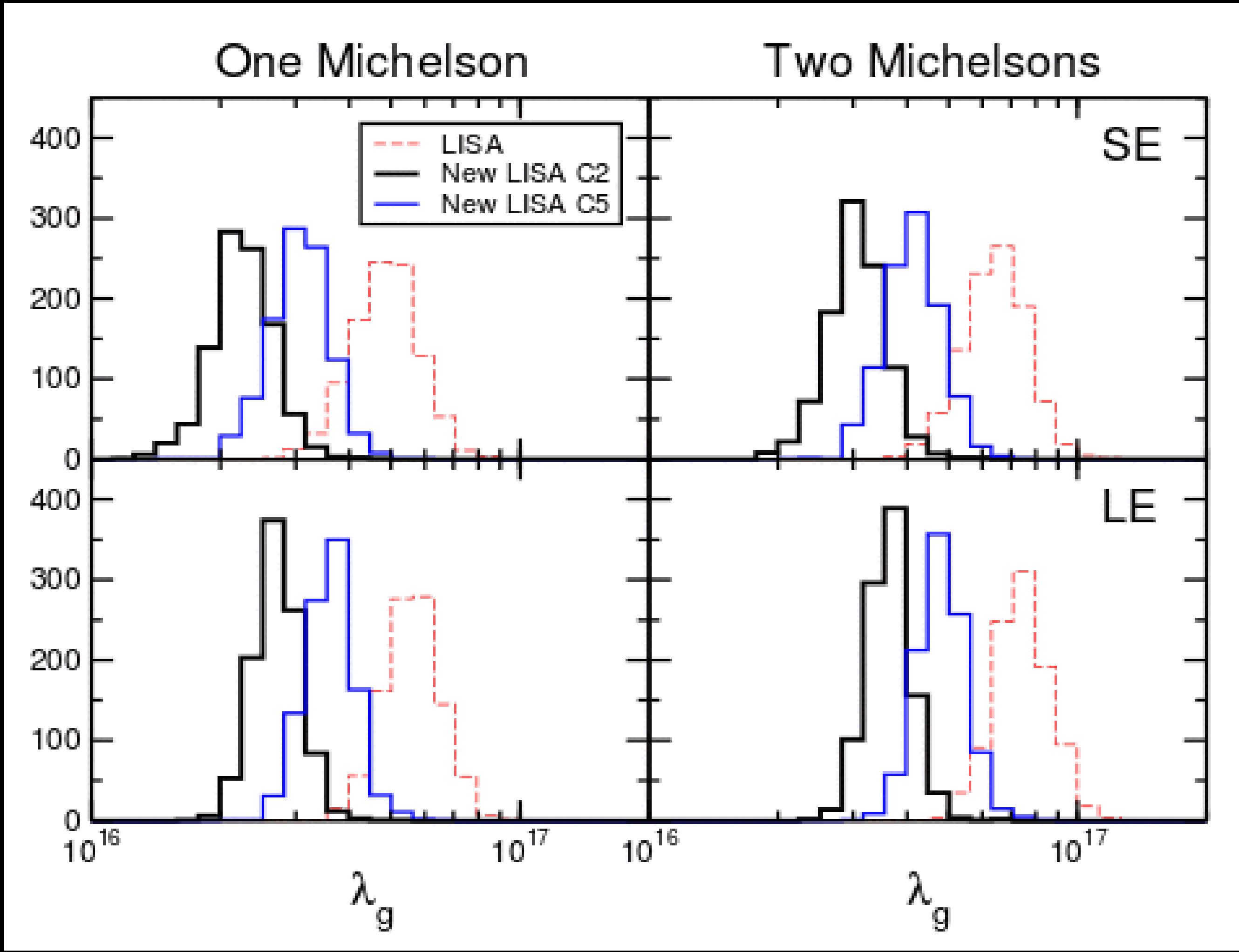


Dynamical bounds on the graviton mass

Massive graviton produces an extra term in the phase evolution that can be disentangled from the GR ones

$$\Psi_{\rm MG}(f) = \Psi_{\rm GR}(f) - \beta_g(\pi M f)^{-1}$$

$$\beta_g \equiv \pi^2 DM/[(1+z)\lambda_g^2]$$



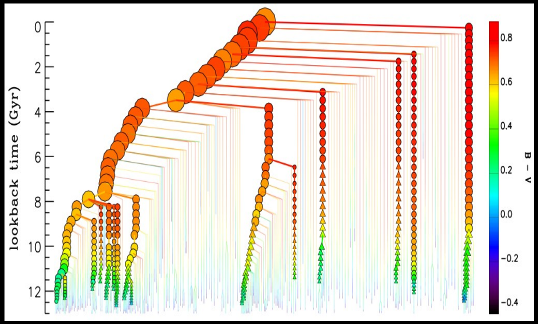
Combining several events will place the strongest dynamical constrain on the Compton lenght (and therefore mass) of the graviton

(Berti Gair AS, 2011)

MBH astrophysics with GW observations

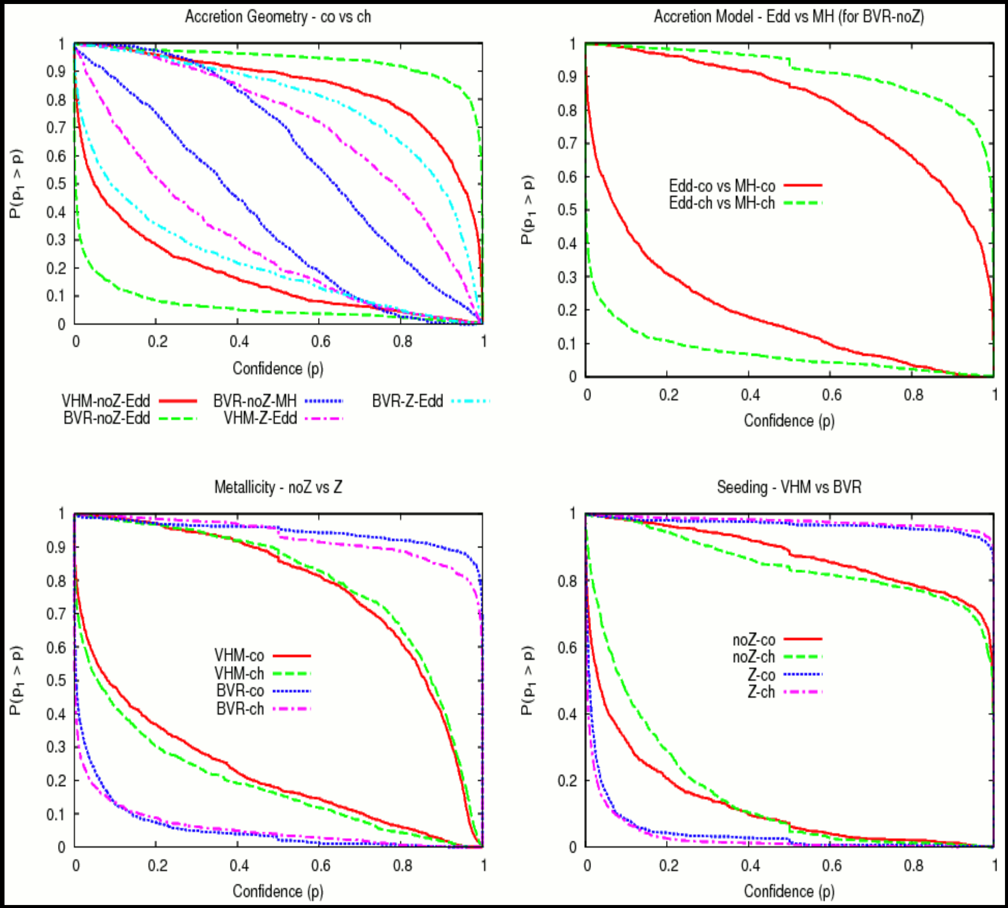
Astrophysical unknowns in MBH formation scenarios

1- **MBH seeding mechanism (heavy vs light seeds)**
2- **Metallicity feedback (metal free vs all metallicities)**
3- **Accretion efficiency (Eddington?)**
4- **Accretion geometry (coherent vs. chaotic)**

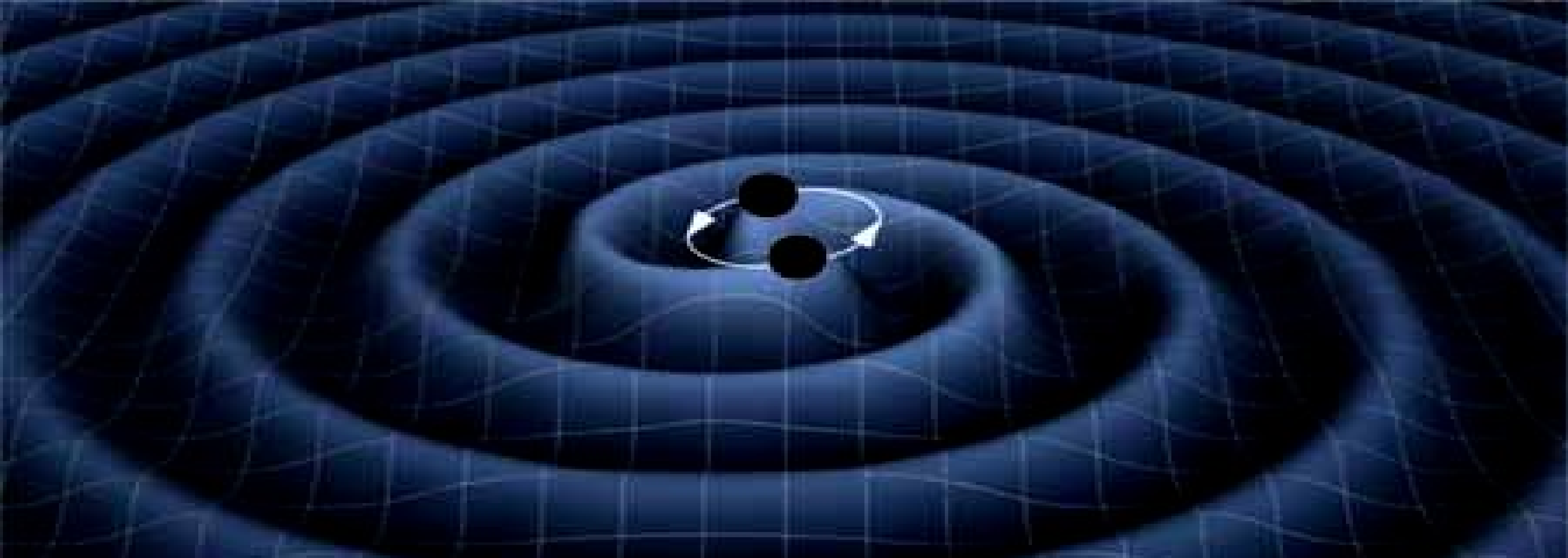


CRUCIAL QUESTION:
Given a set of LISA observation of coalescing MBH binaries, what astrophysical information about the underlying population can we recover?

