Dark Energy Phenomenology: Effective Field Theory approach

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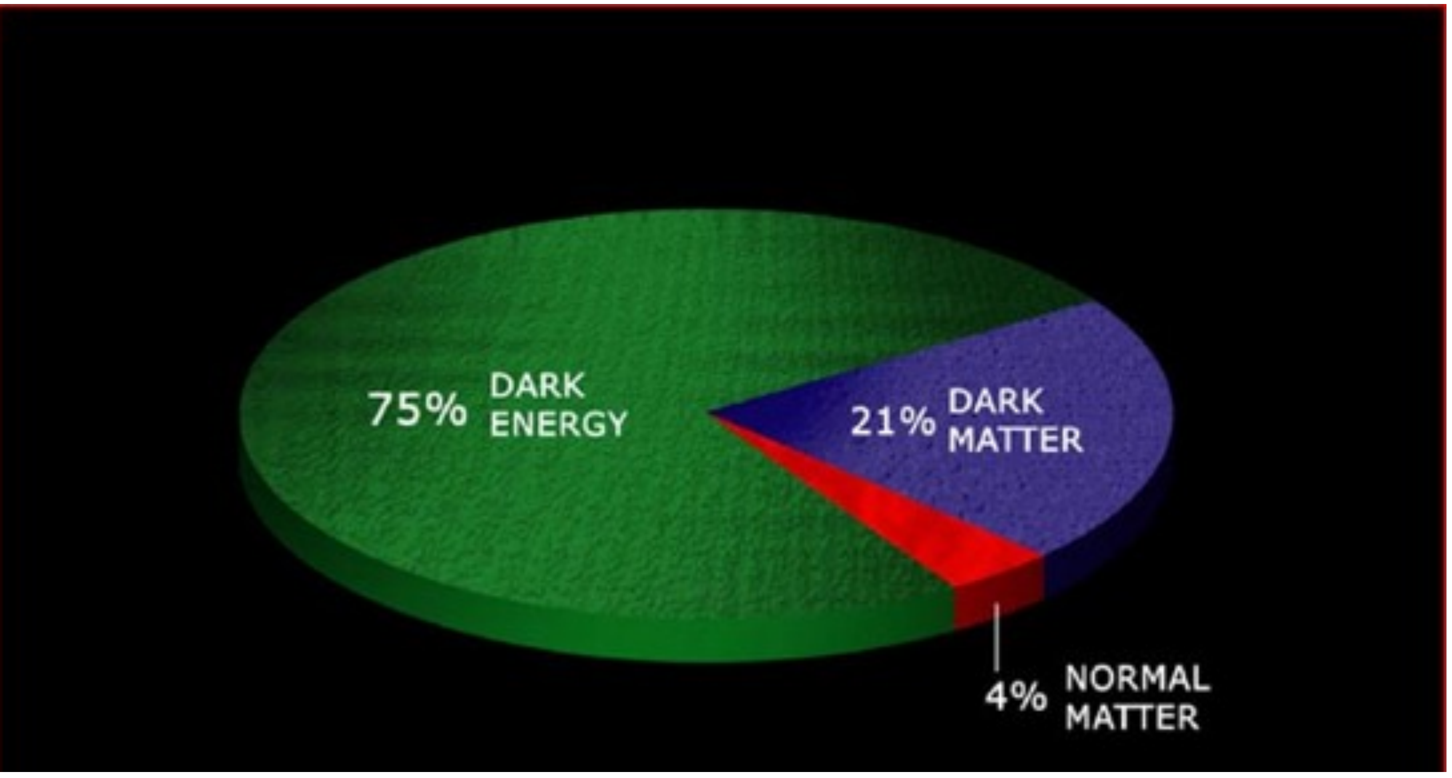
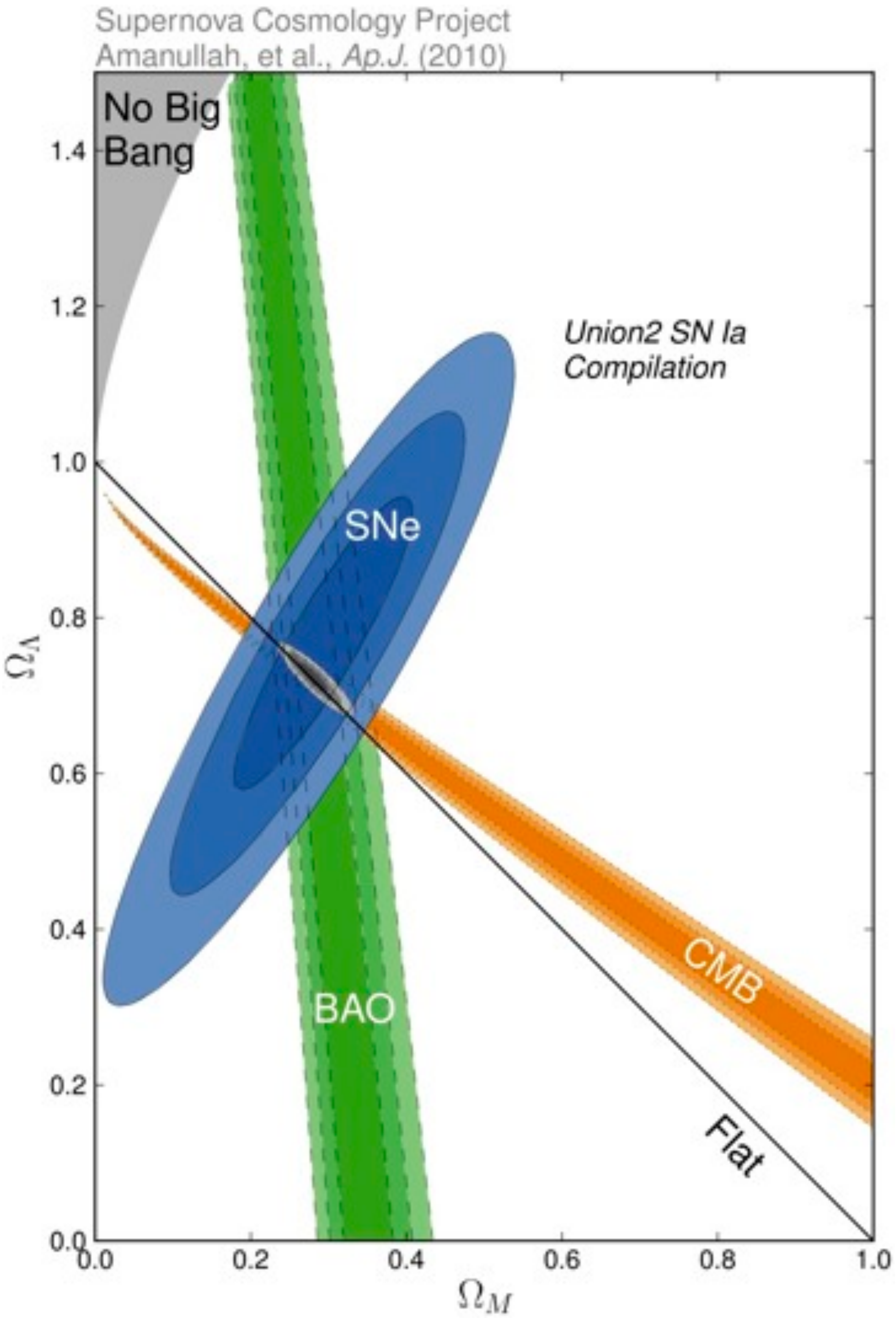
Gubitosi, F. P., Vernizzi, 1210.0201
Gleyzes, Langlois, F.P., Vernizzi, 1304.4840
F. P., F. Vernizzi, 1307.4350
F. P., C. Marinoni, H. Steigerwald 1312.6111
In progress...

