

# Cosmic variance on the local expansion rate

Wessel Valkenburg, Leiden University

at PONT Avignon, 2014

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# Look out!

- Outlook
- The point of this talk:  $H_0 \neq H(0)$
- Variance of  $H(0)$  if you know  $P(k)$
- Variance of  $H(0)$  if you don't know  $P(k)$
- What to do to reduce this systematic error

# What is $H_0$ ?

- Real Universe:  $g_{\mu\nu} = g_{\mu\nu}(\vec{x}, t)$
- High school universe:  $g_{\mu\nu} = g_{\mu\nu}(t) = a(t) \eta_{\mu\nu}$
- High school:  $H_0 = \dot{a}(t) / a(t)$
- Real (still simplistic) Universe:  $\Theta \equiv \nabla_\alpha u^\alpha$

$$G_{\alpha\beta} = 8\pi G_N T_{\alpha\beta} = 8\pi G_N \rho u_\alpha u_\beta$$

$$\nabla_\beta u_\alpha = \frac{1}{3} h_{\alpha\beta} \Theta + \sigma_{\alpha\beta} + \omega_{\alpha\beta}$$

# What do we measure?

- $d_A(z) = \sqrt{[ \text{Surface}(\text{obj}) / \Omega(\text{obj}) ]}$
- $d_L(z) = (1+z)^2 d_A(z)$
- For  $z \ll 1$ , High school universe:  
 $d_L(z) = d_A(z) = z/H_0 + \mathcal{O}(z^2)$
- In reality,  $z = z(\vec{x}, t)$ ,  $d_A = d_A(\vec{x}, t)$  both given by geodesic and geodesic deviation equation.

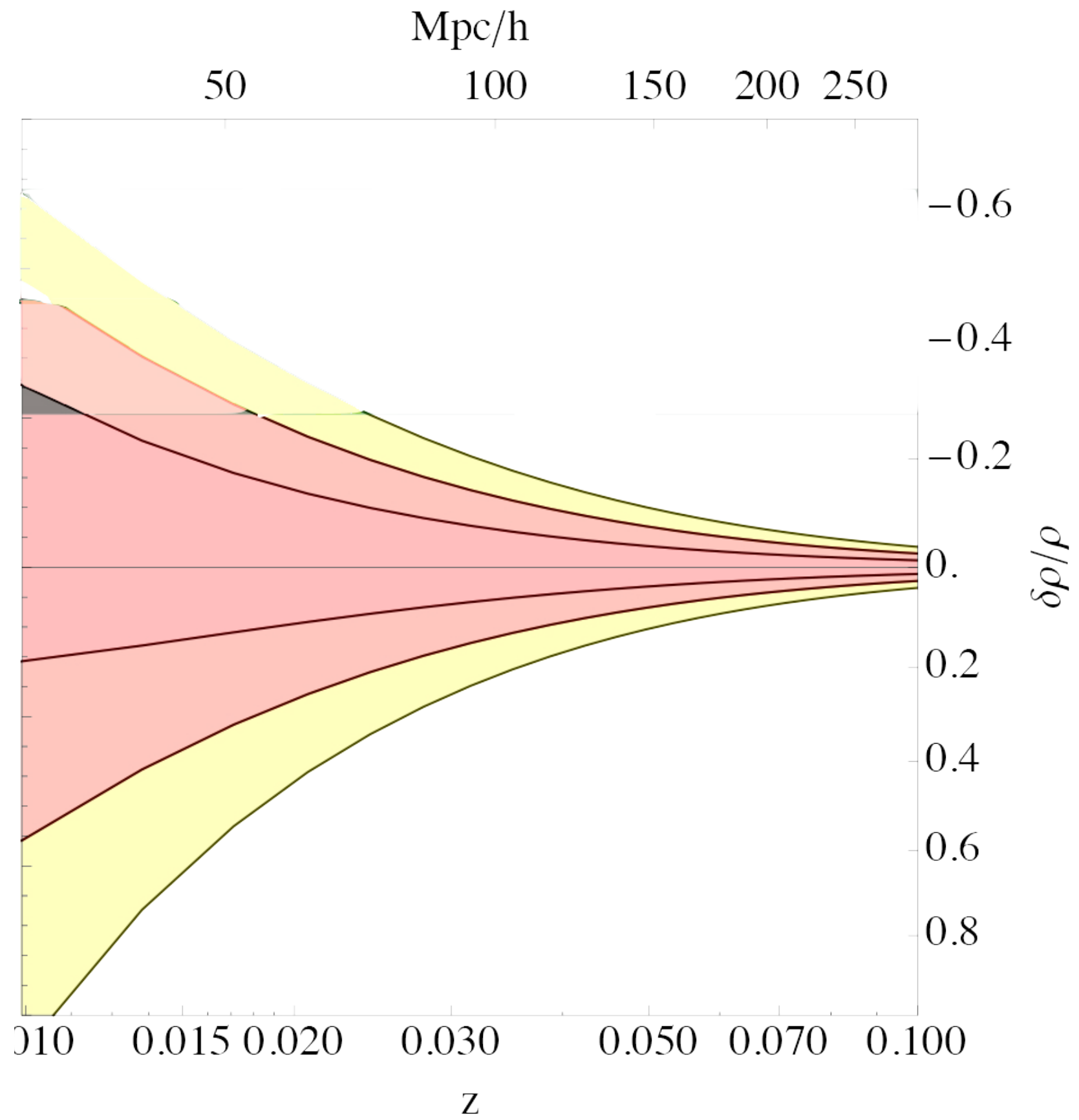
# But the difference is small, right? Right?

- As you would expect, yes.
- But error bars in data are also getting small these days.
- Can no longer ignore perturbations.

