

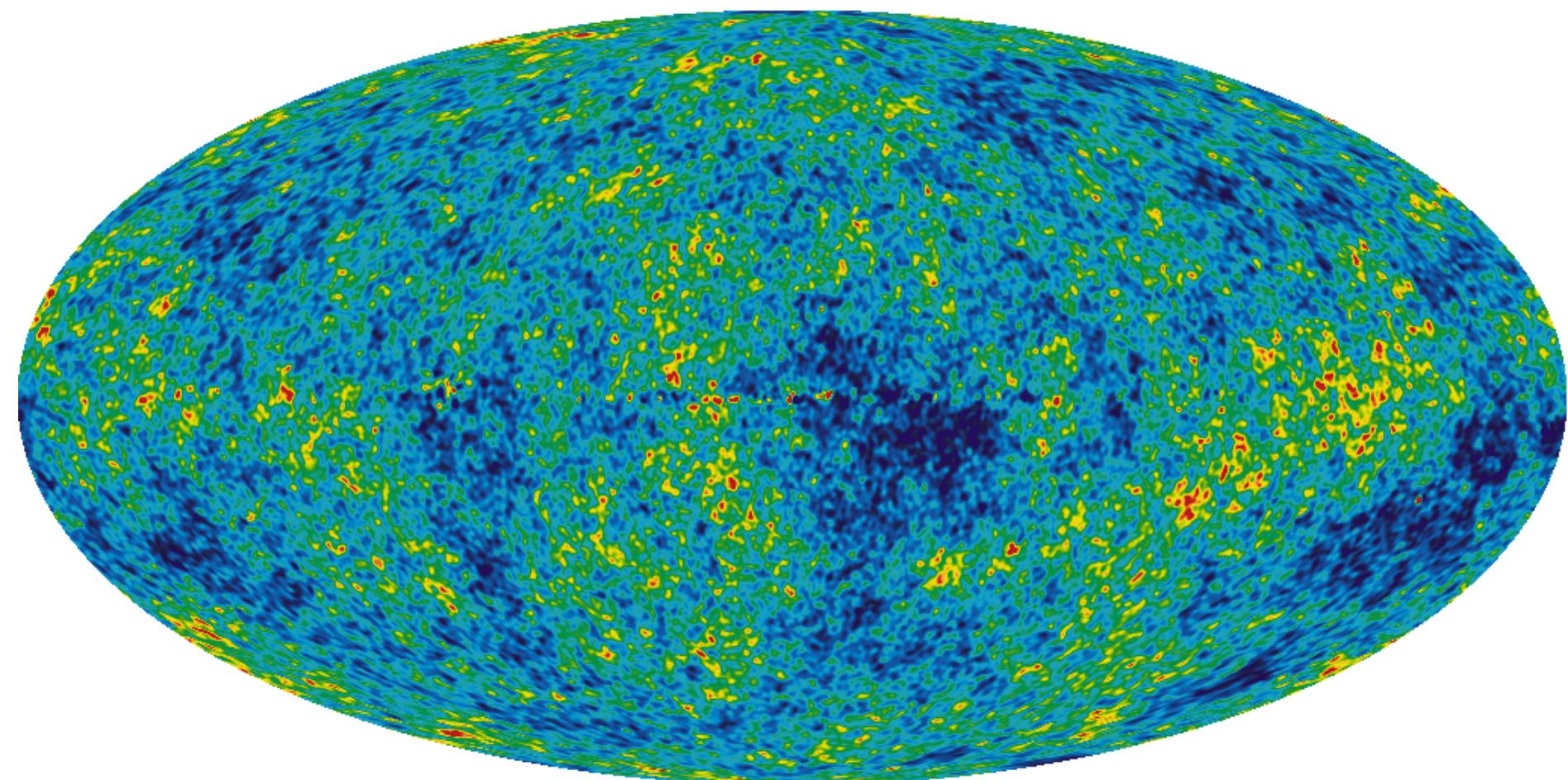
# What's left of non-Gaussianity... *(i) after Planck? (ii) after BICEP2?*

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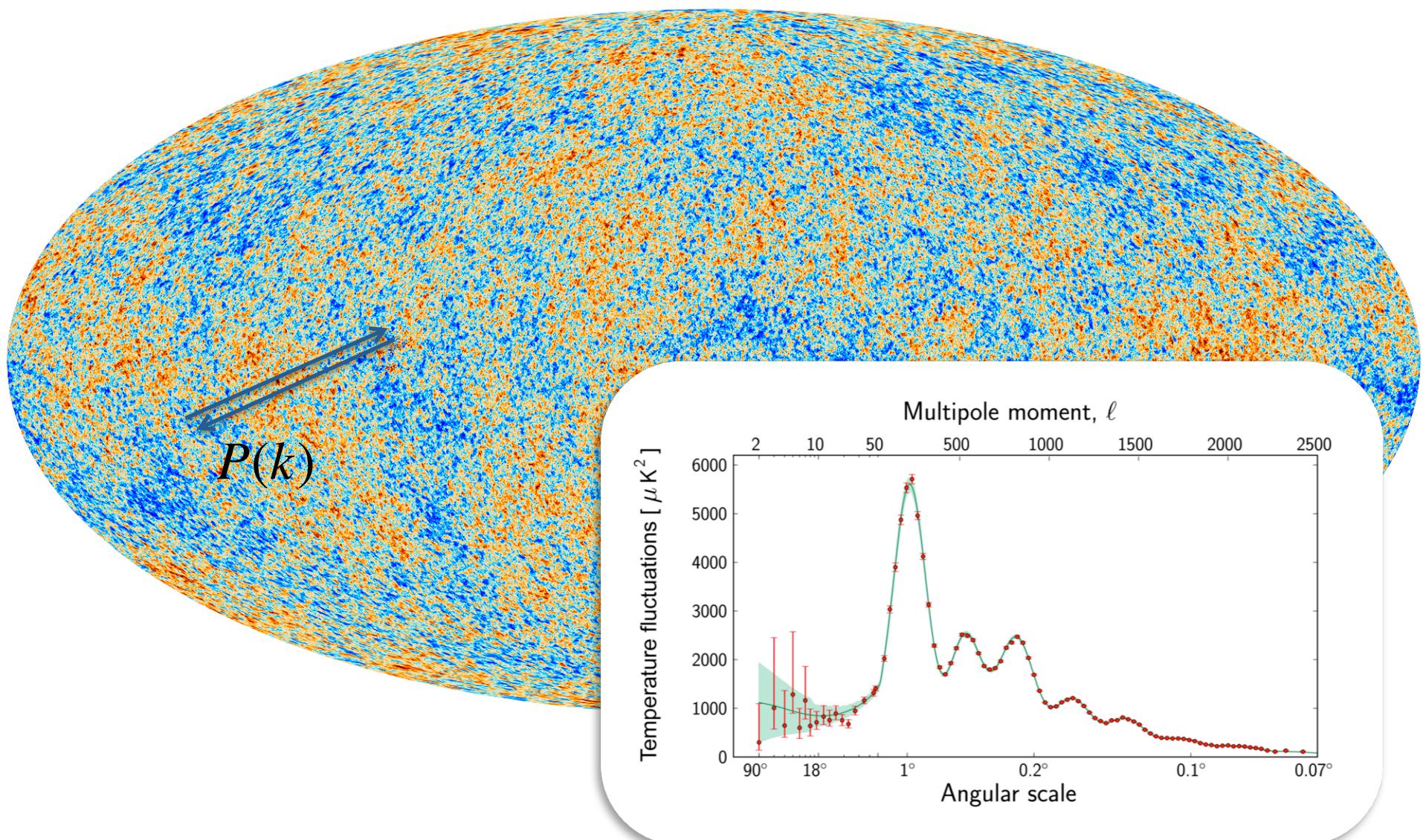
# Conclusions from Planck+BICEP2

- Planck2013
  - not easy to detect non-Gaussianity
  - large non-Gaussianity ( $f_{NL} \gg 10$ ) ruled out
  - need  $|f_{NL}| < 1$  to discriminate inflaton vs alternative models
  - intrinsic+secondary non-Gaussianity (last-scattering and line-of-sight) will be important
- BICEP2014 (*if confirmed*)
  - GUT-scale inflation
  - implies Gaussian (inflaton) perturbations not negligible in slow-roll inflation