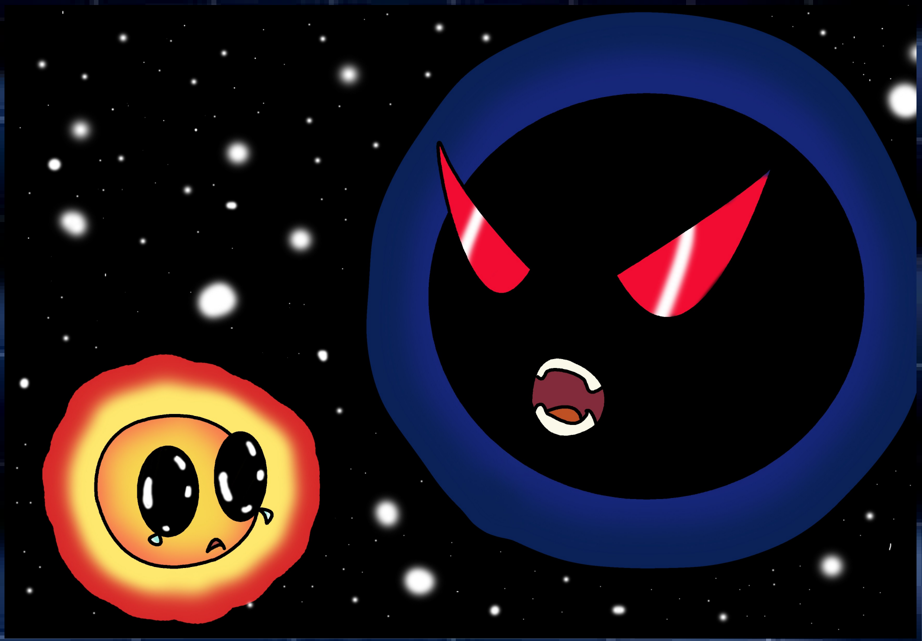
Create catalogues of observed binaries including errors from eLISA observations and compare observations with theoretical models



AS et al. 2011, see also Plowman et al 2011



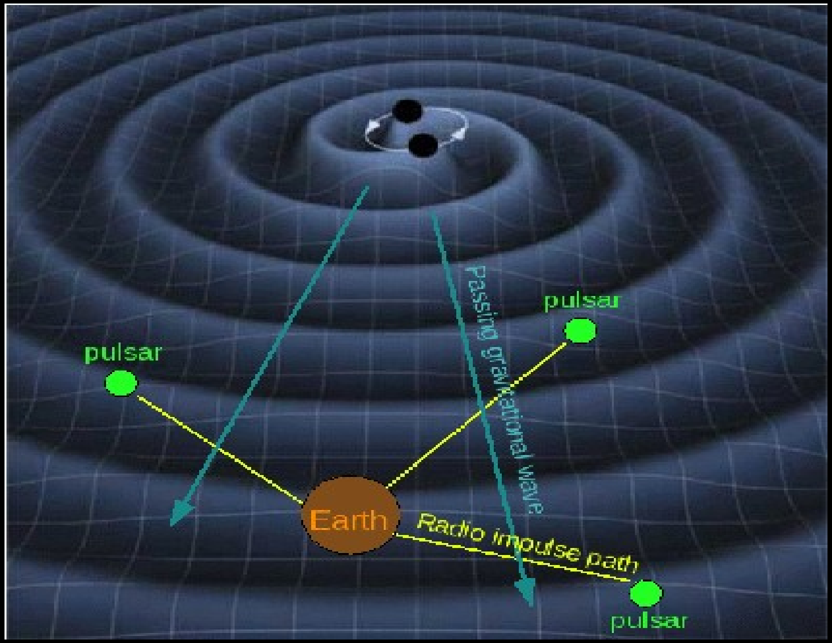
Black hole beasts: PTA



The timing residual R

The GW passage cause a modulation of the MSP frequency

$$\frac{\nu(t)-\nu_0}{\nu_0} = \Delta h_{ab}(t) \equiv h_{ab}(t_{\rm p},\hat{\Omega}) - h_{ab}(t_{\rm ssb},\hat{\Omega})$$



The *residual* in the time of arrival of the pulse is the integral of the frequency modulation over time

$$R(t) = \int_0^T \frac{\nu(t)-\nu_0}{\nu_0}\,dt$$

(Sazhin 1979, Helling & Downs 1983, Jenet et al. 2005, Sesana Vecchio & Volonteri 2009)

$$\begin{aligned} &= \frac{\mathcal{M}^{5/3}}{D}\,[\pi f(t)]^{-1/3} \\ &\simeq 25.7\left(\frac{\mathcal{M}}{10^9\,M_\odot}\right)^{5/3}\left(\frac{D}{100\,\mathrm{Mpc}}\right)^{-1} \\ &\quad\times\left(\frac{f}{5\times10^{-8}\,\mathrm{Hz}}\right)^{-1/3}\ \mathrm{ns} \end{aligned}$$

R~h/(2πf)

Examples of signals

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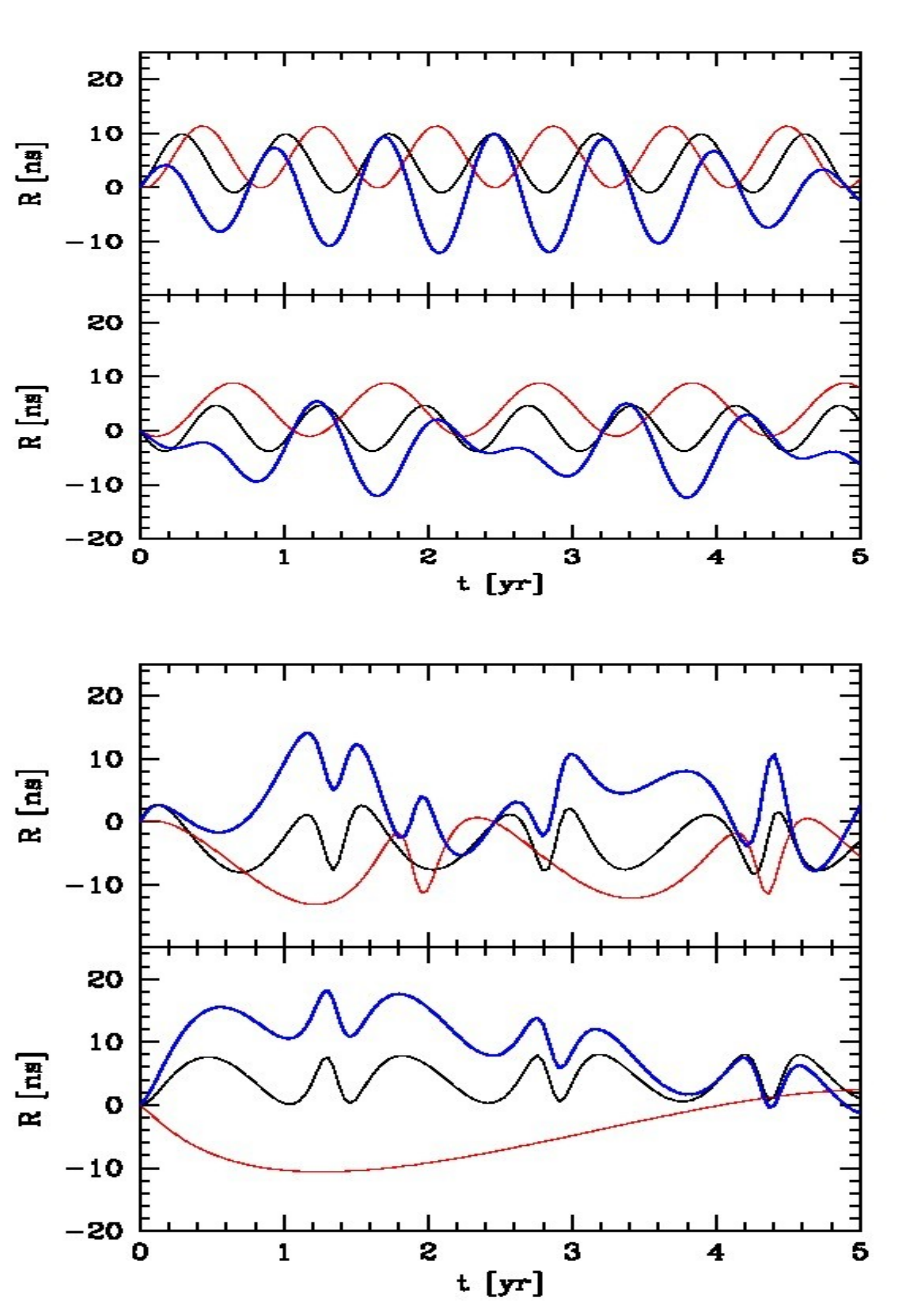