How typical is any  
perturbation?

$$\sigma_L^2 = \int_0^\infty \frac{dk}{k} \Delta_{m0}^2(k) [3j_1(Lk)/Lk]^2$$

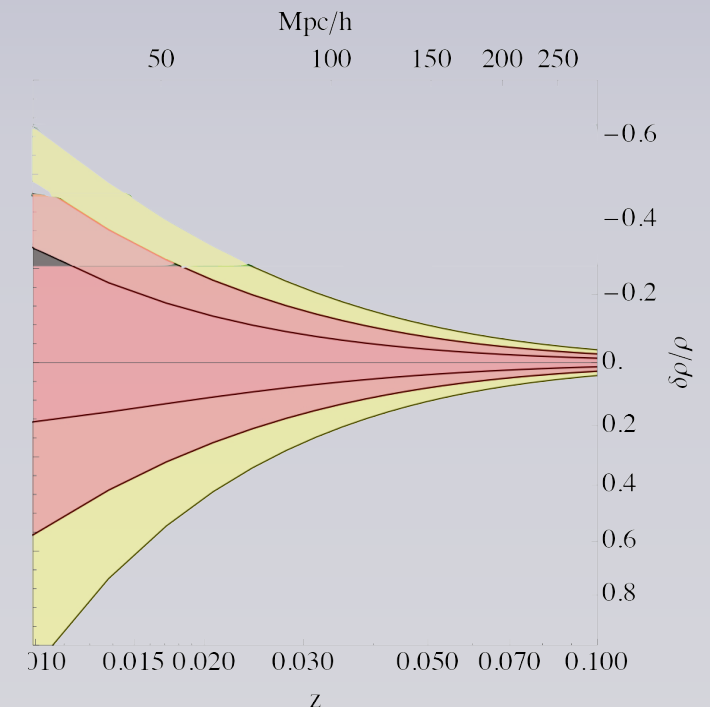
$$P(\delta_0, L) = (\sigma_L \sqrt{2\pi})^{-1} \exp \left[ -\frac{1}{2} (\delta_0 / \sigma_L)^2 \right]$$





# What does that imply?

- This is the probability distribution of matter inside radius  $L$ .
- This is not an observable.
- However, if one observes candles in all directions, and treats all directions equally, one **averages** over all angles.
- Expand the density field in spherical coordinates, and only keep the monopole component.



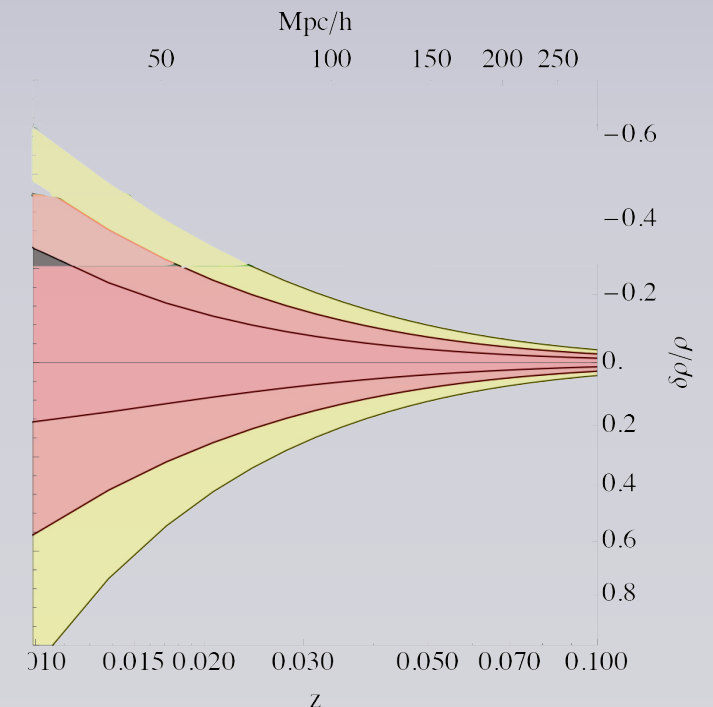
e.g. Romano, Chen (JCAP 2011)

# Advanced high school universe

Lemaître-Tolman-Bondi metric

$$ds^2 = -dt^2 + \frac{a_{\parallel}^2(t, r)}{1 - k(r)r^2} dr^2 + a_{\perp}^2(t, r) r^2 d\Omega^2$$

$$a_{\parallel} = (a_{\perp} \tilde{r})'$$



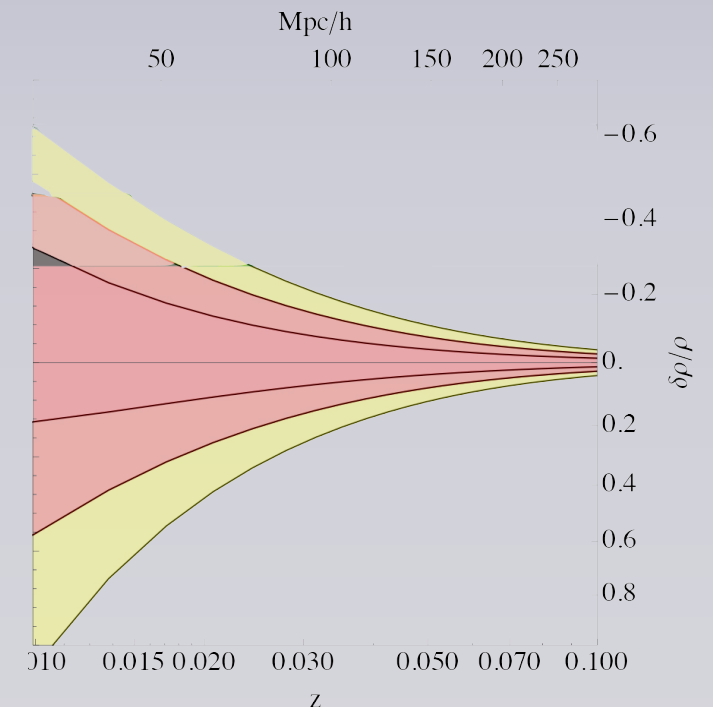
for  $\Lambda$ LTB solutions,  
see [Valkenburg, GERG\(2012\)](#)

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$$H_{\perp} = H_{\perp}(t, r) \equiv \dot{a}_{\perp}/a_{\perp} \quad a_{\parallel} = (a_{\perp} \tilde{r})'$$



