Pressure in QCD at finite μ: then and now

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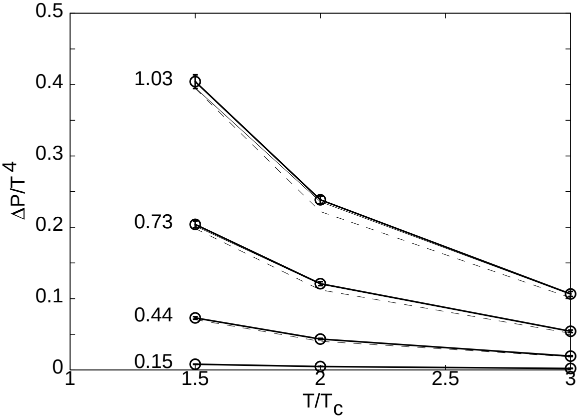
EOS at $\mu \neq 0$



Gavai, SG: Phys.Rev. D68 (2003) 034506

$$\Delta P = P(\mu, T) - P(0, T).$$

The mathematical problem

Perform a series expansion of the pressure in powers of chemical potential

$$\Delta P(\mu_u, \mu_d, T) = \sum_{m,n} \chi_{m,n}(T) \frac{\mu_u^m \mu_d^n}{m!n!}.$$

Does this converge? Can one reconstruct the function? Well studied classical problem. Special complications: few coefficients known, with errors.

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Also, expansion in $z = \mu_B/T$

$$\chi_B(\mu_B, T) = \frac{\partial^2 \Delta P}{\partial \mu_B^2} = \chi_B^0(T) + \frac{T^2}{2!}\chi_B^2(T) z^2 + \frac{T^4}{4!}\chi_B^4(T) z^4 + \cdots$$

Our simulations

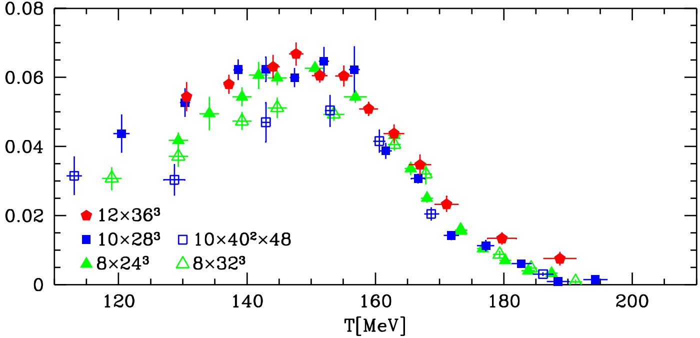
Lattice simulations with $N_f = 2$ staggered quarks and Wilson gauge action. Used $m_\pi \simeq 0.3 m_\rho$; spatial size $L = 4/T$.

Temperature scale, T_c, found by the point at which χ_L peaks. If $T_c \simeq 170$ MeV, then $1/a = 0.7$ GeV, 1 GeV, 1.4 GeV for $N_t = 4$, 6 and 8.

Configurations: 50K+ at each coupling; large number of fermion sources used for determination of fermion traces. Partial statistics reported in: QM 2012, Lattice 2013 **Datta, Gavai, SG: arXiv:1210.6784**

This doubles the statistics reported in Lattice 2013.

On T_c

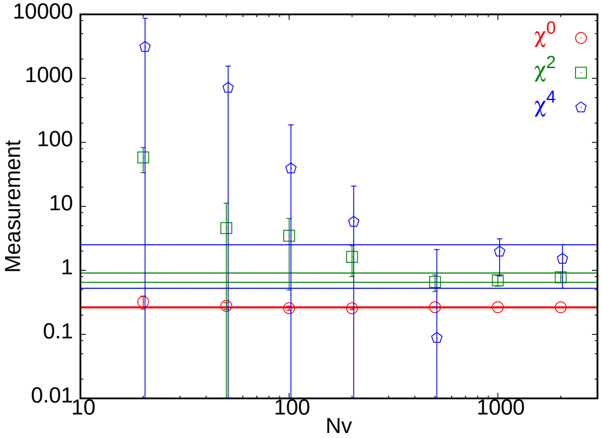


Broad crossover: even with one single measure (figure: chiral susceptibility) T_c uncertain by 20 MeV. Reflected in quoted values.

