Perturbative QCD at finite temperature and density

Purnendu Chakraborty

Variable Energy Cyclotron Centre

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Outline

Asymptotic freedom implies deconfinement

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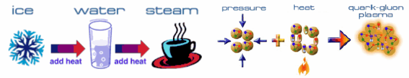
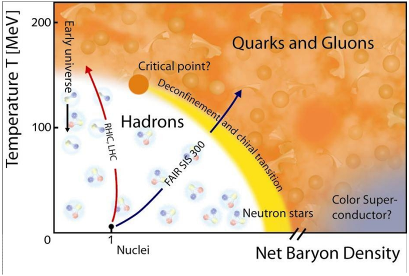
Superdense Matter: Neutrons or Asymptotically Free Quarks?

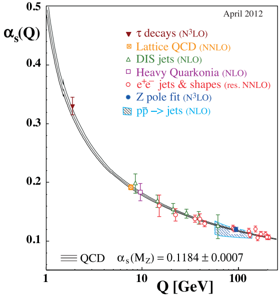
J. C. Collins and M. J. Perry
Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge CB3 9EW, England
(Received 6 January 1975)

We note the following: The quark model implies that superdense matter (found in neutron-star cores, exploding black holes, and the early big-bang universe) consists of quarks rather than of hadrons. Bjorken scaling implies that the quarks interact weakly. An asymptotically free gauge theory allows realistic calculations taking full account of strong interactions.



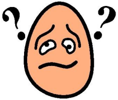


perturbative QCD

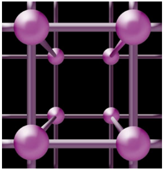


- $\alpha_s \to 0$ for high virtuality, temperature or density

- static + dynamic quantities
- off-equilibrium



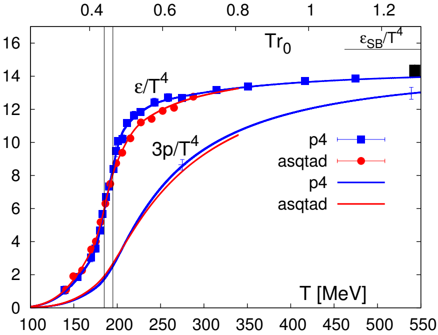
lattice QCD



- non-perturbative at all T

- limited dynamic quantities
- limited off-equilibrium
- no true μ_q calculation

Do we understand this perturbatively?

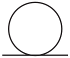


Bazavov et al (2009)

A detour : mishaps with naive PT

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{\lambda}{24}\phi^4$$ ➢ only mass scale is T

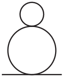
$\mathcal{O}(\lambda)$



$$\Pi = \lambda T^2$$

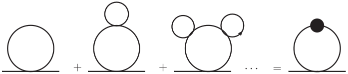
- T^2 correction from hard sector of loop
- radiatively generated soft scale $\sim \sqrt{\lambda} T$
- $\Delta^{-1} = P^2$, so mass correction important for soft external momenta

$\mathcal{O}(\lambda^2)$



$$\Pi \sim (\lambda T^2)\, \lambda T \int \frac{d^3k}{k^4} + \text{finite terms}$$

$\sum_{n=1}^{\infty} \mathcal{O}(\lambda^n)$



$$\Pi^\star = \lambda T^2 \left(1 - \frac{3\sqrt{\lambda}}{\pi}\right)$$

- NLO mass correction is nonanalytic in λ
- only a particular class of diagrams are re-summed

➢ Lesson : Certain reorganization of perturbation theory is necessary for soft momenta $\sim \sqrt{\lambda} T$

Detour : many scales in statistical field theory

➢ Assumption : T is high $\gg$ any intrinsic mass scale of theory and $g \ll 1$

- typical momenta of particles in heat bath $\sim T$ ("hard" scale)
- due to interaction massless particles acquire a mass $\sim gT$ ("soft" scale)
- there may be other scales *e.g* $\sim g^2T$ (magnetic screening length)
- scales are well separated in weak coupling ($T \gg gT \gg g^2T$)

➢ Observation : there are thermal corrections from all orders of PT such that

$$\boxed{\text{Thermal Corrections}} = \frac{g^2T^2}{P^2} \times \boxed{\text{Tree Level}}$$

➢ Lesson : thermal corrections from all scales should be taken into account if the physical quantity under consideration is sensitive to them

QCD thermodynamics : Dimensional reduction

Braaten and Nieto (1995)

$$\mathcal{Z} = e^{-\beta\Omega} = \int [\mathcal{D}A] \, \exp\left(- \int_0^\beta d\tau \int d^3x \, \mathcal{L} \right)$$

- $P_{\rm QCD} = \frac{T}{V} \ln \mathcal{Z}$, and $s = \frac{\partial P}{\partial T}$ etc. etc.