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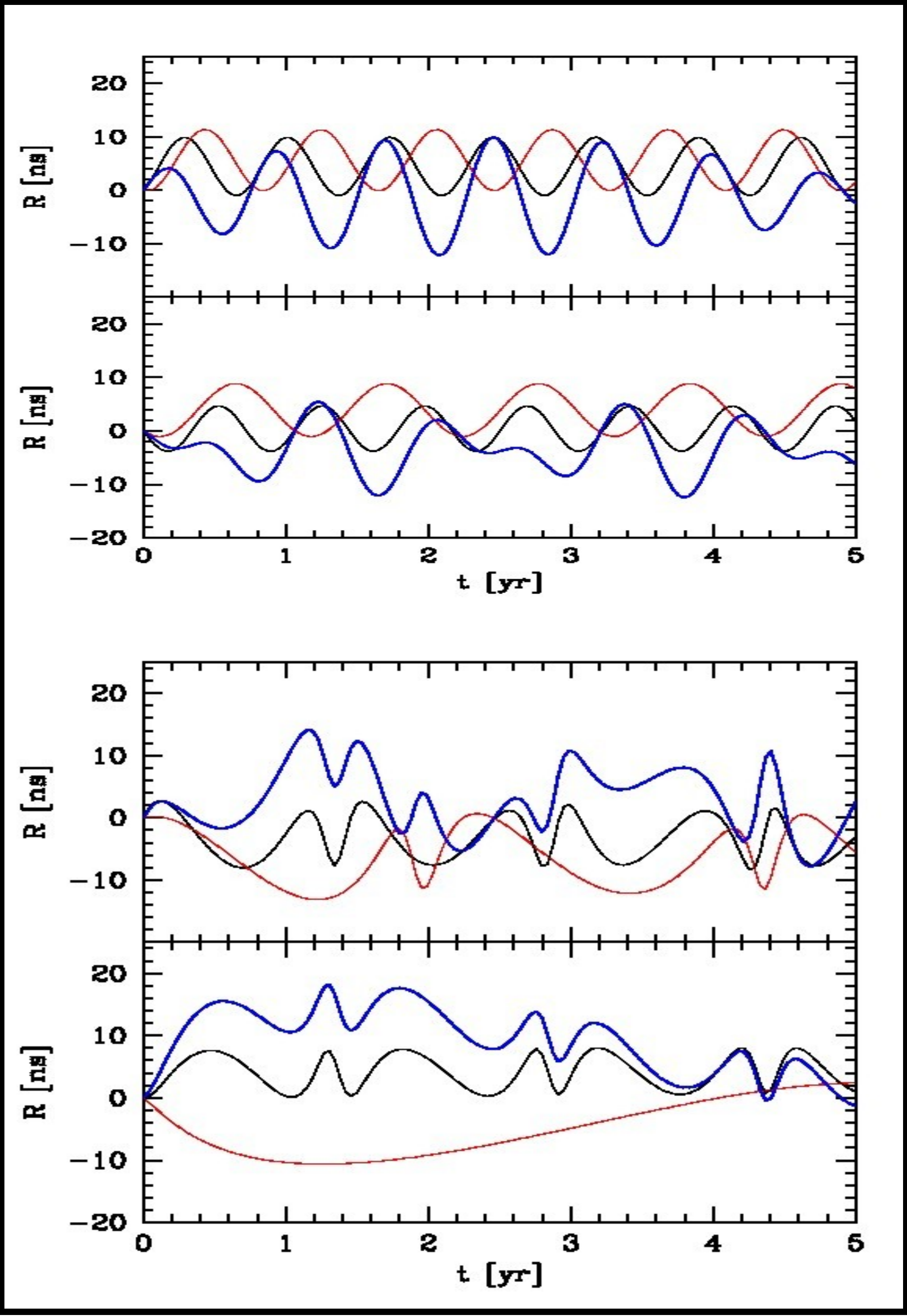
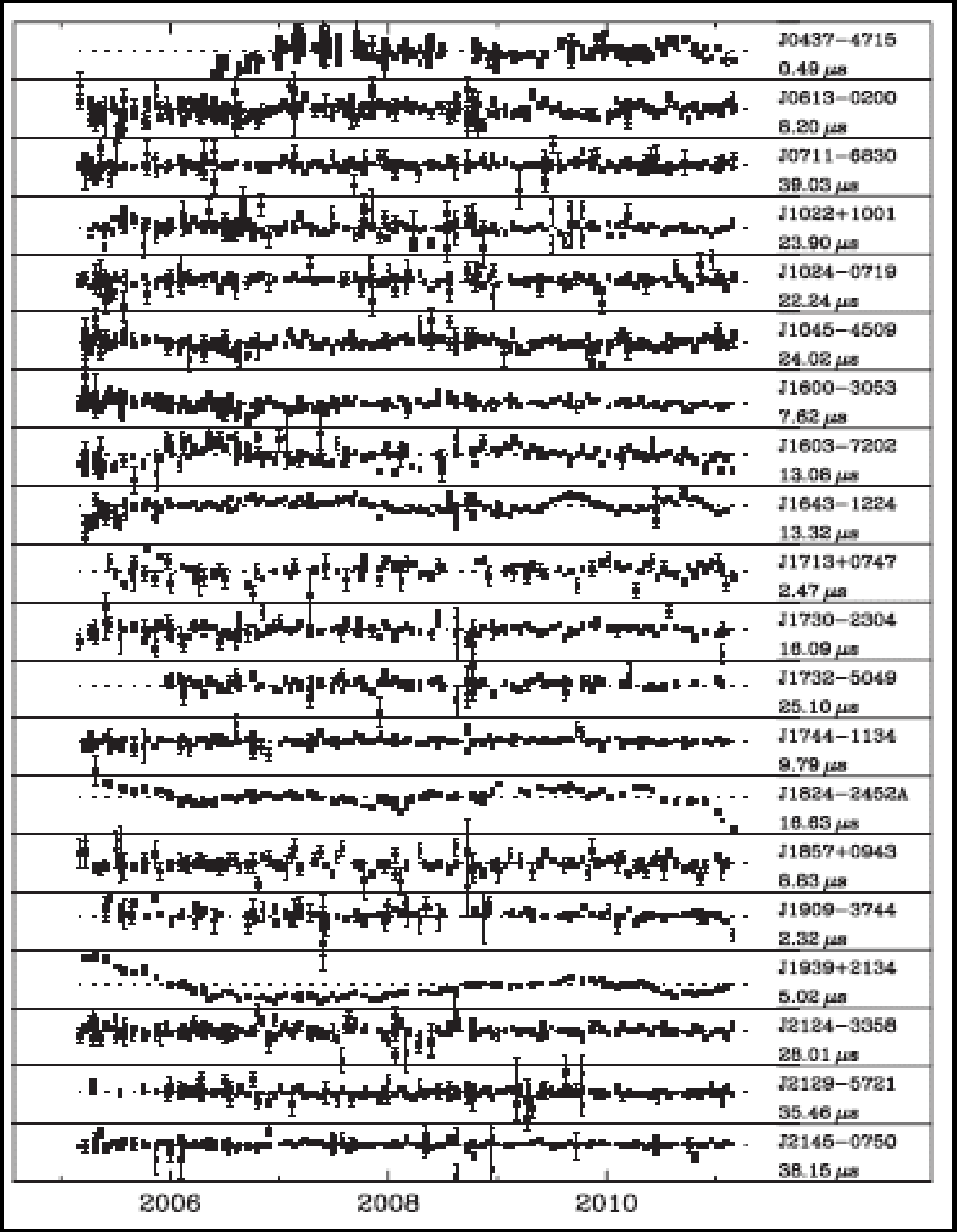
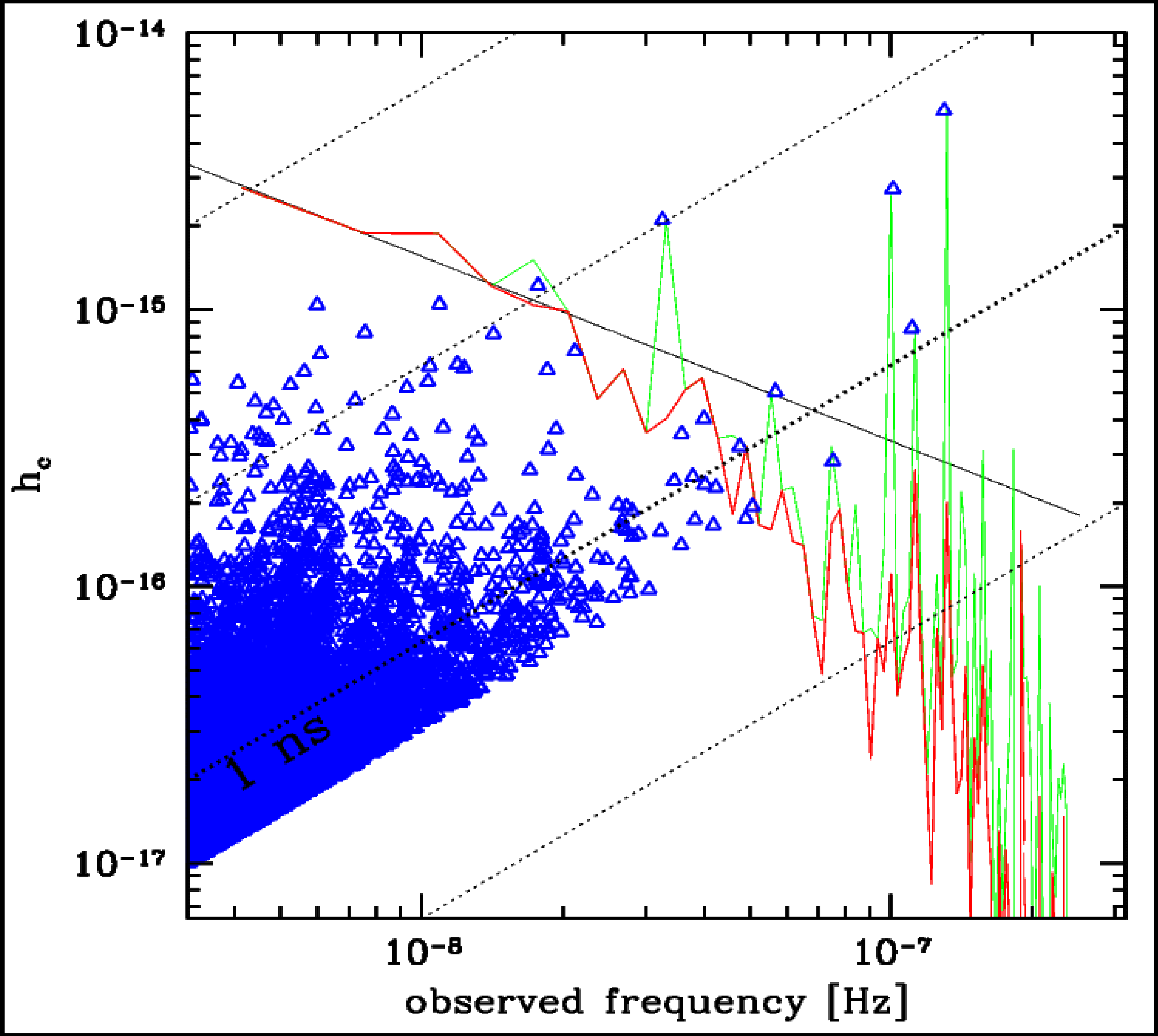