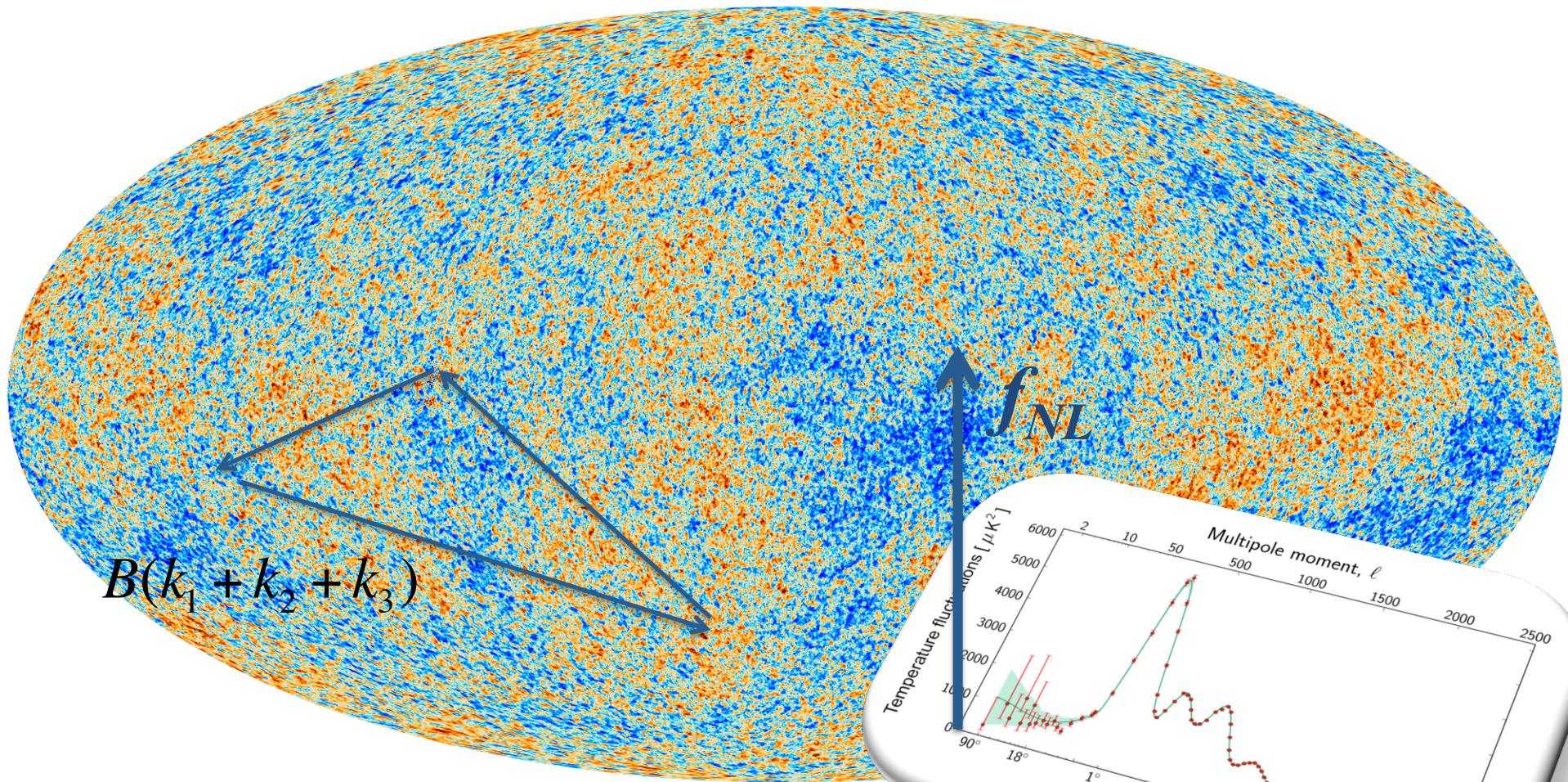
# WMAP2012 standard model of primordial cosmology



# Planck2013 - new standard model of primordial cosmology



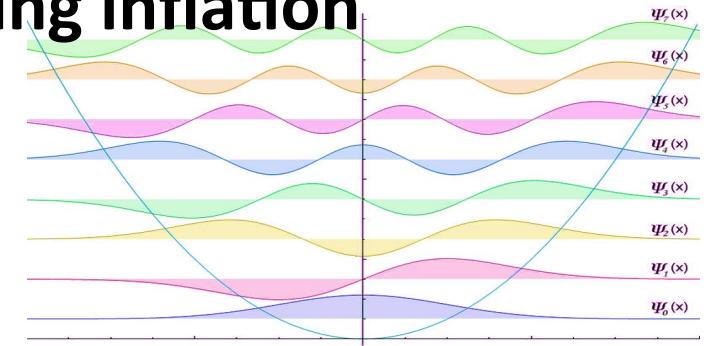
# more information in higher-order correlators...



$$f_{NL} = \frac{B_\xi(k_1, k_2, k_3)}{P(k_1)P(k_2) + P(k_2)P(k_3) + P(k_3)P(k_1)}$$

# Primordial Gaussianity from inflation

- **Quantum field fluctuations during inflation**
  - *ground state of simple harmonic oscillator*
  - *almost free field in almost de Sitter space*
  - *almost scale-invariant and almost Gaussian*
- **Power spectra probe background *dynamics* ( $H, \epsilon, \dots$ )**
$$\langle \zeta_{k_1} \zeta_{k_2} \rangle = (2\pi)^3 P_\zeta(k) \delta^3(k_1 + k_2) , \quad P_\zeta(k) \propto k^{n-4}$$
  - *contain all the information in a Gaussian random field*
  - *but many different models can produce similar power spectra*
- **Higher-order correlations probe *interactions***
$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle = (2\pi)^3 B_\zeta(k_1, k_2, k_3) \delta^3(k_1 + k_2 + k_3)$$
  - *physics+gravity → non-linearity → non-Gaussianity*



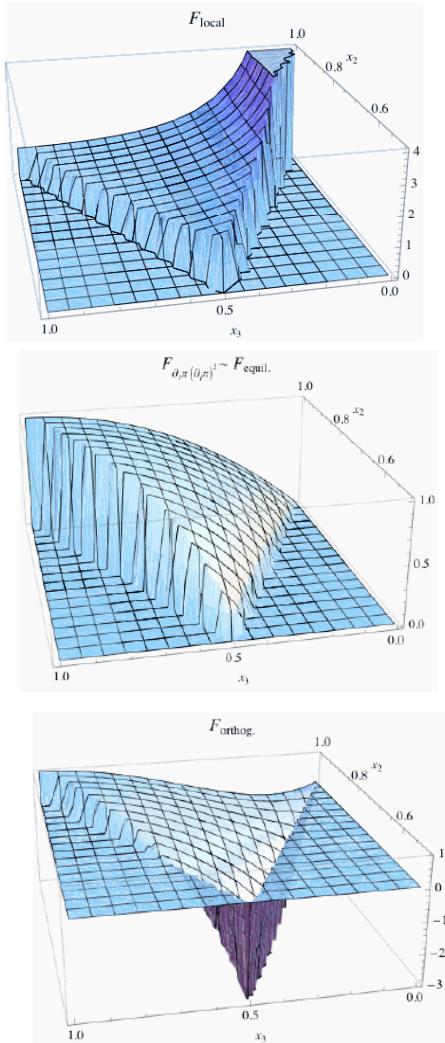
Wikipedia: AllenMcC

# Many possible sources of non-Gaussianity

Initial vacuum	Excited state
Sub-Hubble evolution	Higher-derivative interactions e.g. k-inflation, DBI, Galileons
Hubble-exit	Features in potential Particle production during inflation
Super-Hubble evolution	Self-interactions + gravity
End of inflation	Tachyonic instability
(p)Reheating	Modulated (p)reheating
After inflation	Curvaton decay
	Topological defects
	Magnetic fields
<b>primordial non-Gaussianity</b>	
Primary anisotropies	Last-scattering
Secondary anisotropies	ISW/lensing + foregrounds

# Many possible shapes for bispectra

need templates to extract small nG signals



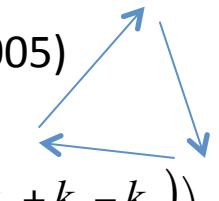
- **local type** (Komatsu&Spergel 2001)
  - local in real space
  - max for squeezed triangles:  $k \ll k', k''$

$$B_\zeta(k_1, k_2, k_3) \propto \left( \frac{1}{k_1^3 k_2^3} + \frac{1}{k_2^3 k_3^3} + \frac{1}{k_3^3 k_1^3} \right)$$



- **equilateral type** (Creminelli et al 2005)
  - peaks for  $k_1 \sim k_2 \sim k_3$

$$B_\zeta(k_1, k_2, k_3) \propto \left( \frac{3(k_1 + k_2 - k_3)(k_2 + k_3 - k_1)(k_3 + k_1 - k_2)}{k_1^3 k_2^3 k_3^3} \right)$$



- **orthogonal type** (Senatore et al 2009)
  - independent of local + equilateral shapes

$$B_\zeta(k_1, k_2, k_3) \propto \left( \frac{81}{k_1 k_2 k_3 (k_1 + k_2 + k_3)^3} \right)$$

- **separable basis** (Ferguson et al 2008)