Aoki, Borsanyi, Dürr, Fodor, Katz, Krieg, Szabo: JHEP 0906 (2009) 088

Select any definition and stick with it: we use Polyakov loop susceptibility.

Numerical errors



Errors depend on number of fermion sources for evaluation of propagator as well as number of gauge configurations. Multiple fermion loops are source hungry.

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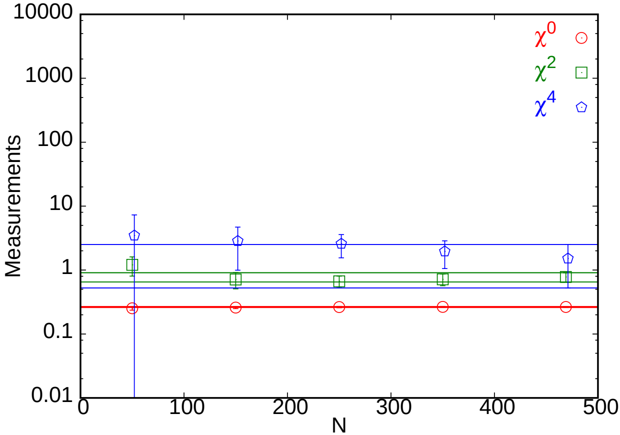

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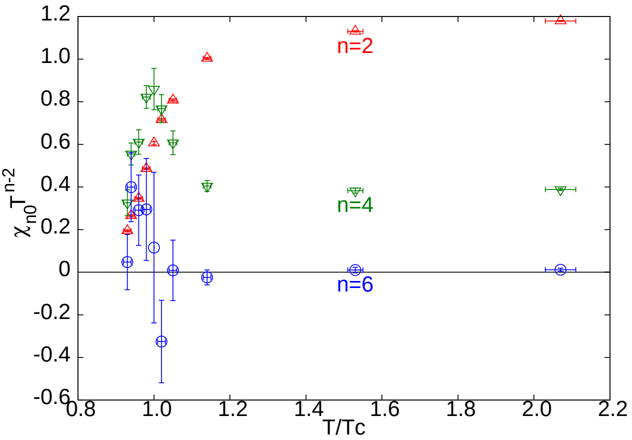
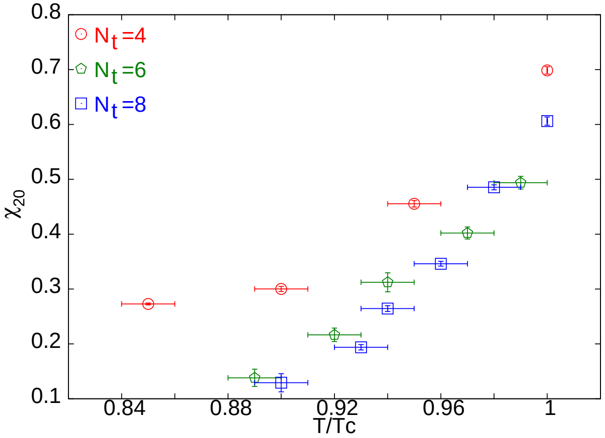
Susceptibilities at $\mu = 0$