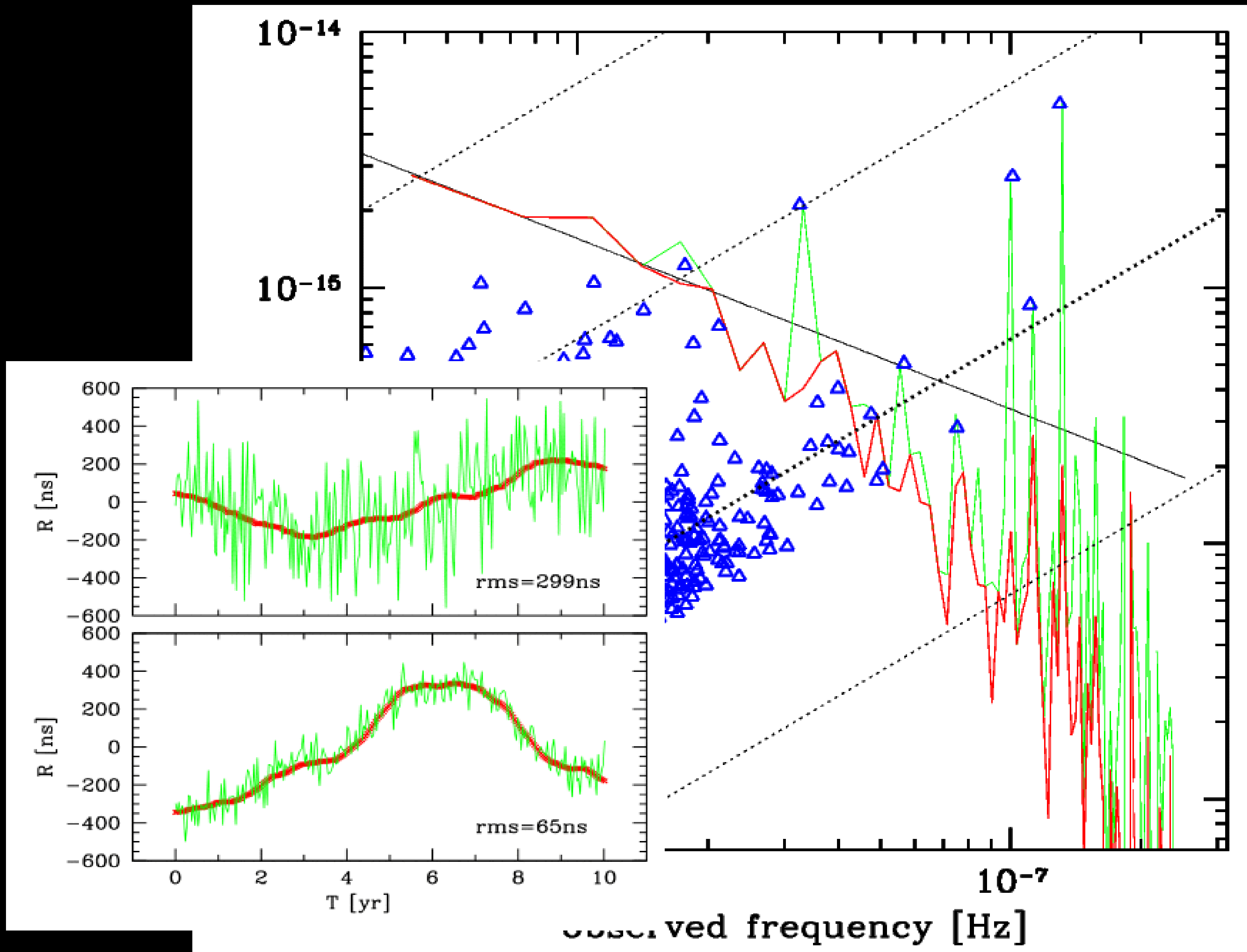
The cruel reality



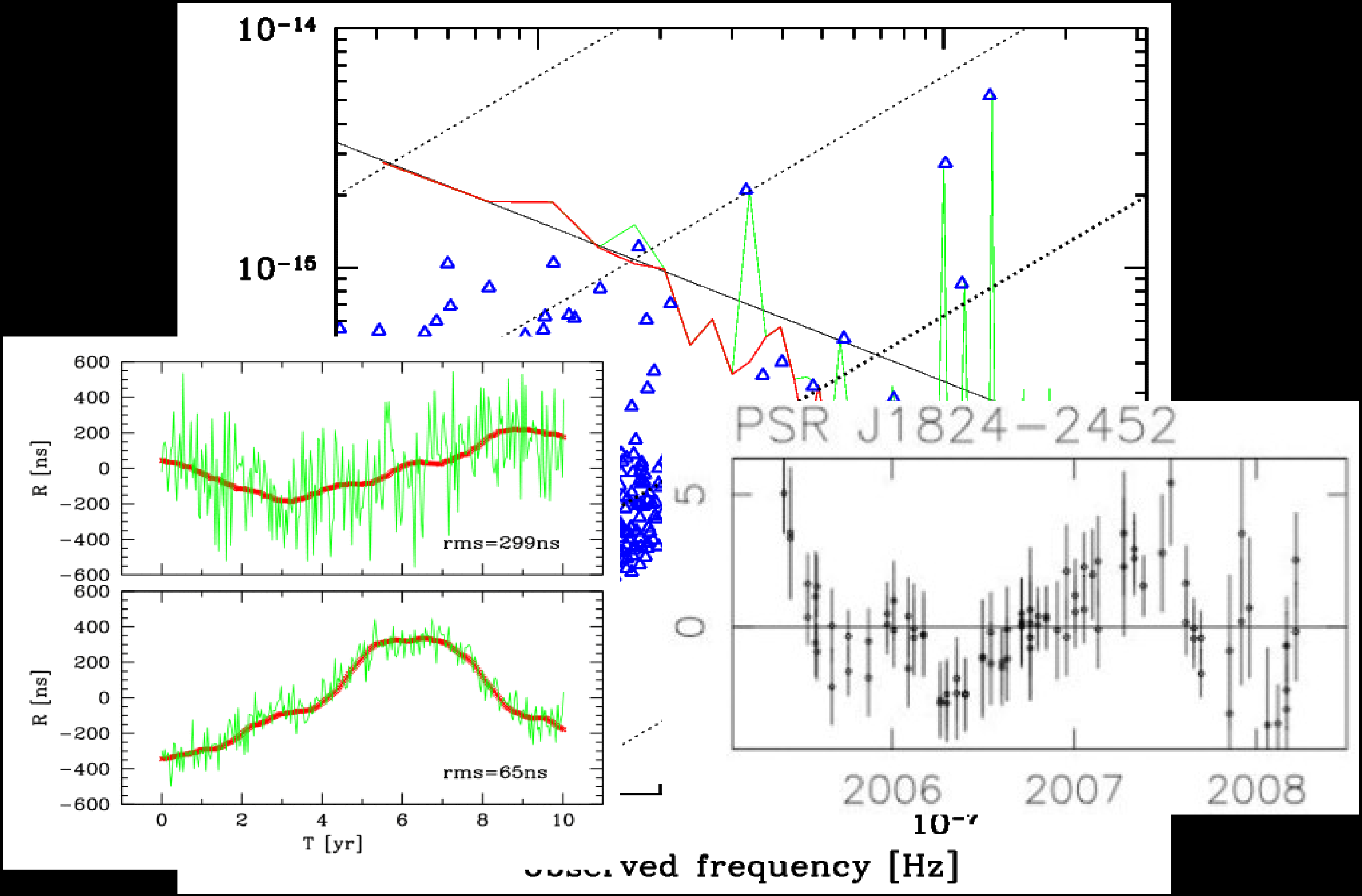
Manchester et al. 2013



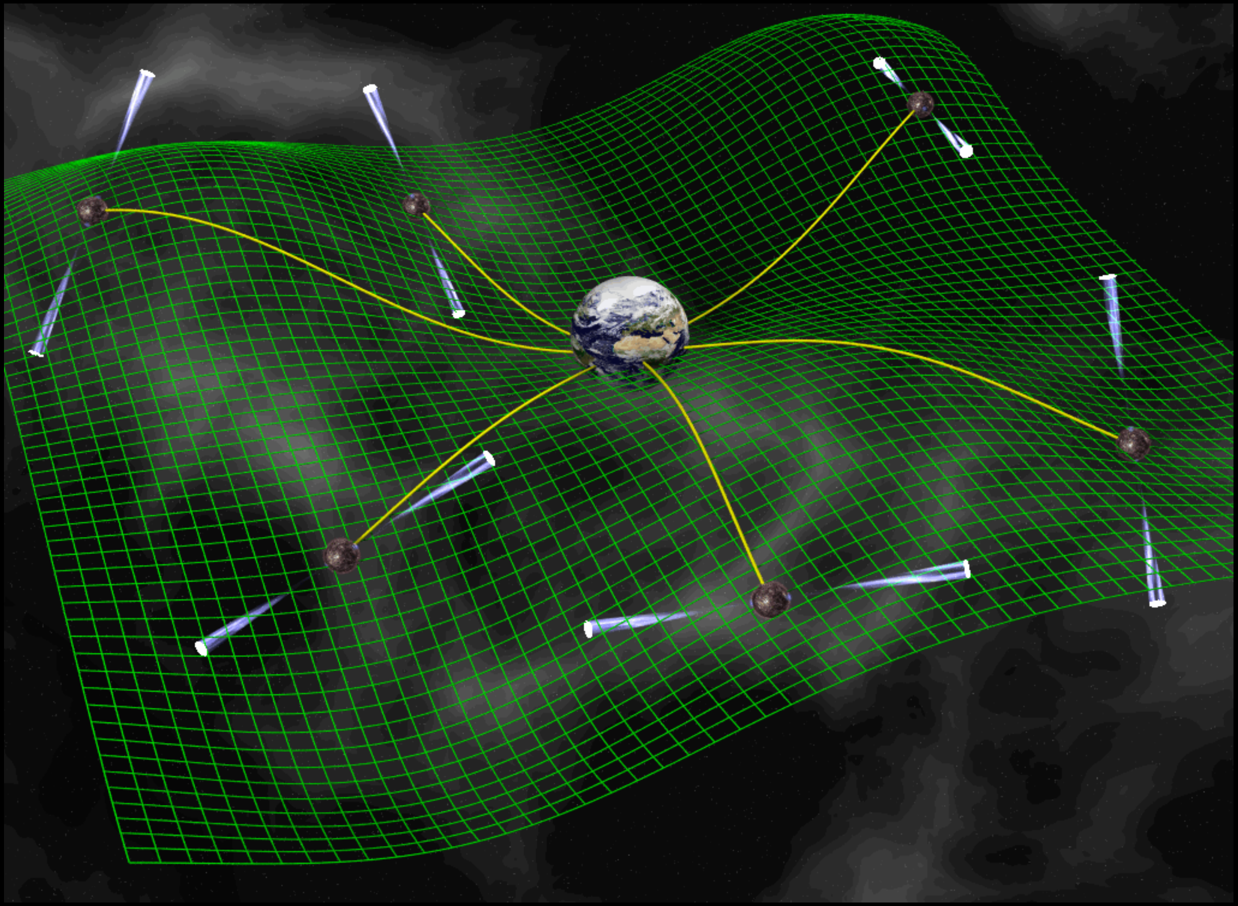
10^{-14}
10^{-15}
10^{-16}
10^{-17}
h_c
1 ns
10^{-8}
10^{-7}
observed frequency [Hz]



10^{-14}
10^{-15}
10^{-7}
...ved frequency [Hz]
R [ns]
T [yr]
rms=299ns
rms=65ns

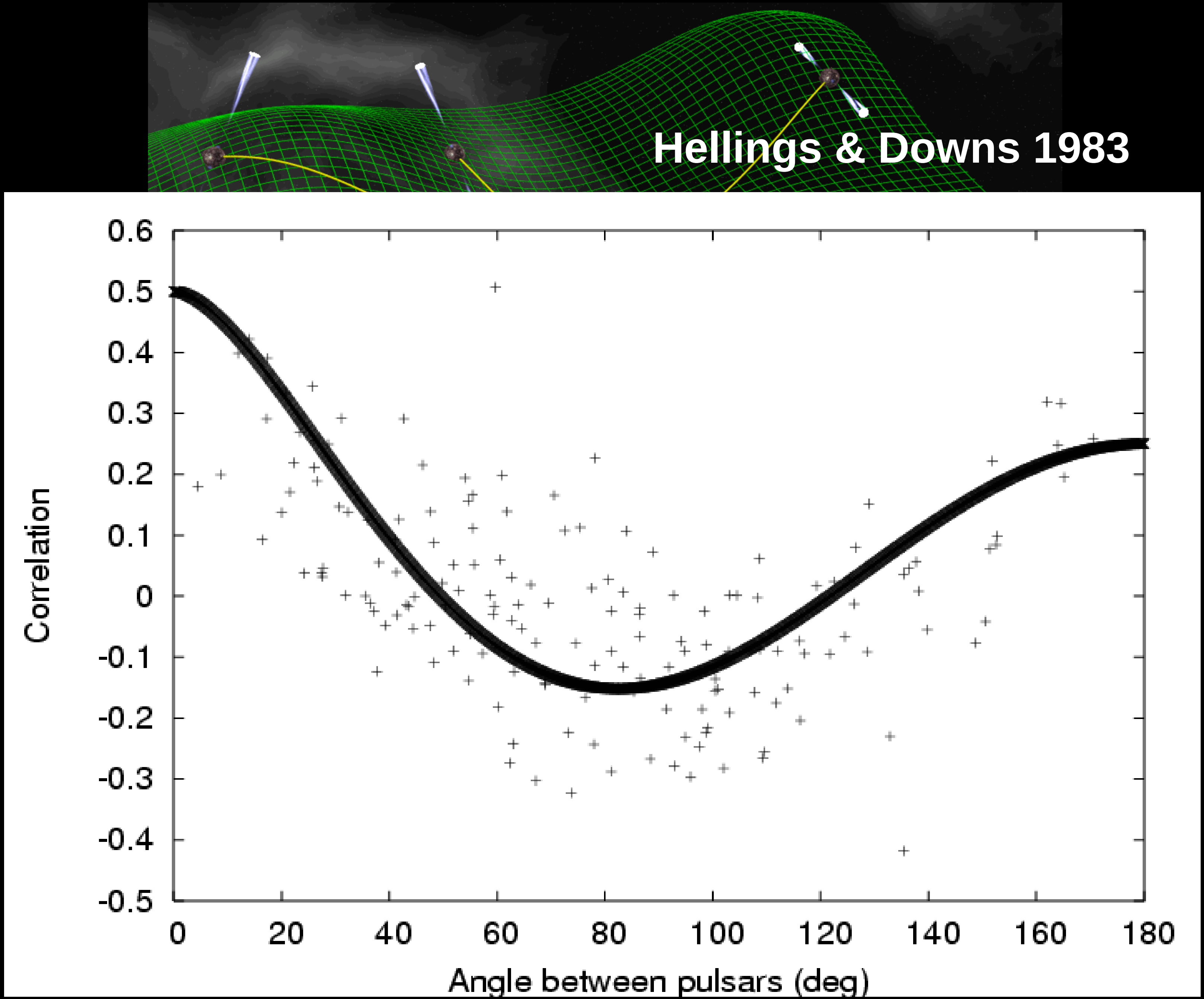


There are, however, many other sources of red noise in pulsar timing: intrinsic spin noise, DM effects, etc.

