Thermodynamics of non-ideal quark gluon plasma using Mayer's cluster expansion method

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- This work investigates the applicability of using the Mayer's cluster expansion method to derive the equation of state (EoS) of the quark-antiquark plasma.

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Equation of state for a non-ideal quark gluon plasma

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Equation of state of a quark-gluon plasma using the Cornell potential

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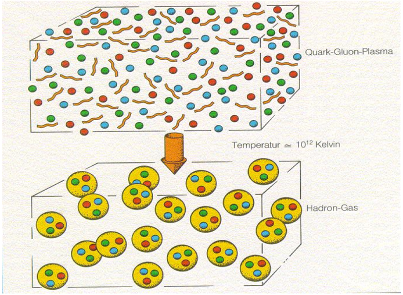
- We only consider the contribution of quarks and antiquarks, since here the gluon are massless and interaction free from with respect to each other.
- The EoS has been studied by using Cornell potential with the effect of screening.
- The possibility of the existence of quarkonium after deconfinement at higher temperature than the critical temperature $T > T_c$ is investigated.
- The EoS has been studied by calculating second and third cluster integrals.

- The number density of quark cluster system at which a non-ideal quark-antiquark plasma condense into cluster of two and three quarks ie, into a fluid mesons [E.g: $\Phi(S\bar{S})$, $J/\Psi(c,\bar{c})$ and $\Upsilon(b,\bar{b})$] and baryons [E.g:$\Omega^{-}(sss)$] using Mayers cluster expansion method is calculated.



Mayer's cluster expansion method

- A systematic method of expansions, in the case of real gases obeying classical statistics, was developed by Mayer and his collaborators and known as the method of cluster expansions.
- The masses of the heavy quarks (charm and bottom) are much larger than the QCD scale parameter ($\sim$200 MeV), the non-relativistic approximation is a good place to start to analyze the system.
- For a high temperature QGP quantum effects can be neglected , we know that the higher the temperature and mass, the smaller the thermal wavelength λ_T and we enter in the range of classical statistical mechanics.

$$\lambda_{\mathrm{T}} = \sqrt{\frac{2\pi}{\mathrm{mT}}}$$

- In this study we deal with the canonical ensemble partition function using the Mayer's cluster expansion method.

$$Q_N(V,T) = \sum_{\{m_l\}}' \left[\prod_{l=1}^{N} \left(\frac{b_l V}{\lambda_l^{3l}} \right)^{m_l} \frac{1}{m_l!} \right]$$

Where, b_l is the cluster integral.

- The evaluation of the primed sum is complicated by the restrictive condition.

$$\sum_{l=1}^{N} l m_l = N$$

Which must be obeyed by every set $\{m_l\}$.

- After the evaluation at zero chemical potential ($\mu = 0$)

$$\sum_{l=1}^{N} \frac{l b_l}{\lambda_l^{3l}} = \frac{N}{V} = n$$

  n is the number density of l-particles forming a l-particle cluster just at the moment when the clustering take place.

- $$\sum_{l=1}^{N} \frac{l v b_l}{\lambda_l^{3l}} = 1$$

- It is seen from the above equation that the lth term, $\frac{l v b_l}{\lambda_l^{3l}}$ ($v = V/N$), of this sum is the fraction of the material in clusters of size l at equilibrium.

- $$\mathrm{PV} = \{\sum_{l=1}^{N} m_l\}\mathrm{T}$$

  The initial non-ideal quark-antiquark plasma has been phase transformed to an ideal system of clusters.

- $$m_l = \left(\frac{b_l V}{\lambda_l^{3l}}\right)$$

  It is the number of *l*-quark clusters (maximum value) during the process of phase transition.

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Equation of state for hot quark-gluon plasma transitions to hadrons with full QCD potential

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- We take l=2 for two particle cluster and l=3 for three particle cluster.
  Then the EoS for two quark cluster and three quark cluster are,
  - $$n_{diquarkcluster} = b_2 \left(\frac{M_2 T}{2\pi} \right)^3$$
  - $$n_{triquarkcluster} = b_3 \left(\frac{M_3 T}{2\pi} \right)^{9/2}$$

Where b_2 and b_3 are the cluster integral, M_2 and M_3 are the masses of the quarks.

Cluster integrals

- $$b_2 = \frac{2\pi}{\lambda^3} \int_0^\infty f_{12} d^3 r_{12}$$
- $$b_3 = 2 b_2^2 + \frac{1}{6} C_3$$
- $$C_3 = \int_0^\infty \int_0^\infty f_{12} f_{13} f_{23} d^3 r_{12} d^3 r_{13}$$

- To treat the nonideal quark plasma, the Mayer two-particle function f_{ij} ,defined by the relationship

$$f_{ij} = e^{-\beta U(r_{ij})} - 1$$

  $i, j = 1, 2, ..., N$
- $U(r_{ij})$ is the Cornell potential between quarks.