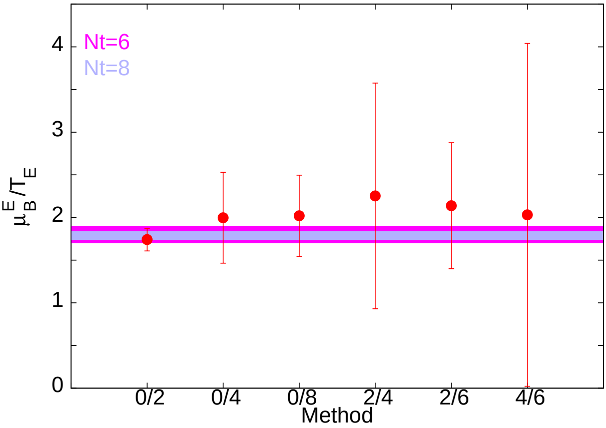


Nearing continuum physics



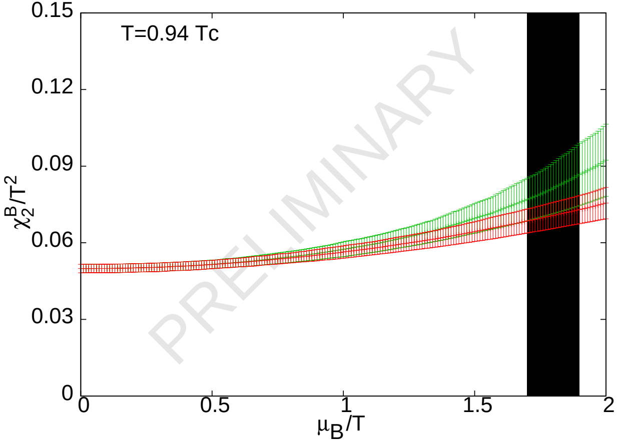
Redoing $N_t = 4$ with more data to check whether there is any improvement.

The radius of convergence



For $N_t = 6$, $\mu_E/T_E = 1.7 \pm 0.1$ **Gavai, SG: 2008**

Must resum a series expansion



Truncated series sum is regular even at the radius of convergence, so is missing something important.

Strategy for finding the pressure

At a critical point

$$\chi_B = \frac{\partial^2(P/T^4)}{\partial z^2} \simeq (z_*^2 - z^2)^{-\psi}.$$

Continuity and finiteness of P at the CEP forces $\psi \leq 1$.

Since

$$m_1(z) = \frac{d \log \chi_B}{dz} \simeq \frac{2\psi z}{z_*^2 - z^2},$$

use the series to estimate the critical exponent. Series for m_1 has one term less than series for χ_B. Accurate results require fine statistical control of at least 3 series coefficients of χ_B: 2 of m_1.

From the Padé approximant to $m_1(z)$, integrate to find χ_B and again twice to find ΔP.

Critical behaviour of m_1

If $\chi_B(z) \simeq (z_* - z)^{-\psi}$, then $m_1 = d \log \chi_B / dz$ has a pole. Series expansion of χ_B gives series for m_1. Resum series into a Padé approximant:

$$[0,1]: \qquad m_1(z) = \frac{c}{z_* - z}$$

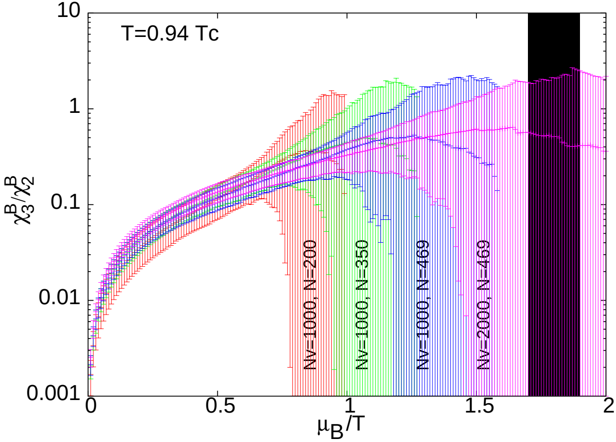
Width of the critical region? If we define it by

$$\left| \frac{m_1(z)}{m_1(0)} \right| > \Lambda,$$

then $|z - z_*| \leq z_*/\Lambda$.
If δ is fractional error in measurement of z_*, then error in Padé?
Easy to check

$$\left| \frac{\Delta m_1}{m_1} \right| > \frac{1}{1 - \Lambda\delta}.$$

Critical slowing down



Widom scaling

Widom scaling for the order parameter gives

$$|\Delta\mu| = |\Delta n|^{\delta} J\left(\frac{|\Delta T|}{|\Delta n|^{1/\beta}}\right),$$