# non-Gaussianity expected from inflation?

- **single-field slow-roll inflaton**

- during conventional slow-roll inflation  $f_{NL}^{local} \approx N''/N'^2 = O(\epsilon) \ll 1$   
Maldacena 2002
- ***adiabatic perturbations***

=>  $\zeta$  constant on large scales => more generally:  $f_{NL}^{local} \propto n_\zeta - 1$

Creminelli & Zaldarriaga 2004

- **super-Hubble evolution**

- ***non-adiabatic perturbations*** during multi-field inflation  
=>  $\zeta \neq$  constant
- at/after end of inflation (curvaton, modulated reheating, etc)

- e.g., **curvaton**  $f_{NL}^{local} \approx 1/\Omega_{\chi, decay}$  Lyth & Wands 2002

- **sub-Hubble interactions**

- e.g. DBI inflation, Galileon fields...
- or coupling to massive fields

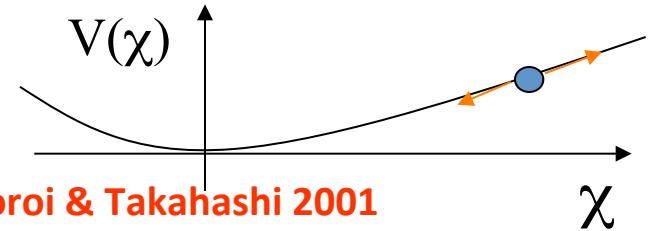
Achucarro et al (2010+); Gao et al (2012)

$$L = (\nabla \varphi)^2 + (\nabla \varphi)^4 + \dots$$

$$\Rightarrow f_{NL}^{equil} \approx 1/c_s^2 \quad \text{Cheung et al 2008}$$

# curvaton scenario:

Linde & Mukhanov 1997; Enqvist & Sloth, Lyth & Wands, Moroi & Takahashi 2001



curvaton  $\chi$  = a weakly-coupled, late-decaying scalar field

- light during inflation ( $m \ll H$ ) hence acquires an almost scale-invariant, **Gaussian distribution of field fluctuations**
- **quadratic energy density** for massive field,  $\rho_\chi = m^2 \chi^2 / 2$
- spectrum of initially **non-adiabatic density perturbations**

$$\zeta_\chi \approx \frac{1}{3} \frac{\delta \rho_\chi}{\rho_\chi} \approx \frac{1}{3} \left( \frac{2\chi \delta \chi + \delta \chi^2}{\chi^2} \right)$$

- transferred to radiation when curvaton decays with some **efficiency**,  $0 < R_\chi < 1$ , where  $R_\chi \approx \Omega_{\chi, \text{decay}}$

$$\zeta = R_\chi \zeta_\chi$$

$$\approx \zeta_G + \frac{3}{4R_\chi} \zeta_G^2 \quad \Rightarrow \quad f_{NL} \approx \frac{5}{4R_\chi}$$

# CMB constraints on non-Gaussianity

- **WMAP constraints**

- Local:  $-3 < f_{\text{NL}} < 77$  (95% CL)

Bennett et al 2012

- Equilateral:  $-5.6 < g_{\text{NL}} / 10^5 < 8.6$

Ferguson et al; Smidt et al 2010

- Orthogonal:  $-121 < f_{\text{NL}} < 223$

- Orthogonal:  $-445 < f_{\text{NL}} < -45$

- **Planck constraints** (Planck XXIV, arXiv:1303.5084)

- Local:  $-9.8 < f_{\text{NL}} < 14.3$  (95% CL)

- $\tau_{\text{NL}} < 2800$

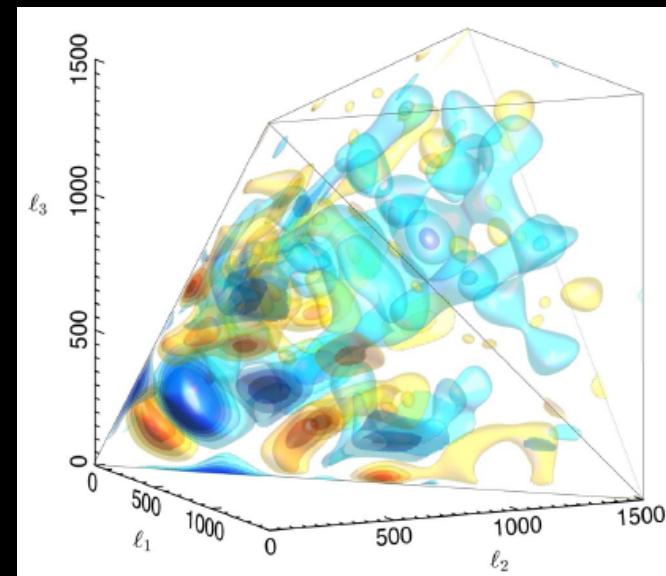
- no bound yet on  $g_{\text{NL}}$

- Equilateral:  $-192 < f_{\text{NL}} < 108$

- Orthogonal:  $-103 < f_{\text{NL}} < 53$

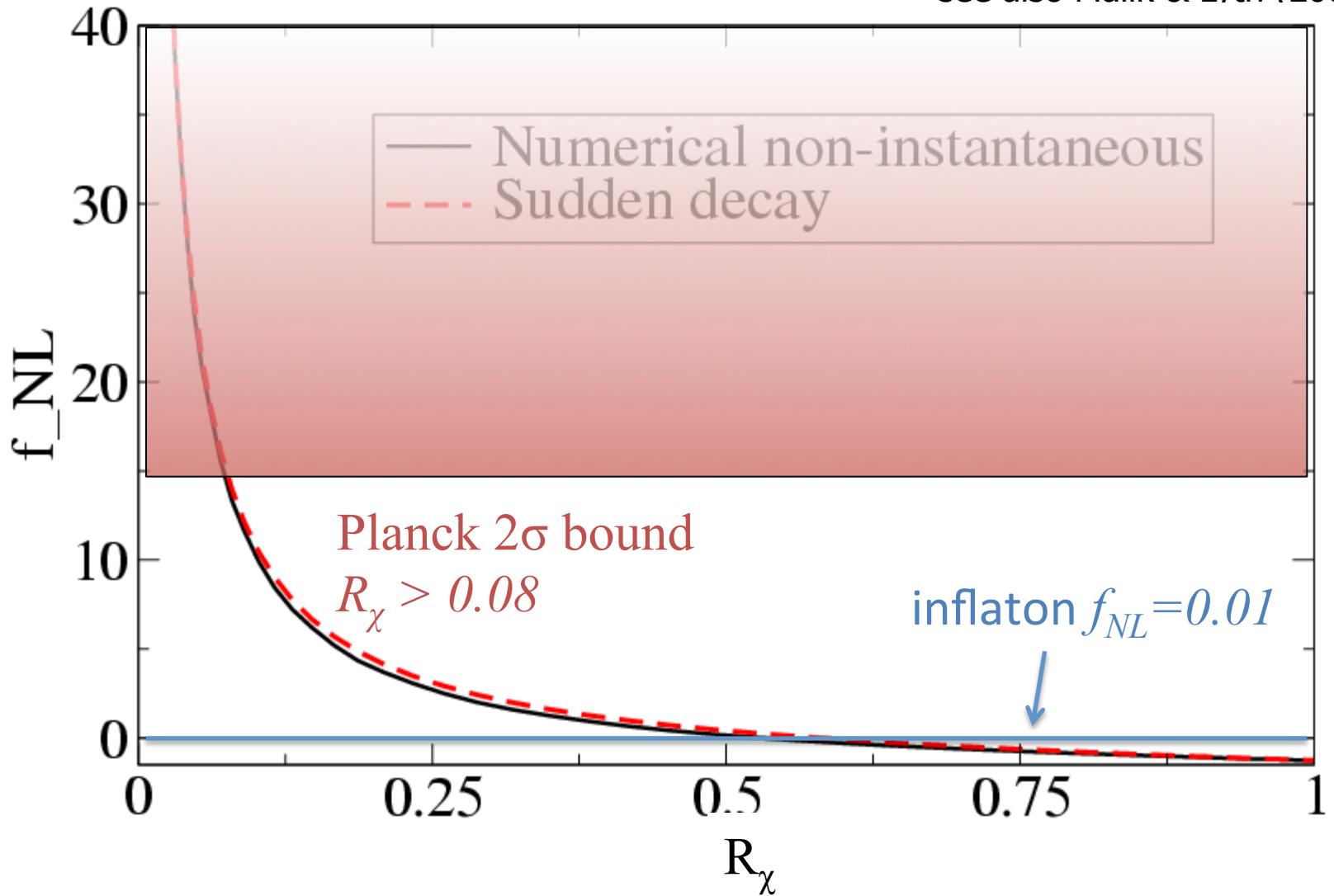
- Many other bispectrum shapes tested

- some evidence for oscillatory features*



# Quadratic curvaton non-linearity parameter

Sasaki, Valiviita & Wands (2006)  
see also Malik & Lyth (2006)



