

# Cosmological constraints on Lorentz Invariance of the Universe

PONT d'Avignon 2014



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in collaboration with Benjamin Audren, Diego Blas,  
Julien Lesgouques, Sergey Sibiryakov

JCAP 1210 (2012) 057 [[arXiv:1209.0464](https://arxiv.org/abs/1209.0464)]  
[arXiv:1404.xxxx](https://arxiv.org/abs/1404.xxxx)

All current data are compatible with  
the  $\Lambda CDM$  model  
(assumes Lorentz Invariance as a  
fundamental property of Nature)



Reasons to question this:

Recent successes of Lorentz-violating  
theory of quantum gravity (Horava' 09)

Lorentz invariance has been tightly  
constrained only in the sector of Standard  
Model particles

$$< 10^{-20}$$

What about other sectors?



For other sectors  
bounds are milder or even don't exist!

**Gravity**

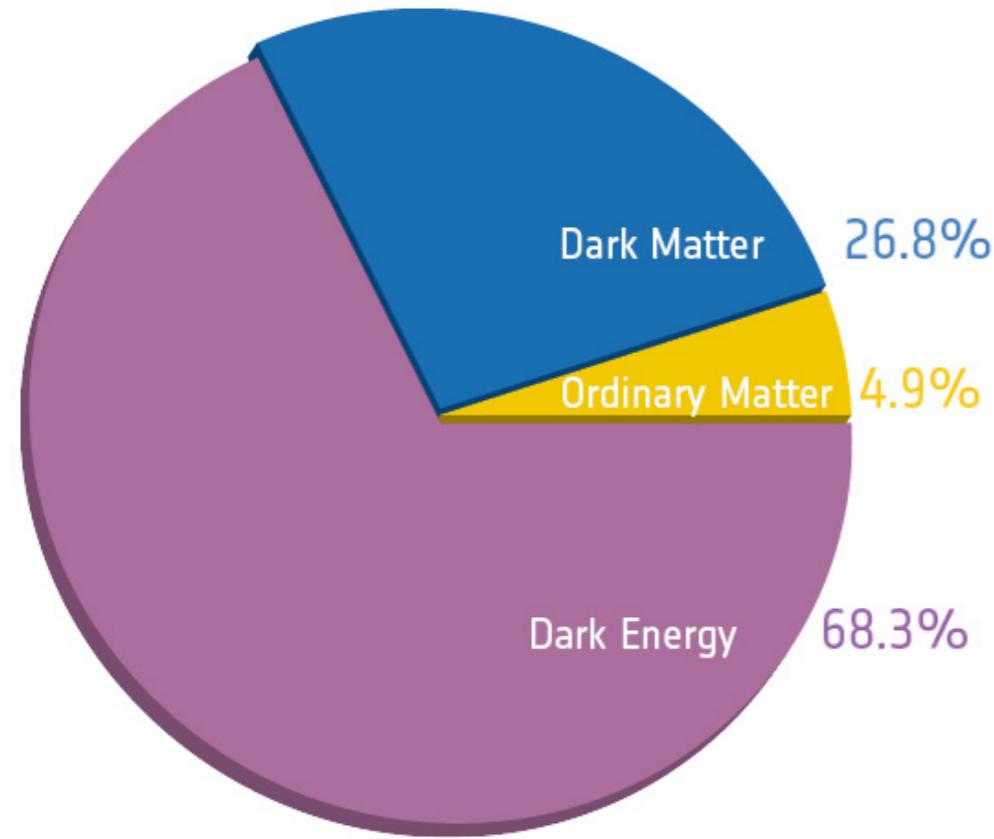
$< 10^{-7}$

**Dark Matter**

???

**Dark Energy**

$< 10^{-2}?$



Given the key role played by LI in modeling Nature, it is essential to test it to the best possible accuracy in all the sectors

# Outline of my talk:



Gravity theory with broken Lorentz Invariance

Physical aspects of Lorentz violation in cosmology:

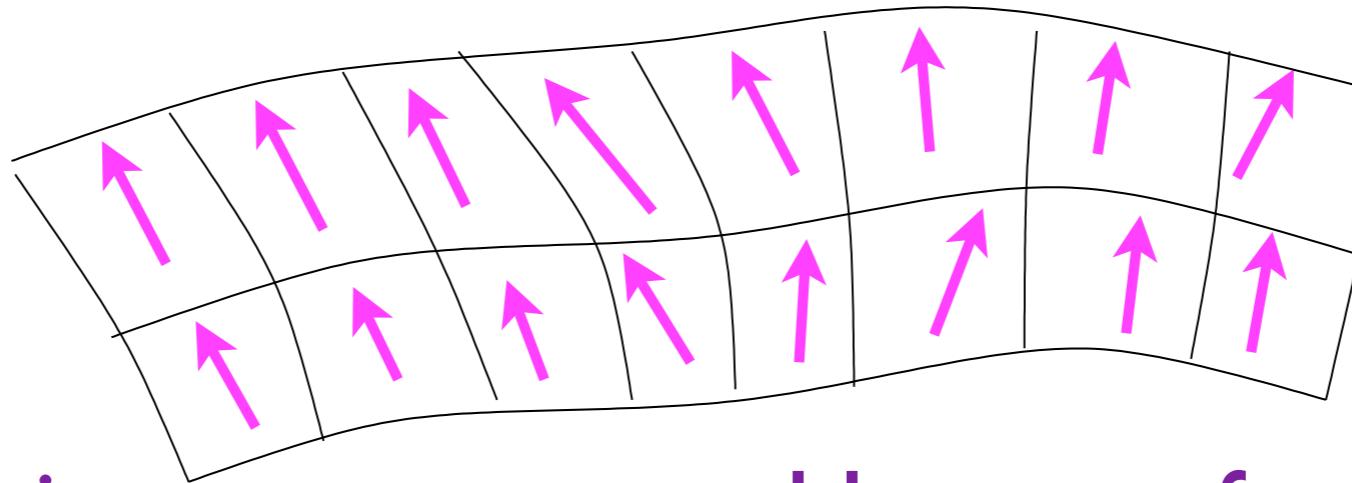
LV →

More structures  
Mimicking Dark Radiation

Observational bounds on LV **in gravity** and **dark matter**

# Breaking Lorentz Invariance

Space-time filled by a preferred **time** direction  
Associated to a time-like unit vector  $u_\mu$



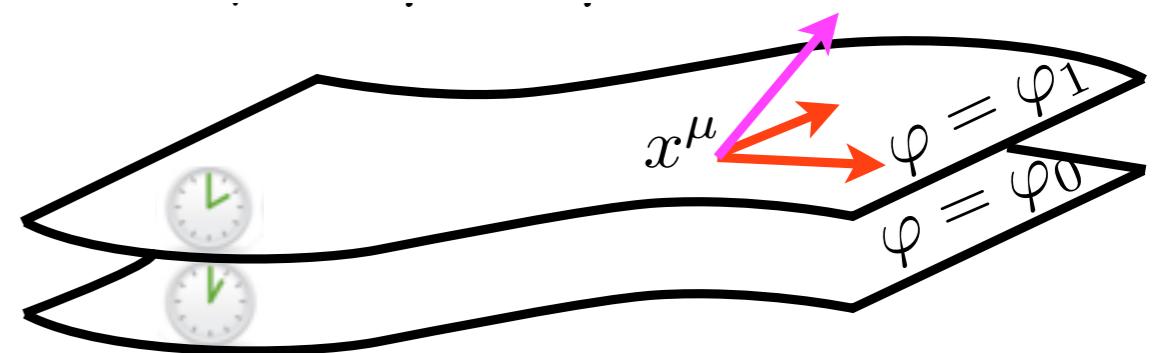
Generic:  
Einstein-aether theory

$$u^\mu u_\mu = 1$$

Jacobson,  
Mattingly' 01

Hypersurface orthogonal:  
Khronon

$$u_\mu = \frac{\partial_\mu \varphi}{\sqrt{(\partial\varphi)^2}}$$



# Gravity Lagrangian:

Einstein-aether:  $\mathcal{L}_{GR} + \mathcal{L}_u$

$$\mathcal{L}_u \sim c_1, c_2, c_3, c_4 M_P^2 (\nabla u_\mu)^2$$

Khronometric:

$$u_\mu = \frac{\partial_\mu \varphi}{\sqrt{(\partial \varphi)^2}}$$

$c_i$  are not independent!

$$\begin{aligned}\lambda &\equiv c_2 \\ \beta &\equiv c_1 + c_3 \\ \alpha &\equiv c_1 + c_4\end{aligned}$$

Both theories have the same scalar and tensor sectors!  
(completely characterized by  $\alpha, \beta, \lambda$ )

Vectors not relevant for CMB-TT and LSS

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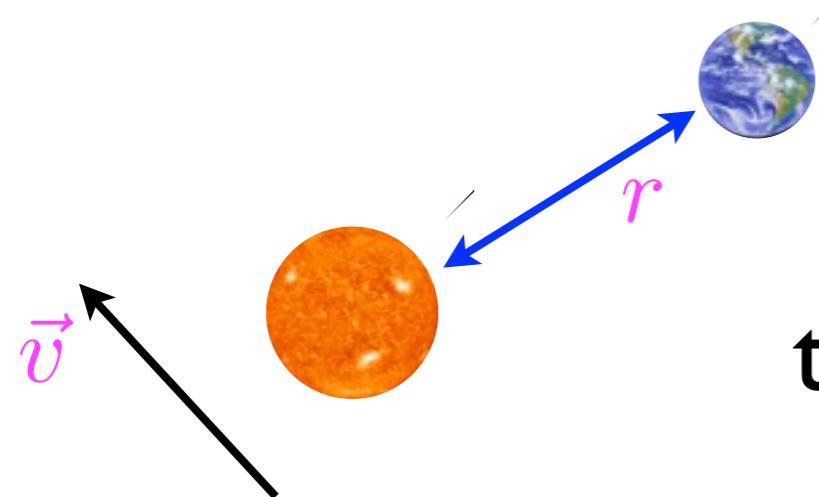
Bottom-left box:  
Low-energy limit of Horava-Lifshitz gravity

$$\Lambda_{IR} \sim \sqrt{\alpha} M_P$$

sectors!

