Hadrons and Multi-hadrons From Lattice QCD

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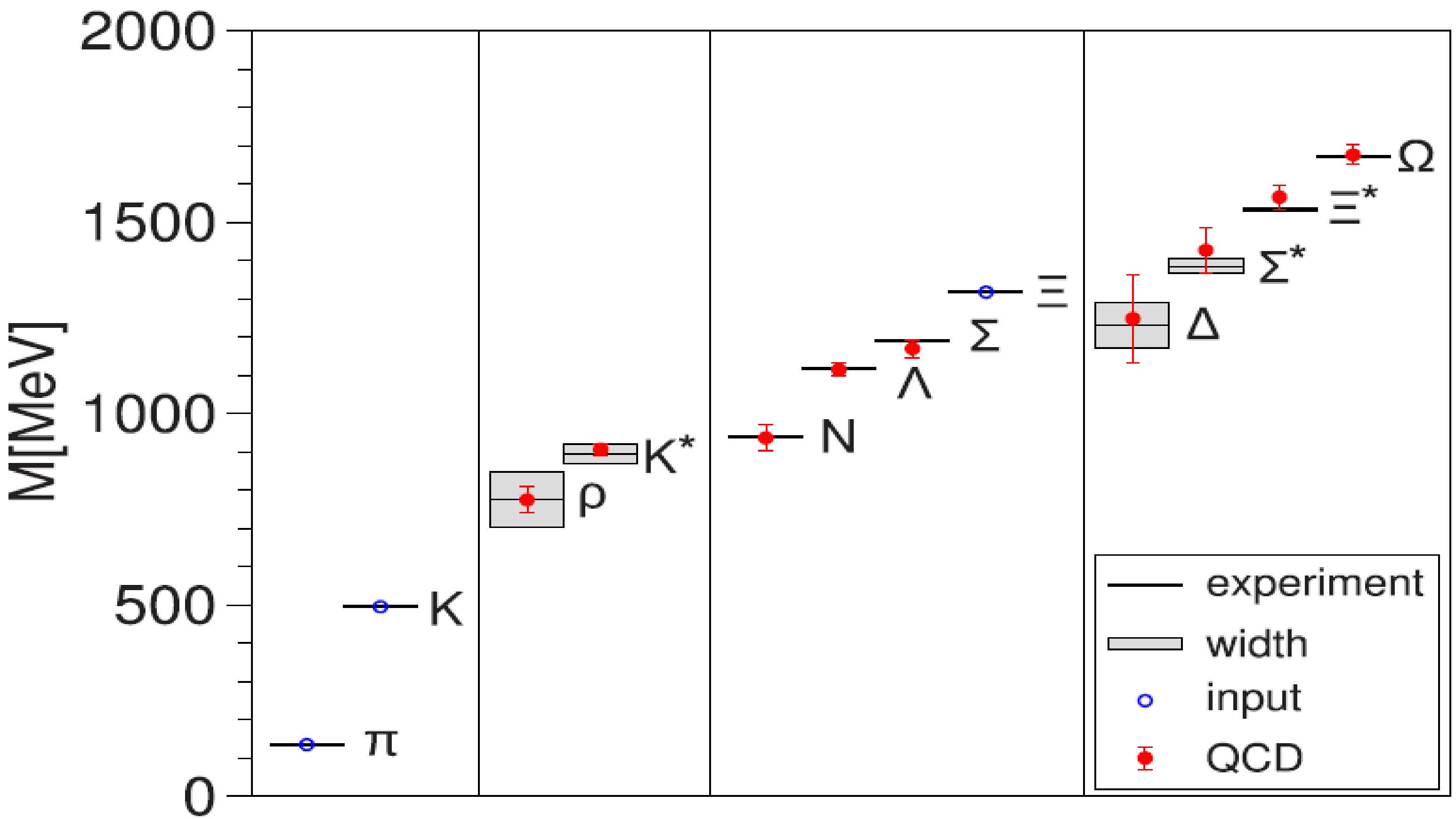


S.Durr et.al, Science 322, 1224 (2008)

Why do we care about Hadron spectra?

- QCD Spectrum ⟺ Physical spectrum ?

  - ✓ Check whether QCD is the correct theory for strong interaction at non-perturbative regime.
  - ✓ Provide information about fundamental as well as effective degrees of freedom.
  - ✓ Necessary for deeper understanding about strong interaction, origin of quark masses, chiral symmetry, confinement etc.
  - ✓ Will contribute to nuclear and astrophysics.