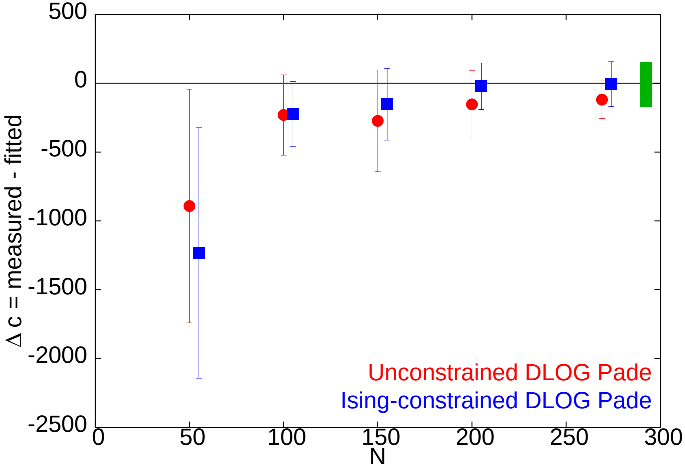
where $\Delta T = T - T_E$ and $\Delta\mu = \mu - \mu_E$. For $\Delta T = 0$ one finds $\Delta n \propto |\Delta\mu|^{1/\delta}$ in the high density phase. Then clearly one has

$$\psi = 1 - \frac{1}{\delta}.$$

For the 3d Ising model, $\delta = 1.49$, so $\psi = 0.79$. In mean field theory one has $\delta = 3$, so $\psi = 0.66$. Our computations consistent with both: cannot distinguish between them yet.

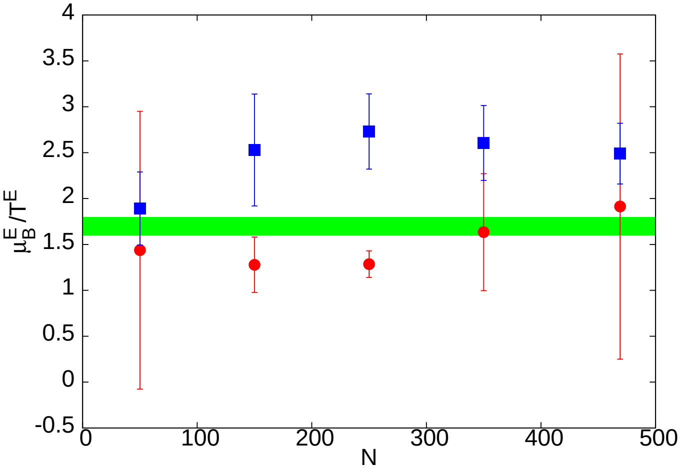
The order parameter could be a mixture of energy density and number density. Then these arguments are limiting cases, and one gets $0.79 \leq \psi \leq 1$ (3d Ising) or $0.66 \leq \psi \leq 1$ (MFT).

Testing the DLOG Pade



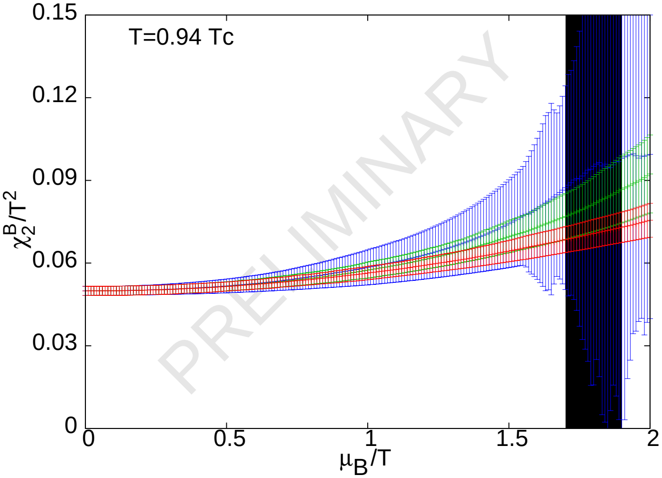
Padé uses 2 terms of the series for m_1. Does it predict the 3rd?

Pole and residue



Position of pole agrees with radius of convergence.

The pressure



Integrated m_1 to find χ_2.