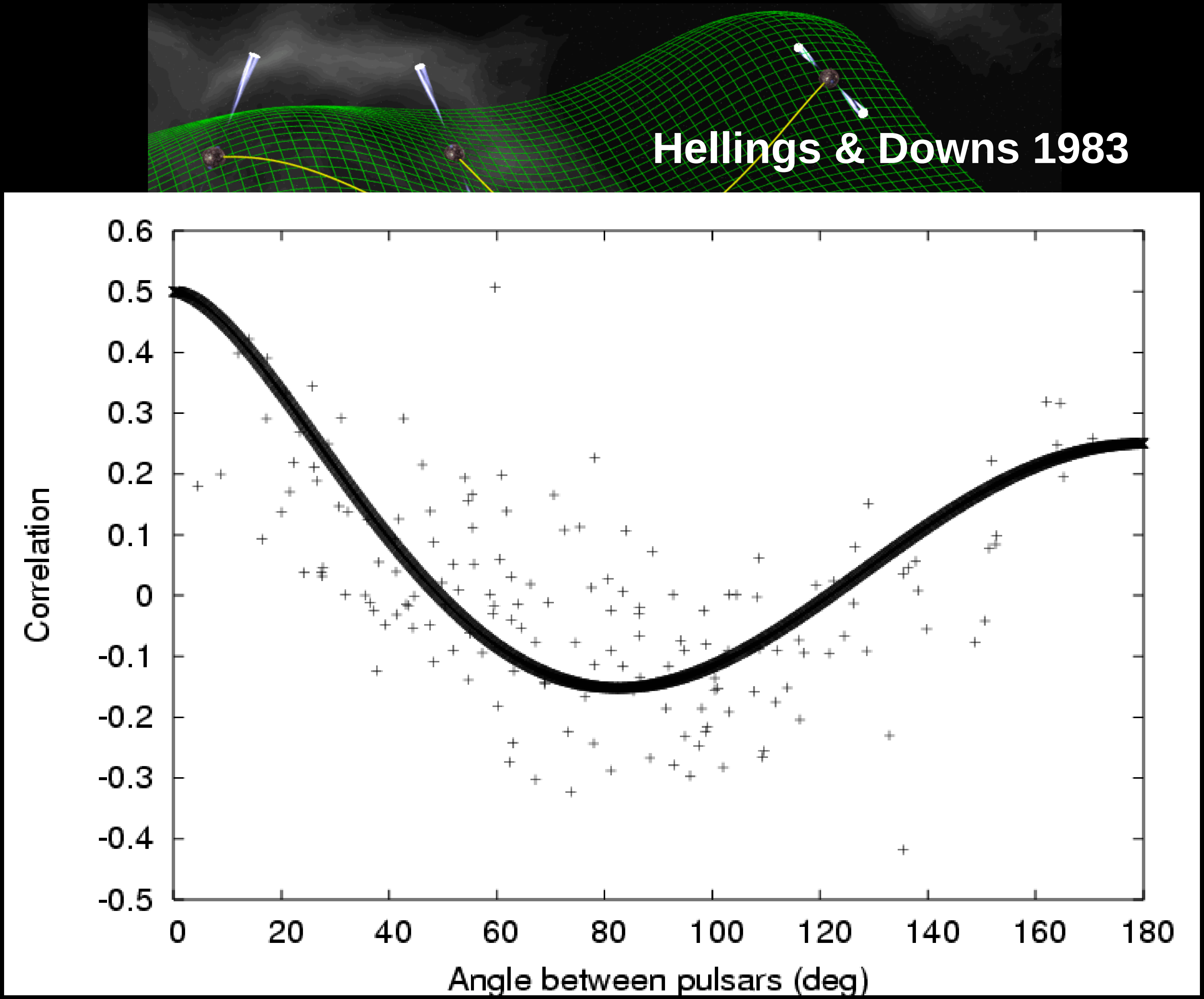


This very red signal has a peculiar correlation pattern among different pairs of pulsars, given by the quadrupolar nature of gravitational waves

Other sources of red noise are uncorrelated!



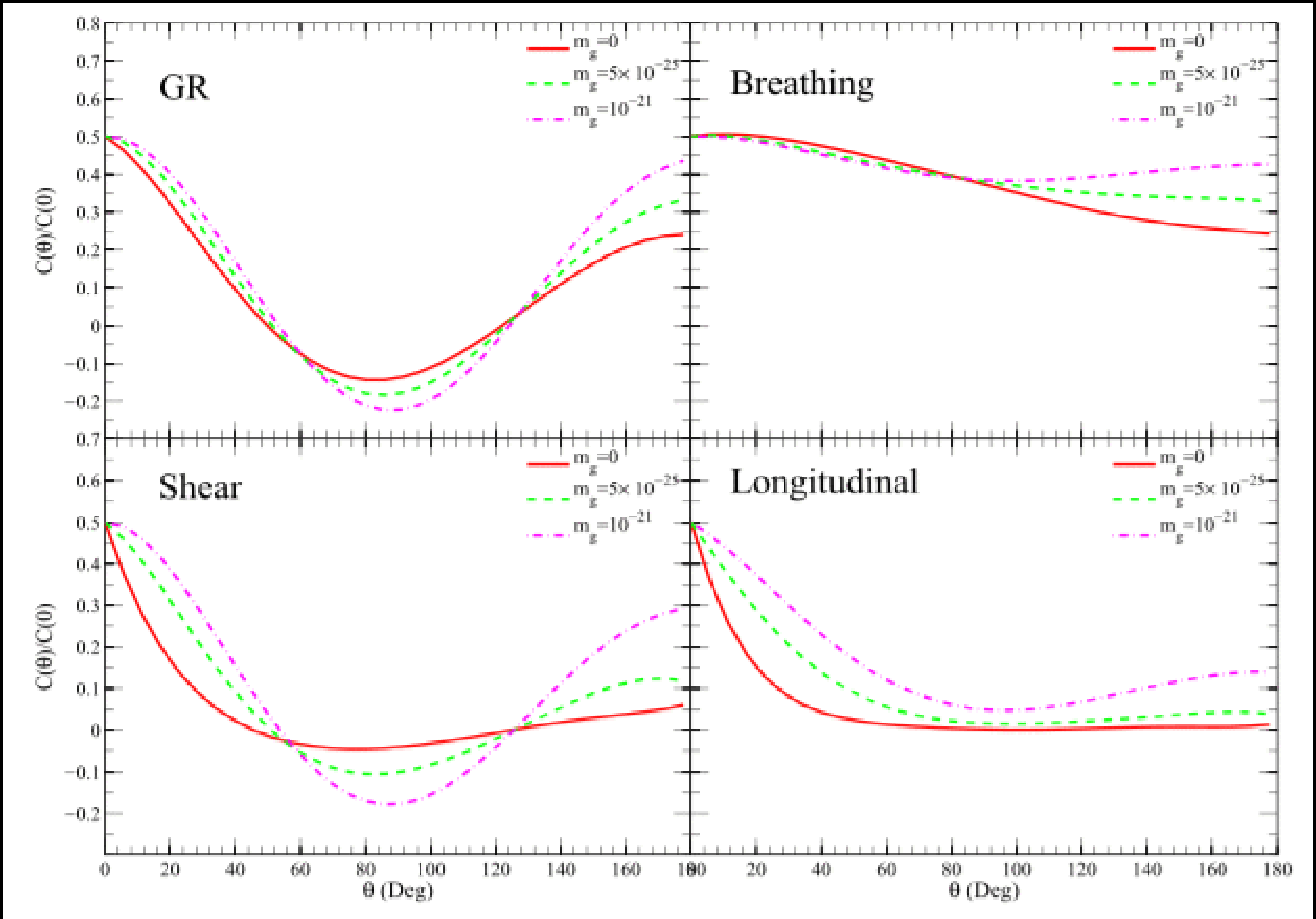
Hellings & Downs 1983
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
Correlation
0
20
40
60
80
100
120
140
160
180
Angle between pulsars (deg)



IT IS ESSENTIAL TO CORRELATE THE SIGNAL OF AS MANY PULSARS AS POSSIBLE

Probing alternative theories of gravity with PTA

Lee 2013



Both the existence of a massive graviton and of extra GW polarizations (other than the plus and cross predicted by GR) modify the correlation pattern in the PTA signal (maybe observable with high SNR with SKA)

The pulsar timing arrays network

EPTA/LEAP (large European array for pulsars)

NanoGrav (north American nHz observatory for gravitational waves)

PPTA (Parkes pulsar timing array)

The pulsar timing arrays network

EPTA/LEAP (large European

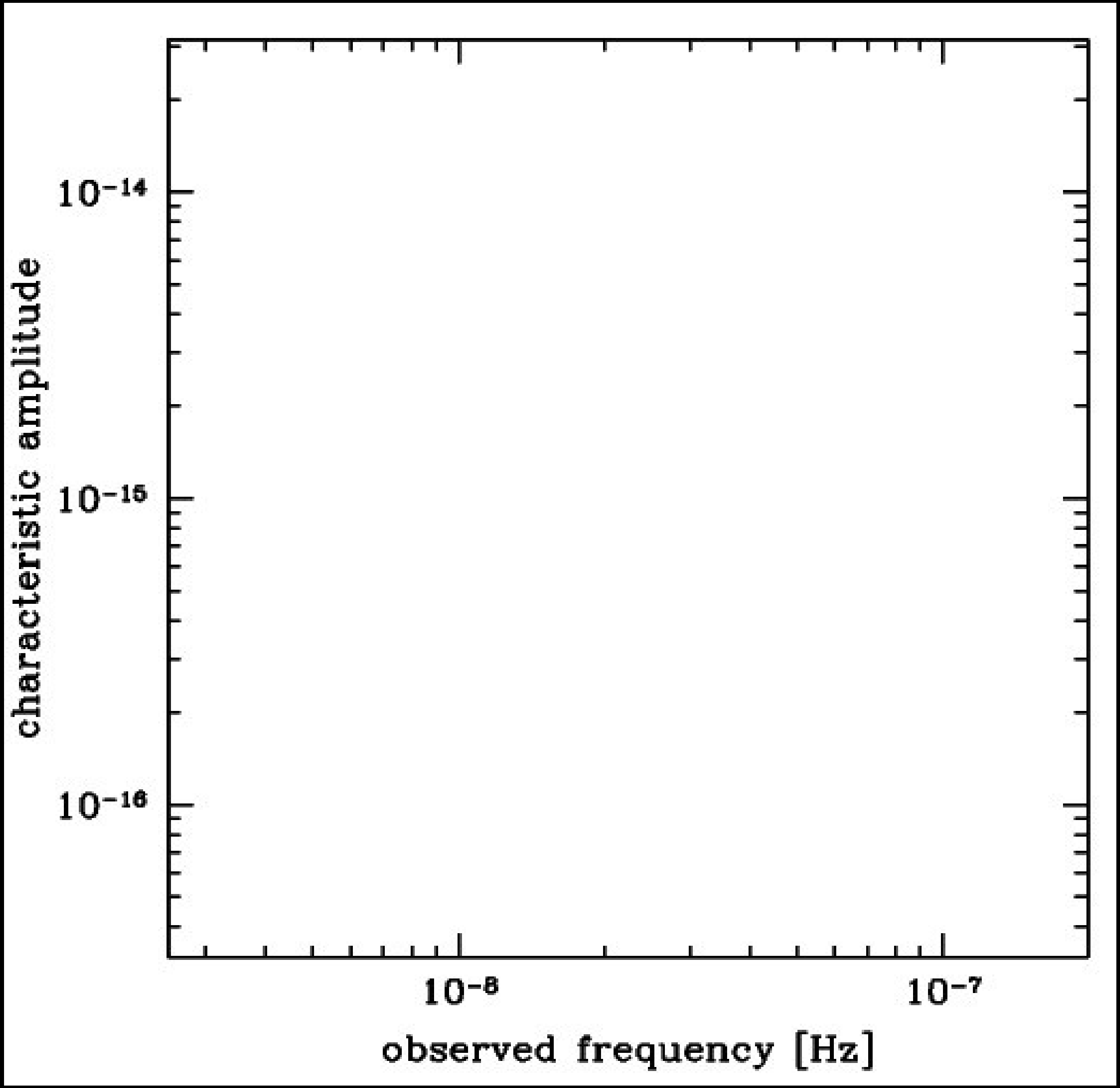
INTERNATIONAL PULSAR TIMING ARRAY

IPTA

rican nHz
onal waves)