The Particle Zoo

Baryons (3-quarks)

p	P_{11}	****
n	P_{11}	****
$N(1440)$	P_{11}	****
$N(1520)$	D_{13}	****
$N(1535)$	S_{11}	****
$N(1650)$	S_{11}	****
$N(1675)$	D_{15}	****
$N(1680)$	F_{15}	****
$N(1700)$	D_{13}	***
$N(1710)$	P_{11}	***
$N(1720)$	P_{13}	****
$N(1900)$	P_{13}	**
$N(1990)$	F_{17}	**
$N(2000)$	F_{15}	**
$N(2080)$	D_{13}	**
$N(2090)$	S_{11}	*
$N(2100)$	P_{11}	*
$N(2190)$	G_{17}	****
$N(2200)$	D_{15}	**
$N(2220)$	H_{19}	****
$N(2250)$	G_{19}	****
$N(2600)$	$I_{1,11}$	***
$N(2700)$	$K_{1,13}$	**

$\Delta(1232)$	P_{33}	****
$\Delta(1600)$	P_{33}	***
$\Delta(1620)$	S_{31}	****
$\Delta(1700)$	D_{33}	****
$\Delta(1750)$	P_{31}	*
$\Delta(1900)$	S_{31}	**
$\Delta(1905)$	F_{35}	****
$\Delta(1910)$	P_{31}	****
$\Delta(1920)$	P_{33}	***
$\Delta(1930)$	D_{35}	***
$\Delta(1940)$	D_{33}	*
$\Delta(1950)$	F_{37}	****
$\Delta(2000)$	F_{35}	**
$\Delta(2150)$	S_{31}	*
$\Delta(2200)$	G_{37}	*
$\Delta(2300)$	H_{39}	**
$\Delta(2350)$	D_{35}	*
$\Delta(2390)$	F_{37}	*
$\Delta(2400)$	G_{39}	**
$\Delta(2420)$	$H_{3,11}$	****
$\Delta(2750)$	$I_{3,13}$	**
$\Delta(2950)$	$K_{3,15}$	**
Λ	P_{01}	****
$\Lambda(1405)$	S_{01}	****
$\Lambda(1520)$	D_{03}	****
$\Lambda(1600)$	P_{01}	***
$\Lambda(1670)$	S_{01}	****
$\Lambda(1690)$	D_{03}	****
$\Lambda(1800)$	S_{01}	***
$\Lambda(1810)$	P_{01}	***
$\Lambda(1820)$	F_{05}	****
$\Lambda(1830)$	D_{05}	****
$\Lambda(1890)$	P_{03}	****
$\Lambda(2000)$		*
$\Lambda(2020)$	F_{07}	*
$\Lambda(2100)$	G_{07}	****
$\Lambda(2110)$	F_{05}	***
$\Lambda(2325)$	D_{03}	*
$\Lambda(2350)$	H_{09}	***
$\Lambda(2585)$		**

Σ^+	P_{11}	****
Σ^0	P_{11}	****
Σ^-	P_{11}	****
$\Sigma(1385)$	P_{13}	****
$\Sigma(1480)$		*
$\Sigma(1560)$		**
$\Sigma(1580)$	D_{13}	*
$\Sigma(1620)$	S_{11}	**
$\Sigma(1660)$	P_{11}	***
$\Sigma(1670)$	D_{13}	****
$\Sigma(1690)$		**
$\Sigma(1750)$	S_{11}	***
$\Sigma(1770)$	P_{11}	*
$\Sigma(1775)$	D_{15}	****
$\Sigma(1840)$	P_{13}	*
$\Sigma(1880)$	P_{11}	**
$\Sigma(1915)$	F_{15}	****
$\Sigma(1940)$	D_{13}	***
$\Sigma(2000)$	S_{11}	*
$\Sigma(2030)$	F_{17}	****
$\Sigma(2070)$	F_{15}	*
$\Sigma(2080)$	P_{13}	**
$\Sigma(2100)$	G_{17}	*
$\Sigma(2250)$		***
$\Sigma(2455)$		**
$\Sigma(2620)$		**
$\Sigma(3000)$		*
$\Sigma(3170)$		*