SS

## Constraints from the visible sector



PPN bounds  $|\alpha, \beta, \lambda| < 10^{-7}$   
can be avoided for  
the special choice of parameters:

**Khronometric:**  $\alpha = 2\beta$

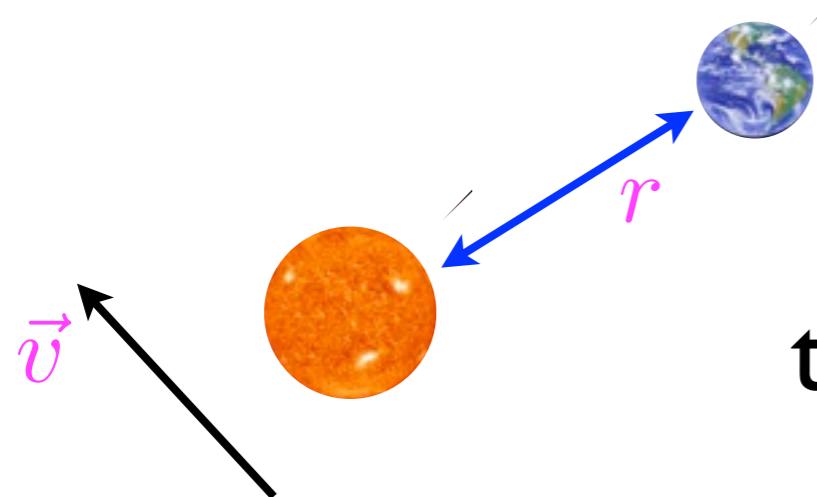
**Einstein-aether:**  $\alpha = -(\beta + 3\lambda)$

Constraints from GW emission in  
binary systems:

$$|\alpha, \beta, \lambda| \lesssim 0.01$$



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Khron  
Einst

Can be improved with  
cosmology !



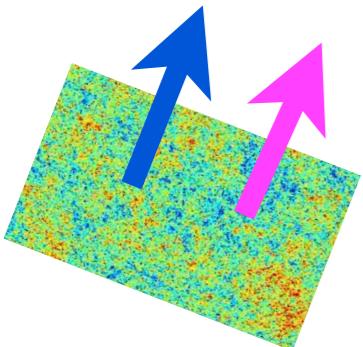
# Relativistic cosmology

$$G_{\mu\nu} = \frac{1}{M_P^2} T_{\mu\nu}^{SM} + \frac{1}{M_P^2} T_{\mu\nu}^{dm, LV} + \frac{1}{M_P^2} T_{\mu\nu}^{aether} + \Lambda g_{\mu\nu}$$

$$T_{\mu\nu}^{dm, LV} = T_{\mu\nu}^{dm} + \textcolor{red}{Y} \cdot \rho_{[dm]} O(u_\mu v_\nu^{[dm]})$$

D.Blas, MI, S.Sibiryakov' 12

**Background:** Homogeneous and isotropic  
(preferred foliation aligned with CMB frame)



$$\begin{aligned} ds^2 &= g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a(t)^2 dx^i dx^i \\ u_\mu &= (u_0(t), 0, 0, 0) = v_\mu \quad , \quad \rho(t) \end{aligned}$$

Friedmann equations almost not modified!

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_c}{3} \rho_m$$

$$G_c = \frac{1}{8\pi M_P^2 [1 + 3\lambda/2 + \beta/2]}$$

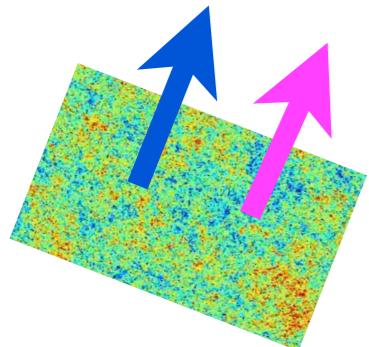
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$$ds^2 = g_{\mu\nu} dx^\mu$$
$$u_\mu = (u_0(t), 0)$$