☛ theory becomes 3-dimensional as $T \to \infty$

- propagator for massless fields $\Delta = \frac{1}{\omega_n^2 + k^2}$, $\omega_n^B = 2\pi n T$, $\omega_n^f = (2n+1)\pi T$

☛ all non-static modes become heavy as $T \to \infty$ and are removed from the spectrum

- Let's successively integrate out the heavy modes from the theory. First let's do it for modes $\sim \pi T$

$$P_{\rm QCD} = P_T + \frac{T}{V} \ln \int \mathcal{D}A_0^a \mathcal{D}A_i^a \, \exp\left(- \int d^3x \, \mathcal{L}_E \right)$$

$$\mathcal{L}_E = \frac{1}{2} {\rm Tr}\, F_{ij}^2 + {\rm Tr}\, [D_i, A_0]^2 + m_E^2 {\rm Tr} A_0^2 + \lambda_E^{(1)} {\rm Tr}\, \left(A_0^2\right)^2 + \lambda_E^{(2)} {\rm Tr}\, \left(A_0^4\right) + \cdots$$

$$F_{ij} = \frac{i}{g_E} [D_i, D_j] \,, \;\; D_i = \partial_i - g_E A_i$$

$$m_E \sim gT, \; g_E \sim g^2 T, \; \lambda_E^{(i)} \sim g^4 T$$

QCD Thermodynamics : Dimensional Reduction

- Next integrate out modes $\sim gT$

$$P_{\rm QCD} = P_T + P_E + \underbrace{\frac{T}{V}\ln\int \mathcal{D}A_i^a\, \exp\left(-\int d^3x\,\mathcal{L}_M\right)}_{P_M}$$

$$\mathcal{L}_M = \frac{1}{2}{\rm Tr}\, F_{ij}^2 + \cdots$$

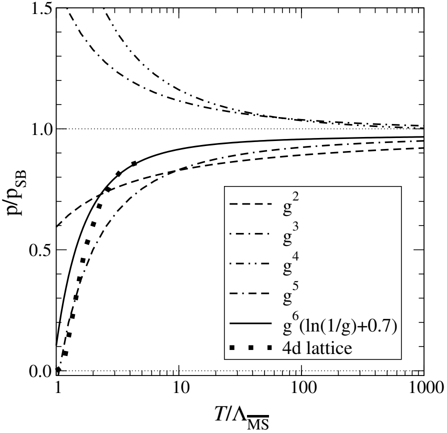
$$F_{ij} = \frac{i}{g_M}\,[D_i, D_j]\,,\;\; D_i = \partial_i - g_M A_i, \quad g_M \sim gT$$

- $$\frac{P_{QCD}(T)}{T^4} = c_0 + \underbrace{c_2\, g^2}_{'78} + \underbrace{c_3\, g^3}_{'79} + \underbrace{c_{4'}\, g^4 \ln g}_{'83} + \underbrace{c_4\, g^4}_{'94} + \underbrace{c_5\, g^5}_{'95} + \underbrace{c_{6'}\, g^6 \ln g}_{'03} + \underbrace{c_6\, g^6}_{?} + \cdots$$

- $P_T \rightsquigarrow g^2, g^4$ (normal perturbative behavior)
- $P_E \rightsquigarrow g^3, g^5$
- $P_M \rightsquigarrow g^6$ (non-perturbative physics)

QCD Thermodynamics

Kajantie et al (2003)

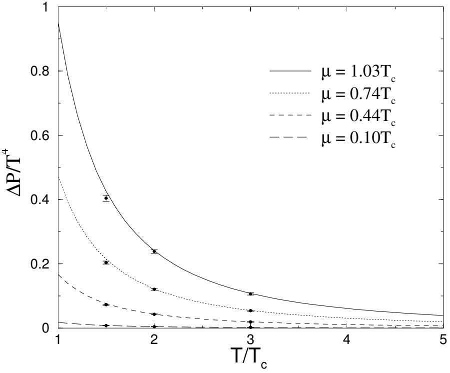


lattice data : Boyd et al (1996)