Ξ^0	P_{11}	****
Ξ^-	P_{11}	****
$\Xi(1530)$	P_{13}	****
$\Xi(1620)$		*
$\Xi(1690)$		***
$\Xi(1820)$	D_{13}	***
$\Xi(1950)$		***
$\Xi(2030)$		***
$\Xi(2120)$		*
$\Xi(2250)$		**
$\Xi(2370)$		**
$\Xi(2500)$		*
Ω^-		****
$\Omega(2250)^-$		***
$\Omega(2380)^-$		**
$\Omega(2470)^-$		**

Λ_c^+	****
$\Lambda_c(2595)^+$	***
$\Lambda_c(2625)^+$	***
$\Lambda_c(2765)^+$	*
$\Lambda_c(2880)^+$	***
$\Lambda_c(2940)^+$	***
$\Sigma_c(2455)$	****
$\Sigma_c(2520)$	***
$\Sigma_c(2800)$	***
Ξ_c^+	***
Ξ_c^0	***
$\Xi_c'^+$	***
$\Xi_c'^0$	***
$\Xi_c(2645)$	***
$\Xi_c(2790)$	***
$\Xi_c(2815)$	***
$\Xi_c(2930)$	*
$\Xi_c(2980)$	***
$\Xi_c(3055)$	**
$\Xi_c(3080)$	***
$\Xi_c(3123)$	*
Ω_c^0	***
$\Omega_c(2770)^0$	***
Ξ_{cc}^+	*
Λ_b^0	***
Σ_b	***
Σ_b^*	***
Ξ_b^0, Ξ_b^-	***
Ω_b^-	***

Mesons (2-quarks)

LIGHT UNFLAVORED ($S = C = B = 0$)

	$I^G(J^{PC})$		$I^G(J^{PC})$
• $\pi^\pm$	$1^-(0^-)$	• $\pi_2(1670)$	$1^-(2^{-+})$
• π^0	$1^-(0^{-+})$	• $\phi(1680)$	$0^-(1^{--})$
• η	$0^+(0^{-+})$	• $\rho_3(1690)$	$1^+(3^{--})$
• $f_0(600)$	$0^+(0^{++})$	• $\rho(1700)$	$1^+(1^{--})$
• $\rho(770)$	$1^+(1^{--})$	$a_2(1700)$	$1^-(2^{++})$
• $\omega(782)$	$0^-(1^{--})$	• $f_0(1710)$	$0^+(0^{++})$
• $\eta'(958)$	$0^+(0^{-+})$	$\eta(1760)$	$0^+(0^{-+})$
• $f_0(980)$	$0^+(0^{++})$	• $\pi(1800)$	$1^-(0^{-+})$
• $a_0(980)$	$1^-(0^{++})$	$f_2(1810)$	$0^+(2^{++})$
• $\phi(1020)$	$0^-(1^{--})$	$X(1835)$	$?^?(?^{-+})$
• $h_1(1170)$	$0^-(1^{+-})$	• $\phi_3(1850)$	$0^-(3^{--})$
• $b_1(1235)$	$1^+(1^{+-})$	$\eta_2(1870)$	$0^+(2^{-+})$
• $a_1(1260)$	$1^-(1^{++})$	• $\pi_2(1880)$	$1^-(2^{-+})$
• $f_2(1270)$	$0^+(2^{++})$	$\rho(1900)$	$1^+(1^{--})$
• $f_1(1285)$	$0^+(1^{++})$	$f_2(1910)$	$0^+(2^{++})$
• $\eta(1295)$	$0^+(0^{-+})$	• $f_2(1950)$	$0^+(2^{++})$
• $\pi(1300)$	$1^-(0^{-+})$	$\rho_3(1990)$	$1^+(3^{--})$
• $a_2(1320)$	$1^-(2^{++})$	• $f_2(2010)$	$0^+(2^{++})$
• $f_0(1370)$	$0^+(0^{++})$	$f_0(2020)$	$0^+(0^{++})$
$h_1(1380)$	$?^-(1^{+-})$	• $a_4(2040)$	$1^-(4^{++})$
• $\pi_1(1400)$	$1^-(1^{-+})$	• $f_4(2050)$	$0^+(4^{++})$
• $\eta(1405)$	$0^+(0^{-+})$	$\pi_2(2100)$	$1^-(2^{-+})$
• $f_1(1420)$	$0^+(1^{++})$	$f_0(2100)$	$0^+(0^{++})$
• $\omega(1420)$	$0^-(1^{--})$	$f_2(2150)$	$0^+(2^{++})$
$f_2(1430)$	$0^+(2^{++})$	$\rho(2150)$	$1^+(1^{--})$
• $a_0(1450)$	$1^-(0^{++})$	$\phi(2170)$	$0^-(1^{--})$
• $\rho(1450)$	$1^+(1^{--})$	$f_0(2200)$	$0^+(0^{++})$
• $\eta(1475)$	$0^+(0^{-+})$	$f_J(2220)$	$0^+(2^{++}$ or 4
• $f_0(1500)$	$0^+(0^{++})$	$\eta(2225)$	$0^+(0^{-+})$
$f_1(1510)$	$0^+(1^{++})$	$\rho_3(2250)$	$1^+(3^{--})$
• $f_2'(1525)$	$0^+(2^{++})$	• $f_2(2300)$	$0^+(2^{++})$
$f_2(1565)$	$0^+(2^{++})$	$f_4(2300)$	$0^+(4^{++})$
$\rho(1570)$	$1^+(1^{--})$	$f_0(2330)$	$0^+(0^{++})$
$h_1(1595)$	$0^-(1^{+-})$	• $f_2(2340)$	$0^+(2^{++})$
• $\pi_1(1600)$	$1^-(1^{-+})$	$\rho_5(2350)$	$1^+(5^{--})$
$a_1(1640)$	$1^-(1^{++})$	$a_6(2450)$	$1^-(6^{++})$
$f_2(1640)$	$0^+(2^{++})$	$f_6(2510)$	$0^+(6^{++})$
• $\eta_2(1645)$	$0^+(2^{-+})$	OTHER LIGHT	
• $\omega(1650)$	$0^-(1^{--})$	Further States	
• $\omega_3(1670)$	$0^-(3^{--})$		