Nobel Prize in Physics 2011

The Universe is accelerating!





Beyond the Cosmological Constant...

Naturalness problem (perhaps $\Lambda = 0$ is better than $\Lambda \sim (10^{-3}\mathrm{eV})^4$)

Coincidence problem (why $\rho_{\mathrm{DE}} \sim \rho_{\mathrm{M}}$ now?!).

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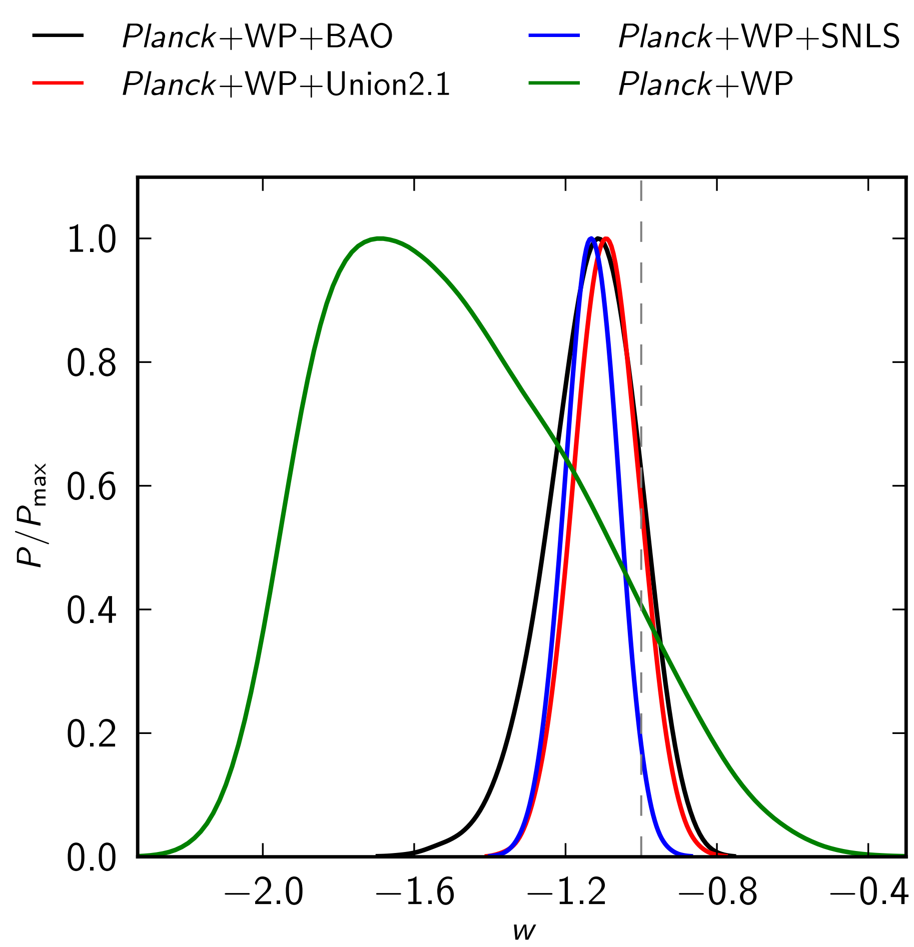
DATA...?



$w = -1.24^{+0.18}_{-0.19}$ (95%; *Planck*+WP+H_0)

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Naturalness problem (perhaps $\Lambda = 0$ is better than $\Lambda \sim (10^{-3}\mathrm{eV})^4$)

Coincidence problem (why $\rho_{\mathrm{DE}} \sim \rho_{\mathrm{M}}$ now?!).

DATA...?

PAN-STARRS1 (1310.3828): $w = -1.18 \pm 0.07$

SDSSII & SNLS (1401.4064): $w = -1.02 \pm 0.05$

(talk by M. Betoule on friday)

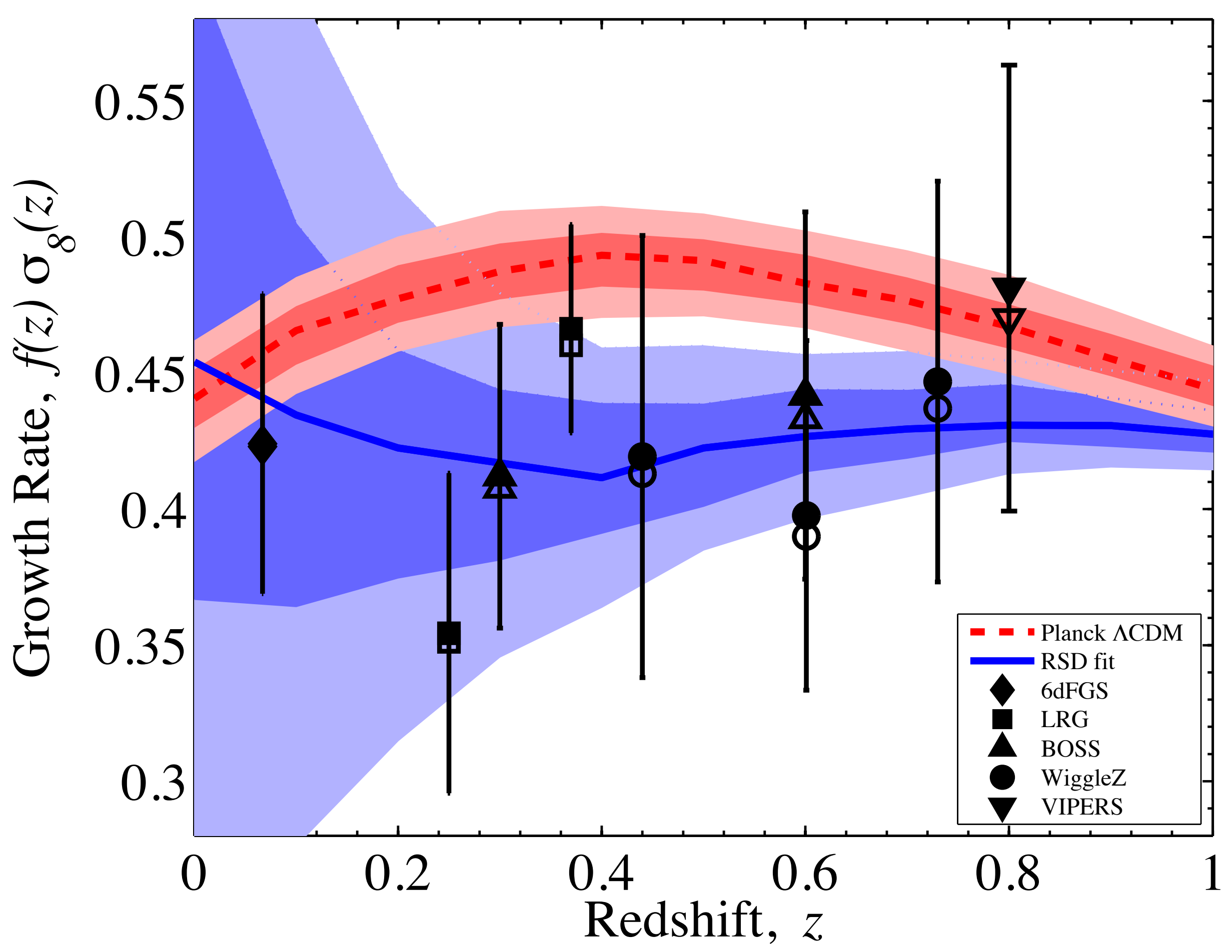
Beyond the Cosmological Constant...