PPTA (Parke

Where we stand: theory vs observations



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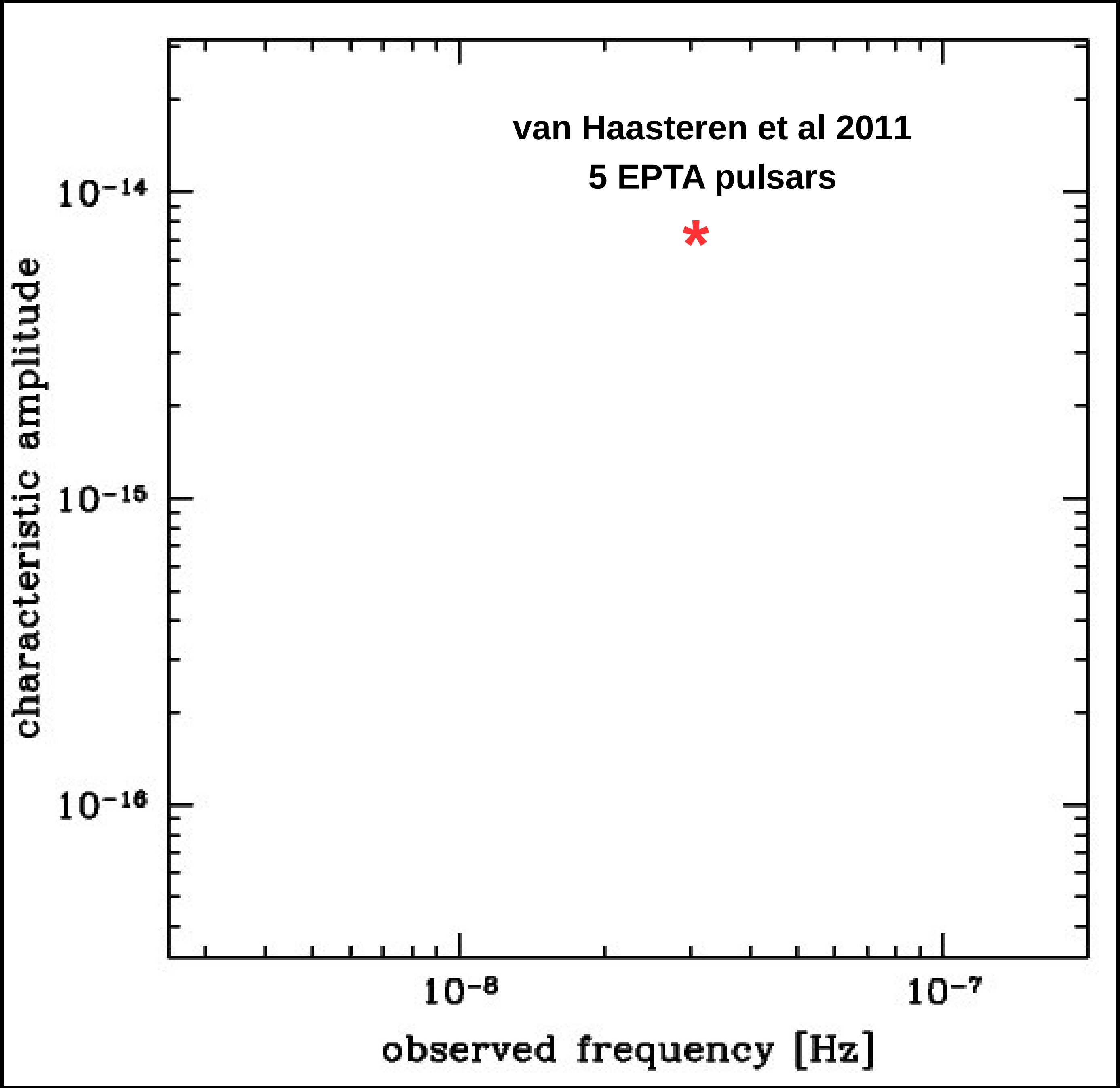
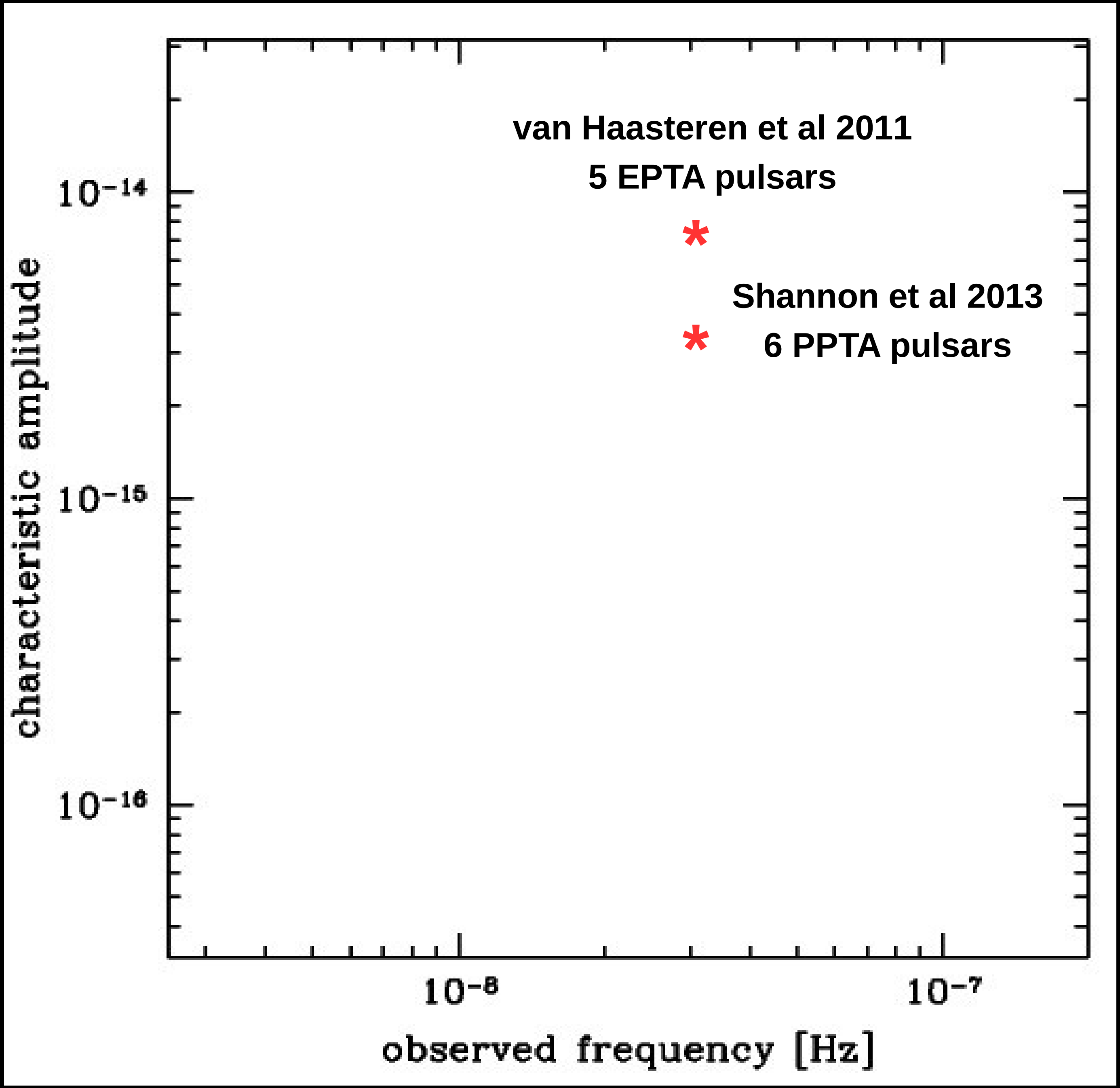


Where we stand: theory vs observations



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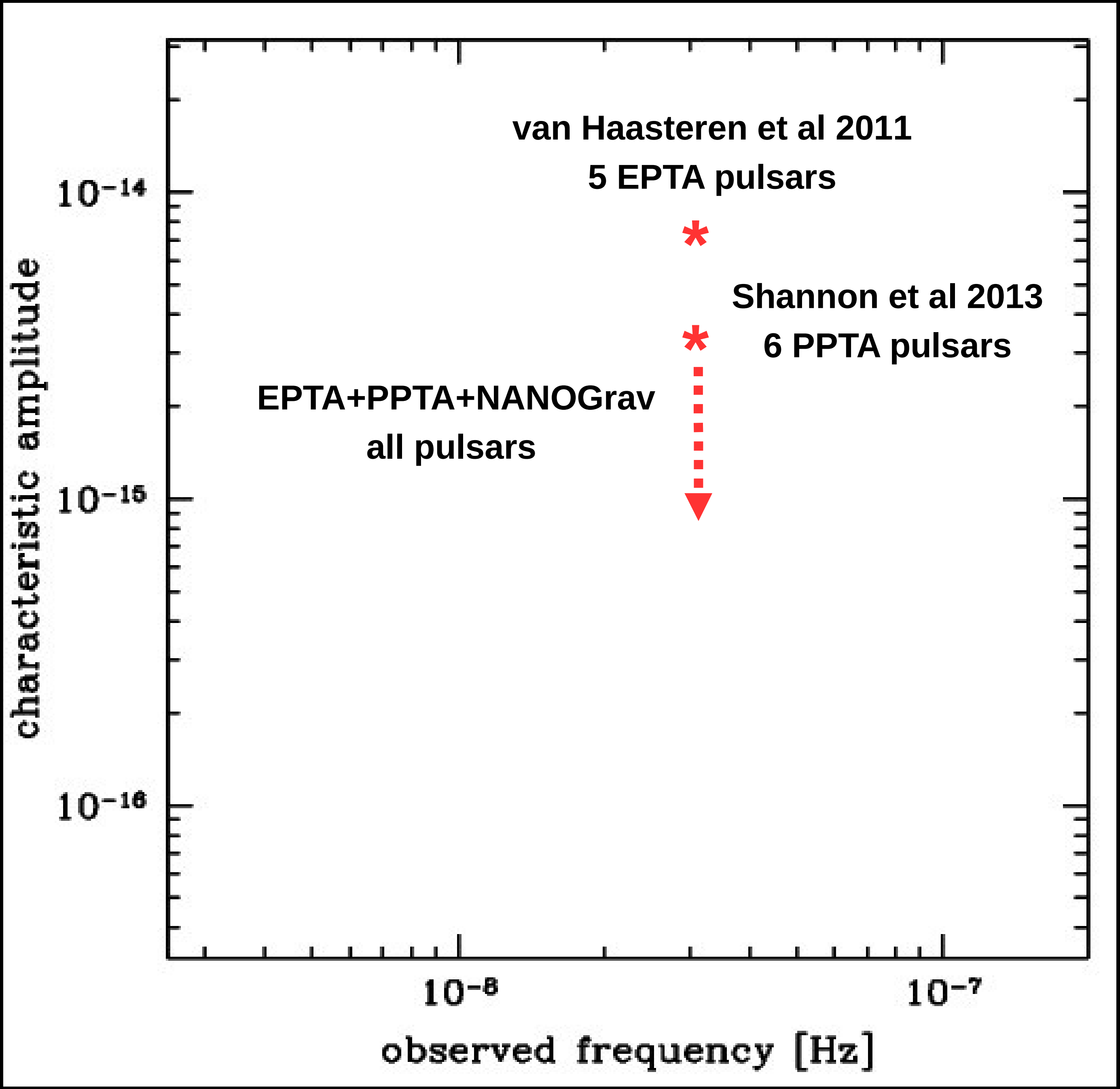