STRANGE ($S = \pm1, C = B = 0$)

	$I(J^P)$
• $K^\pm$	$1/2(0^-)$
• K^0	$1/2(0^-)$
• K_S^0	$1/2(0^-)$
• K_L^0	$1/2(0^-)$
$K_0^*(800)$	$1/2(0^+)$
• $K^*(892)$	$1/2(1^-)$
• $K_1(1270)$	$1/2(1^+)$
• $K_1(1400)$	$1/2(1^+)$
• $K^*(1410)$	$1/2(1^-)$
• $K_0^*(1430)$	$1/2(0^+)$
• $K_2^*(1430)$	$1/2(2^+)$
$K(1460)$	$1/2(0^-)$
$K_2(1580)$	$1/2(2^-)$
$K(1630)$	$1/2(?^?)$
$K_1(1650)$	$1/2(1^+)$
• $K^*(1680)$	$1/2(1^-)$
• $K_2(1770)$	$1/2(2^-)$
• $K_3^*(1780)$	$1/2(3^-)$
• $K_2(1820)$	$1/2(2^-)$
$K(1830)$	$1/2(0^-)$
$K_0^*(1950)$	$1/2(0^+)$
$K_2^*(1980)$	$1/2(2^+)$
• $K_4^*(2045)$	$1/2(4^+)$
$K_2(2250)$	$1/2(2^-)$
$K_3(2320)$	$1/2(3^+)$
$K_5^*(2380)$	$1/2(5^-)$
$K_4(2500)$	$1/2(4^-)$
$K(3100)$	$?^?(?^{??})$

CHARMED ($C = \pm1$)

	$I(J^P)$
• $D^\pm$	$1/2(0^-)$
• D^0	$1/2(0^-)$
• $D^*(2007)^0$	$1/2(1^-)$
• $D^*(2010)^\pm$	$1/2(1^-)$
$D_0^*(2400)^0$	$1/2(0^+)$
$D_0^*(2400)^\pm$	$1/2(0^+)$
• $D_1(2420)^0$	$1/2(1^+)$
$D_1(2420)^\pm$	$1/2(?^?)$
$D_1(2430)^0$	$1/2(1^+)$
• $D_2^*(2460)^0$	$1/2(2^+)$
• $D_2^*(2460)^\pm$	$1/2(2^+)$
$D^*(2640)^\pm$	$1/2(?^?)$

CHARMED, STRANGE ($C = S = \pm1$)

	$I(J^P)$
• $D_s^\pm$	$0(0^-)$
• $D_s^{*\pm}$	$0(?^?)$
• $D_{s0}^*(2317)^\pm$	$0(0^+)$
• $D_{s1}(2460)^\pm$	$0(1^+)$
• $D_{s1}(2536)^\pm$	$0(1^+)$
• $D_{s2}(2573)^\pm$	$0(?^?)$
$D_{s1}(2700)^\pm$	$0(1^-)$