Naturalness problem (perhaps $\Lambda = 0$ is better than $\Lambda \sim (10^{-3}\mathrm{eV})^4$)

Coincidence problem (why $\rho_{\mathrm{DE}} \sim \rho_{\mathrm{M}}$ now?!).

Macaulay et al. '13

DATA...?



Perturbation sector:
less growth than expected...?

See also Hiranya's talk yesterday

Beyond the Cosmological Constant...

$$DE \neq \Lambda$$

$$\Downarrow$$

There is a new propagating degree of freedom in the theory ϕ

- There is `no shortage' of dark energy and modified gravity models (>5000 papers on Spires)

- EUCLID and BigBoss will be sensitive to dynamical properties of DE

- Need for a Unifying and Effective description of DE

Ideally...

- A limited number of effective operators, each one responsible for an observable dynamical feature (e.g. flavor-changing neutral currents in physics beyond Standard Model)

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$$S[\phi, g_{\mu\nu}, \Psi_m]$$

Background

$$\phi = \phi_0(t)$$
$$ds^2 = -dt^2 + a^2(t)dx^2$$
$$\rho_m = \rho_m(t)$$

Expand in perturbations

$$\delta\rho_m(t,\vec{x}) \Leftrightarrow \delta\phi(t,\vec{x})$$

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Ideally...

- A limited number of operators, each one responsible for an observable dynamical feature (e.g. flavor-changing neutral currents in beyond Standard Model physics)

Hint:

Most DE models reduce, in their relevant regimes, to scalar tensor-theories

$$S = \int d^4x \sqrt{-g} \left[\frac{M^2}{2} f(\phi) R - \frac{1}{2} (\partial\phi)^2 - V(\phi) + \mathcal{F}[\phi, g^{\mu\nu}] \right] + S_m[g_{\mu\nu}, \Psi_m]$$

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One possible strategy: (Weinberg `08, Park, Zurek and Watson `10, Bloomfield and Flanagan `11)

Apply covariant EFT to explore $\mathcal{F}[\phi, g^{\mu\nu}]$: field/derivative expansion

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- Expansion in number of fields is not necessarily meaningful

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C

gan `11)

A

ion

$$V = V_1\phi + V_2\phi^2 + V_3\phi^3 + V_4\phi^4$$
$$= V_2\delta\phi^2 + V_3\phi_0(t)\delta\phi^2 + 6V_4\phi_0^2(t)\delta\phi^2$$

All terms potentially important in cosmological perturbation theory!

H

- Expansion in number of fields is not necessarily meaningful

Hint:

Most DE models reduce, in their relevant regimes, to scalar tensor-theories

$$S = \int d^4x\sqrt{-g}\left[\frac{M^2}{2}f(\phi)R - \frac{1}{2}(\partial\phi)^2 - V(\phi) + \mathcal{F}[\phi, g^{\mu\nu}]\right] + S_m[g_{\mu\nu}, \Psi_m]$$

One possible strategy: (Weinberg `08, Park, Zurek and Watson `10, Bloomfield and Flanagan `11)

Apply covariant EFT to explore $\mathcal{F}[\phi, g^{\mu\nu}]$: field/derivative expansion

However:

- Expansion in number of fields is not necessarily meaningful

- Only halfway through the work to be done (background first + expand..)

EFT: a theory for the relevant low-energy d.o.f.

QCD $\Rightarrow$ nucleons and pions

EW theory $\Rightarrow$ 3 massive vector bosons, 1 ``Higgs''...

Cosmology $\Rightarrow$ Cosmological Perturbations!

They (re-)enter the horizon

1) Small in amplitude (expansion in number of cosmological perturbations)

2) Large in size (expansion in number of derivatives)

Unitary Gauge in Cosmology

The Effective Field Theory of Inflation (Creminelli et al. `06, Cheung et al. `07)

Main idea: scalar degrees of freedom are `eaten' by the metric. Ex:

$$\phi(t,\vec{x}) \to \phi_0(t) \quad (\delta\phi = 0) \qquad -\frac{1}{2}\partial\phi^2 \to -\frac{1}{2}\dot{\phi}_0^2(t)\ g^{00}$$

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Our Recipe for Dark Energy: (Gubitosi, F.P., Vernizzi 2012)

1) Assume WEP (universally coupled metric $S_m[g_{\mu\nu}, \Psi_i]$)

2) Write the most generic action for $g_{\mu\nu}$ compatible with the residual un-broken symmetries (3-diff).

The Action

$$S = \int d^4x \sqrt{-g}\, \frac{M^2(t)}{2}\, \left[R - 2\lambda(t) - 2\mathcal{C}(t) g^{00}\right.$$

The most generic action written in terms of $g_{\mu\nu}$ compatible with the residual symmetry of spatial diffeomorphisms

The Action

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The most generic action written in terms of $g_{\mu\nu}$ compatible with the residual symmetry of spatial diffeomorphisms

Genuine 4-dim covariant terms are still allowed, but will in general be multiplied by functions of time cause time translations are broken

The Action

$$S = \int d^4x \sqrt{-g} \, \frac{M^2(t)}{2} \, \left[R - 2\lambda(t) - 2\mathcal{C}(t) g^{00}\right.$$

The most generic action written in terms of $g_{\mu\nu}$ compatible with the residual symmetry of spatial diffeomorphisms

General functions of time are allowed

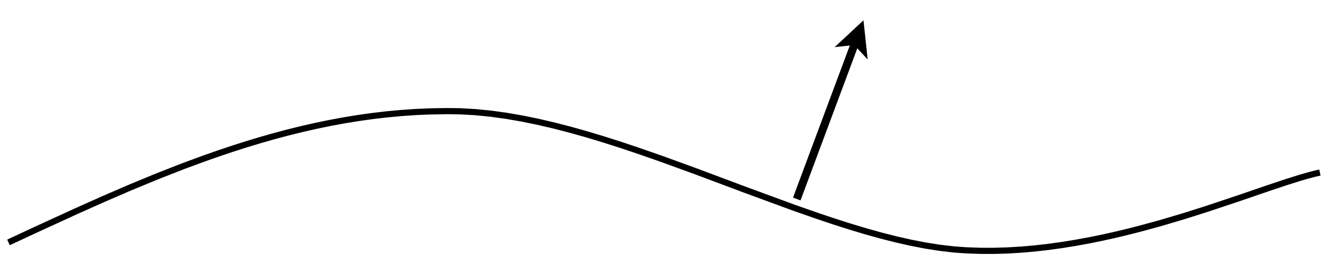
The Action

$$S = \int d^4x \sqrt{-g}\, \frac{M^2(t)}{2} \left[R - 2\lambda(t) - 2\mathcal{C}(t) g^{00} \right.$$

The most generic action written in terms of $g_{\mu\nu}$ compatible with the residual symmetry of spatial diffeomorphisms

...as well as tensors with free `0' indices

Essentially: contractions with $n_\mu = -\dfrac{\partial_\mu \phi}{\sqrt{-(\partial\phi^2)}}$



The Action

$$S = \int d^4x \sqrt{-g}\,\frac{M^2(t)}{2}\,\left[R - 2\lambda(t) - 2\mathcal{C}(t)g^{00}\right.$$

Any arbitrarily complicate action with one scalar d.o.f. will reduce to this in Unitary gauge, plus terms that start explicitly quadratic in the perturbations