# Lesson #1: Planck2013

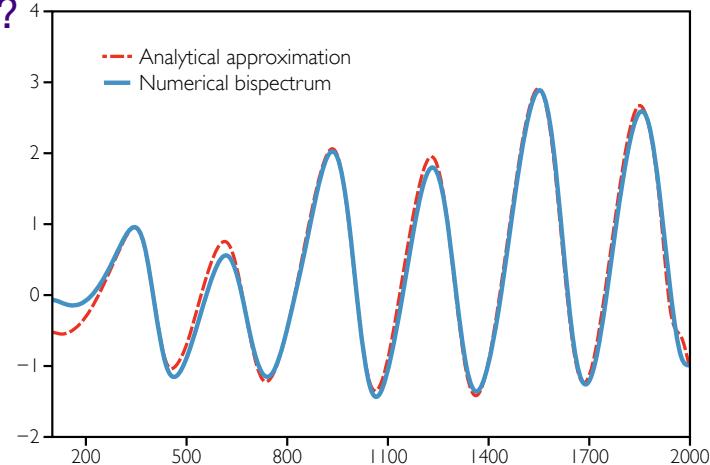
- Primordial non-Gaussianity is not going to be so easy to find!
  - $|f_{NL}| > 1$  would be a good way to falsify inflaton mechanism
  - need  $|f_{NL}| < 1$  to disprove curvaton-type mechanisms

# Lesson #2: Planck2013

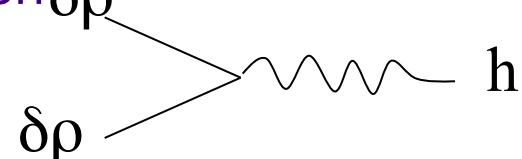
- Intrinsic (last-scattering) and secondary nG (line-of-sight) are important for  $f_{NL}=O(1)$ 
  - *see talk this afternoon by Filippo*

# Intrinsic bispectrum of CMB

- Existing non-Gaussianity templates based on non-linear primordial perturbations + linear Boltzmann codes (CMBfast, CAMB, etc)
- Second-order general relativistic Boltzmann codes now available
  - Pitrou (2010): CMBquick in Mathematica:  $f_{NL} \sim 5?$
  - Huang & Vernizzi (2012) (Paris)
  - Pettinari, Fidler et al (Portsmouth)
  - Su, Lim & Shellard (Cambridge & London)

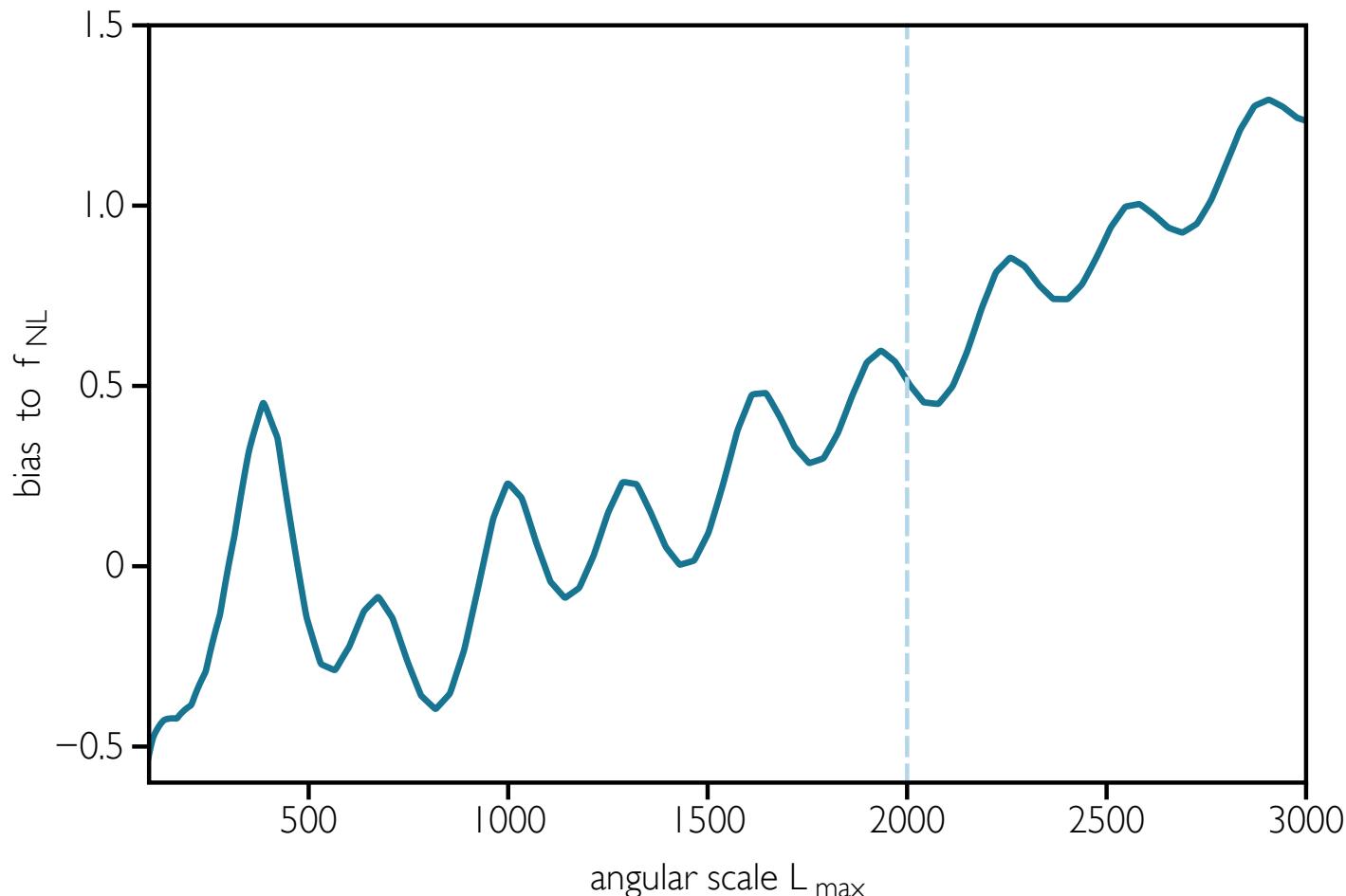


- need templates to identify intrinsic + secondary non-Gaussianity
- testing interactions at recombination
  - e.g., gravitational wave production



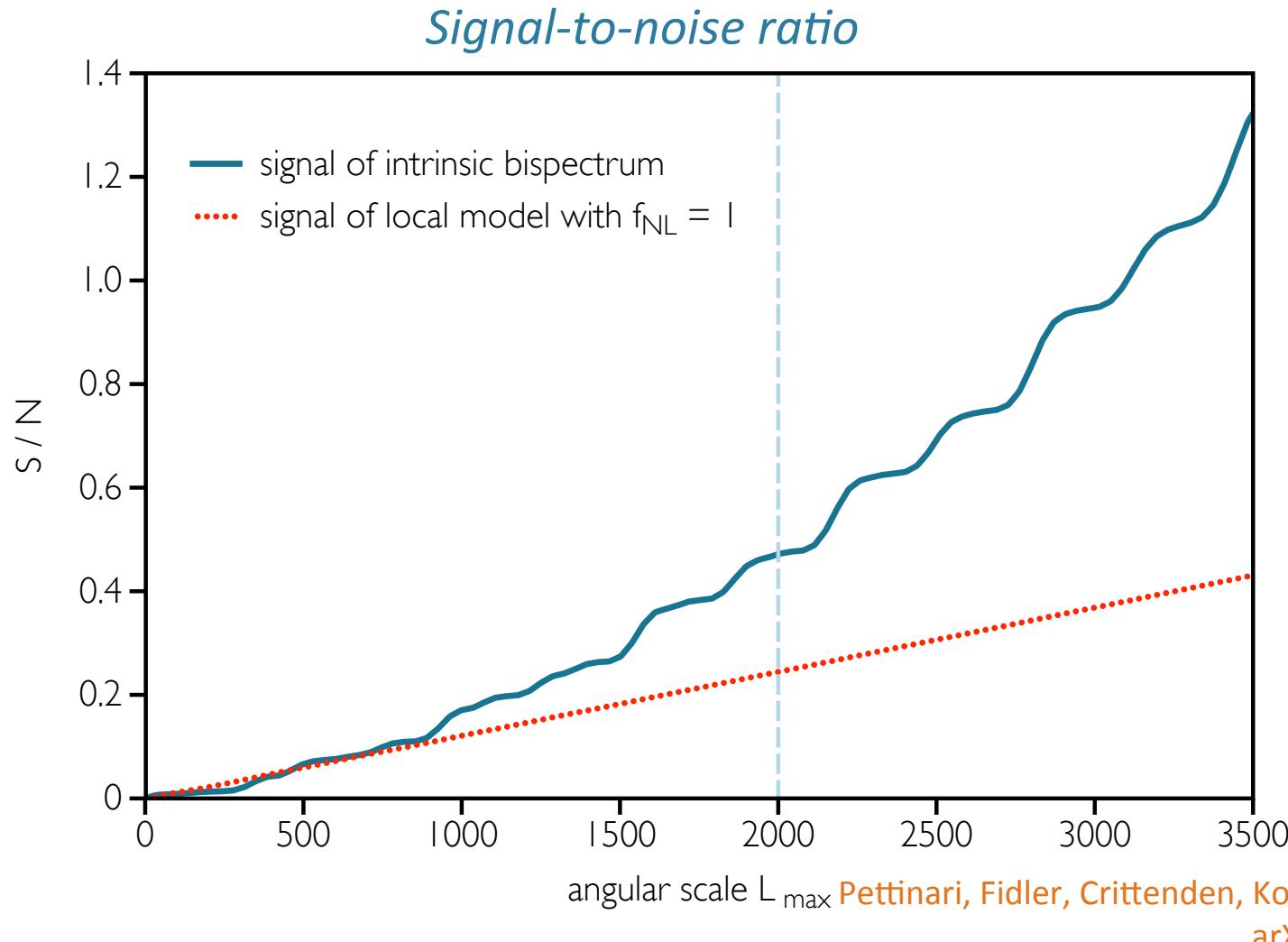
# Intrinsic bias on local non-Gaussianity