BOTTOM ($B = \pm1$)

• $B^\pm$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^\pm/B^0$ ADMIXTURE	
• $B^\pm/B^0/B_s^0/b$ baryon ADMIXTURE	
V_{cb} and V_{ub} CKM Matrix Elements	
• B^*	$1/2(1^-)$
$B_J^*(5732)$	$?(?^?)$
• $B_1(5721)^0$	$1/2(1^+)$
• $B_2^*(5747)^0$	$1/2(2^+)$

BOTTOM, STRANGE ($B = \pm1, S = \mp1$)

• B_s^0	$0(0^-)$
• B_s^*	$0(1^-)$
• $B_{s1}(5830)^0$	$1/2(1^+)$
• $B_{s2}^*(5840)^0$	$1/2(2^+)$
$B_{sJ}^*(5850)$	$?(?^?)$

BOTTOM, CHARMED ($B = C = \pm1$)

• $B_c^\pm$	$0(0^-)$

$c\bar{c}$

	$I^G(J^{PC})$
• $\eta_c(1S)$	$0^+(0^{-+})$
• $J/\psi(1S)$	$0^-(1^{--})$
• $\chi_{c0}(1P)$	$0^+(0^{++})$
• $\chi_{c1}(1P)$	$0^+(1^{++})$
• $h_c(1P)$	$?^?(1^{+-})$
• $\chi_{c2}(1P)$	$0^+(2^{++})$
• $\eta_c(2S)$	$0^+(0^{-+})$
• $\psi(2S)$	$0^-(1^{--})$
• $\psi(3770)$	$0^-(1^{--})$
• $X(3872)$	$0^?(?^{?+})$
$\chi_{c2}(2P)$	$0^+(2^{++})$
$X(3940)$	$?^?(?^{??})$
• $X(3945)$	$?^?(?^{?+})$
• $\psi(4040)$	$0^-(1^{--})$
• $\psi(4160)$	$0^-(1^{--})$
$X(4160)$	$?^?(?^{??})$
• $X(4260)$	$?^?(1^{--})$
$X(4360)$	$?^?(1^{--})$
• $\psi(4415)$	$0^-(1^{--})$
$X(4660)$	$?^?(1^{--})$

$b\bar{b}$

$\eta_b(1S)$	$0^+(0^{-+})$
• $\Upsilon(1S)$	$0^-(1^{--})$
• $\chi_{b0}(1P)$	$0^+(0^{++})$
• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $\chi_{b2}(1P)$	$0^+(2^{++})$
• $\Upsilon(2S)$	$0^-(1^{--})$
$\Upsilon(1D)$	$0^-(2^{--})$
• $\chi_{b0}(2P)$	$0^+(0^{++})$
• $\chi_{b1}(2P)$	$0^+(1^{++})$
• $\chi_{b2}(2P)$	$0^+(2^{++})$
• $\Upsilon(3S)$	$0^-(1^{--})$
• $\Upsilon(4S)$	$0^-(1^{--})$
• $\Upsilon(10860)$	$0^-(1^{--})$
• $\Upsilon(11020)$	$0^-(1^{--})$

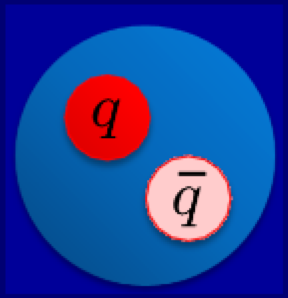
NON $q\bar{q}$ CANDIDATES

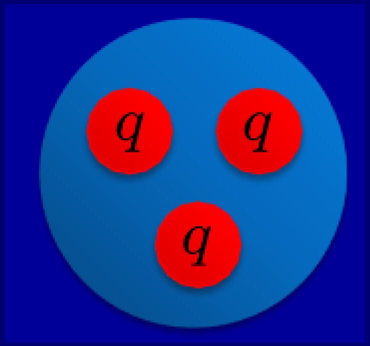
NON $q\bar{q}$ CANDIDATES

HADRON SPECTRUM ...PDG

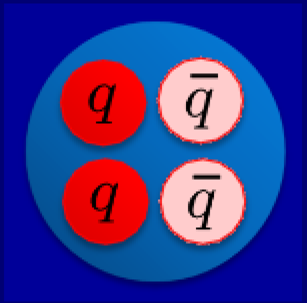
Type of Hadrons

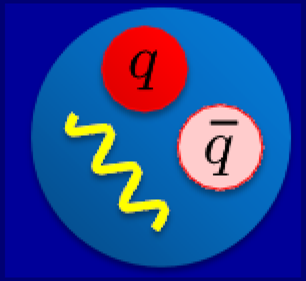
- Normal hadrons :
  - Two quark state (meson)
  - Three quark state (baryon)