Gubitosi, F. P., Vernizzi, 12

The Action

$$S = \int d^4x \sqrt{-g}\,\frac{M^2(t)}{2}\,\Big[R - 2\lambda(t) - 2\mathcal{C}(t)g^{00}$$

$$+\,\mu_2^2(t)(\delta g^{00})^2 - \mu_3(t)\,\delta K \delta g^{00} + \epsilon_4(t)\left(\delta K^{\mu}{}_{\nu}\,\delta K^{\nu}{}_{\mu} - \delta K^2 + \frac{R^{(3)}\,\delta g^{00}}{2}\right) + \ldots\Big]$$

The Action

Background (expansion history)

$$S = \int d^4x \sqrt{-g}\, \frac{M^2(t)}{2} \left[R - 2\lambda(t) - 2\mathcal{C}(t) g^{00} \right.$$

$$\left. + \mu_2^2(t)(\delta g^{00})^2 - \mu_3(t)\,\delta K \delta g^{00} + \epsilon_4(t) \left(\delta K^{\mu}{}_{\nu}\, \delta K^{\nu}{}_{\mu} - \delta K^2 + \frac{R^{(3)}\, \delta g^{00}}{2} \right) + \ldots \right]$$

(linear) perturbations

Time-dependent couplings

The Power of the EFT of DE

$$S = \int d^4x \sqrt{-g}\, \frac{M^2(t)}{2}\, \Big[R - 2\lambda(t) - 2\mathcal{C}(t) g^{00}$$
$$+ \mu_2^2(t)(\delta g^{00})^2 - \mu_3(t)\, \delta K \delta g^{00} + \epsilon_4(t) \left(\delta K^{\mu}{}_{\nu}\, \delta K^{\nu}{}_{\mu} - \delta K^2 + \frac{R^{(3)}\, \delta g^{00}}{2} \right) + \ldots \Big]$$

- Clear separation: background v.s. perturbed sector
- Expansion in number of cosmological perturbations
- Expansion in number of derivatives
- Observables, stability etc.

Examples

$$S = \int d^4x \sqrt{-g}\, \frac{M^2(t)}{2}\, \Big[R - 2\lambda(t) - 2\mathcal{C}(t) g^{00}$$

$$+ \mu_2^2(t)(\delta g^{00})^2 - \mu_3(t)\, \delta K \delta g^{00} + \epsilon_4(t) \left(\delta K^\mu{}_\nu\, \delta K^\nu{}_\mu - \delta K^2 + \frac{R^{(3)}\, \delta g^{00}}{2}\right) + \dots\Big]$$

Examples

const.

$$S = \int d^4x \sqrt{-g}\, \frac{M^2(t)}{2} \left[R - 2\lambda(t) - 2\mathcal{C}(t) g^{00} \right.$$

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Quintessence

Examples

$$S = \int d^4x \sqrt{-g}\, \frac{M^2(t)}{2} \left[R - 2\lambda(t) - 2\mathcal{C}(t) g^{00} \right.$$
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Non-minimally coupled scalar field
(Brans-Dicke, f(R) etc.)

Examples

const.

$$S = \int d^4x \sqrt{-g} \, \frac{M^2(t)}{2} \, \Big[R - 2\lambda(t) - 2\mathcal{C}(t) g^{00}$$

$$+ \mu_2^2(t)(\delta g^{00})^2 - \mu_3(t)\, \delta K \delta g^{00} + \epsilon_4(t) \left(\delta K^\mu{}_\nu \, \delta K^\nu{}_\mu - \delta K^2 + \frac{R^{(3)}\, \delta g^{00}}{2} \right) + \dots \Big]$$

K-essence

(Amendariz-Picon et al., 2000)

$$S = \int d^4x \sqrt{-g} P(\phi, X) \qquad X \equiv g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

Examples

$$S = \int d^4x \sqrt{-g} \frac{M^2(t)}{2} \Big[R - 2\lambda(t) - 2\mathcal{C}(t) g^{00}$$

$$+ \mu_2^2(t)(\delta g^{00})^2 - \mu_3(t)\, \delta K \delta g^{00} + \epsilon_4(t) \left(\delta K^\mu{}_\nu \, \delta K^\nu{}_\mu - \delta K^2 + \frac{R^{(3)}\, \delta g^{00}}{2} \right) + \dots \Big]$$

"Galilean Cosmology" (Chow and Khoury, 2009)

$$S = \int d^4x \sqrt{-g} \left[\frac{M^2}{2} e^{-2\phi/M} R - \frac{r_c^2}{M} (\partial\phi)^2 \Box\phi \right]$$

Gleyzes, Langlois, F.P., Vernizzi, 1304.4840

Examples

$$S = \int d^4x \sqrt{-g} \frac{M^2(t)}{2} \Big[R - 2\lambda(t) - 2\mathcal{C}(t) g^{00} + \mu_2^2(t)(\delta g^{00})^2 - \mu_3(t)\,\delta K \delta g^{00} + \epsilon_4(t)\left(\delta K^\mu{}_\nu\, \delta K^\nu{}_\mu - \delta K^2 + \frac{R^{(3)}\,\delta g^{00}}{2}\right) + \dots\Big]$$

"Generalized Galileons" (≡ Horndeski)

(Deffayet et al., 2011)

$$\mathcal{L}_2 = A(\phi, X)\,,$$

$$\mathcal{L}_3 = B(\phi, X)\Box\phi\,,$$

$$\mathcal{L}_4 = C(\phi, X)R - 2C_{,X}(\phi, X)\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right],$$

$$\mathcal{L}_5 = D(\phi, X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi + \frac{1}{3}D_{,X}(\phi, X)\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right],$$

Examples

Gleyzes, Langlois, F.P., Vernizzi, 1304.4840

$$S = \int d^4x \sqrt{-g}\, \frac{M^2(t)}{2} \Big[R - 2\lambda(t) - 2\mathcal{C}(t) g^{00}$$
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$\epsilon_4(t)$ $\tilde{\epsilon}_4(t)$

Beyond Horndeski (linear)

The most general (linear) theory without higher derivatives on the propagating degree of freedom

Gleyzes, Langlois, F.P., Vernizzi... TOMORROW?

Beyond Horndeski (full, non-linear)

- Equations of motion with higher derivatives
- Only two derivatives on the true propagating degree of freedom
- No ghosts
- Interesting phenomenology (modified Jeans phenomenon)

Stability and Observables

F. P., C. Marinoni, H. Steigerwald 1312.6111
and in progress...

$$S = \int d^4x \sqrt{-g}\, \frac{M^2(t)}{2}\, \Big[R - 2\lambda(t) - 2\mathcal{C}(t) g^{00}$$

$$+ \mu_2^2(t)(\delta g^{00})^2 - \mu_3(t)\, \delta K \delta g^{00} + \epsilon_4(t) \left(\delta K^\mu{}_\nu\, \delta K^\nu{}_\mu - \delta K^2 + \frac{R^{(3)}\, \delta g^{00}}{2} \right) + \dots \Big]$$

$$\lambda(t), \quad \mathcal{C}(t), \quad \mu(t) \equiv \frac{dM^2(t)}{dt} \quad \left\{ \begin{array}{l} \bar{w}(t) \\ \mu(t) \end{array} \right.$$

Expansion History

$$\left. \begin{array}{l} \mu(t) \\ \mu_3(t) \\ \epsilon_4(t) \end{array} \right\}$$

Growth rate, lensing etc.

$$\mu_2^2(t)$$

Unconstrained

Stability

$$S_\pi = \int a^3(t) M^2(t) \left[A\left(\mu, \mu_2^2, \mu_3, \epsilon_4\right) \dot{\pi}^2 + B\left(\mu, \mu_3, \epsilon_4\right) \frac{(\vec{\nabla}\pi)^2}{a^2} \right] + \text{lower order in derivatives.}$$