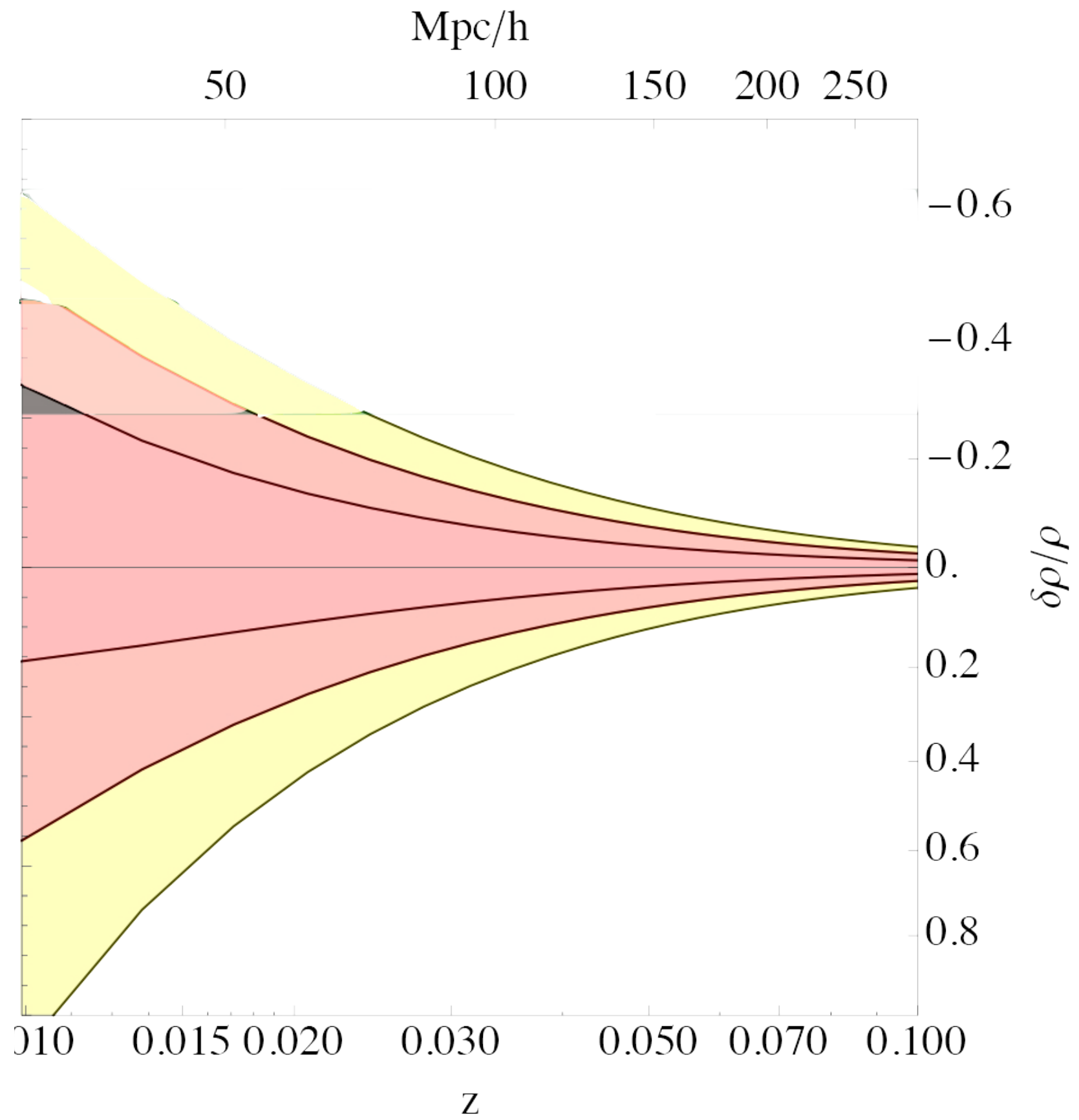
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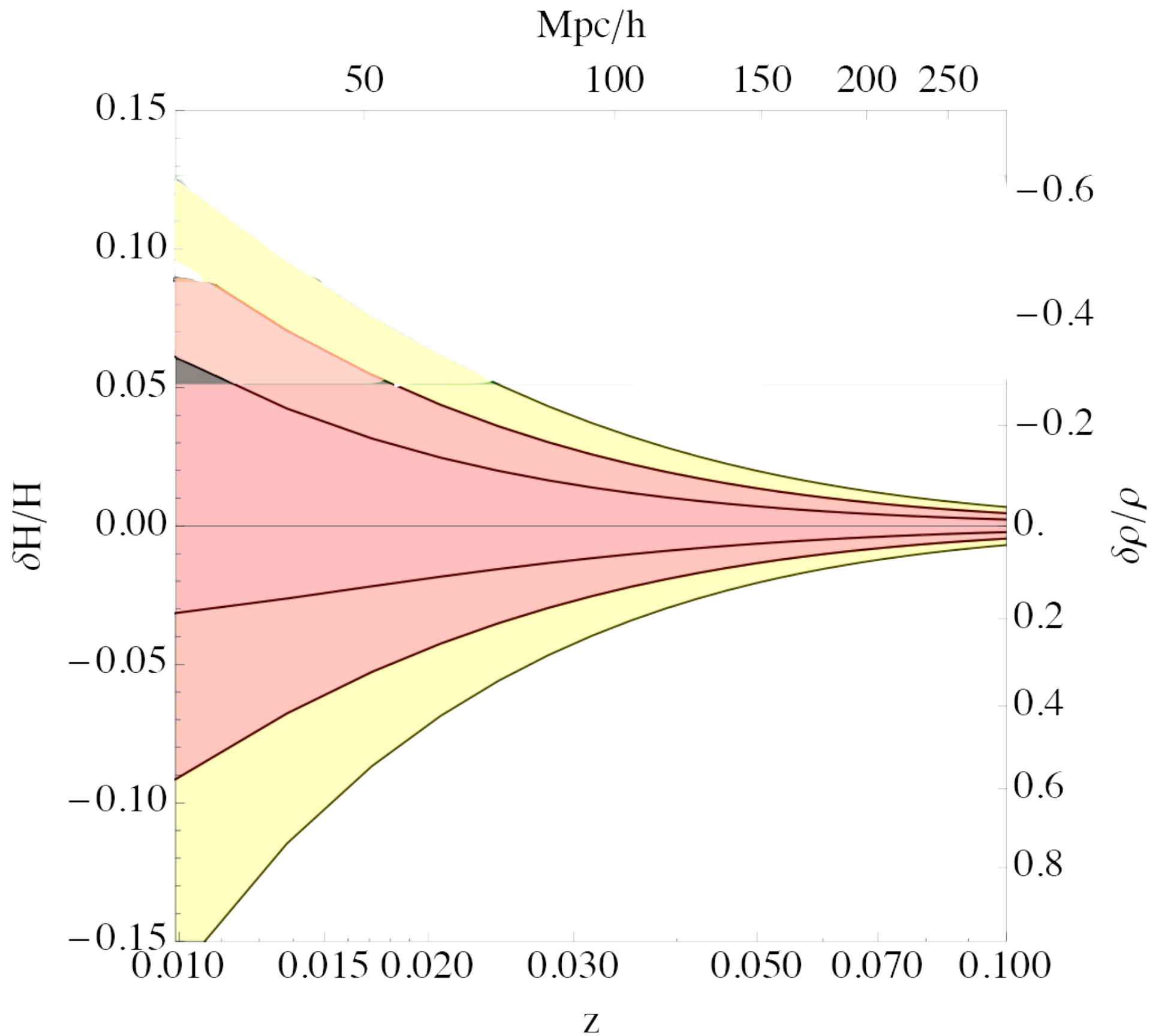
$$\frac{H_{\perp}^2}{H_{\perp 0}^2} = \Omega_m a_{\perp}^{-3} + \Omega_k a_{\perp}^{-2} + \Omega_{\Lambda}$$