*Effects at recombination small compared to Planck sensitivity*



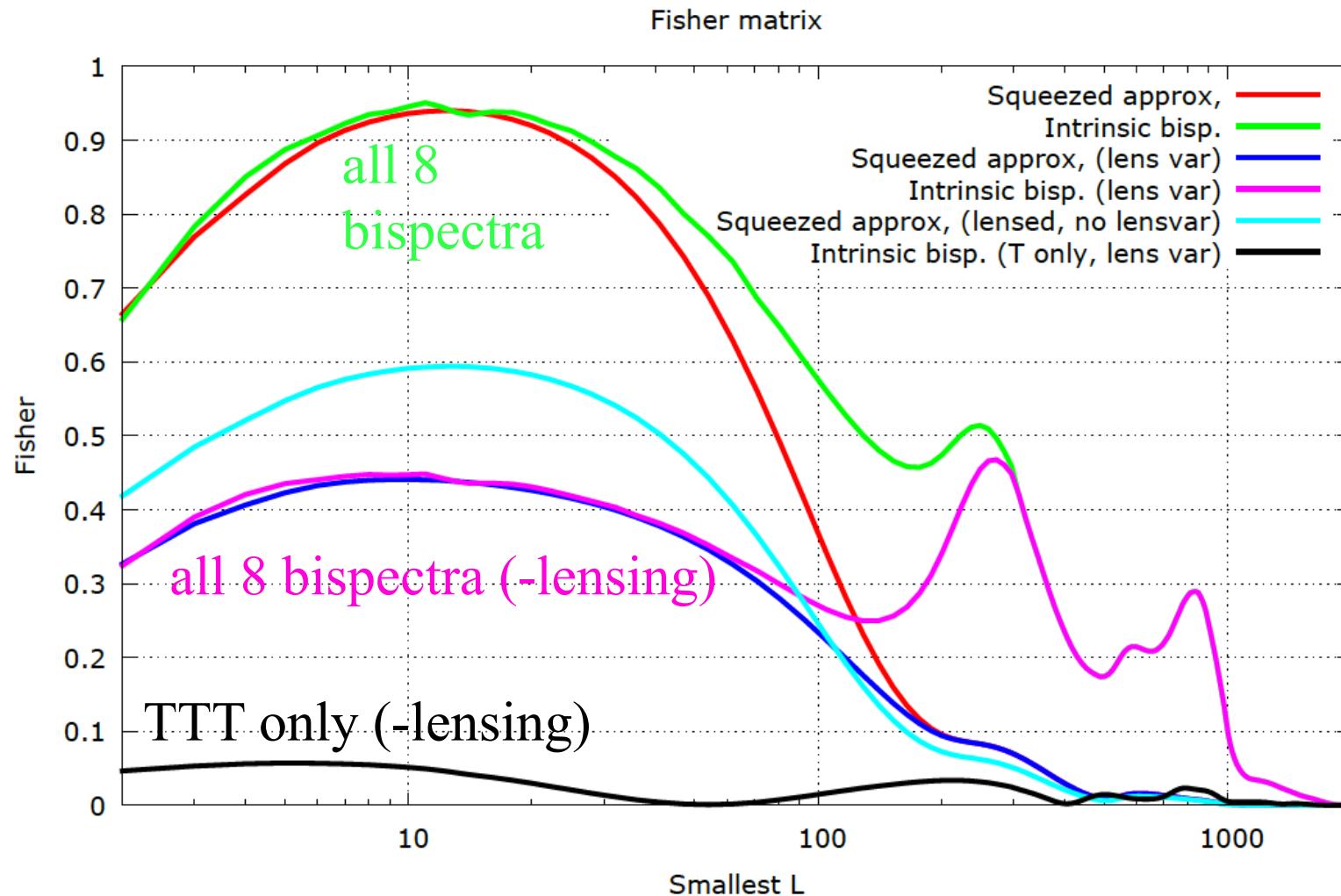
# Detecting the intrinsic bispectrum

even using the correct template, hard to see the intrinsic TTT signal with Planck

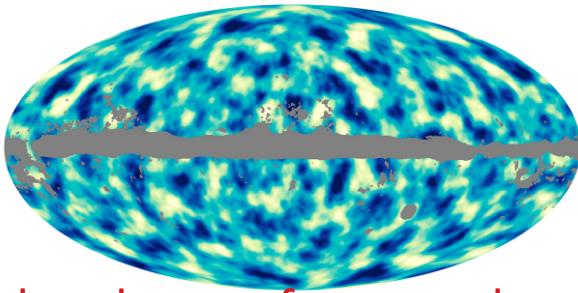


# intrinsic bispectrum with polarisation

correlating TTT, TTE, TEE, TBB, etc, improves signal



# Lensing-ISW



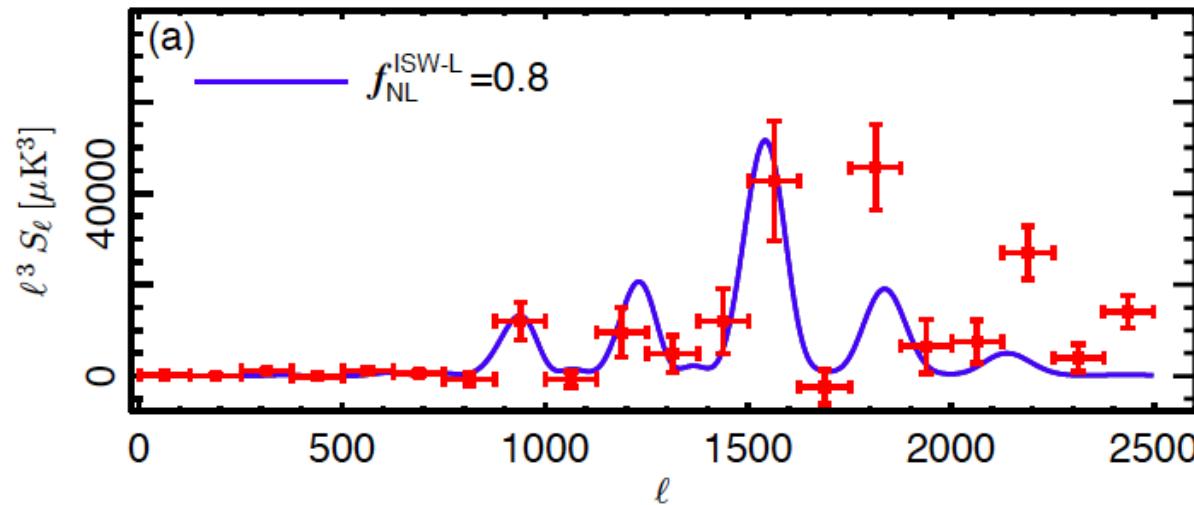
Goldberg & Spergel 1999  
Seljak and Zaldarriaga 1999  
Lewis, Challinor & Hanson 2011  
Lewis 2012

Secondary bispectrum already seen from two late time gravitational effects:  
gravitational lensing and the integrated Sachs-Wolfe effect.

In the ISW effect, large scale over-densities and under-densities appear hotter or colder as photons can gain or lose energy passing through their time dependent potentials.

These potentials correlate to where gravitational lensing (seen on smaller scales) is largest or smallest, creating mode coupling.

Significant bias, local  $f_{NL}$  of order 7, seen in the Planck analysis.