QCD Thermodynamics

$$\Delta P = P\left(T,\mu\right) - P\left(T,0\right), \quad \mu \leq 2\pi T$$

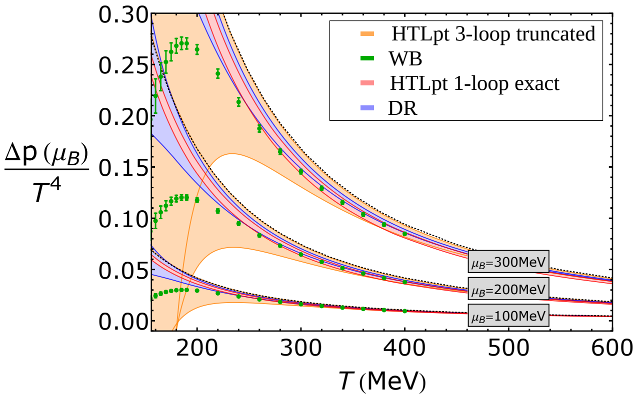
Vuorinen (2003)



lattice data : Gavai, Gupta (2003)

QCD Thermodynamics

Mogliacci, Andersen, Su, Strickland, Vuorinen (2013)



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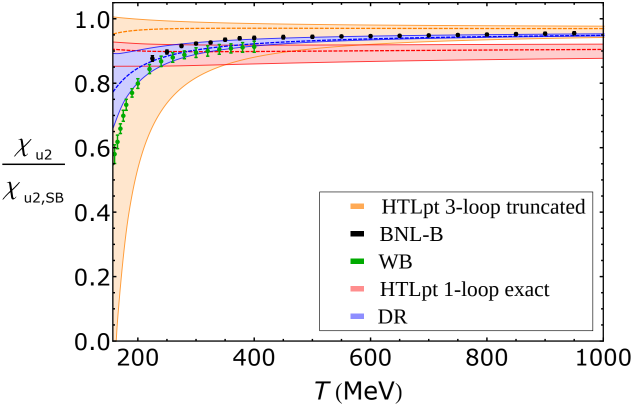

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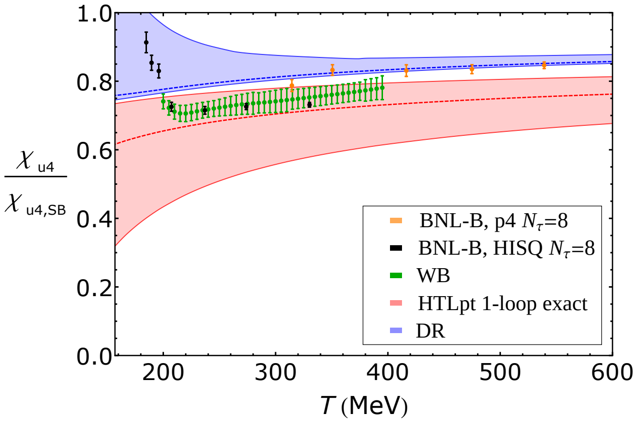
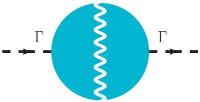


Electromagnetic emission



☛ correlation functions $\langle J_\Gamma(x) J_\Gamma(0)\rangle_T$ are useful tools to probe microscopic dynamics. For example, consider vector channel

- rate of $l_+ l_-$ pair production

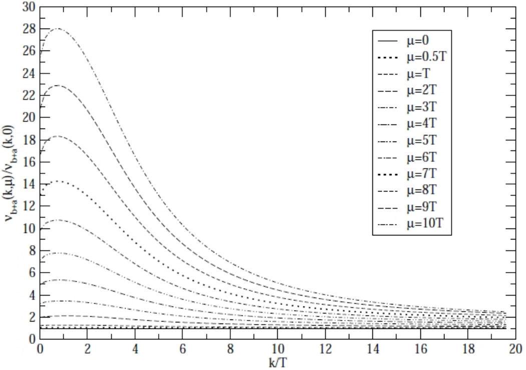
$$\frac{dR}{d^4x d^4P} = -\frac{\alpha}{12\pi^4 M^2}\frac{1}{e^{E/T}-1}\mathrm{Im}\,\Pi_\mu^{R,\mu}(E,p) \qquad \left[M^2 = E^2 - p^2 > 0\right]$$