Friedmann equ

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_c}{3} \rho_m$$

different from

$$G_N = \frac{1}{8\pi M_P^2 (1 - \alpha/2)}$$

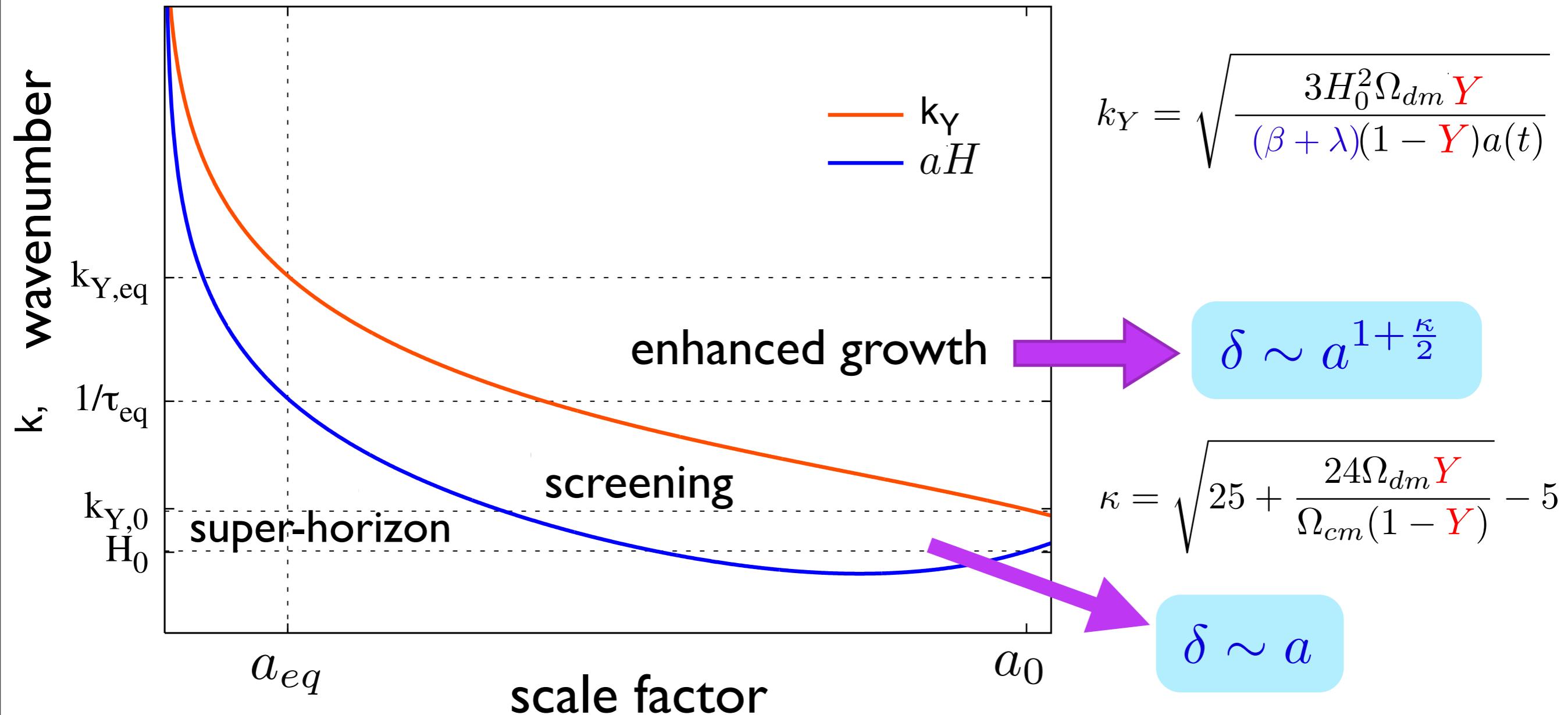


$$G_c = \frac{1}{8\pi M_P^2 [1 + 3\lambda/2 + \beta/2]}$$

# Effects on perturbations: LV in DM

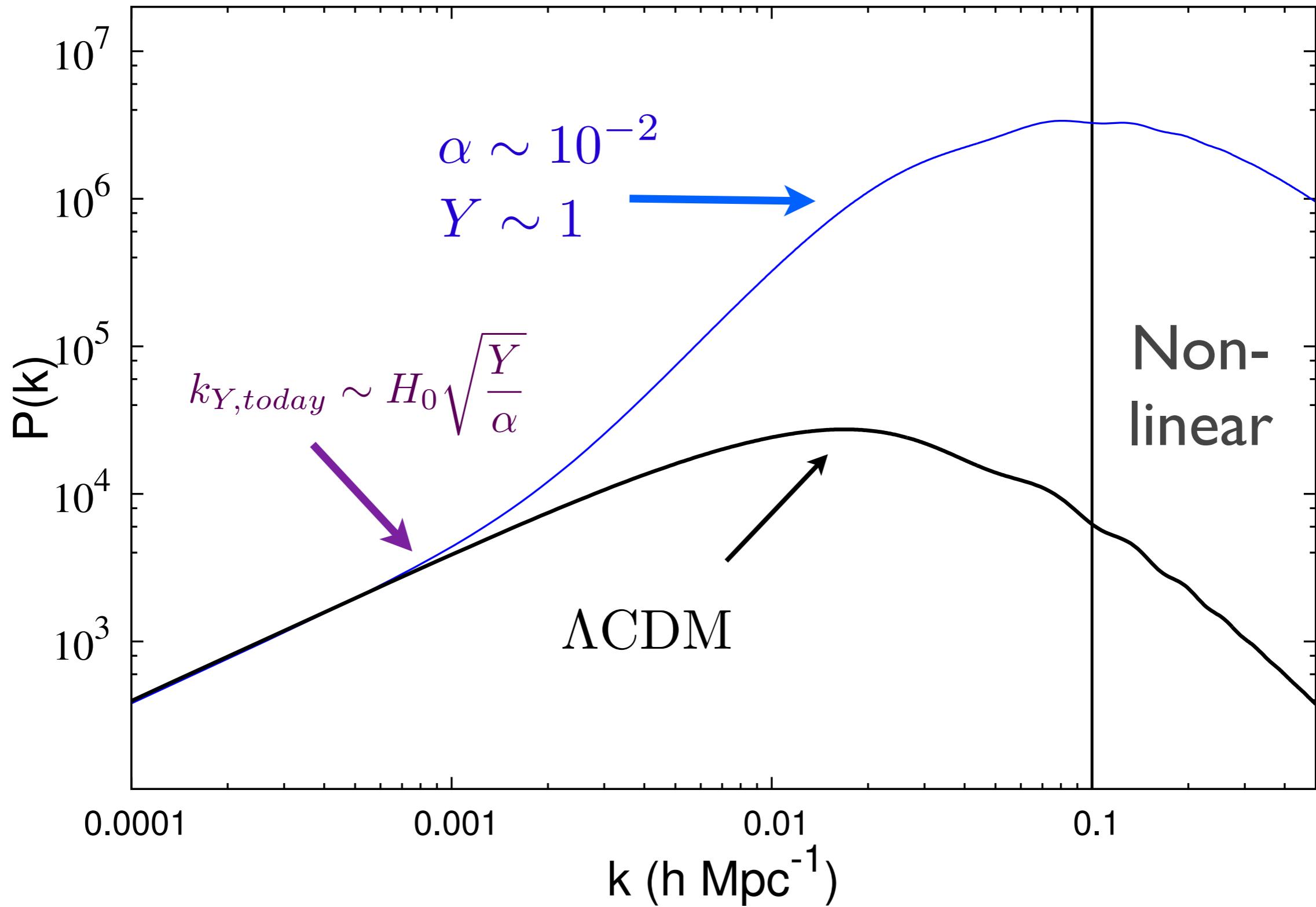
$$\rho(x, t) \equiv \rho(t)(1 + \delta(x, t))$$

Screening horizon

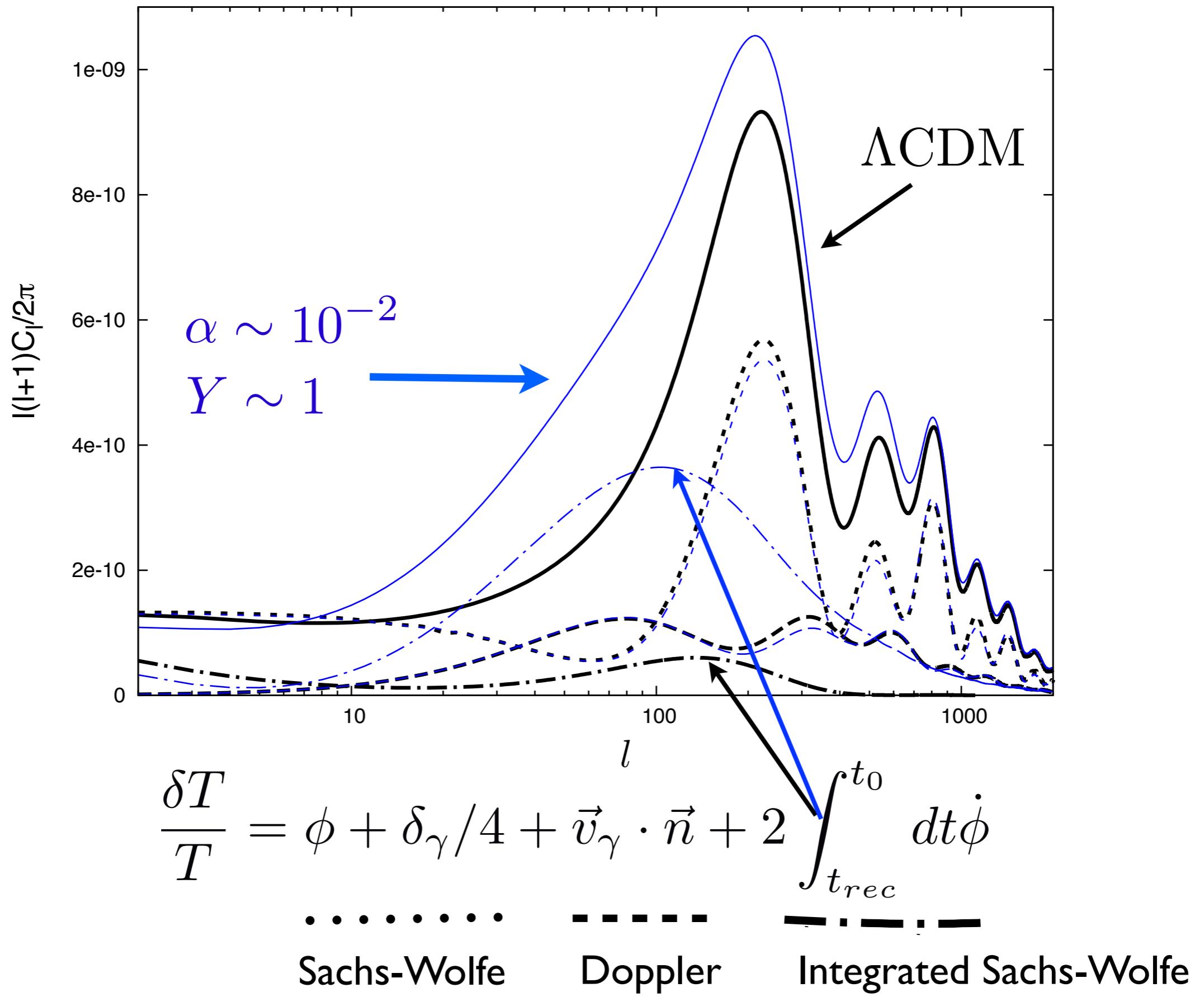


# LV in DM: Matter power spectrum

$$\langle \delta(k)\delta(k') \rangle \equiv \delta^{(3)}(k+k')P(k)k^3$$



# LV in DM: Cosmic microwave background



# LV in gravity: effects on perturbations

I) Modified Poisson equation:

$$k^2 \phi = -4\pi G_N a^2 [\Sigma \rho_i \delta_i + \delta \rho_{aether}] \quad H^2 = 8\pi G_c \rho / 3$$

different from  $G_c$ ,  $\frac{G_N}{G_c} - 1 = \frac{\alpha + \beta + 3\lambda}{2} + O(2)$

DM, baryons

matter domination

$$\delta \sim \tau^{(-1 + \sqrt{1 + 24 \frac{G_N}{G_c}})/2}$$

+ Solar system constraints

**Khronometric** ( $\alpha = 2\beta$ )

$$\frac{G_N}{G_c} - 1 = \frac{3(\beta + \lambda)}{2} + O(2) > 0$$

**Einstein-Aether:**

$$\frac{G_N}{G_c} - 1 = O(2)$$

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II) Anisotropic stress: aether