Cornell potential between quarks

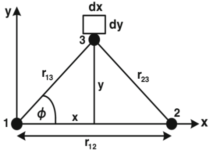
- Contains both linear and Coulomb terms, i.e. both confining and nonconfining terms
- Include the effect of Screening.
- $$U(r,T) = -\sigma r_D(1 - e^{-\frac{r}{r_D}}) + \frac{\alpha}{r} e^{-\frac{r}{r_D}}$$

  Where the coupling constant α, string tension σ and the screening length r_D are temperature dependent.

- Consider the clustering of quark-antiquark pairs to mesons, that we need second cluster integral.

$$b_2(T) = 2\pi \int_{r=0}^{\infty} [1 - e^{[-\beta(\sigma r_D + \frac{\alpha}{r})e^{-\frac{r}{r_D}}]}] r^2 dr$$

- We try to evaluate C_3 numerically based on bipolar coordinate integration, by fixing the positions of the particle 1 and 2 and particle 3 takes all possible position and also using the technique of Jacobian transformation.



- Rotation of the element of area *dxdy* about the *x* axis sweeps out $d^3 r_{13}$, so that

$$d^3 r_{13} = 2\pi y dx dy$$

- The coordinates *x* and *y* can be transformed interms of r_{13} and r_{23} and using the Jacobian transformation, we get

$$d^3 r_{13} = \frac{2\pi r_{13} r_{23}}{r_{12}} dr_{13} dr_{23}$$

$$C_3(T) = 8\pi^2 \int_{r=0}^{\infty} r_{12}^2 f(r_{12}) \int_{r=0}^{\infty} r_{13}^2 f(r_{13})$$

$$\int_{r=-1}^{1} f(\sqrt{r_{12}^2 + r_{13}^2 - 2r_{12}r_{13}\mu}) d\mu dr_{12} dr_{13}$$

$$\mu = \cos(\phi)$$

Clustering of quarks:

For $\phi(S\bar{S})$ clusters.

The mass of the strange quark, $M_S = 150 MeV$

Using the EoS for diquark cluster.

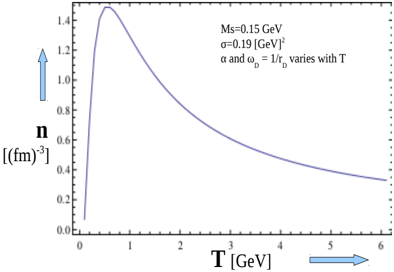


Figure: Number density (n) as a function of temperature (T) for the clustering of two quarks (strange quark).

Clustering of quarks:

For $C\bar{C}$ and $\Omega^{-}(SSS)$ clusters.

The mass of the charm quark, $M_C = 1.32 GeV$

Using the EoS for diquark and triquark cluster.

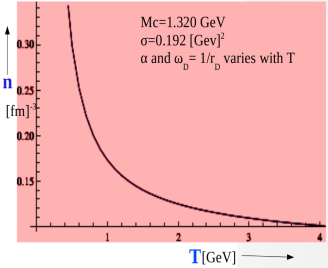


Figure 1 : Number density (n) of charmonium (two charm quark cluster) at various Temperature (T): A large reduction of the number density is found near the critical temperature T_C=228 MeV.

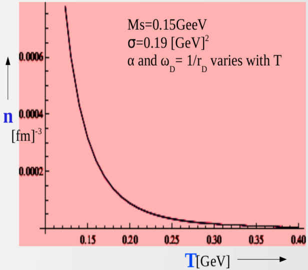


Figure 2 : Number density (n) of three strange quark cluster at various Temperature (T): Using b_2 and b_3 , A large reduction of the number density is found near the critical temperature T_C=160 MeV.

- The main advantage of above mentioned method is that we can apply the classical particle picture to the quarks and investigate phase transitions in a quark-antiquark plasma.
- We made EoS that relate particle number density (n) at various temperature (T) for two and three heavy quark clustering.
- The equation of state found here shows the occurrence of heavy quarkonium at T_c =150-200 MeV.
- When combining our result with the statistical bootstrap model and other available works, we observed a pronounced maximum of number density close to the critical temperature.

- To incorporate lighter quarks we should extend our work into relativistic and quantum regime.
- There is also a possibility to incorporating fourth cluster integral in cluster of four particle droplet (four-quark matter called $Z_C(3900)$).
- The clustering of quarks with different masses is in principle possible and is the subject of investigation at present.

References

- J. E. Mayer and M. G. Mayer, Statistical Mechanics (1940).
- V.M. Bannur, phys. Lett. B 362, 7, 10 (1995).
- K.M. Udayanandan, P. Sethumadhavan, and V.M. Bannur, Phys. Rev. C 76, 044908 (2007).
- Bijan Sheikholeslami-Sabzevari, Phys.Rev. C 65, 054904 (2002).
- Artit Hutem , Sutee Boonchui, J Math.Chem. 50, 1262-1276(2012).
- M. Ablikim et al. (BESIII Collaboration), Phys. Rev. Lett. 110, 252001 (2013).
- R.K. Pathria, Statistical Mechanics Pergamon, Oxford, (1988).

Thank you for your attention!