- rate of γ production

$$E\frac{dR}{d^3p} = -\frac{\alpha}{8\pi^3}\frac{1}{e^{E/T}-1}\mathrm{Im}\,\Pi_\mu^{R,\mu}(E=p)$$

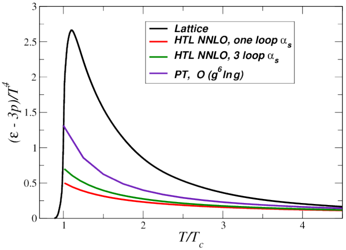
Photon production from baryon rich plasma

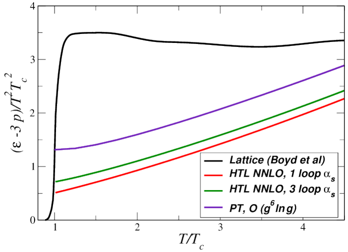
- $2 \rightarrow 2$: Traxler, Vija and Thoma (1995), Dumitru, Rischke, Stoecker, Greiner (1993))
- $2 \rightarrow 3$: Gervais and Jeon (2012))



Gervais and Jeon (2012)

What is the power of deconfinement?

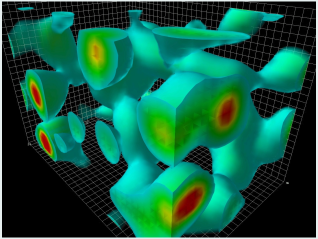




- large deviation from perturbation theory near T_c
- unusual power correction

QCD vacuum is not empty

Action density of gluodynamics



D. B. Leinweber, hep-lat/0004025

- $s_g = \frac{1}{2}\sum_x \sum_{\mu\nu} \mathrm{Tr}\, G^2_{\mu\nu}(x)$
  field fluctuations do not vanish in the ground state of QCD but are strongly correlated inside domains distributed over space and time
- $v = 2.4\,\mathrm{fm} \times 2.4\,\mathrm{fm} \times 3.6$ fm
  sufficient to hold a couple of protons

Dilational current is not conserved in QCD

$$\partial_\mu S^\mu = T^\mu_\mu = \frac{\beta(g)}{2g} G^2_{\mu\nu} + \sum_q (1 + \gamma_m(g))\, m_q \bar{q} q$$

$$T_0^{\mu\nu} = -\epsilon_v g^{\mu\nu}$$

- condensates of mass dimension $d \sim \Lambda^d$, Λ is QCD scale

Melting of condensates

Chiral condensate

- restoration of chiral symmetry above T_c $\Rightarrow$ $\bar{q}q = 0$

Color electromagnetic correlators for $T \neq 0$

- Lorentz symmetry broken by the choice of heatbath $\Rightarrow$ electric and magnetic fluctuations are different

$$\left\langle g^2 \mathrm{Tr}\left[\mathcal{E}_i(x)\, S(x,0)\, \mathcal{E}_j(0)\, S(0,x)\right]\right\rangle_T = \delta_{ij}\left[\mathcal{D}^{\mathrm{E}} + \mathcal{D}_1^{\mathrm{E}} + x_4^2 \frac{\partial \mathcal{D}_1^{\mathrm{E}}}{\partial x_4^2}\right] + x_i x_j \frac{\partial \mathcal{D}_1^{\mathrm{E}}}{\partial \mathbf{x}^2}$$

$$\left\langle g^2 \mathrm{Tr}\left[\mathcal{B}_i(x)\, S(x,0)\, \mathcal{B}_j(0)\, S(0,x)\right]\right\rangle_T = \delta_{ij}\left[\mathcal{D}^{\mathrm{B}} + \mathcal{D}_1^{\mathrm{B}} + \mathbf{x}^2 \frac{\partial \mathcal{D}_1^{\mathrm{B}}}{\partial \mathbf{x}^2}\right] - x_i x_j \frac{\partial \mathcal{D}_1^{\mathrm{B}}}{\partial \mathbf{x}^2}$$