# Large-scale structure constraints on $f_{\text{NL}}$

- constraints on local  $f_{\text{NL}}$  from scale-dependent bias in galaxy power spectrum (Dalal et al 2007; Verde & Matarrese 2008):
  - $-37 < f_{\text{NL}} < 20$  (95% CL) Giannantonio et al 2013
  - $-4.5 < g_{\text{NL}} / 10^5 < 1.6$  [cross-correlating SDSS+NVSS]
- constraints on local  $g_{\text{NL}}$  may be possible from scale-dependent bias in galaxy bispectrum (Tasinato, Tellarini, Ross & Wands 2014)

# Intrinsic nG in large-scale structure

(Bruni, Hidalgo & Wands, submitted)

- amplitude of matter density growing mode on large scales:  $\delta(t,x)=C(x)D_+(t)+\dots$ 
  - $C(x)$  determines abundance of collapsed halos (and hence galaxies)
  - set by gravitational potential,  $\phi$ , in *Newton gravity*
    - *linear* Poisson constraint
    - $\rightarrow$  Gaussian potential implies Gaussian density field
  - set by intrinsic curvature,  $R$ , in *Einstein gravity*
    - *non-linear* relation between  $R$  and primordial metric  $\zeta$
    - $\rightarrow$  intrinsic non-Gaussianity characteristic of GR

$$f_{NL}^{GR} = -\frac{5}{3} \quad , \quad g_{NL}^{GR} = -\frac{50}{3} \quad , \quad h_{NL}^{GR} = -\frac{125}{81} \quad \dots$$

# Planck power spectrum constraints

slow roll parameters  $\varepsilon = -\dot{H}/H^2$ ,  $\eta_i = m_i^2/3H^2$

- **Inflaton**

$$P_\zeta \approx \frac{1}{8\pi^2 \varepsilon} \left( \frac{H_*}{M_{Pl}} \right)^2$$

tilt:  $n - 1 = -6\varepsilon + 2\eta_\varphi$

$$P_T \approx \frac{2}{\pi^2} \left( \frac{H_*}{M_{Pl}} \right)^2$$

tensor-scalar ratio:  $r_T = 16\varepsilon$

- **Curvaton**

$$P_\zeta \approx \frac{R_\chi^2}{9\pi^2} \left( \frac{H_*}{\langle \chi_* \rangle} \right)^2$$

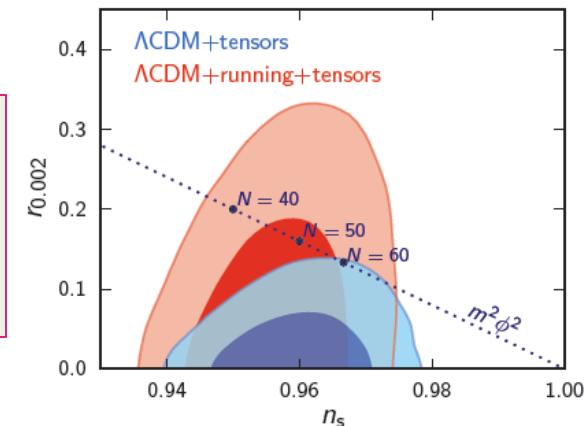
tilt:  $n - 1 = -2\varepsilon + 2\eta_\chi$

$$P_T \approx \frac{2}{\pi^2} \left( \frac{H_*}{M_{Pl}} \right)^2$$

tensor-scalar:  $r_T = 18\langle \chi_* \rangle^2 / R_\chi^2 M_{Pl}^2 < 16\varepsilon$

- Planck2013:
  - $n = 0.9624 \pm 0.0075$
  - $r_T < 0.12$

David Wands



# Different Planck2013 perspectives

## Inflaton viewpoint

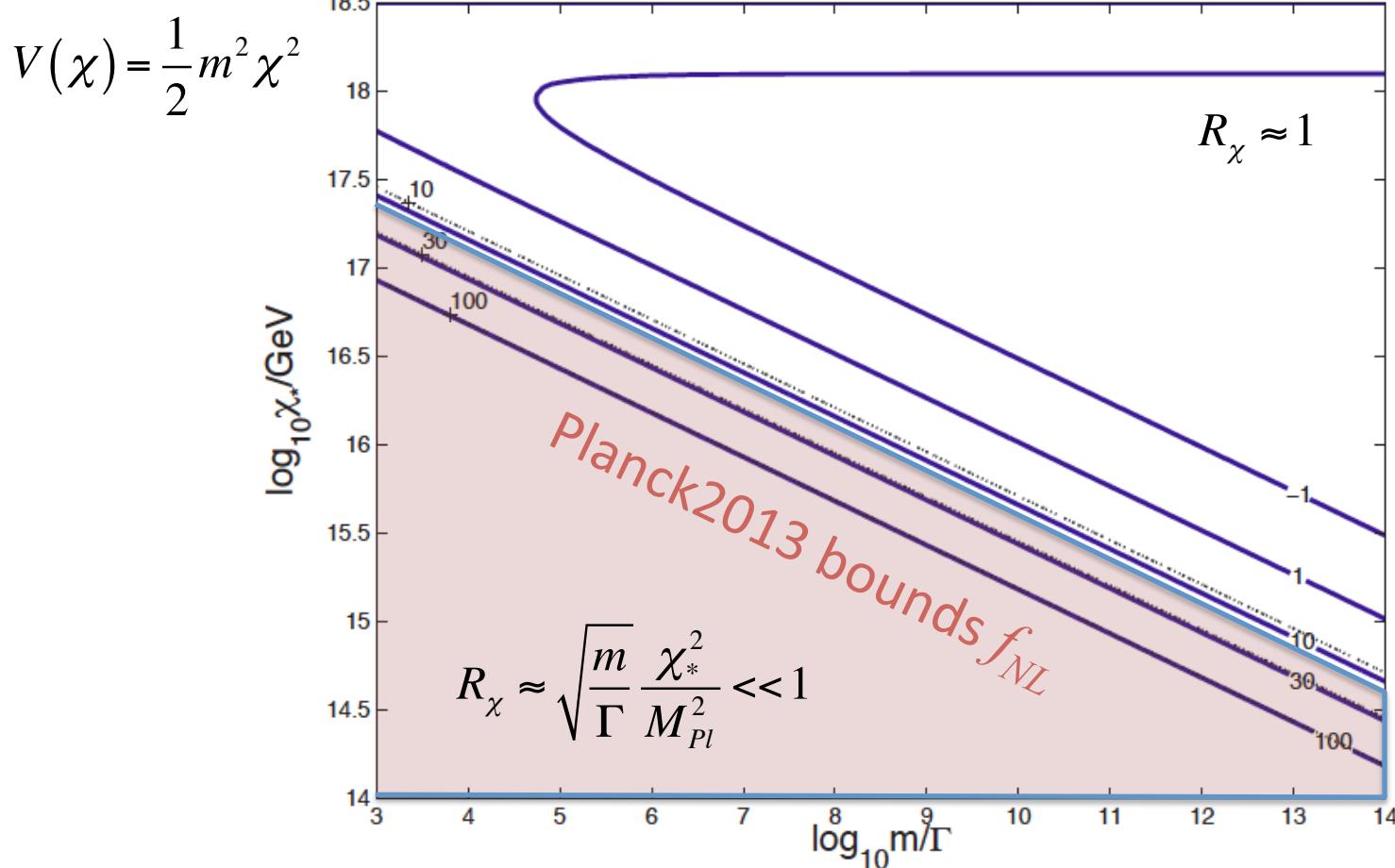
- Tensor-scalar ratio bounds  $\varepsilon < 0.01$
- Scalar tilt  $n=0.96$  then favours  $\eta_\phi < 0$ 
  - E.g., axion e.g., axion  $V(\phi)=\Lambda^4(1-\cos(\phi/f))$

## Curvaton viewpoint

- Tensor-scalar ratio bounds  $H_{\text{inf}}$  (*model-independent*), not  $\varepsilon$
- Scalar-tilt implies either
  - Large-field inflation:  $\varepsilon \approx 0.02$  and  $\eta_\chi \approx 0$ 
    - e.g.,  $\lambda\phi^4$  inflation (with  $N \approx 60$ )
  - Small-field with  $\eta_\chi < 0$  and  $\varepsilon \ll 0.02$ 
    - e.g., axion  $V(\chi)=\Lambda^4(1-\cos(\chi/f))$