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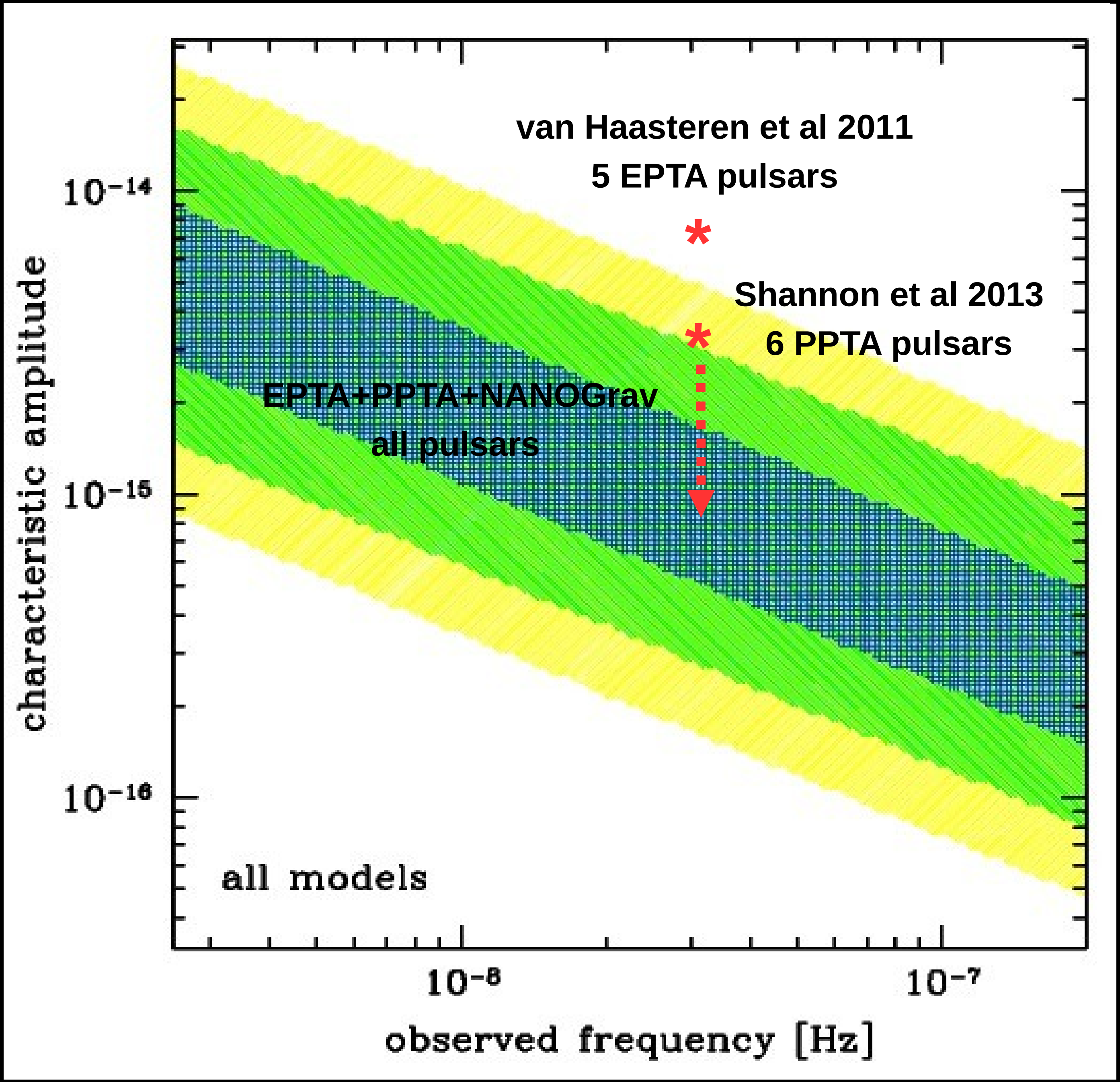

AS et al. 2008; AS et al. 2009; AS 2013

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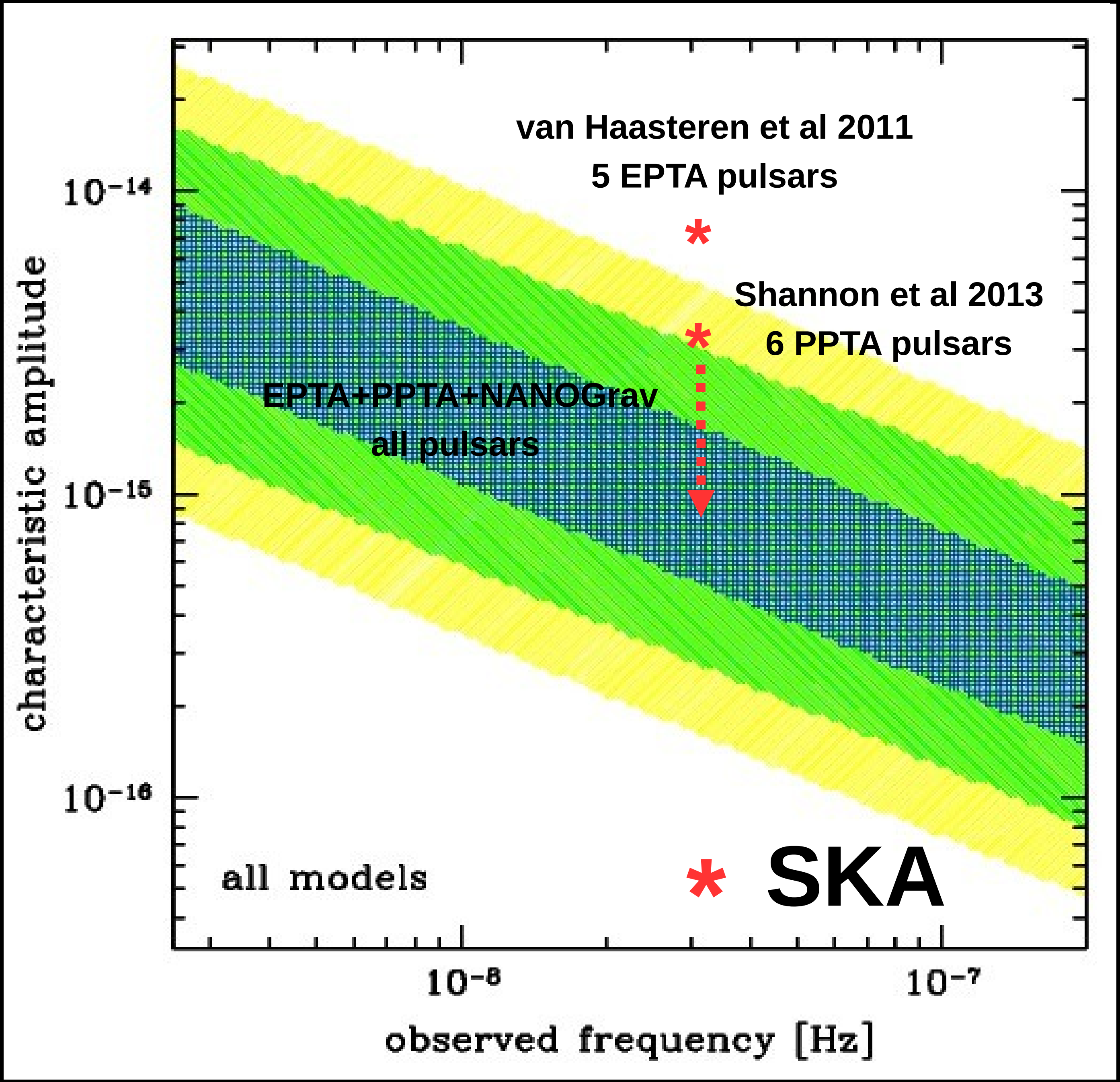

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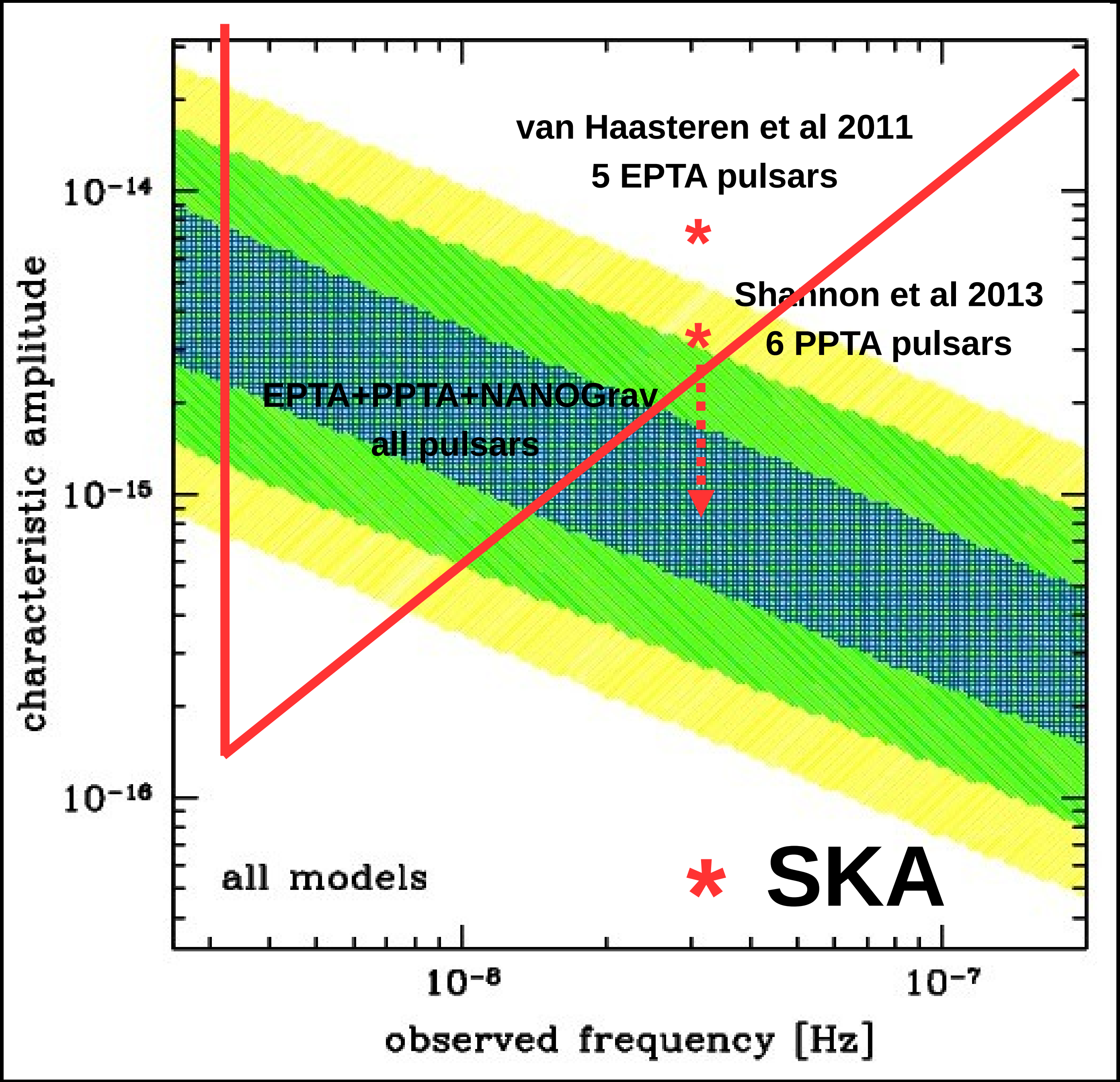