$$ds^2 = a(t)^2 [(1 + 2\phi)dt^2 - \delta_{ij}(1 - 2\psi)dx^i dx^j]$$

$$\phi - \psi = O(\beta)$$

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DM,

matter

Kin

$$\frac{G_N}{G_c} - 1 = -\frac{1}{2} + O(2) > 0$$

$$\frac{G_N}{G_c} - 1 = O(2)$$

Aether - effective relativistic dof,  
undergoes free-streaming

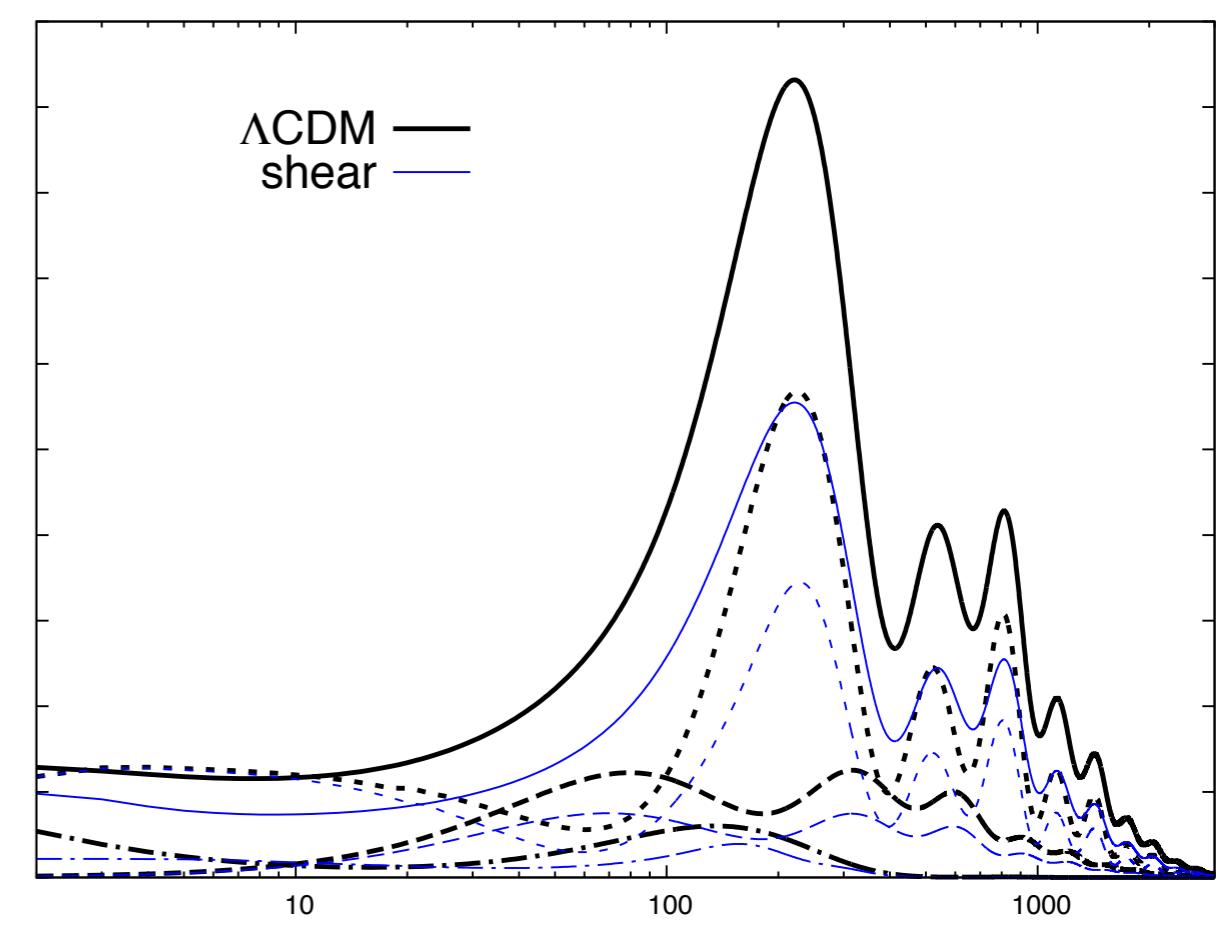
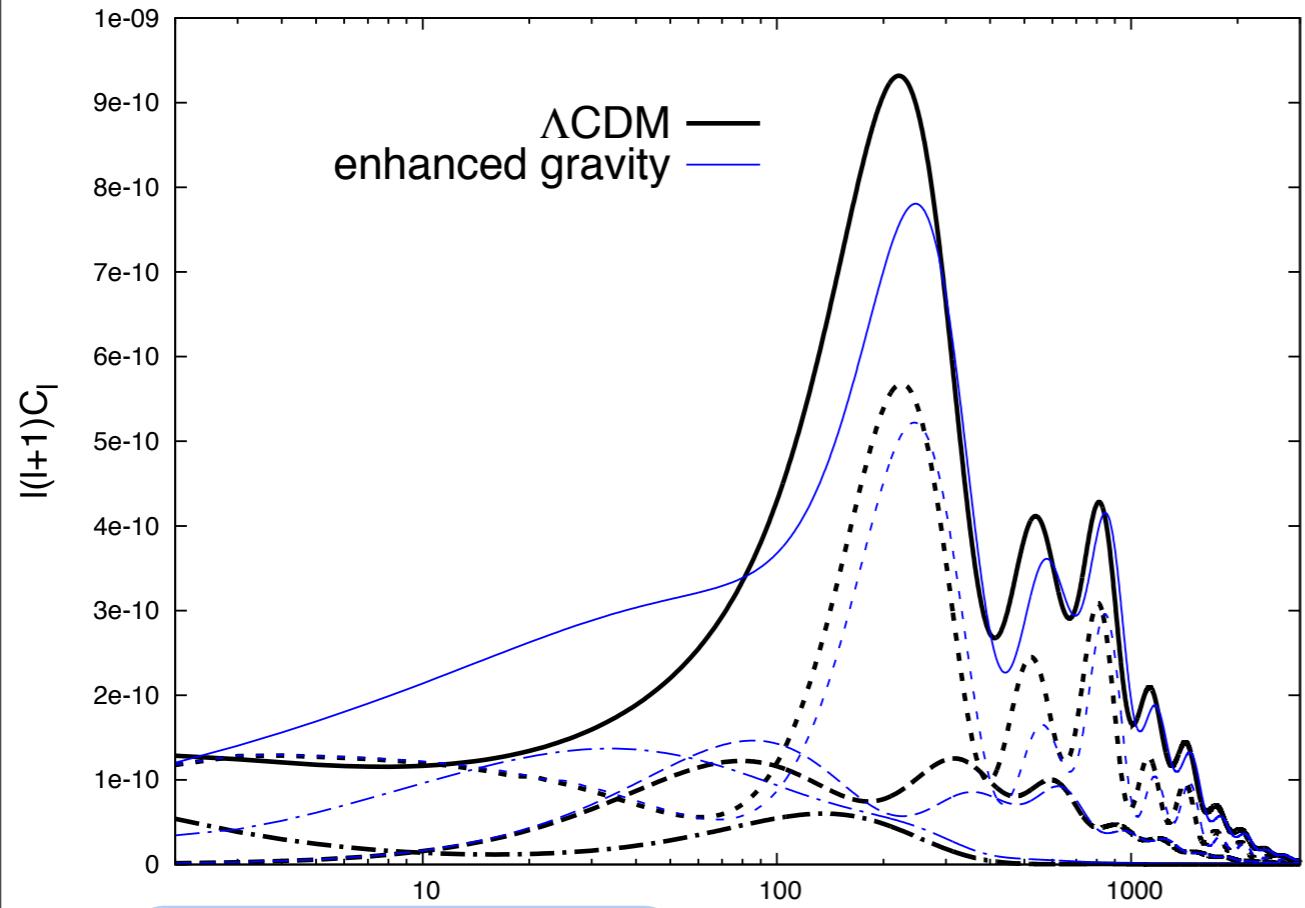
II) Anisotropic stress: aether

$$ds^2 = a(t)^2 [(1 + 2\phi)dt^2 - \delta_{ij}(1 - 2\psi)dx^i dx^j]$$



$$\phi - \psi = O(\beta)$$

# LV in gravity: effects in CMB



$$\frac{G_N}{G_c} - 1 \sim 1 \quad \beta = 0 \quad Y = 0$$

$$\beta \sim 1 \quad \frac{G_N}{G_c} - 1 = 0 \quad Y = 0$$

$$\frac{\delta T}{T} = \phi + \delta_\gamma/4 + \vec{v}_\gamma \cdot \vec{n} + 2 \cdots \cdots \cdots \cdots \cdots \cdots$$