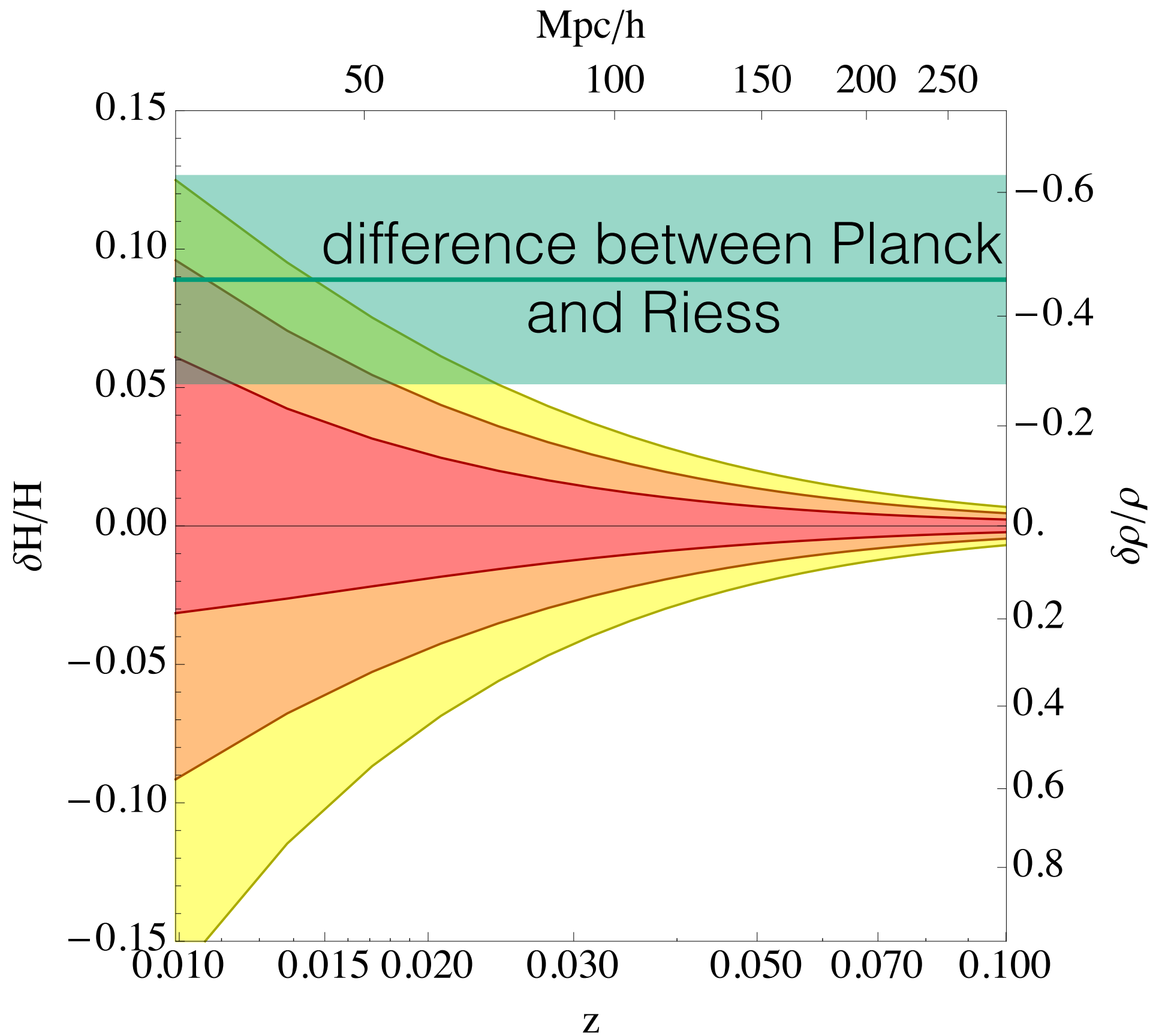
$$\Omega_m(r) = \frac{m(r)}{H_{\perp 0}^2}, \quad \Omega_k(r) = -\frac{k}{H_{\perp 0}^2}, \quad \Omega_{\Lambda}(r) = \frac{\Lambda}{3H_{\perp 0}^2}$$

# Solving dynamics and geodesics..

- $\Delta H \sim \Delta \rho$
- Obviously, at first order,  $\Delta d_A(z) \sim -\Delta H$
- So  $H_{\text{obs}} = H_{\text{inferred}} = \lim_{z \rightarrow 0} \langle z \rangle / \langle d_A(z) \rangle \sim \Delta \rho$





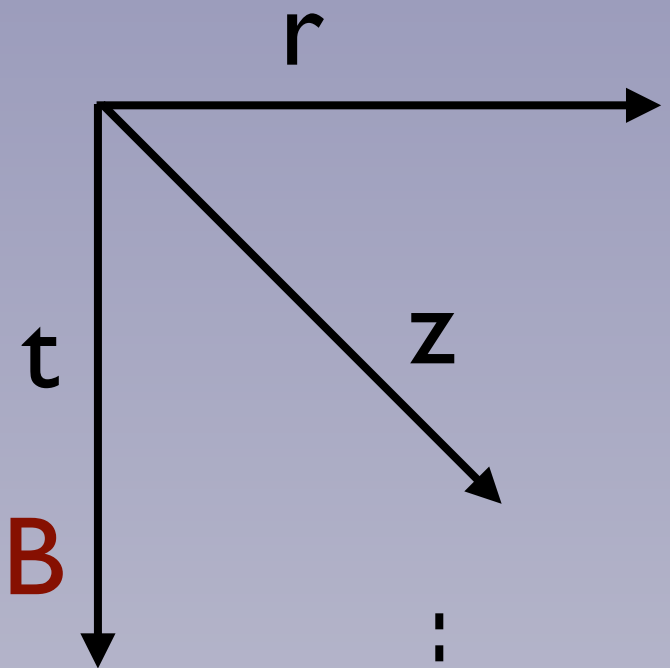


# This was if you know $P(k)$

- But do we really?
- Yes: CMB.
- But CMB is not here. It is at  $z=1100$ .  
(Same reasoning for  $P_{gg}(k)$ )