AS et al. 2008; AS et al. 2009; AS 2013

Summary:

Gravitational waves from massive black hole binaries will be extremely useful tools in cosmology and fundamental physics

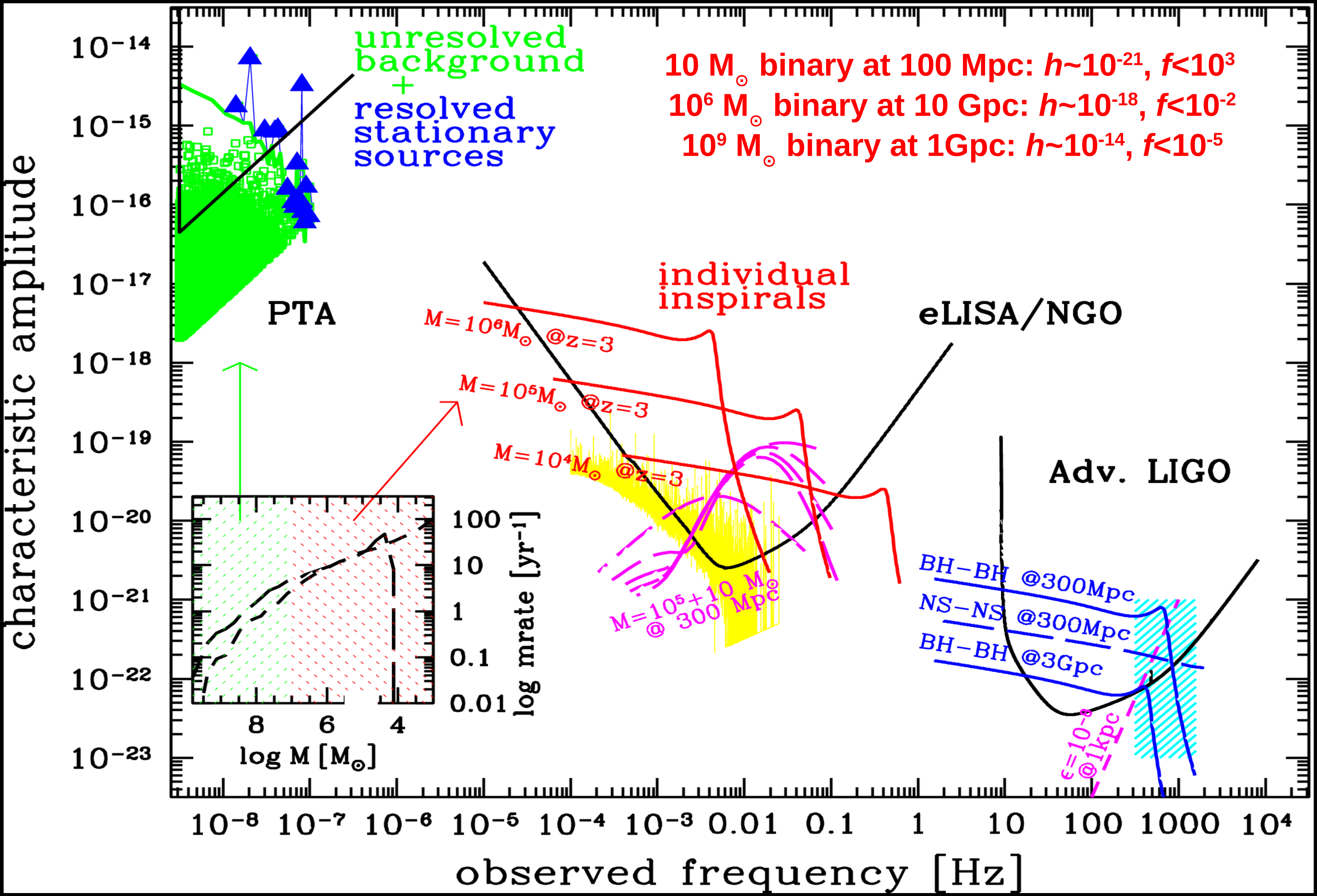
eLISA observations can in principle constrain H_0 to 1% and w to 10% (competitive and independent measurement)

eLISA will place the strongest dynamical bound on the graviton mass ($\lambda_c > 10^{17}$km)

MBHB merger rate will tell us about the formation and evolution of seed Bhs (maybe useful to discriminate the nature of Dark Matter?)

PTA might also place useful constrains on the nature of gravity by setting limit on the graviton mass and on extra GW polarizations

Coverage of the GW spectrum



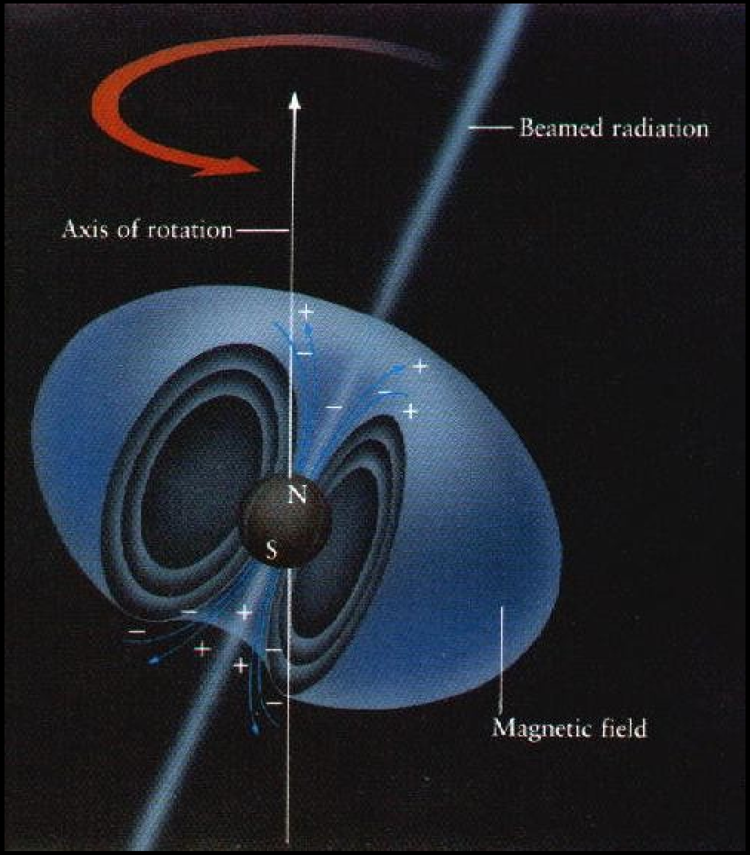
AS 2013

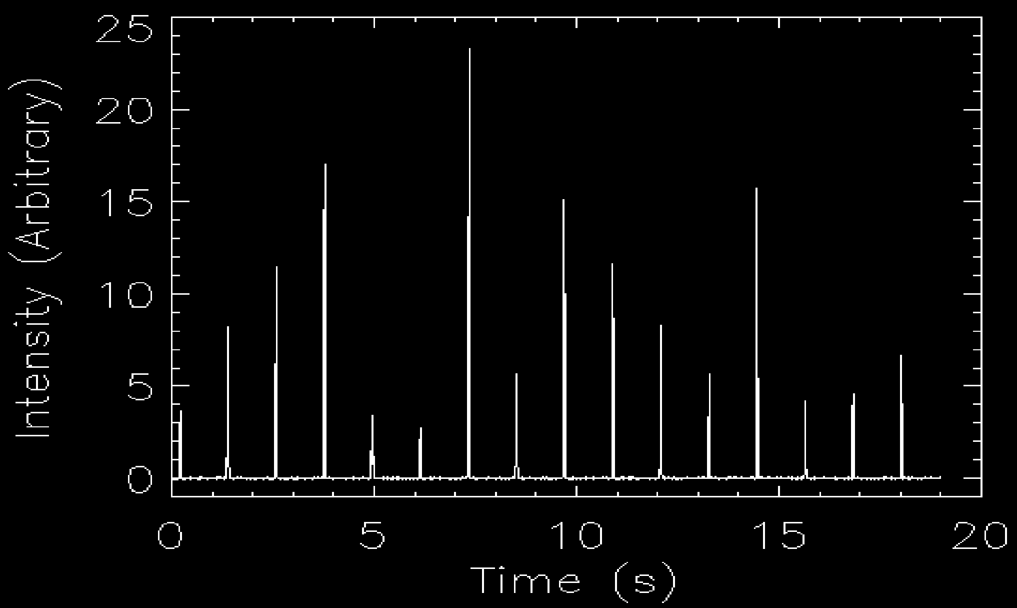
What is pulsar timing?

Pulsars are neutron stars that emit regular burst of radio radiation

Pulsar timing is the process of measuring the time of arrival (TOA) of each individual pulse and then subtracting off the expected time of arrival given a physical model for the system.

1- Obseve a pulsar and measure the TOA of each pulse





2-Determine the model which best fits the TOA data

$$t_{\mathrm{e}}^{\mathrm{psr}} = t_{\mathrm{a}}^{\mathrm{obs}} - \Delta_{\odot} - \Delta_{\mathrm{IS}} - \Delta_{\mathrm{B}}.$$

The emission time at the pulsar is converted to the observed time at the Earth modelling several time delays due to:

- -coordinate transformations
- -GR effects (e.g. Shapiro delay, PN binary dynamics)
- -Propagation uncertainties (e.g. Atmospheric delay, ISM dispersion)

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3-Calculate the timing residual *R*

$$R = \mathrm{TOA} - \mathrm{TOA}_m$$

If your model is perfect, then $R=0$. *R* contains all the uncertainties related to the signal propagation and detection plus the effect of unmodelled physics, like -possibly- ***gravitational waves***

Heuristic scalings

We want compact accelerating systems
Consider a BH binary of mass M, and semimajor axis a

$$h \sim \frac{R_S}{a}\frac{R_S}{r} \sim \frac{(GM)^{5/3}(\pi f)^{2/3}}{c^4 r}$$

In astrophysical scales

$$h \sim 10^{-20}\,\frac{M}{M_\odot}\,\frac{\mathrm{Mpc}}{D}$$

$$f \sim \frac{c}{2\pi R_s} \sim 10^4\,\mathrm{Hz}\,\frac{M_\odot}{M}$$

10 $M_\odot$ binary at 100 Mpc: $h \sim 10^{-21}$, $f < 10^{3}$

10^6 $M_\odot$ binary at 10 Gpc: $h \sim 10^{-18}$, $f < 10^{-2}$

10^9 $M_\odot$ binary at 1Gpc: $h \sim 10^{-14}$, $f < 10^{-5}$