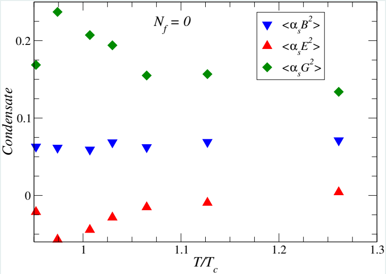
- $\mathcal{D}^E$ and $\mathcal{D}^B$ are related to temporal (σ_τ) and the spatial (σ_s) string tensions

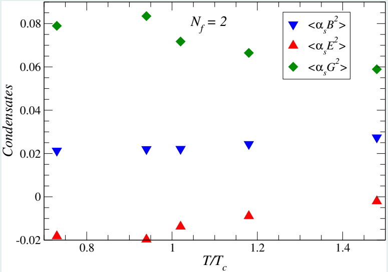
$$\sigma_\tau = \frac{1}{2}\int d^2x\, \mathcal{D}_E(x) \qquad \sigma_s = \frac{1}{2}\int d^2x \mathcal{D}_B(x)$$

- $\mathcal{D}_1^E$, $\mathcal{D}_1^B$ contribute to perimeter or Coulomb terms

Melting of condensates

It is electric deconfinement and magnetic confinement above T_c





@ D'Elia *et al*, PRD 67 (2003).

- σ_τ vanishes between $T_c < T < 2T_c$. At high T, $\langle E^2 \rangle_T$ comes from perimeter terms
- $\langle B^2 \rangle_T$ changes little across deconfinement and beyond $T \geq 2T_c$

$$\sqrt{\sigma_s} \simeq \sqrt{\sigma_3} = c_\sigma g^2 T$$

- scale of non-perturbative physics in hot QCD changes from Λ near T_c to $g^2 T$ for $T \geq 2T_c$

Operator Product Expansion

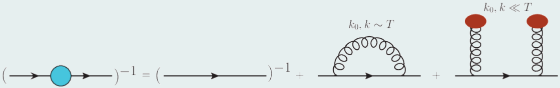
$$
\begin{aligned}
i\int dx\,\mathcal{T}\left(j(x)j(0)\right) &= C_I\left(\alpha_s\left(x\right)\right)I+\sum_n C_n\left(\alpha_s(x)\right)O_n \\
C_I, C_n &= \text{perturbatively computable Wilson coefficients} \\
O_n &= \text{non-perturbative local operators, } e.g,\ G^2_{\mu\nu}, \bar{\psi}\psi
\end{aligned}
$$

Aim : To use OPE to analytically resolve the structures of QCD propagators and correlation functions above T_c

Quasi quarks above T_c

Nonperturbative quark propagation

Schafer, Thoma (1999), Chakraborty, Mustafa(2012), Chakraborty ((2013))

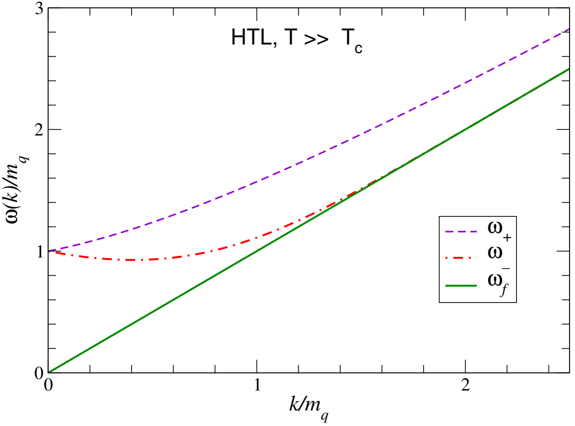


$$\Sigma(\omega, p) = -a(\omega, p)\, \not{P} - b(\omega, p)\, \gamma_0 \,,$$

$$S(p_0, p) = \frac{\gamma_0 - \vec{\gamma}\cdot\hat{p}}{2D_+(p_0, p)} + \frac{\gamma_0 + \vec{\gamma}\cdot\hat{p}}{2D_-(p_0, p)}$$