No ghost: A>0

No gradient instabilities: B>0

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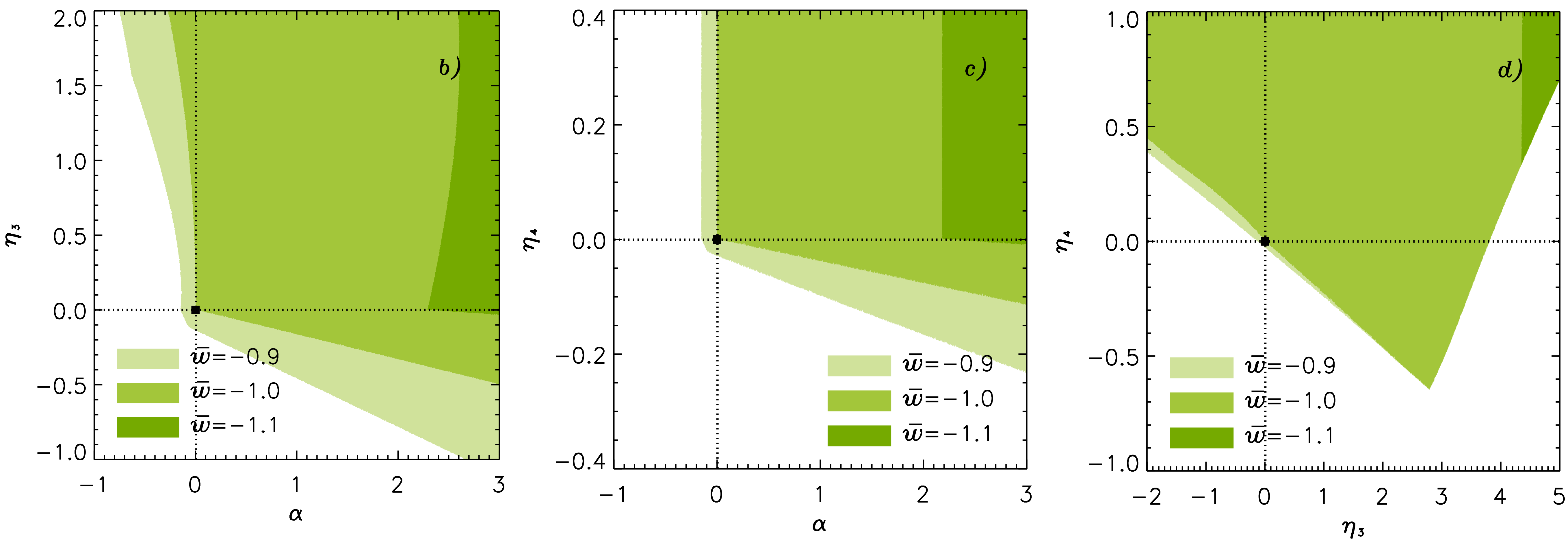
No ghost: A>0

No gradient instabilities: B>0

$\mu_2^2 = 0$



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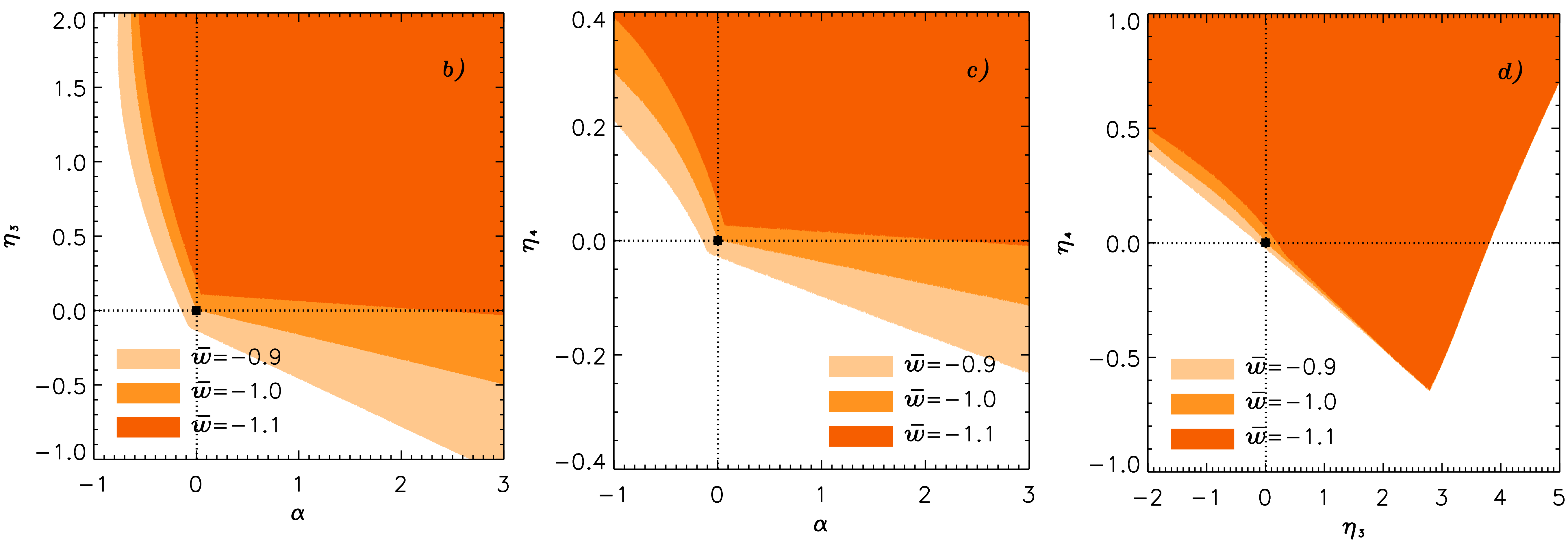
No ghost: A>0

No gradient instabilities: B>0

$$\mu_2^2 \gg H^2$$



F. P., C. Marinoni, H. Steigerwald 1312.6111

Growth rate

$$G_{\rm eff}(t) = G_{\rm eff}(\mu, \mu_3, \epsilon_4)$$

$$f \equiv \frac{d\ln\delta}{d\ln a} = \Omega_m^{\gamma_0+\gamma_1 \ln(\Omega_m)}$$