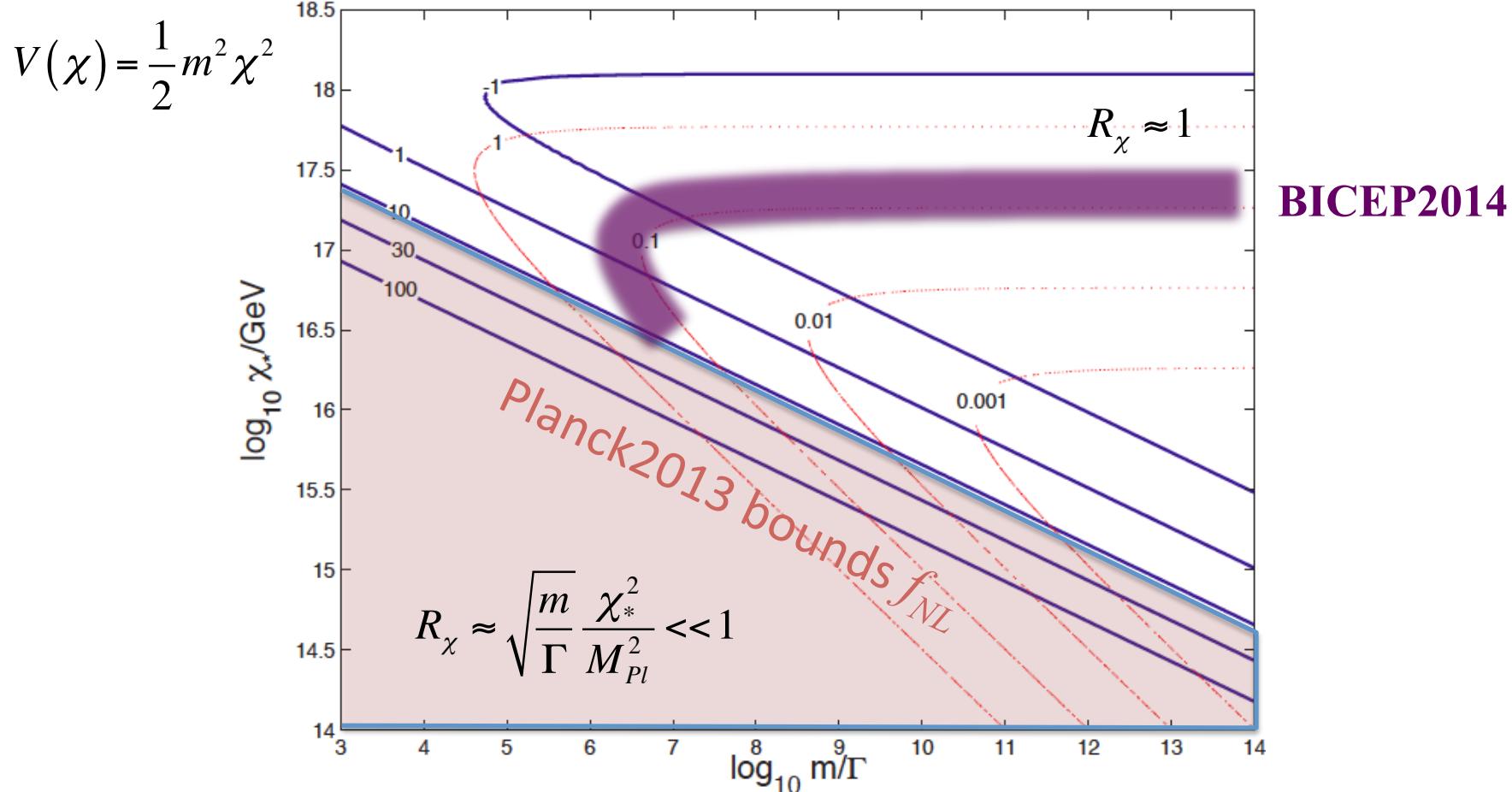
# $f_{NL}$ bounds on quadratic curvaton

Fonseca & Wands (2011)  
see also Nakayama et al (2010)



# $f_{NL}$ bounds on quadratic curvaton + $r_T = P_T/P_\zeta$

Fonseca & Wands (2011)  
see also Nakayama et al (2010)



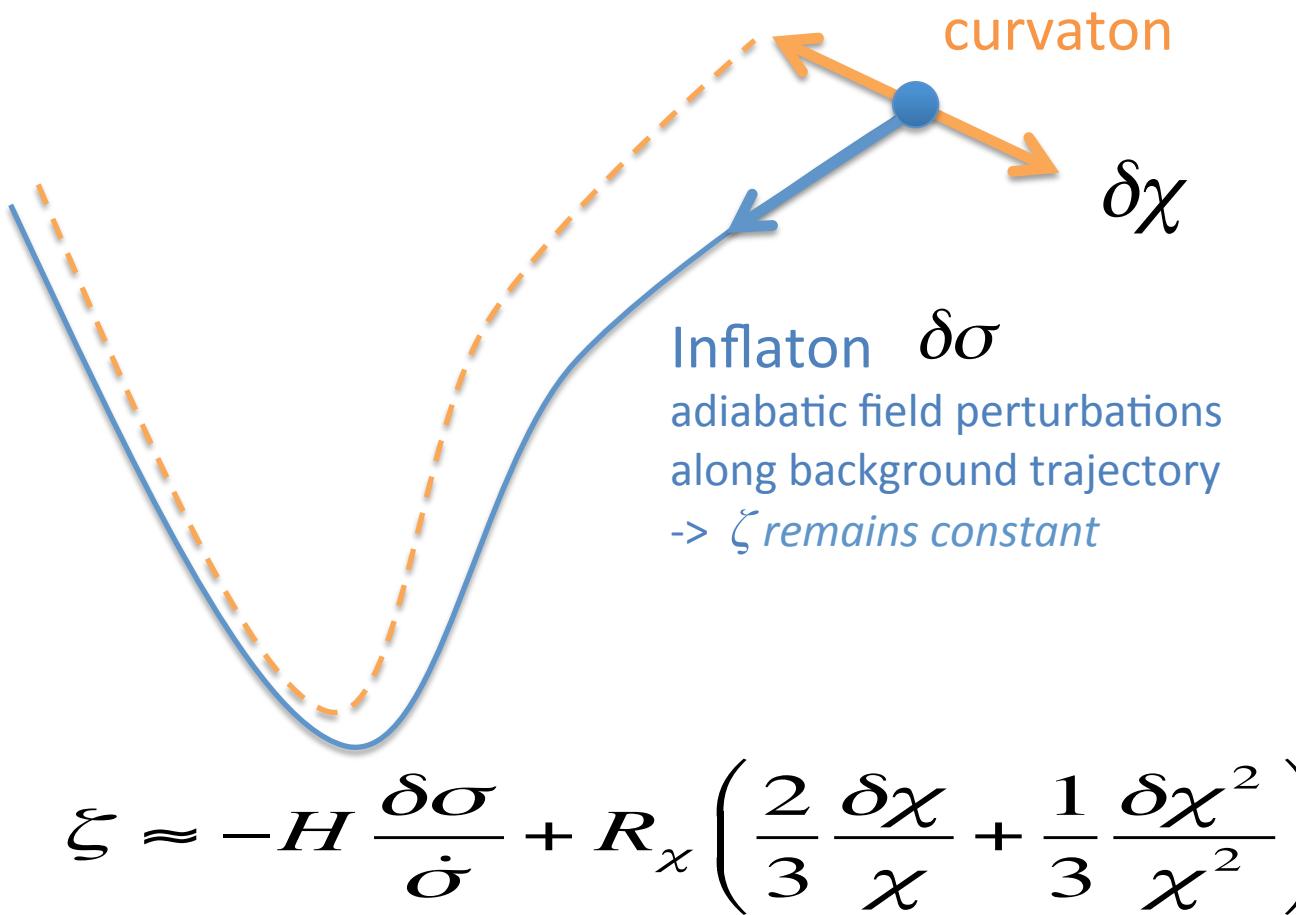
# Lesson #3: BICEP2014

- BICEP2 (if confirmed)  $P_T \approx 0.16 P_\zeta$ 
  - adiabatic inflaton perturbations  $P_{\zeta^*} = P_T / 16\varepsilon$  cannot be arbitrarily small in slow-roll\* inflation ( $\varepsilon < 1$ )
  - for curvaton+inflaton where  $w_\chi = \text{curvaton/total}$   
 $r_T = 16\varepsilon(1-w_\chi) \approx 0.16 \Rightarrow \varepsilon \geq 0.01$
  - e.g., if  $\varepsilon = 0.02$  then  $w_\chi \approx 0.5$
  - primordial perturbations from inflaton,  $\zeta_*$ , are always *Gaussian* (Maldacena, 2002)