Sachs-Wolfe

Doppler

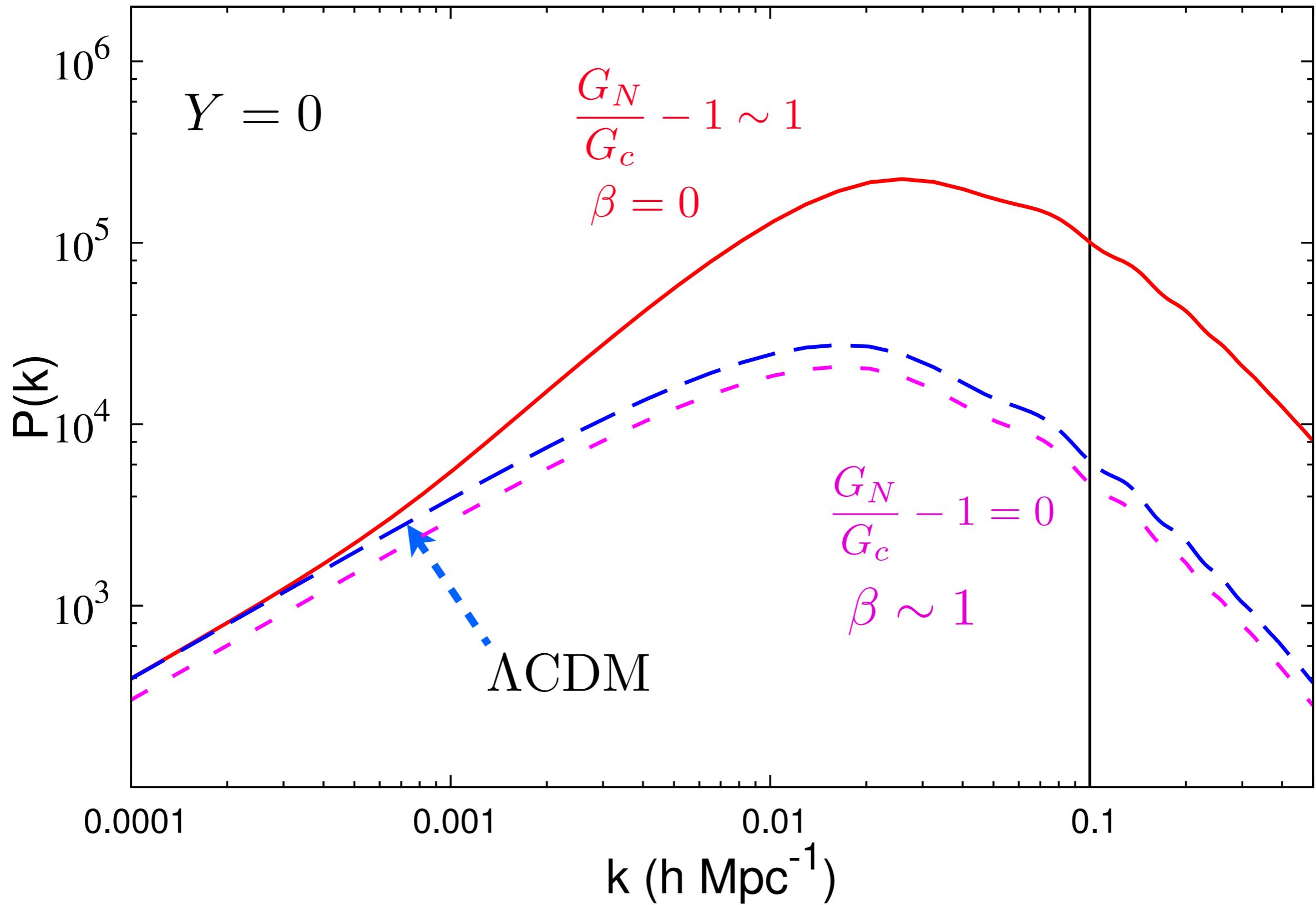
$$\int_{t_{rec}}^{t_0} dt \dot{\phi}$$

Integrated Sachs-Wolfe

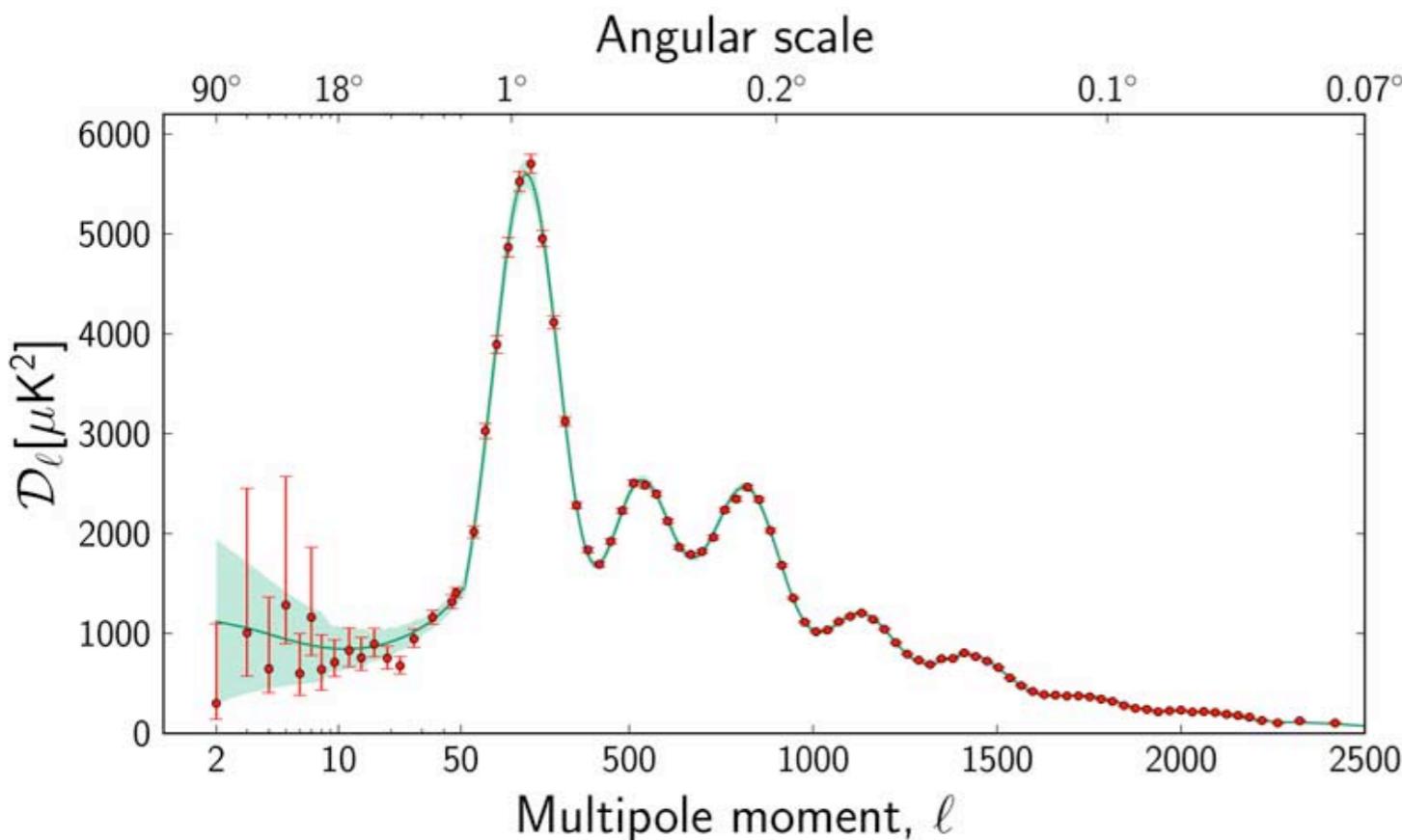
B.Audren, D.Blas,  
J.Lesgourgues,  
S.Sibiryakov 13'

# LV in gravity: effects in MPS

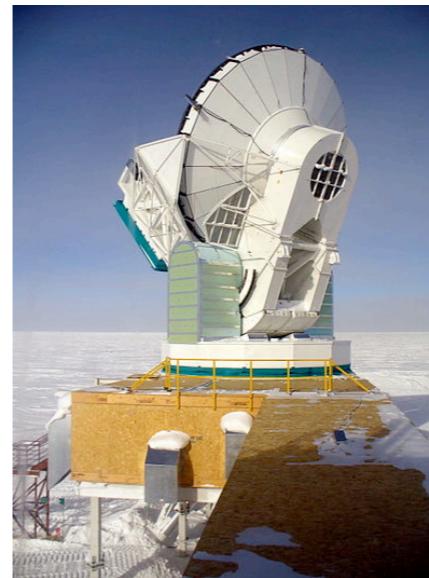
$$\langle \delta(k)\delta(k') \rangle \equiv \delta^{(3)}(k+k')P(k)k^3$$