# Light cone



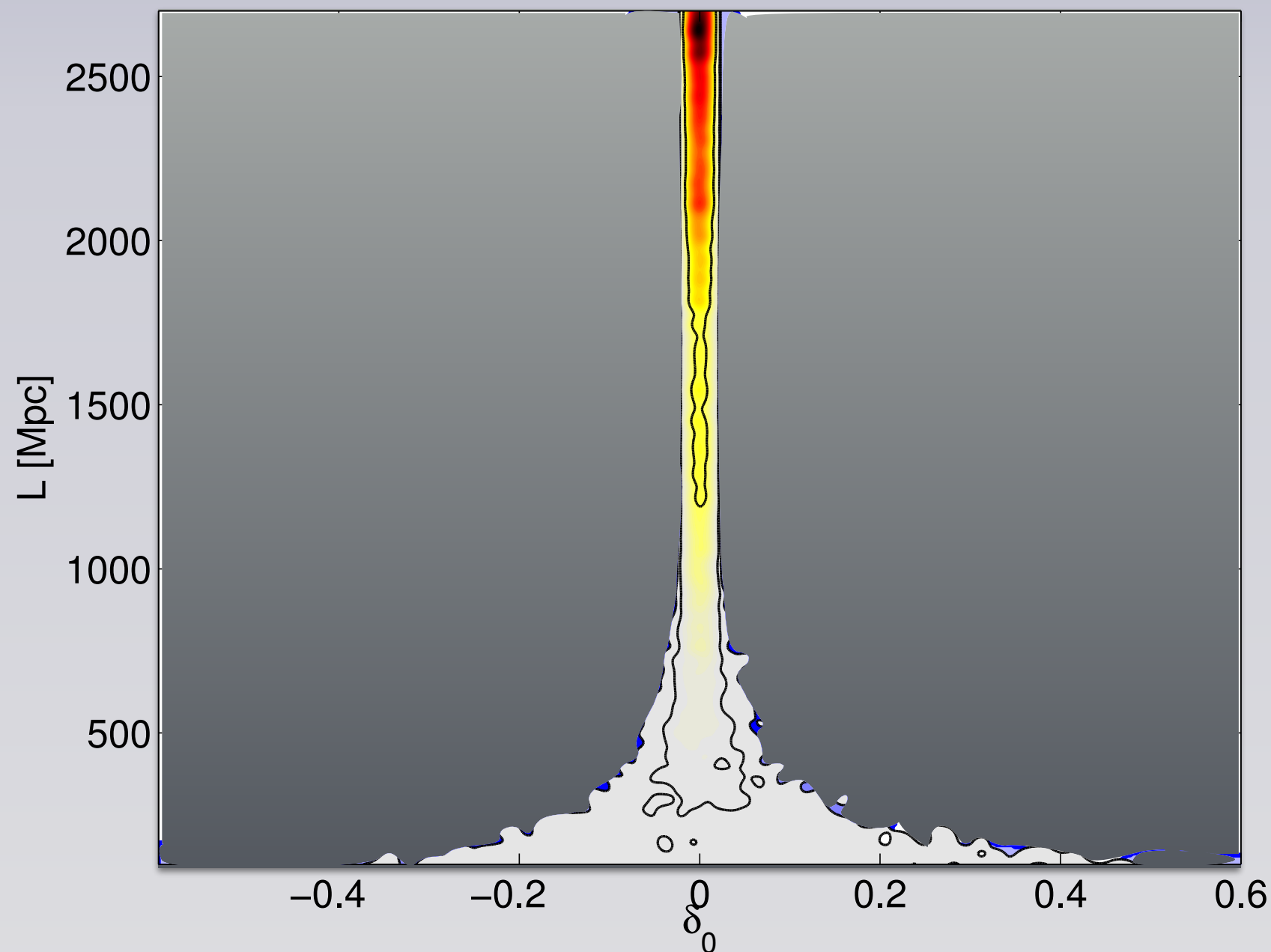
Us: isotropic CMB

Our past light cone

Our world line

Cosmic Microwave Background

# Copernican Prior



From WMAP's CMB:  
temperature  
fluctuations predict  
probability of density  
perturbation today

Fitting a cosmology with  $\Omega_m$ ,  $\Omega_\Lambda$ ,  $H_0$ ,  $A_s$ ,  $n_s$ ,  $\tau$ ,  $L$ ,  $\delta_\theta$   
give expected  $L$  and  $\delta_0$