\**curvaton-type mechanisms do not require slow-roll inflaton*

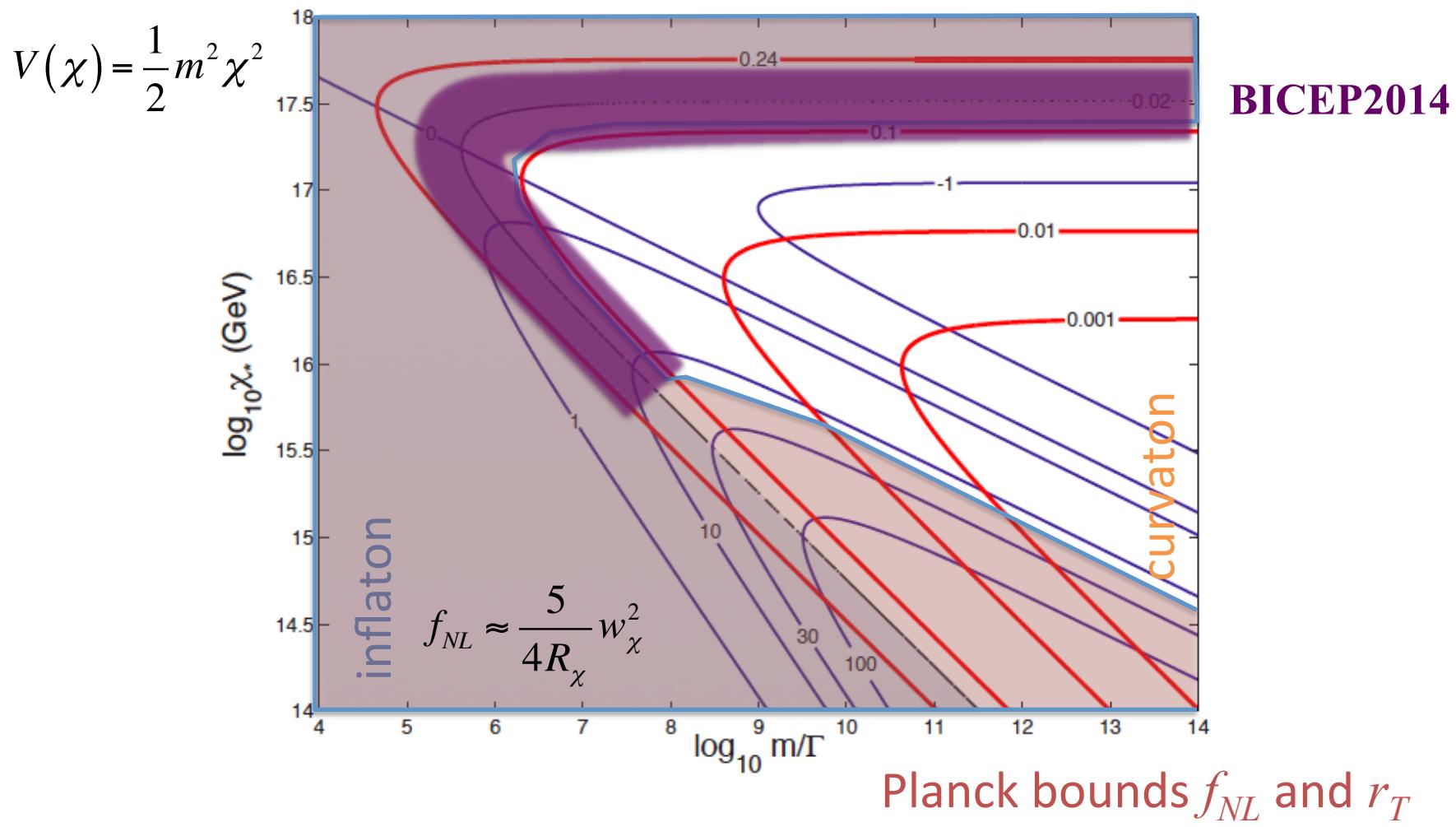
# Local non-Gaussianity from non-adiabatic fluctuations

curvaton, also multi-field inflation, modulated reheating, inhomogeneous end of inflation...



# $f_{NL}$ + $r_T$ quadratic curvaton + inflaton: for $\varepsilon=0.02$

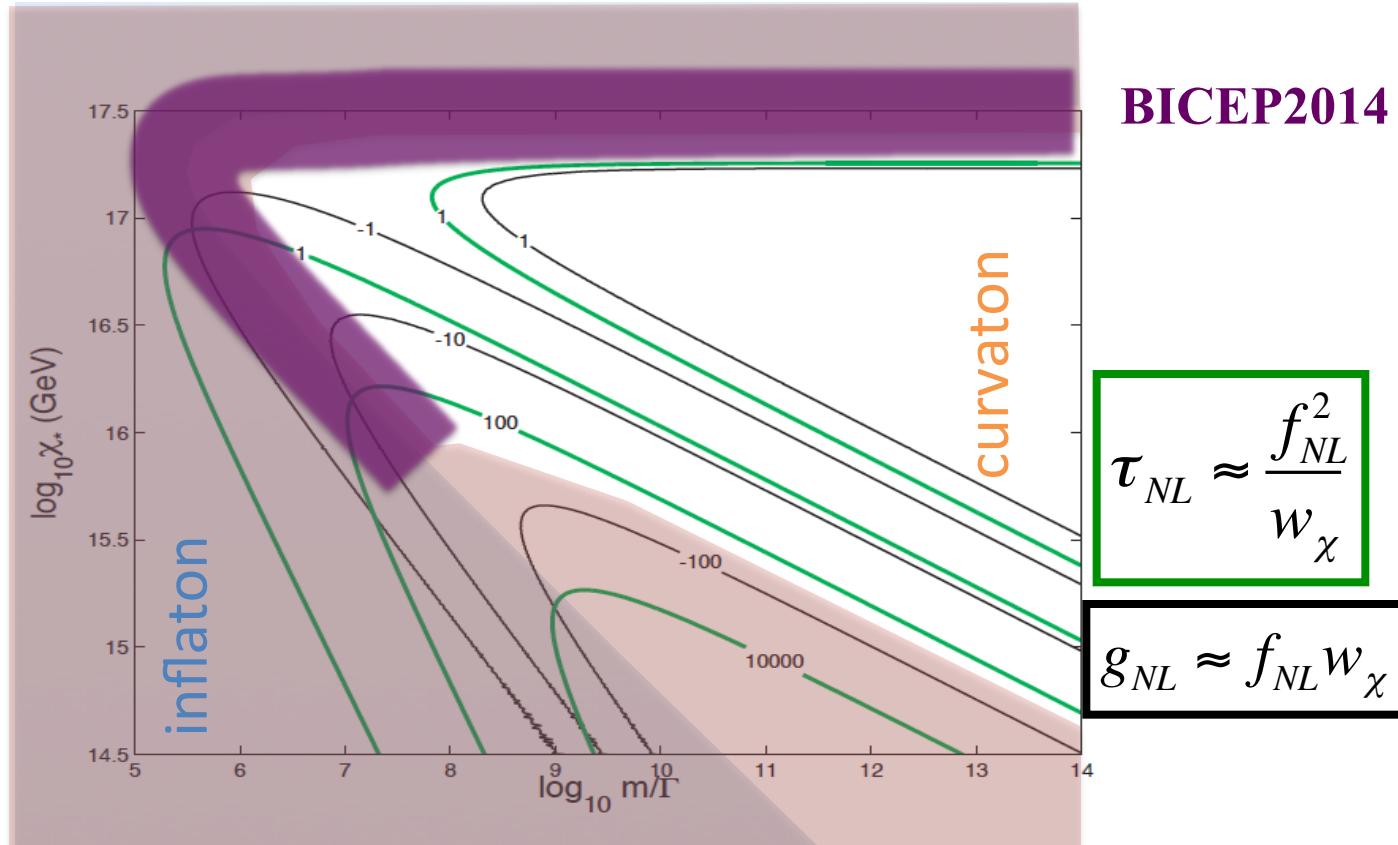
Fonseca & Wands (2012)



# trispectrum quadratic curvaton + inflaton: for $\varepsilon=0.02$

Fonseca & Wands (2012)

$$V(\chi) = \frac{1}{2} m^2 \chi^2$$



Planck bounds  $f_{NL}$  and  $r_T$

# How to measure $w_\chi$ ?

- trispectrum ( $\tau_{NL}$ , Suyama-Yamaguchi inequality)

$$w_\chi = \frac{36}{25} \frac{f_{NL}^2}{\tau_{NL}}$$

- tensor tilt ( $n_T = -2\varepsilon$ )

$$1 - w_\chi = \frac{r_T}{16\varepsilon} = -\frac{r_T}{8n_T}$$

# Non-Gaussian outlook:

- **Complementary to power spectrum constraints**
  - requires multiple fields and/or unconventional physics
  - detection of primordial non-Gaussianity would kill textbook single-field slow-roll inflation models
- **Planck2013 not a game-changer**
  - large local fNL is dead...
  - ...long-live fNL = O(1)
  - intrinsic non-Gaussianity is important
- **but BICEP2014 could be pivotal**
  - tensor modes -> non-negligible inflaton (Gaussian) perturbations
- **need more data**
  - Planck2014, SPT, etc + large-scale structure surveys
- **non-Gaussianity has been detected**
  - need to disentangle primordial and generated non-Gaussianity