# Observational data



Planck



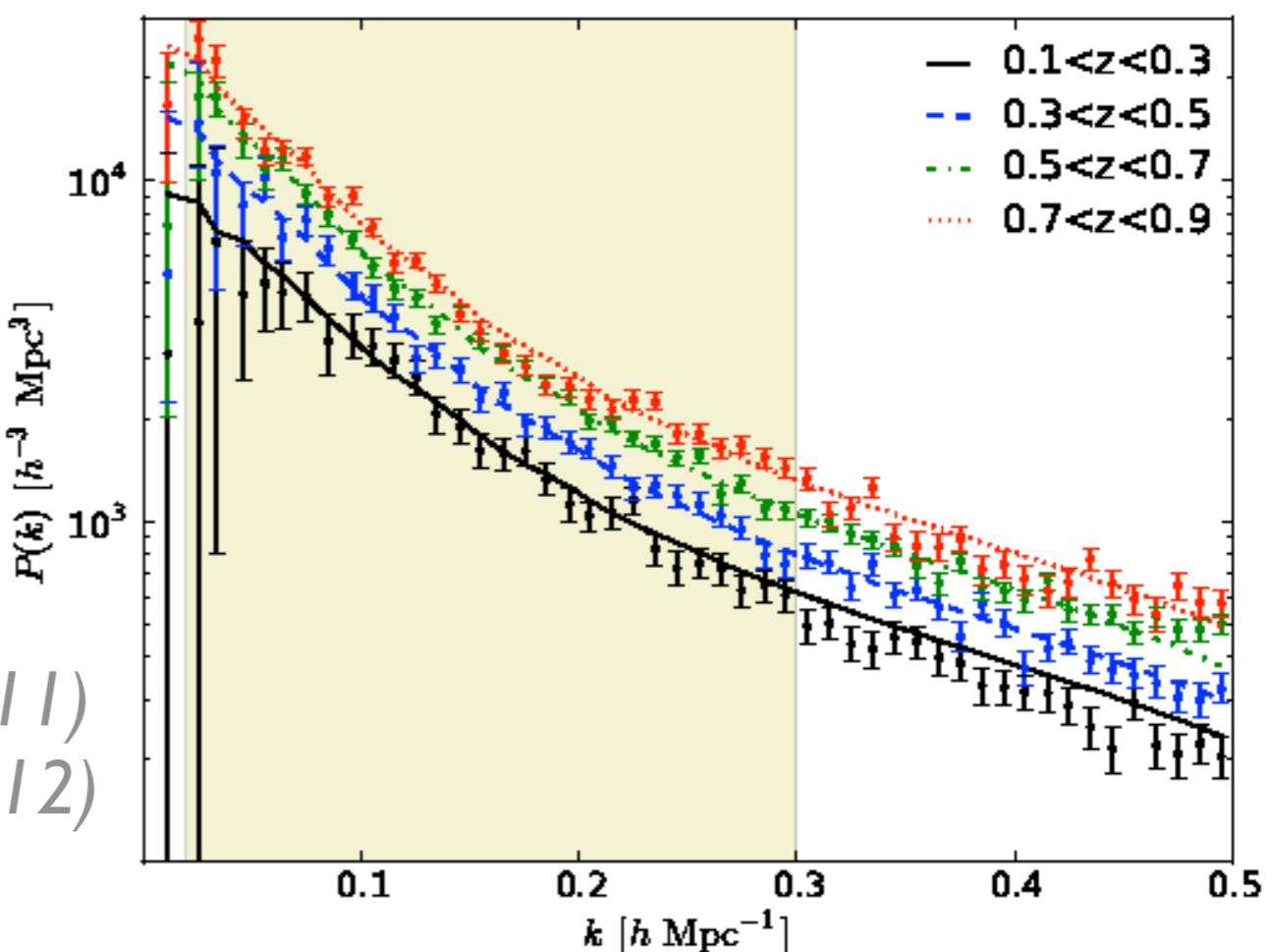
SPT



WiggleZ survey



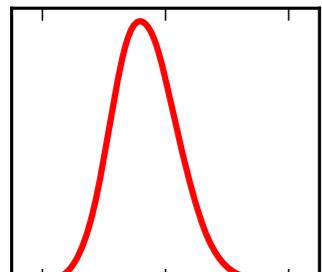
use CLASS Blas, Lesgourgues, Tram (2011)  
and MONTE PYTHON Audren et. al. (2012)



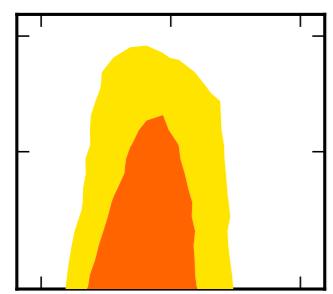
# Cosmological bounds

95% CL upper limits

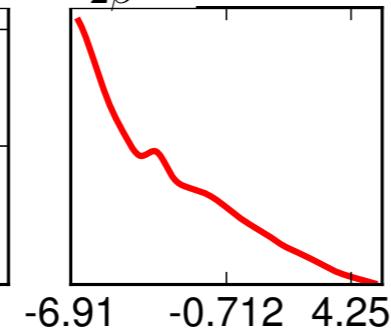
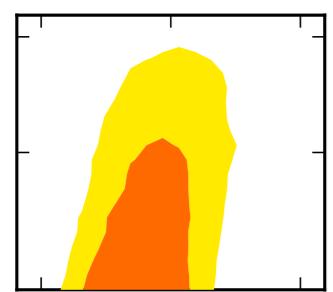
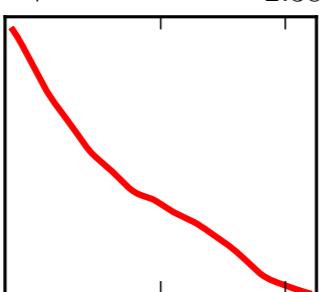
$$H_0 = 67.9^{+0.961}_{-1.15}$$



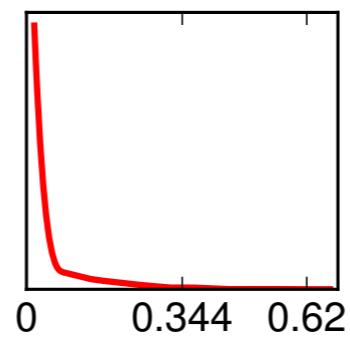
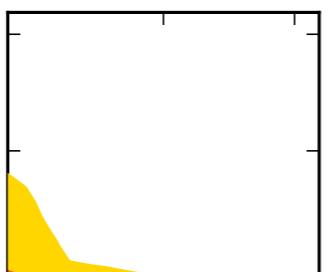
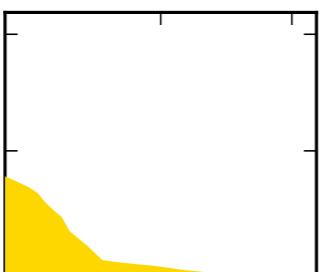
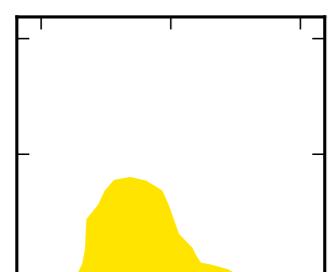
$$\ln \beta = -9.14^{+0.73}_{-2.38}$$



$$\ln \frac{\beta + \lambda}{2\beta} = -3.49^{+0.000878}_{-3.42}$$



$$Y = 0.045^{+0.00815}_{-0.045}$$



PPN Khronometric  $\alpha = 2\beta$

$$\beta < 0.004$$

$$\beta + \lambda < 2 \cdot 10^{-4}$$

$$Y < 0.2$$

PPN Einstein-aether

$$3\lambda = -(\alpha + \beta)$$

$$\alpha < 0.004$$

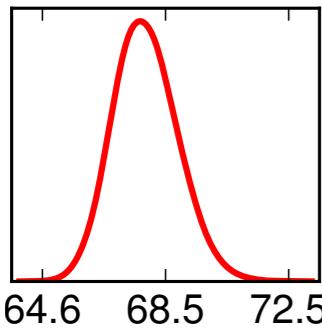
$$\beta < 0.003$$

$$Y < 0.6$$

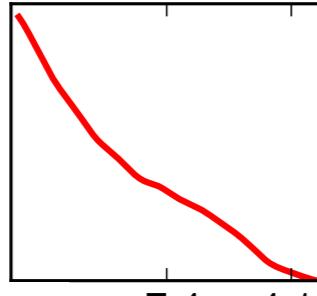
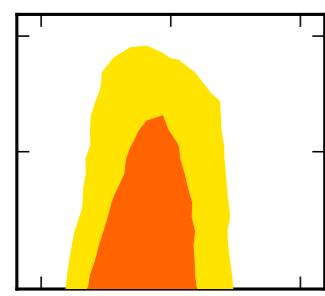
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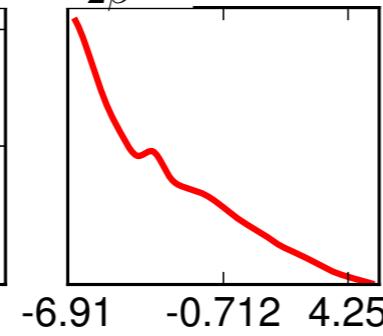
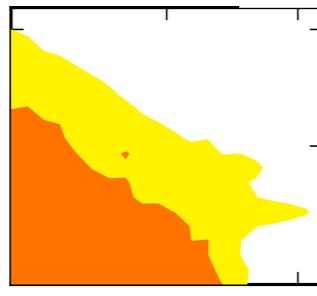
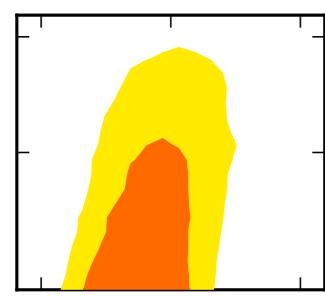
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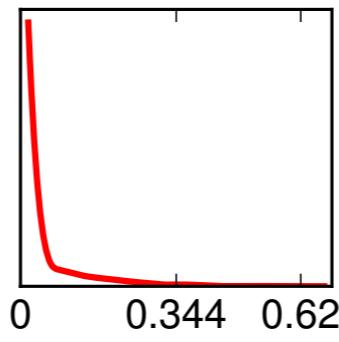
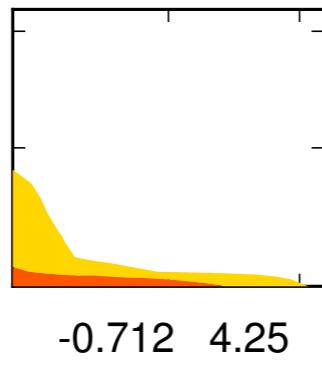
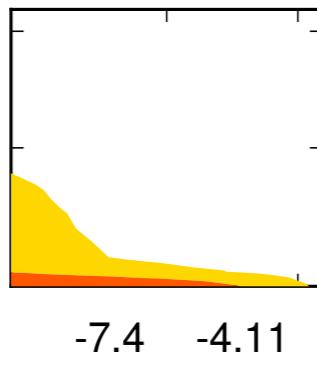
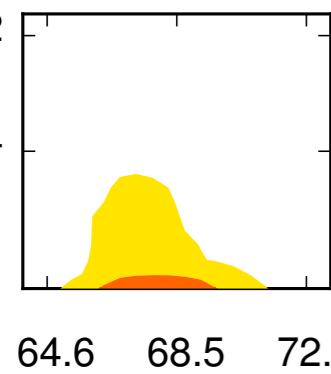
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$$\ln \frac{\beta + \lambda}{2\beta} = -3.49^{+0.000878}_{-3.42}$$



$$Y = 0.045^{+0.00815}_{-0.045}$$



Correlation with  $H_0$   
Khronon resembles dark radiation for  
some LV parameters !