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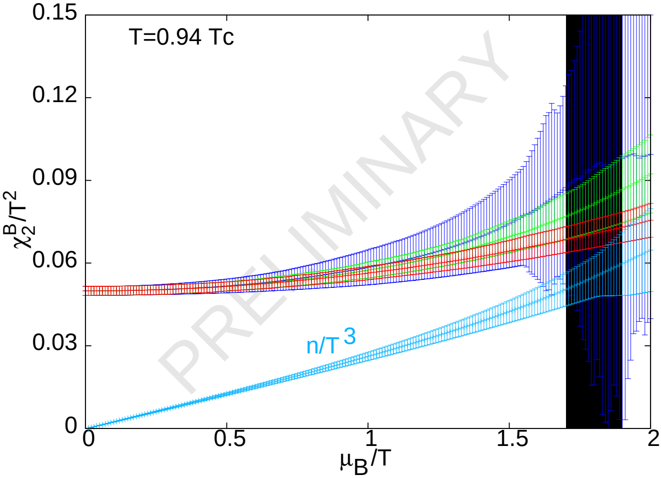

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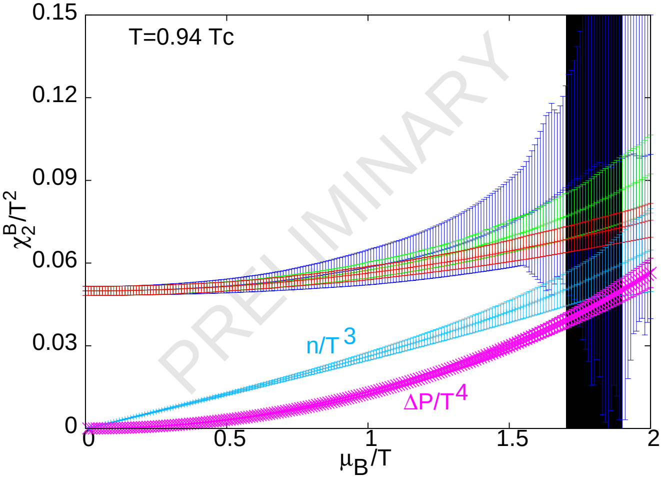

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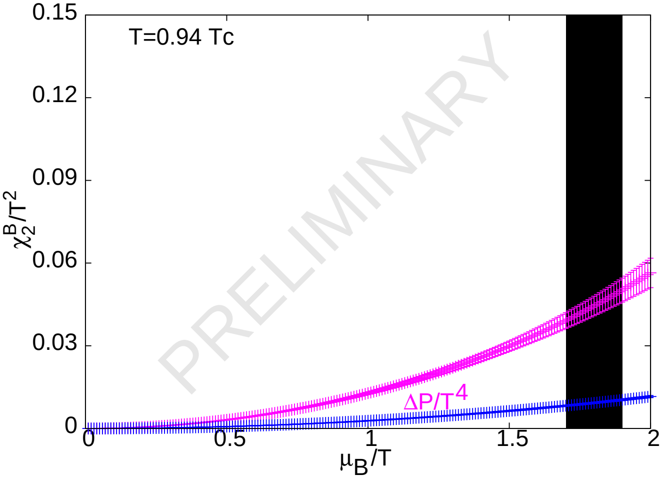

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Critical point and the pressure

- QNS require huge CPU expenses; we have up to the 8th order. Momentum cutoff of 0.7 GeV, 1 GeV and 1.4 GeV. Able to see the approach to the renormalized values: $T^E \simeq 0.94\, T_c$, $\mu_B^E / T^E \simeq 1.7$.
- When the series diverges then ΔP at finite μ_B cannot be obtained from a partial resummation of the series.
- Since $\chi_B \simeq |\mu_B - \mu_B^E|^{-\psi}$, the ratio $m_1 = \chi'_B/\chi_B$ has a simple pole. Resum the series expansion into a simple pole. Integrate this to find χ_B and ΔP. First results for pressure at finite μ_B are reported.
- Lattice uses m_1 along a path of constant T and varying μ_B. Event-to-event fluctuations of baryon number can measure m_1 along the freezeout curve.