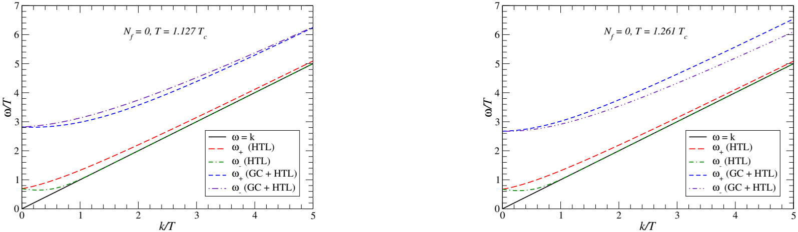
- $\omega_\pm(p)$ be the zeroes of $D_\pm(p)$
- $\omega_+(p)$: chirality/helicity $= +1$
- $\omega_-(p)$: chirality/helicity = -1

Quasi quarks above T_c : High Temperature (HTL) approximation



Nonperturbative effects dominate quark propagation near T_c

Quark propagation and melting gluon condensate



☛ in a non-perturbative background ω_+, ω_- have just the opposite behavior compared to high T limit

Bifurcation of the propagating modes

$$\omega_{\pm}(k) \sim m_q \pm \# \frac{\langle E^2 \rangle_T}{\left(\langle E^2 \rangle_T + \langle B^2 \rangle_T\right)} \qquad (k/m_q \ll 1)$$

☛ there could exist a temperature T_p which may loosely be called "vacuum persistence temperature" such that non-perturbative vacuum condensates influence quasiparticle properties to a large extent

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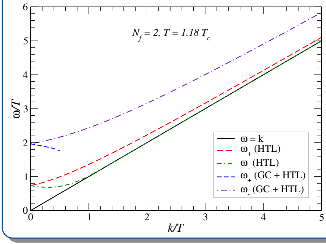

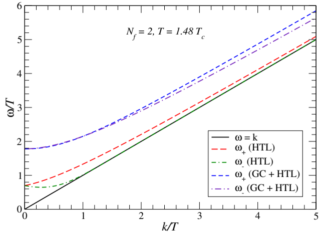

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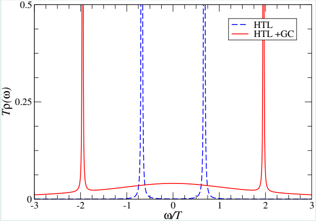
Nonperturbative effects dominate quark propagation near T_c

Quark propagator at $p = 0$

$$\gamma_0 S_F(\omega, 0) = \frac{R_+}{-\omega + m_q} + \frac{R_-}{-\omega - m_q} + \underbrace{\frac{\Gamma_+}{-\omega - i\gamma_q} + \frac{\Gamma_-}{-\omega - i\gamma_q}}_{\text{not in HTL}}$$

Quark spectral function

$$\rho(\omega, 0) = \frac{1}{2\pi}\Im\left(\gamma^0 S_F(\omega, 0)\right)$$

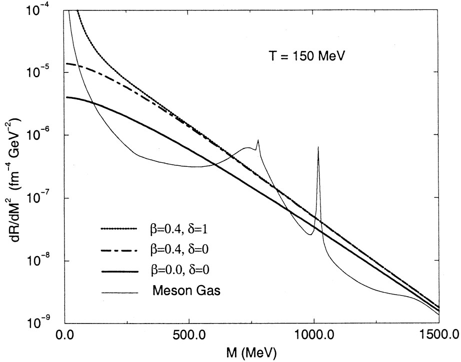


- $\frac{\Gamma_\pm}{R_\pm} \sim \mathcal{O}(1)$ near T_c and damped modes are as important as propagating modes
- quasiparticle modes do not saturate sum rule

$$0.5 \lesssim \sum_{\pm} \int d\omega\, \rho_{qp}(\omega) \lesssim 1.0$$

Dilepton emission from gluon condensate

Lee, Wirstam, Zahed, Hansson (1999)



$\beta = \langle \frac{\alpha_s}{\pi} A_4^2 \rangle / T^2, \quad \alpha \times (200\mathrm{MeV})^4 = \langle \frac{\alpha_s}{\pi} E^2 \rangle = \langle \frac{\alpha_s}{\pi} B^2 \rangle$

☛ low mass dilepton enhancement from melting gluon condensates

Summary

- within its limitations, perturbation theory works quite well in describing equation state of hot and dense QCD at suprisingly low temperature
- dynamic and off-equlibrium quantities can similarly be computed at finite density
- OPE can used to resolve the non-perturbative structure of QGP near T_c