PPN Einstein-aether

$$3\lambda = -(\alpha + \beta)$$

$$\alpha < 0.004$$

$$\beta < 0.003$$

$$Y < 0.6$$

## Conclusions:



Lorentz violation is a consistent framework to test deviations from  $\Lambda$ CDM



Consequences of LV in cosmology:  
accelerated growth of structures

+

additional cosmic stress



Bounds at the level  $\text{few} \times 10^{-3}$  on LV in gravity  
and  $(0.01 - 0.1)$  on LV in Dark Matter  
(depending on LV in gravity)

## Outlook:



Nonlinear structure formation, ‘DM problems’

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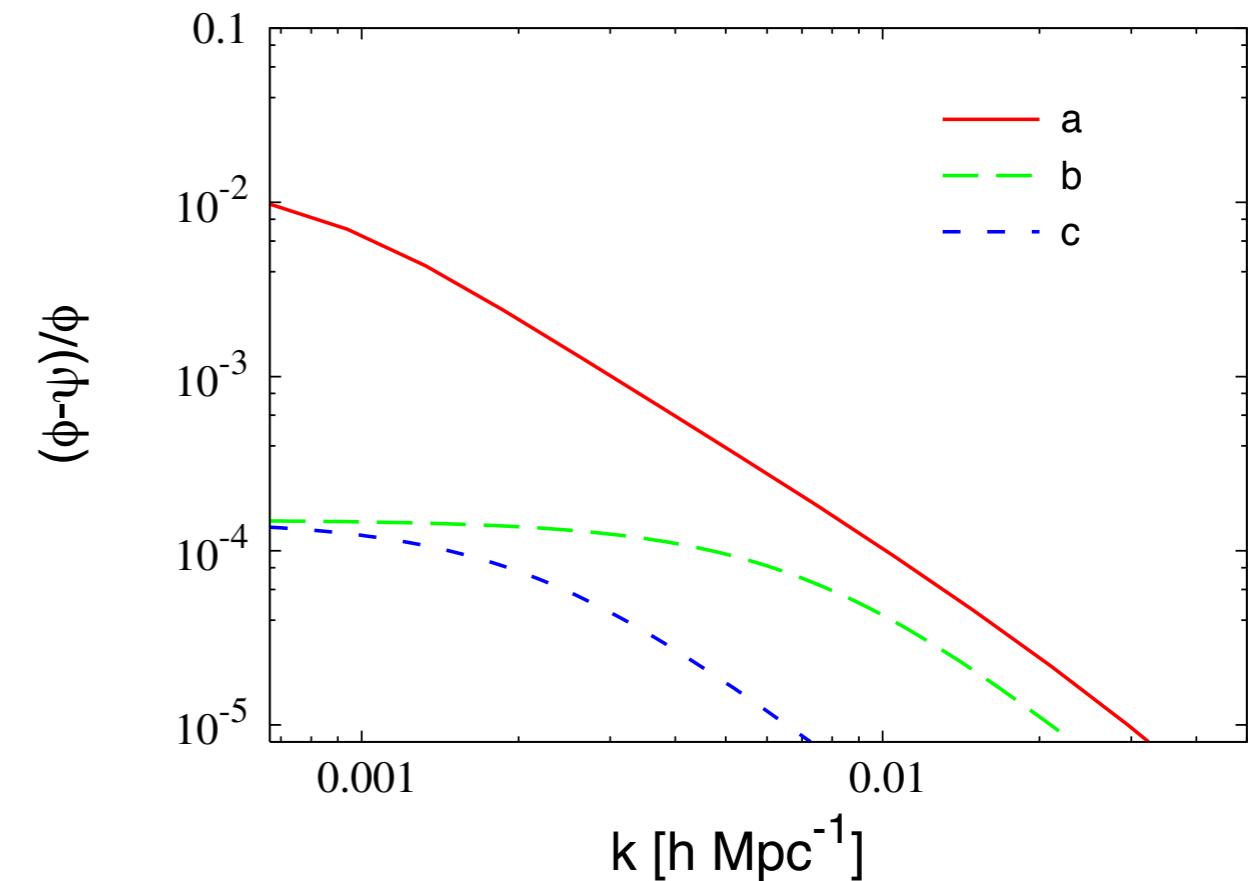
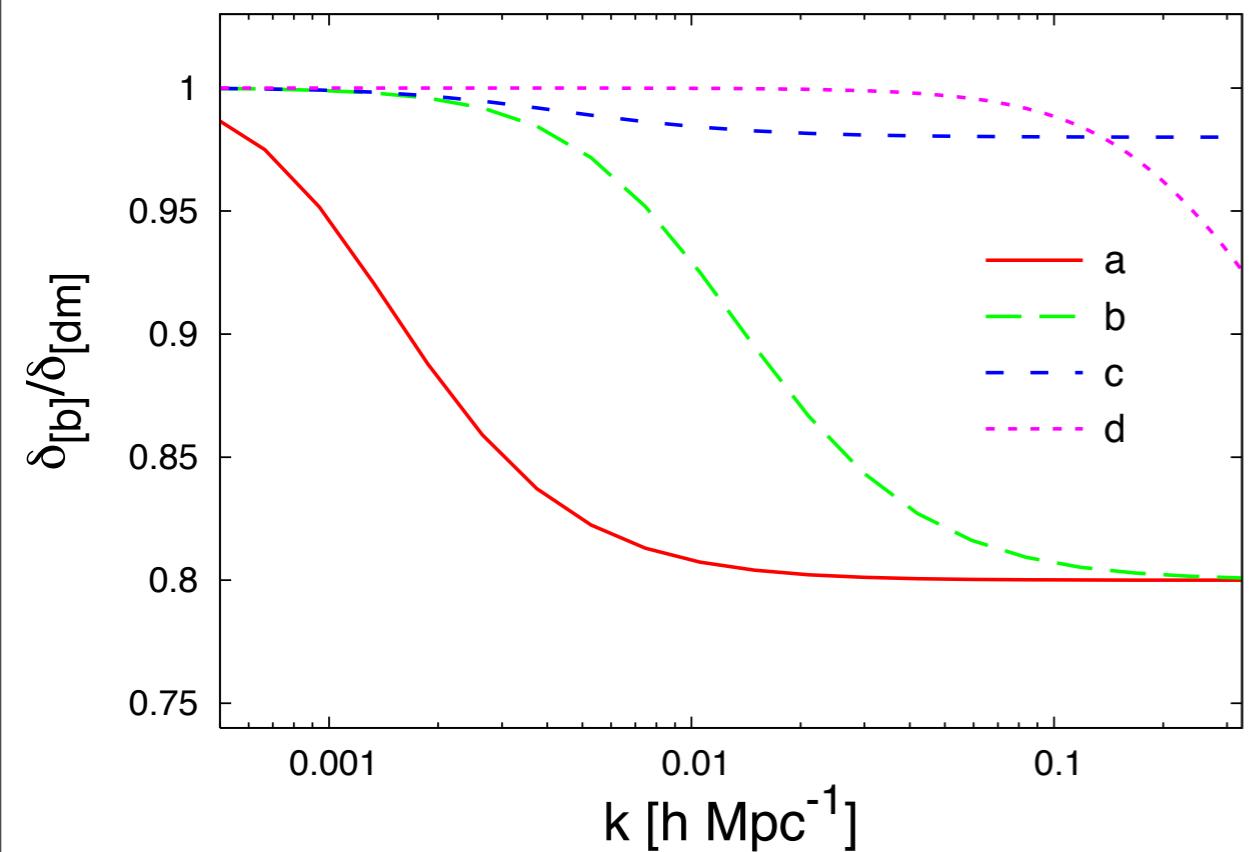
Thank you for your attention!

(depending on LV in gravity)

## Outlook:

- \* Nonlinear structure formation, ‘DM problems’

# Baryonic bias and anisotropic stress:



	$\alpha$	$\beta$	$\lambda$	$Y$	$k_{Y,0} (\mathrm{h \, Mpc}^{-1})$	$k_{Y,eq} (\mathrm{h \, Mpc}^{-1})$
a	$2 \cdot 10^{-2}$	$10^{-2}$	$10^{-2}$	0.2	$9.2 \cdot 10^{-4}$	$6.5 \cdot 10^{-2}$
b	$2 \cdot 10^{-4}$	$10^{-4}$	$10^{-4}$	0.2	$9.1 \cdot 10^{-3}$	0.65
c	$2 \cdot 10^{-4}$	$10^{-4}$	$10^{-4}$	0.02	$2.6 \cdot 10^{-3}$	0.18
d	$10^{-7}$	0	$10^{-7}$	0.2	0.41	29

## Gravity action: $g_{\mu\nu}, u^\mu$

Einstein-aether:

$$S_{\text{ae}} \equiv -\frac{M_0^2}{2} \int d^4x \sqrt{-g} \left[ R + K^{\mu\nu}{}_{\sigma\rho} \nabla_\mu u^\sigma \nabla_\nu u^\rho + l(u_\mu u^\mu - 1) \right]$$

$$K^{\mu\nu}{}_{\sigma\rho} \equiv c_1 g^{\mu\nu} g_{\sigma\rho} + c_2 \delta_\sigma^\mu \delta_\rho^\nu + c_3 \delta_\rho^\mu \delta_\sigma^\nu + c_4 u^\mu u^\nu g_{\sigma\rho}$$

Khronometric:  $u_\mu = \frac{\partial_\mu \varphi}{\sqrt{(\partial\varphi)^2}}$  

$c_i$  are not independent!