Steigerwald, Bel Marinoni 1403.0898

F. P., C. Marinoni, H. Steigerwald 1312.6111

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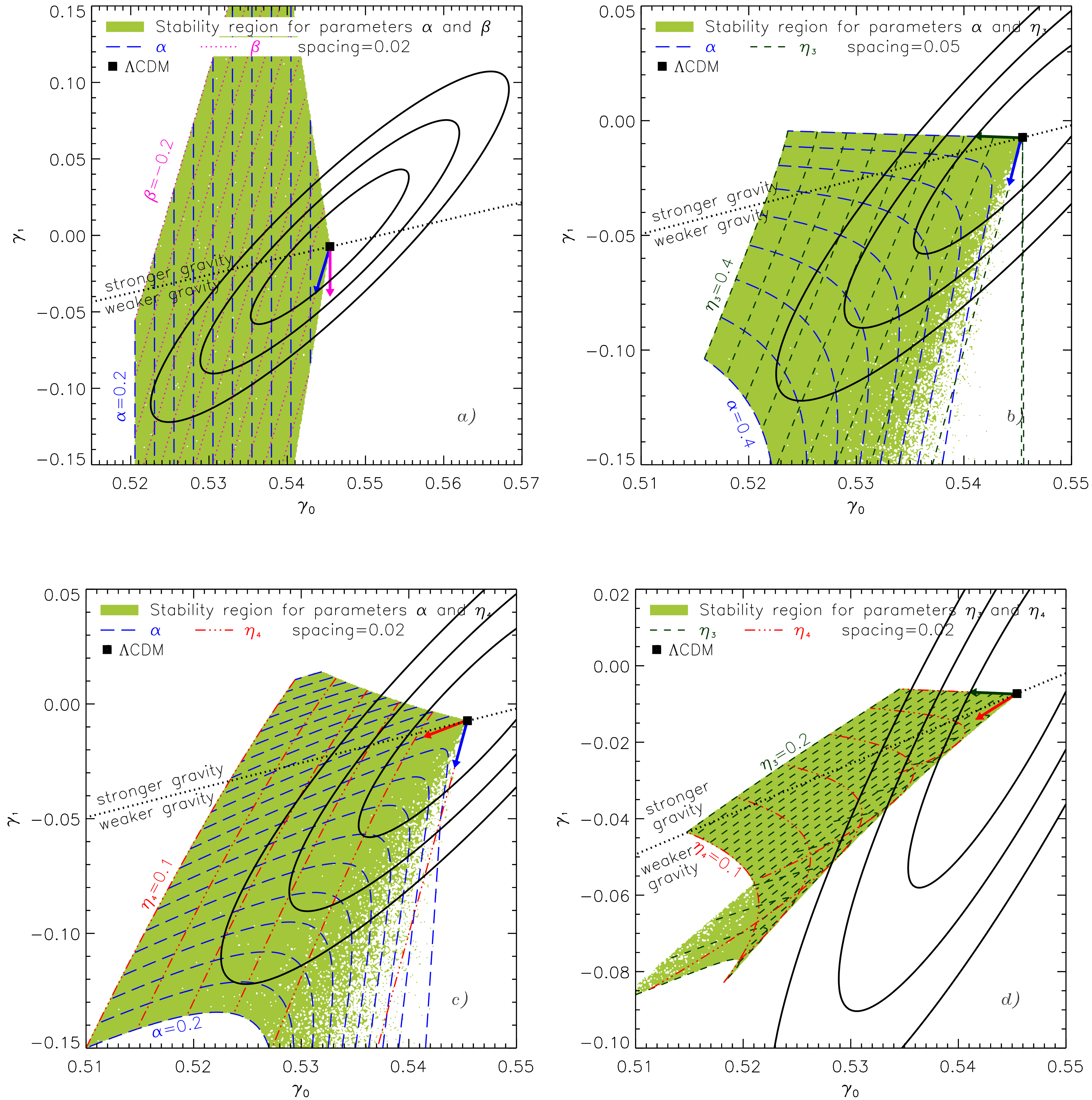
Steigerwald, Bel Marinoni 1403.0898

Non trivial result:

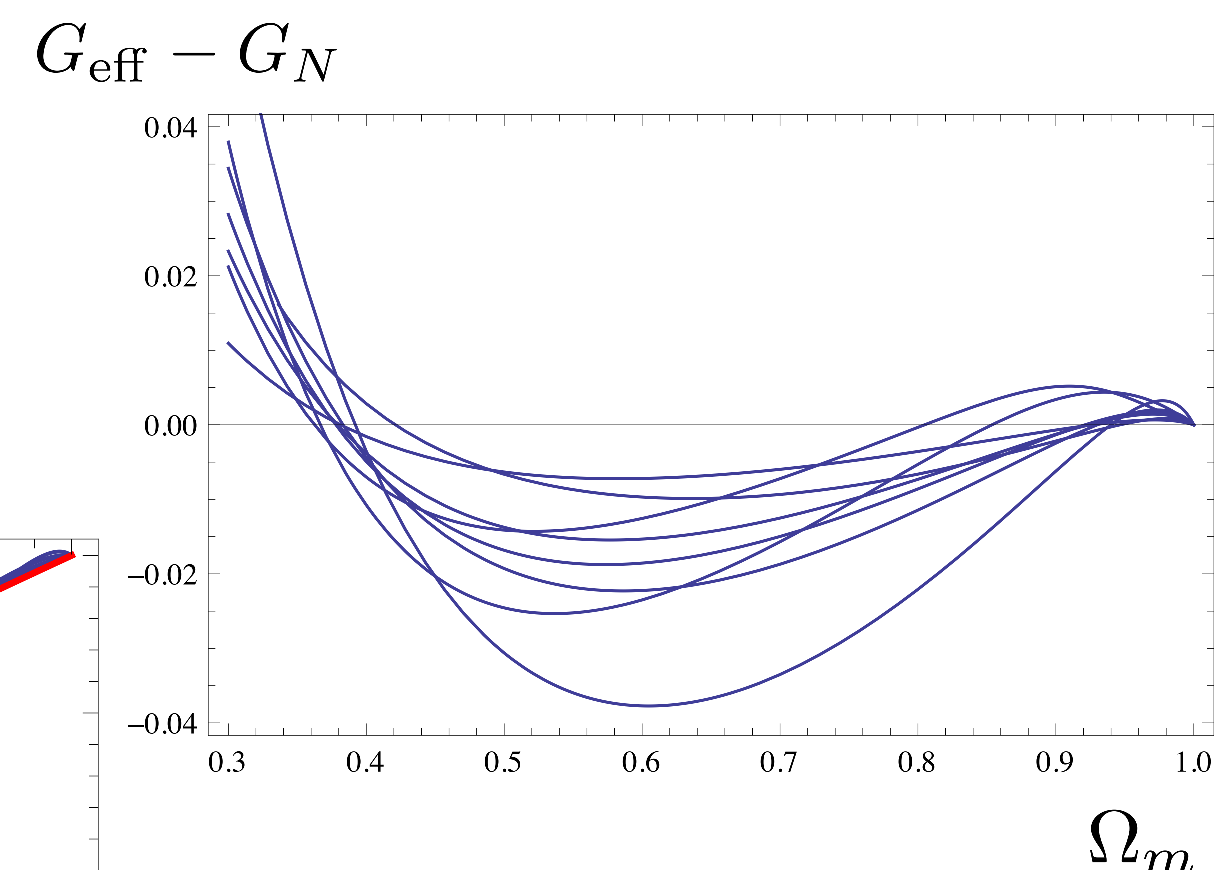
$$\gamma_0 < \gamma_0(\Lambda CDM)$$

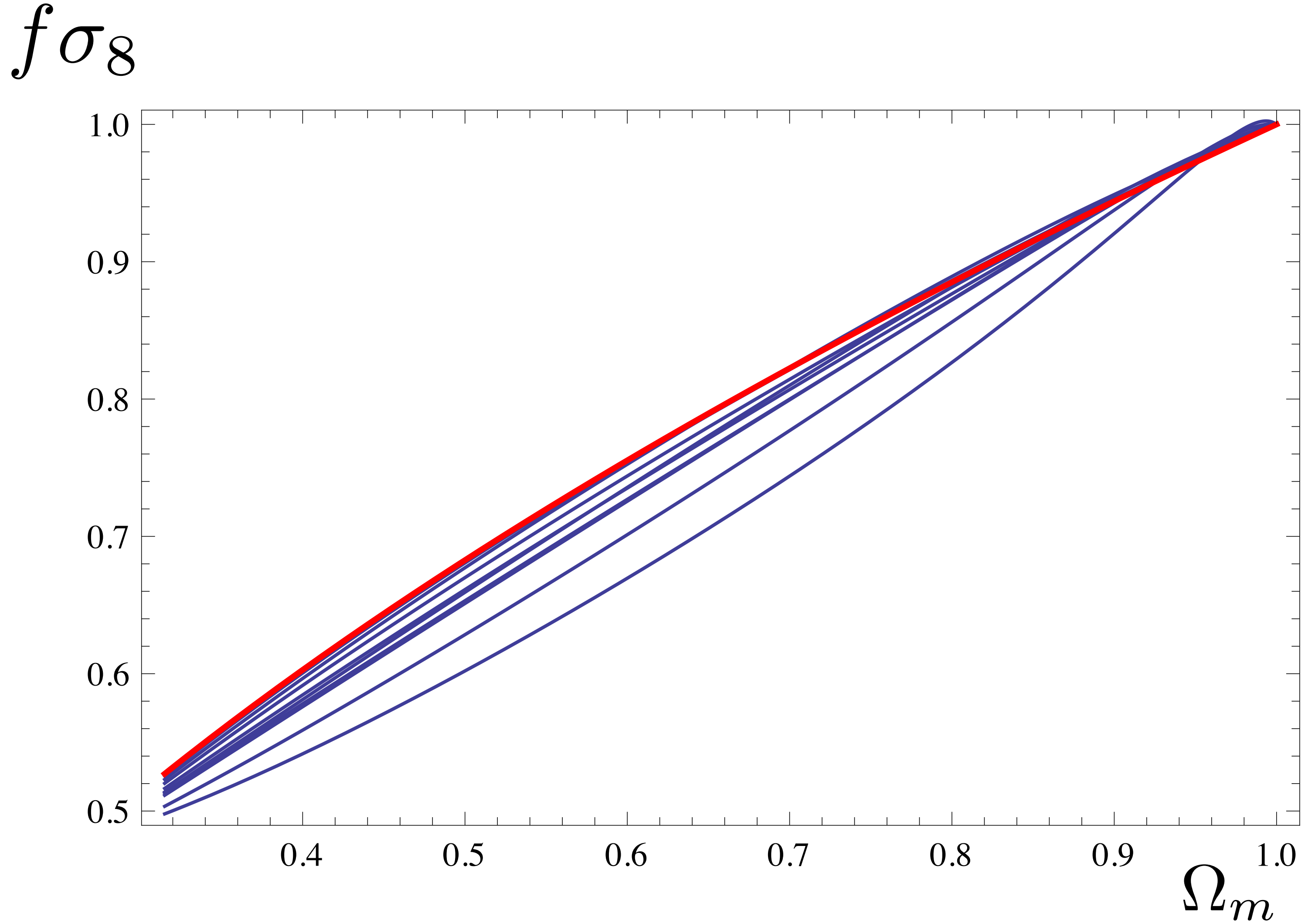


H. Steigerwald, C. Marinoni, F. P. in preparation

Growth rate (preliminary)

Modified gravity: less growth than LCDM?





Conclusions

- Unifying framework for dark energy/modified gravity
- Effective language: cosmological perturbations as the relevant d.o.f.
- Systematic way to address stability (e.g. stable violations of NEC)
- Observational constraints and forecasts: much work in progress

$$S_\pi = \int a^3 M^2 \left\{ \left[(\mathcal{C} + 2\mu_2^2)(1+\epsilon_4) + \frac{3}{4}(\mu - \mu_3)^2 \right] \dot{\pi}^2 \right.$$

$$\left. - \left[(\mathcal{C} + \frac{\mathring{\mu}_3}{2} - \dot{H}\epsilon_4 + H\mathring{\epsilon}_4)(1+\epsilon_4) - (\mu - \mu_3)\left(\frac{\mu - \mu_3}{4(1+\epsilon_4)} - \mu - \mathring{\epsilon}_4 \right) \right] \frac{(\vec{\nabla}\pi)^2}{a^2} \right\}$$

$$4\pi G_{\rm eff} = \frac{1}{2M^2} \frac{2\mathcal{C} + 2(\mu + \mathring{\epsilon}_4)^2 + \mathring{\mu}_3 - 2\dot{H}\epsilon_4 + 2H\mathring{\epsilon}_4 + 3(a/k)^2\mathcal{A}}{(1+\epsilon_4)^2[2\mathcal{C} + \mathring{\mu}_3 - 2\dot{H}\epsilon_4 + 2H\mathring{\epsilon}_4] + 2(1+\epsilon_4)(\mu + \mathring{\epsilon}_4)(\mu - \mu_3) - (\mu - \mu_3)^2/2 + 3(a/k)^2\mathcal{A}'}$$

$$\mathring{\mu}_3 \equiv \dot{\mu}_3 + \mu\mu_3 + H\mu_3, \qquad \mathring{\epsilon}_4 \equiv \dot{\epsilon}_4 + \mu\epsilon_4 + H\epsilon_4$$

$$\mathcal{A} \equiv 2\dot{H}\mathcal{C} - \dot{H}\mathring{\mu}_3 + \ddot{H}(\mu - \mu_3) - 2H\dot{H}\mu_3 - 2H^2(\mu^2 + \dot{\mu}), \qquad \mathcal{A}' \equiv (1+\epsilon_4)^2\mathcal{A}$$

The Action

$$S = \int d^4x \sqrt{-g}\left[\frac{M^2}{2} f(t) R - \Lambda(t) - c(t) g^{00}\right] + S_{DE}^{(2)}$$

Enough for background equations:

$$\begin{aligned} c &= \frac{1}{2}(-\ddot{f} + H\dot{f})M^2 + \frac{1}{2}(\rho_D + p_D) \\ \Lambda &= \frac{1}{2}(\ddot{f} + 5H\dot{f})M^2 + \frac{1}{2}(\rho_D - p_D) \end{aligned}$$

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Generally Related to post-newtonian parameters

The Action

$$S = \int d^4x \sqrt{-g}\left[\frac{M^2}{2} f(t) R - \Lambda(t) - c(t) g^{00}\right] + S^{(2)}_{DE}$$

Enough for background equations:

$$\begin{aligned} c &= \frac{1}{2}(-\ddot{f} + H\dot{f})M^2 + \frac{1}{2}(\rho_D + p_D) \\ \Lambda &= \frac{1}{2}(\ddot{f} + 5H\dot{f})M^2 + \frac{1}{2}(\rho_D - p_D) \end{aligned}$$

Generally Related to post-newtonian parameters

“Bare” Planck Mass

The Action

$$S = \int d^4x\sqrt{-g}\left[\frac{M^2}{2}f(t)R - \Lambda(t) - c(t)g^{00}\right] + S_{DE}^{(2)}$$

Enough for background equations:

$$c \;=\; \frac{1}{2}(-\ddot{f} + H\dot{f})M^2 + \frac{1}{2}(\rho_D + p_D)$$

$$\Lambda \;=\; \frac{1}{2}(\ddot{f} + 5H\dot{f})M^2 + \frac{1}{2}(\rho_D - p_D)$$

$$H^2 = \frac{1}{3fM^2}(\rho_m + \rho_D)$$

$$\dot{H} = -\frac{1}{2fM^2}(\rho_m + \rho_D + p_m + p_D)$$

Generally Related to post-newtonian parameters

"Bare" Planck Mass

Defined by the modified Friedman equations

The Action

$$S = \int d^4x\sqrt{-g}\left[\frac{M^2}{2}f(t)R - \Lambda(t) - c(t)g^{00}\right] + S_{DE}^{(2)}$$

Enough for background equations:

$$c = \frac{1}{2}(-\ddot{f} + H\dot{f})M^2 + \frac{1}{2}(\rho_D + p_D)$$

$$\Lambda = \frac{1}{2}(\ddot{f} + 5H\dot{f})M^2 + \frac{1}{2}(\rho_D - p_D)$$

Generally Related to post-newtonian parameters

"Bare" Planck Mass

Defined by the modified Friedman equations

Matter + Dark matter (in practice $\rho_m \propto a^{-3}$)

$$H^2 = \frac{1}{3fM^2}(\rho_m + \rho_D)$$

$$\dot{H} = -\frac{1}{2fM^2}(\rho_m + \rho_D + p_m + p_D)$$

Mixing with gravity 1: Brans-Dicke

$$S=\int\sqrt{-g}\left(\frac{M^2}{2}fR-\Lambda-cg^{00}+\frac{M_2^4}{2}(\delta g^{00})^2-\frac{\bar{m}_1^3}{2}\delta g^{00}\delta K+\frac{1}{2}T^{\mu\nu}\delta g_{\mu\nu}\right)$$