CMB: **WMAP**

BAO: **SDSS, 6dFGS, WiggleZ**

SN: **SNLS**

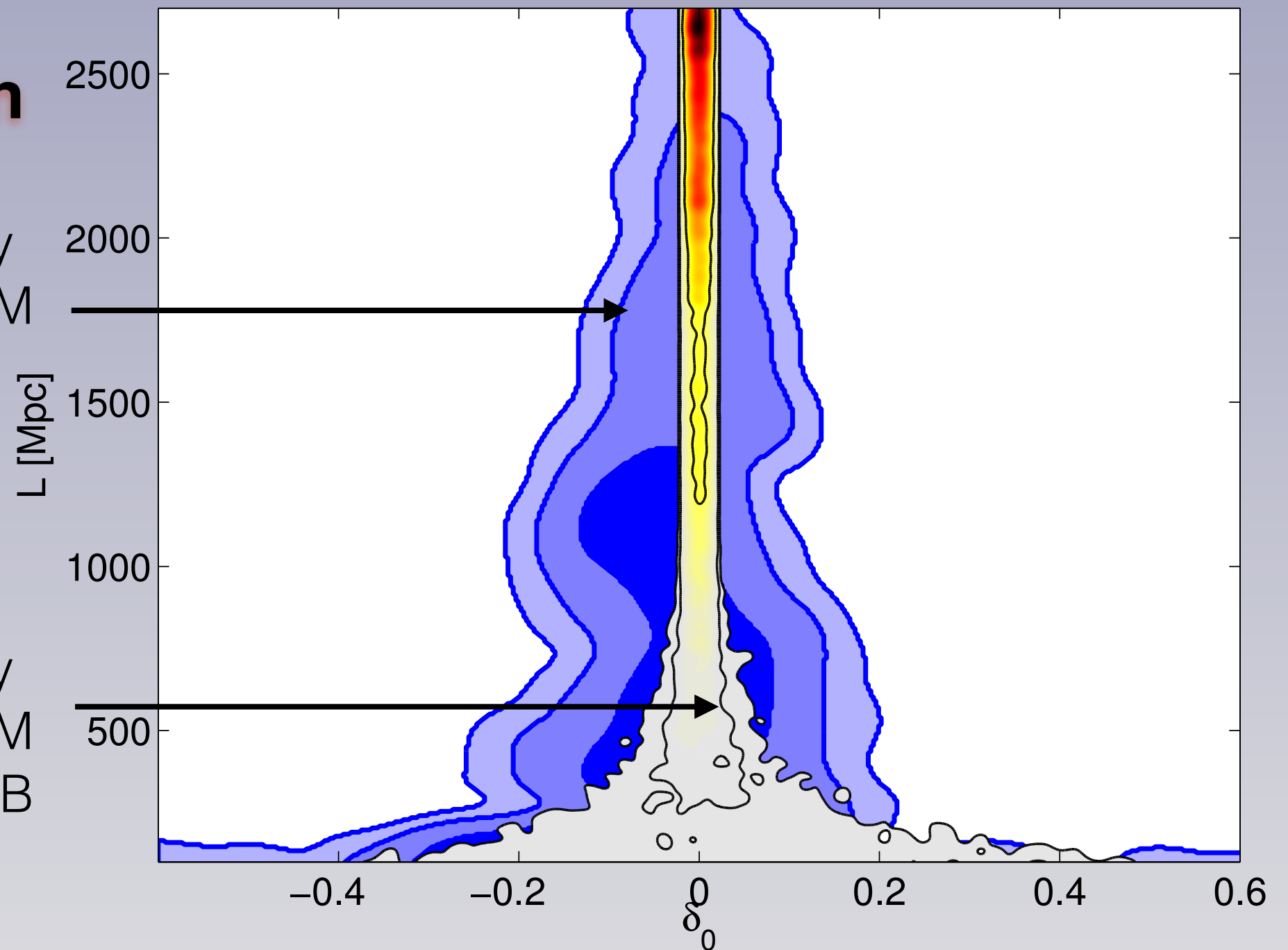
H0: **Freedman**

Probability of density  
perturbation in  $\Lambda$ CDM  
around us given  
observations

**//**

Probability of density  
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spectrum

Observations vs.  
prior/bias



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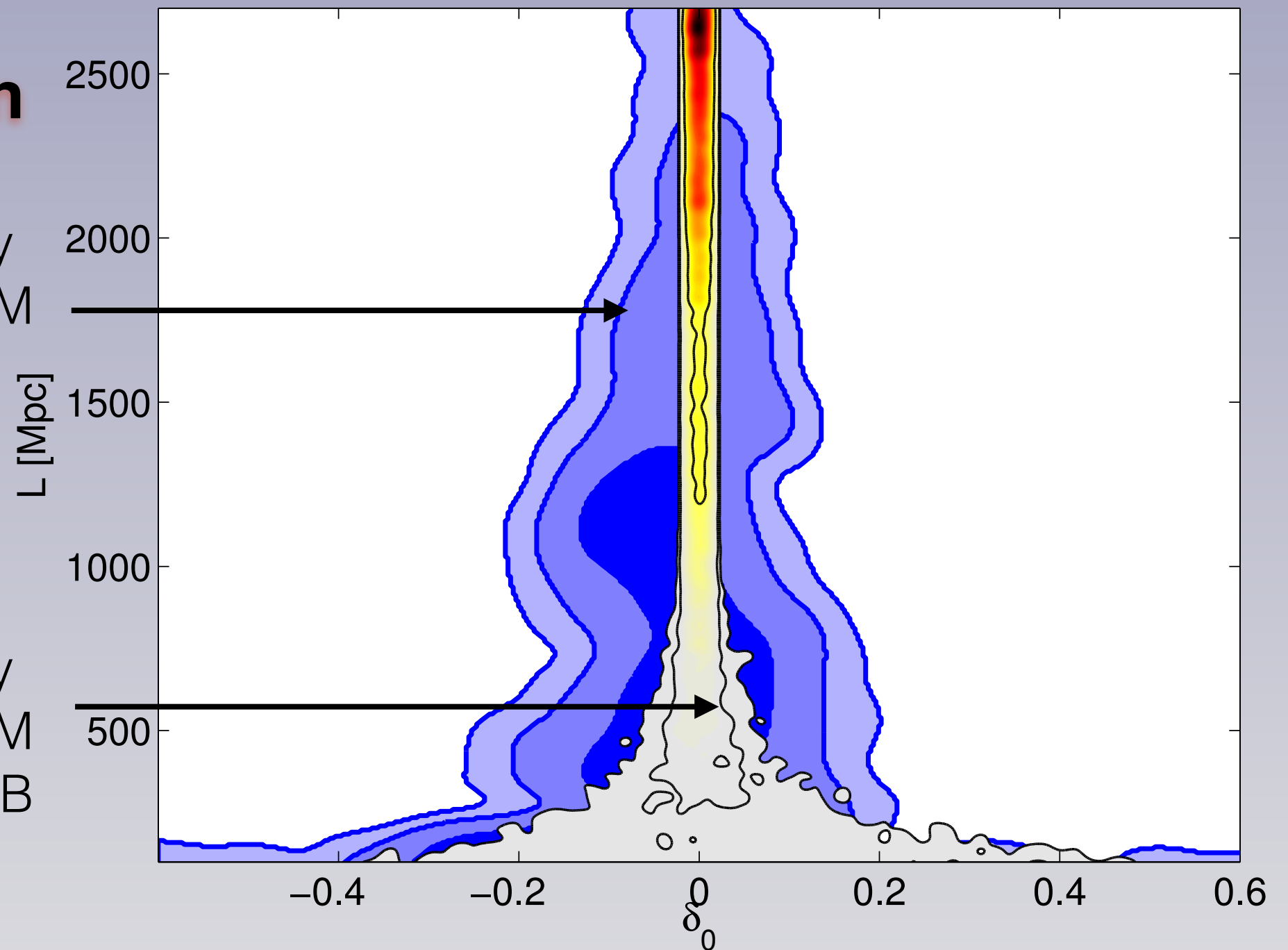
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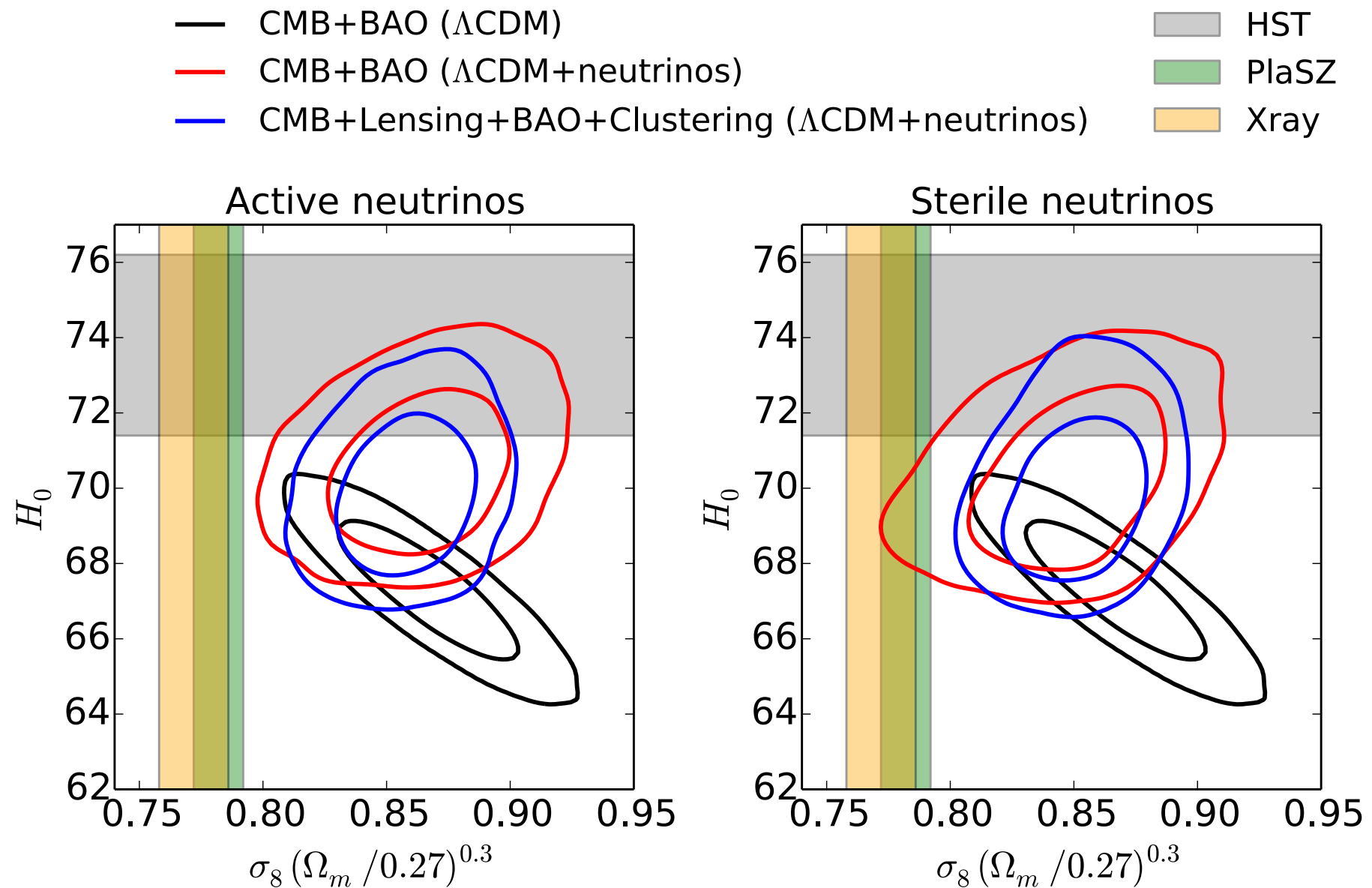
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Roughly a factor of 2—3 in surface in probability space