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Vectors not relevant for CMB-TT and LSS

# Dark matter

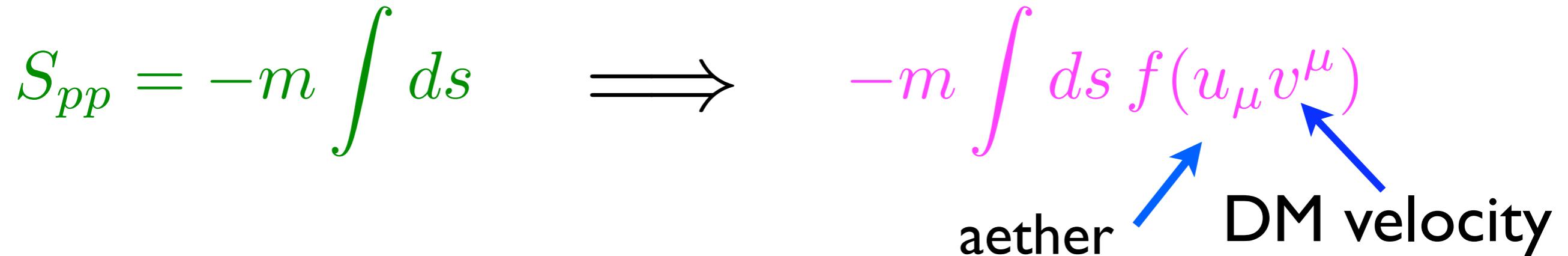
Is non-relativistic ([small velocities](#)).  
Is it possible to test its Lorentz invariance?

[Yes!](#)

# Generalized point particle action

$$S_{pp} = -m \int ds \quad \Longrightarrow \quad -m \int ds f(u_\mu v^\mu)$$

aether      DM velocity



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Newtonian limit:  $u^i, v^i$  - small

$$S = \int d^4x \left[ M_P^2 \phi \Delta \phi + \frac{M_P^2 \alpha}{2} u^i \Delta u^i \right] + \int d^4x \rho \left[ \frac{(v^i)^2}{2} - \phi - Y \frac{(u^i - v^i)^2}{2} \right]$$

DM density       $f'(1)$

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DM density       $f'(1)$

- all effects are encoded in one parameter  $Y$
- modified inertial mass (coefficient in front of  $(v^i)^2$ )

# Generalized point particle action

$$S_{pp} = -m \int ds \quad \Longrightarrow \quad -m \int ds f(u_\mu v^\mu)$$

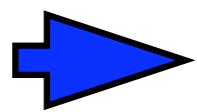
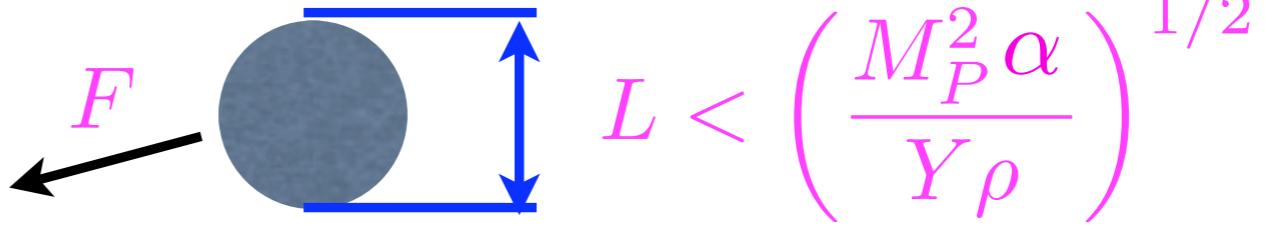
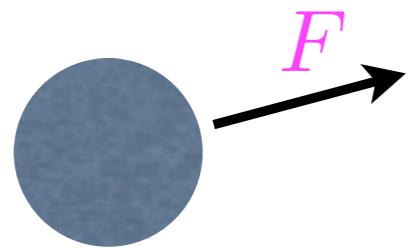
aether      DM velocity

Newtonian limit:  $u^i, v^i$  - small

$$S = \int d^4x \left[ M_P^2 \phi \Delta \phi + \frac{M_P^2 \alpha}{2} u^i \Delta u^i \right] + \int d^4x \rho \left[ \frac{(v^i)^2}{2} - \phi - Y \frac{(u^i - v^i)^2}{2} \right]$$

DM density       $f'(1)$

- all effects are encoded in one parameter  $Y$
- modified inertial mass (coefficient in front of  $(v^i)^2$ )
- effective potential for aether in matter  $m_{eff} = \frac{Y\rho}{\alpha M_P^2}$



$$F = \frac{F_N}{(1 - Y)}$$

$$m_{\text{inert}} = m_{\text{grav}} (1 - Y)$$

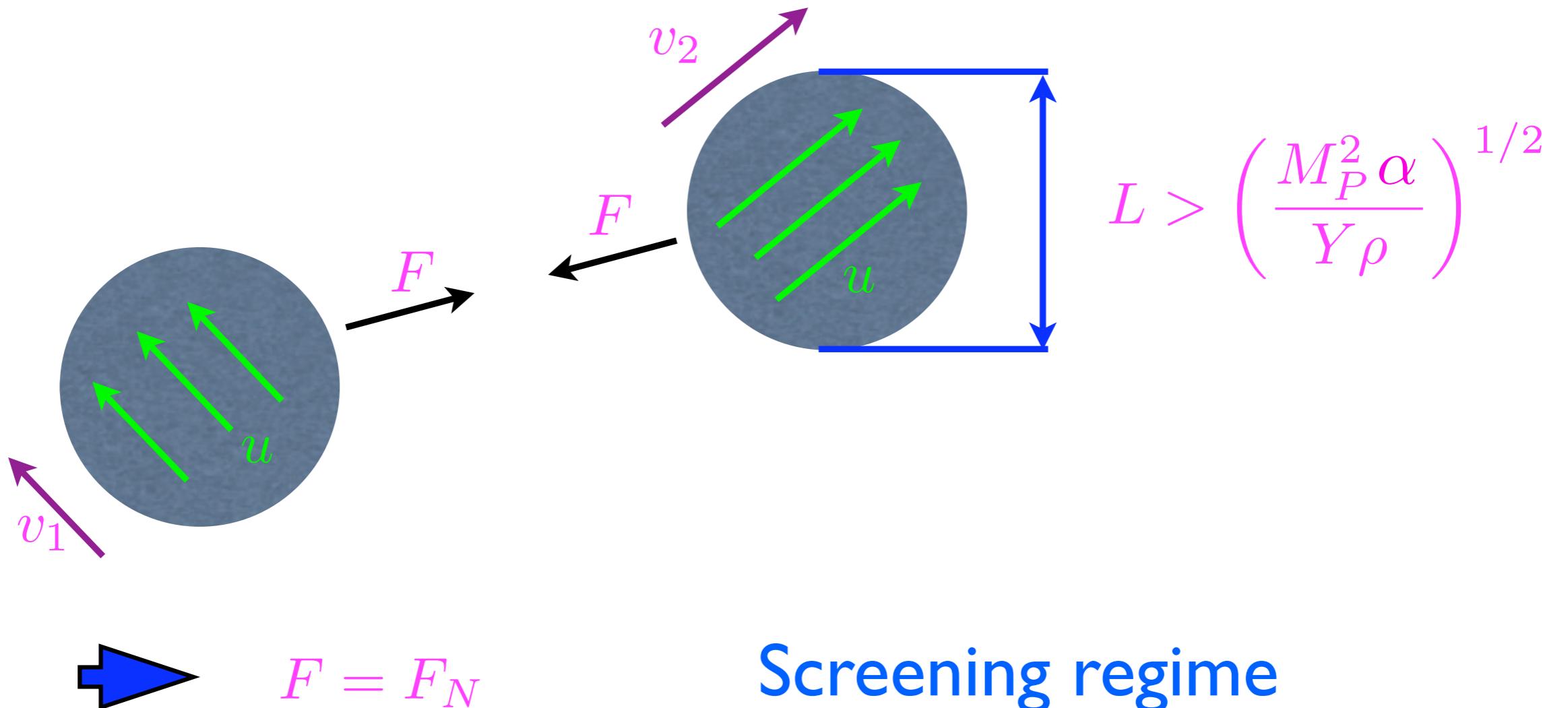
## Accelerated Jeans instability !

$$\delta \propto \tau^\gamma,$$

$$\gamma = \frac{1}{6} \left[ -1 + \sqrt{\frac{25 - Y}{1 - Y}} \right]$$

$$\frac{\delta\rho}{\rho}$$

density contrast



Standard growth of structures  
 chameleon-like mechanism

$$\delta \propto \tau^{2/3}$$