Apply Stueckelberg and go to Newtonian Gauge

$$ds^2=-(1+2\Phi)dt^2+a^2(1-2\Psi)\delta_{ij}dx^i dx^j$$

$$S\overset{\text{kinetic}}{=}\int M^2 f\left[-3\dot{\Psi}^2-2\vec{\nabla}\Phi\vec{\nabla}\Psi+(\vec{\nabla}\Psi)^2+c\,\dot{\pi}^2-c(\vec{\nabla}\pi)^2+3(\dot{f}/f)\dot{\Psi}\dot{\pi}+(\dot{f}/f)\vec{\nabla}\pi(\vec{\nabla}\Phi-2\vec{\nabla}\Psi)\right]$$

Mixing

Mixing with gravity 1: Brans-Dicke

$$S = \int \sqrt{-g}\left(\frac{M^2}{2} f R - \Lambda - c g^{00} + \frac{M_2^4}{2}(\delta g^{00})^2 - \frac{\bar{m}_1^3}{2}\delta g^{00}\delta K + \frac{1}{2}T^{\mu\nu}\delta g_{\mu\nu}\right)$$

Apply Stueckelberg and go to Newtonian Gauge

$$ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)\delta_{ij}dx^i dx^j$$

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De-mixing = conformal transformation

$$\Phi_E = \Phi + \frac{1}{2}(\dot{f}/f)\pi$$

$$\Psi_E = \Psi - \frac{1}{2}(\dot{f}/f)\pi$$

Mixing

Mixing with gravity 1: Brans-Dicke

$$S = \int \sqrt{-g}\left(\frac{M^2}{2} fR - \Lambda - cg^{00} + \frac{M_2^4}{2}(\delta g^{00})^2 - \frac{\bar{m}_1^3}{2}\delta g^{00}\delta K + \frac{1}{2}T^{\mu\nu}\delta g_{\mu\nu}\right)$$

Apply Stueckelberg and go to Newtonian Gauge

$$ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)\delta_{ij}dx^i dx^j$$

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Newtonian limit $\longrightarrow$

$$1-\gamma \equiv \frac{\Phi-\Psi}{\Phi} = \frac{M^2\dot{f}^2/f}{2(c+M^2\dot{f}^2/f)}$$

anisotropic stress

$$G_{\text{eff}} = \frac{1}{8\pi M^2 f}\frac{c+M^2\dot{f}^2/f}{c+\frac{3}{4}M^2\dot{f}^2/f}$$

dressed Newton constant

Mixing with gravity 2:

(Cf. braiding: Deffayet et al., 2010)

$$f(t) = 1$$

$$S = \int \sqrt{-g}\left(\frac{M^2}{2} f R - \Lambda - c g^{00} + \frac{M_2^4}{2}(\delta g^{00})^2 - \frac{\bar{m}_1^3}{2}\delta g^{00}\delta K + \frac{1}{2}T^{\mu\nu}\delta g_{\mu\nu}\right)$$

Apply Stueckelberg and go to Newtonian Gauge

$$ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)\delta_{ij}dx^i dx^j$$

$$S^{\text{kinetic}} = \int M^2\left[-3\dot{\Psi}^2 - 2\vec{\nabla}\Phi\vec{\nabla}\Psi + (\vec{\nabla}\Psi)^2\right] + c\dot{\pi}^2 - c(\vec{\nabla}\pi)^2 - 3\bar{m}_1^3\dot{\Psi}\dot{\pi} - \bar{m}_1^3\vec{\nabla}\Phi\vec{\nabla}\pi$$

Mixing

De-mixing ≠ conformal transformation

$$\Phi_E = \Phi + \frac{\bar{m}_1^3}{2M^2}\pi$$

$$\Psi_E = \Psi + \frac{\bar{m}_1^3}{2M^2}\pi$$

Mixing with gravity 2:

(Cf. braiding: Deffayet et al., 2010)

$$f(t) = 1$$

$$S = \int \sqrt{-g}\left(\frac{M^2}{2} f R - \Lambda - c g^{00} + \frac{M_2^4}{2}(\delta g^{00})^2 - \frac{\bar{m}_1^3}{2}\delta g^{00}\delta K + \frac{1}{2}T^{\mu\nu}\delta g_{\mu\nu}\right)$$

Apply Stueckelberg and go to Newtonian Gauge

$$ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)\delta_{ij}dx^i dx^j$$

$$S \overset{\text{kinetic}}{=} \int M^2\left[-3\dot{\Psi}^2 - 2\vec{\nabla}\Phi\vec{\nabla}\Psi + (\vec{\nabla}\Psi)^2\right] + c\dot{\pi}^2 - c(\vec{\nabla}\pi)^2 - 3\bar{m}_1^3\dot{\Psi}\dot{\pi} - \bar{m}_1^3\vec{\nabla}\Phi\vec{\nabla}\pi$$

Mixing

Mixing with gravity 2:

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Apply Stueckelberg and go to Newtonian Gauge

$$ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)\delta_{ij}dx^i dx^j$$

$$S \overset{\text{kinetic}}{=} \int M^2\left[-3\dot{\Psi}^2 - 2\vec{\nabla}\Phi\vec{\nabla}\Psi + (\vec{\nabla}\Psi)^2\right] + c\dot{\pi}^2 - c(\vec{\nabla}\pi)^2 - 3\bar{m}_1^3\dot{\Psi}\dot{\pi} - \bar{m}_1^3\vec{\nabla}\Phi\vec{\nabla}\pi$$

Mixing

Speed of Sound of DE

$$c_s^2 = \frac{c + \frac{1}{2}(H\bar{m}_1^3 + \dot{\bar{m}}_1^3) - \frac{1}{4}\bar{m}_1^6/M^2}{c + \frac{3}{4}\bar{m}_1^6/M^2}$$

Mixing with gravity 2:

(Cf. braiding: Deffayet et al., 2010)

$$f(t) = 1$$

$$S = \int \sqrt{-g}\left(\frac{M^2}{2} fR - \Lambda - cg^{00} + \frac{M_2^4}{2}(\delta g^{00})^2 - \frac{\bar{m}_1^3}{2}\delta g^{00}\delta K + \frac{1}{2}T^{\mu\nu}\delta g_{\mu\nu}\right)$$

Apply Stueckelberg and go to Newtonian Gauge

$$ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)\delta_{ij}dx^i dx^j$$

$$S \overset{\text{kinetic}}{=} \int M^2\left[-3\dot{\Psi}^2 - 2\vec{\nabla}\Phi\vec{\nabla}\Psi + (\vec{\nabla}\Psi)^2\right] + c\dot{\pi}^2 - c(\vec{\nabla}\pi)^2 - 3\bar{m}_1^3\dot{\Psi}\dot{\pi} - \bar{m}_1^3\vec{\nabla}\Phi\vec{\nabla}\pi$$

$$1 - \gamma = \frac{\Phi - \Psi}{\Phi} = 0$$

NO anisotropic stress

Newtonian limit $\longrightarrow$

$$G_{\text{eff}} = \frac{1}{8\pi M^2 f}\left(1 - \frac{\bar{m}_1^3}{4cM^2}\right)^{-1}$$

dressed Newton constant