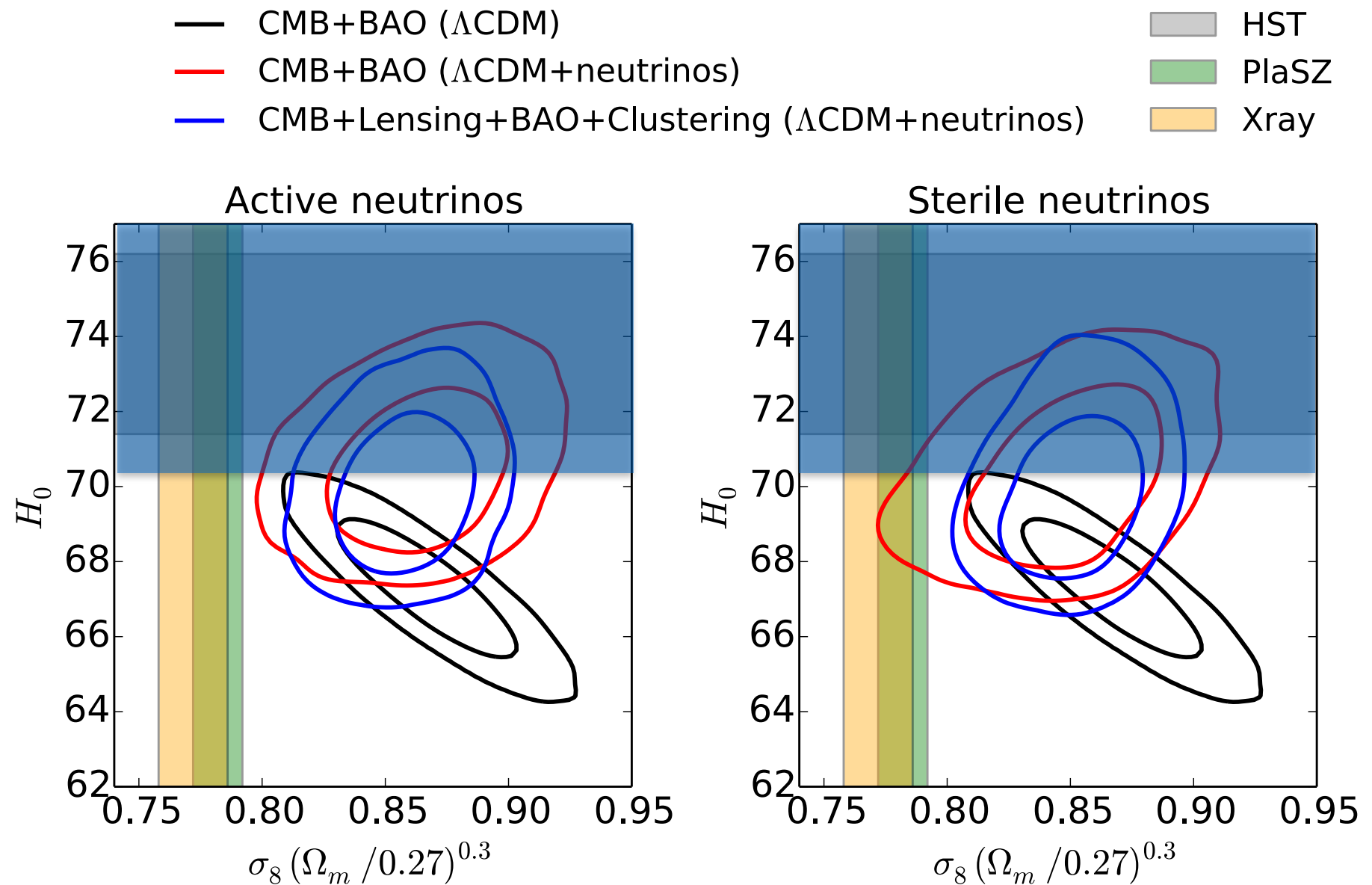
## ***A new cosmic concordance?***



Bayesian Evidence does not support massive sterile neutrino model even when combining conflicted datasets

Leistedt, HVP, Verde (to be submitted)

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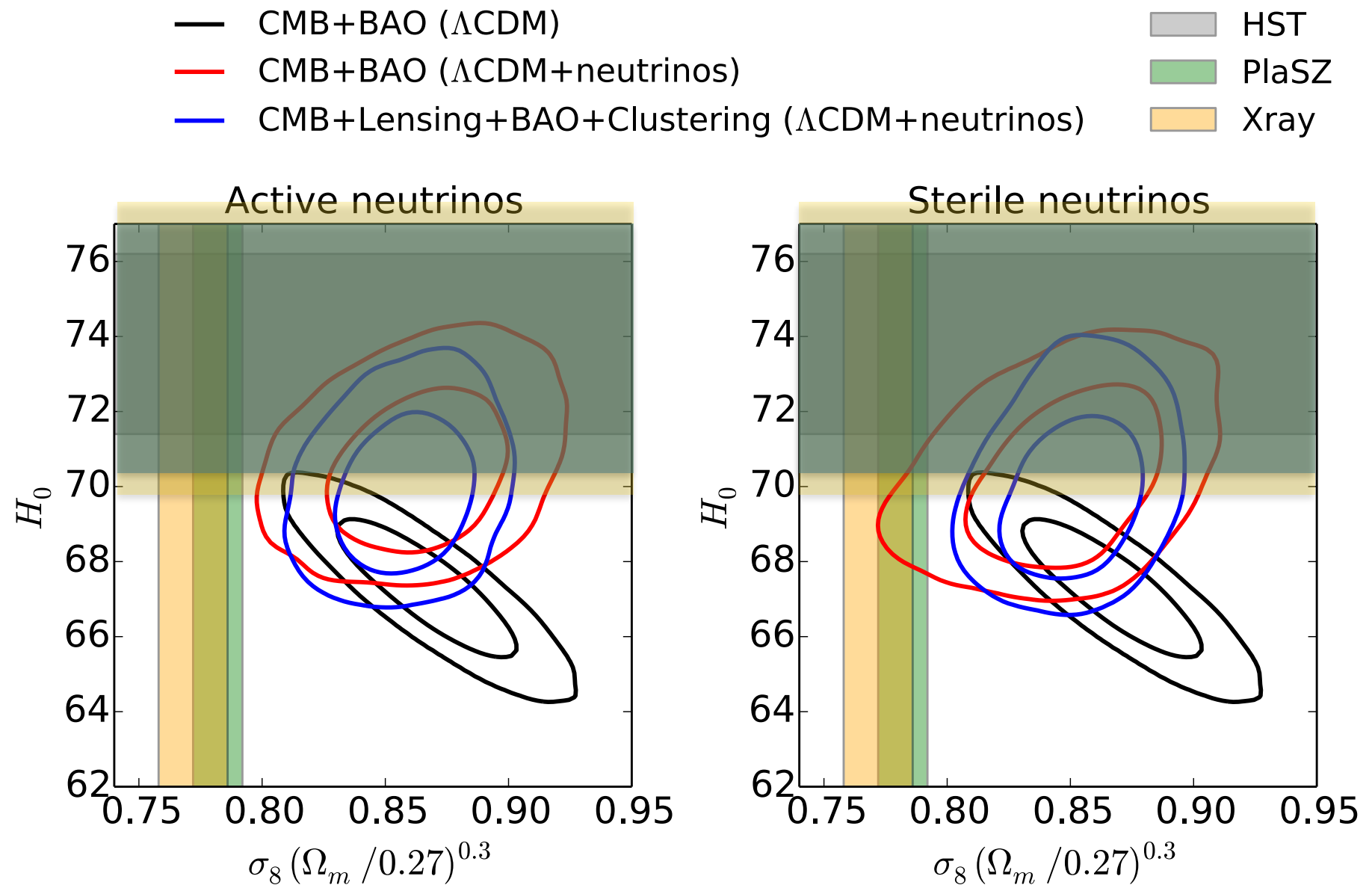


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# One step further

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- So far looked at full angular average. But no observer sees infinitely many sources.
- Take into account poisson noise from individual candles:

[arXiv:1401.7973](https://arxiv.org/abs/1401.7973)

## The value of $H_0$ in the inhomogeneous Universe

Ido Ben-Dayan<sup>1</sup>, Ruth Durrer<sup>2</sup>, Giovanni Marozzi<sup>2</sup> and Dominik J. Schwarz<sup>3</sup>

<sup>1</sup>*Deutsches Elektronen-Synchrotron DESY, Theory Group, D-22603 Hamburg, Germany*

<sup>2</sup>*Université de Genève, Département de Physique Théorique and CAP,  
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<sup>3</sup>*Fakultät für Physik, Universität Bielefeld, Postfach 100131, 33501 Bielefeld, Germany*

(Dated: February 6, 2014)

Local measurements of the Hubble expansion rate are affected by structures like galaxy clusters or voids. Here we present a first fully relativistic treatment of this effect, studying how clustering modifies the mean distance (modulus)-redshift relation and its dispersion. The best estimates of the local expansion rate stem from supernova observations at small redshifts ( $0.01 < z < 0.1$ ). It is interesting to compare these local measurements with global fits to data from cosmic microwave background anisotropies. In particular, we argue that cosmic variance (i.e. the effects of the local structure) is of the same order of magnitude as the current observational errors and must be taken into account in all future local measurements of the Hubble expansion rate.

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# Summary

- $H_0$  is not what it seems
- Age of universe (distance to CMB) is not same as local expansion rate
- Local expansion rate subject to local physics
- Effect quantified: extra
  - 1% with known  $P(k)@r=0$  + infinite observation #,
  - 3% with no known  $P(k)@r=0$  but use LTB constraints
  - 5% with known  $P(k)@r=0$  + current  $H_0$  observation #