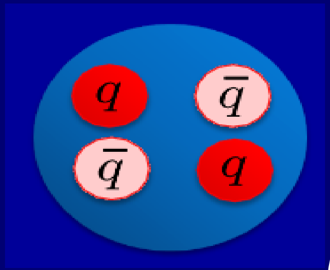


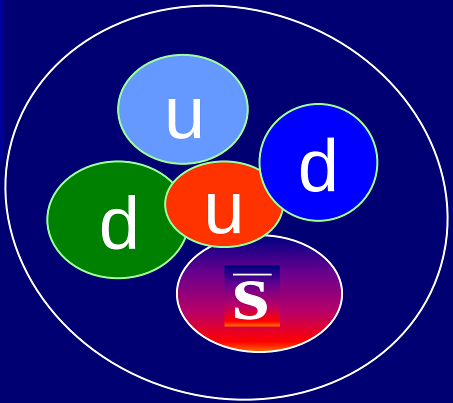


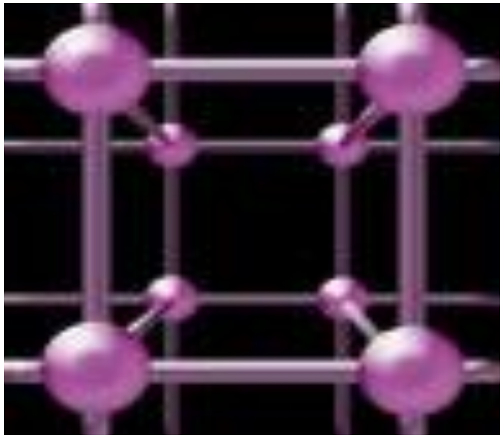
- Exotic Hadrons







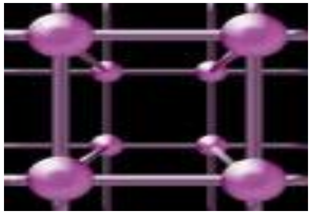




How to calculate an observable??

From statistical mechanical correlation functions

Observables



Quark Jungle Gym

$$< \hat{O} > = \mathrm{Lim}_{\beta\to\infty} \frac{1}{Z} \mathrm{Tr}[e^{-\beta H} \hat{O}(U,\bar{\psi},\psi)]$$

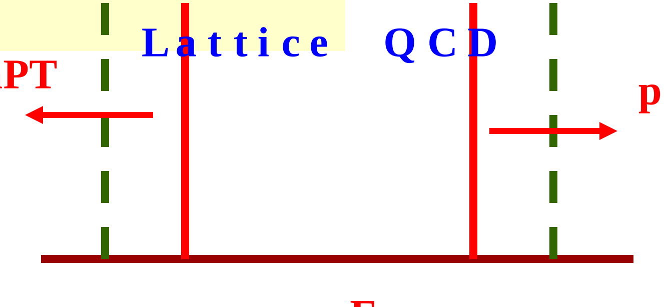
$$= \mathrm{Lim}_{\beta\to\infty} \frac{\int DU\, D\bar{\psi}\, D\psi\, O[U,\bar{\psi},\psi]\, e^{-S_g[U]-S_F[U,\bar{\psi},\psi]}}{\int DU\, D\bar{\psi}\, D\psi\, e^{-S_g[U]-S_F[U,\bar{\psi},\psi]}}$$

Integrating out the Grassmann variables is possible since $S_F = \bar{\psi} D \psi$

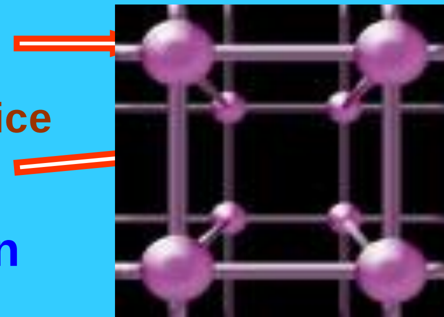
$$< \hat{O} > = \frac{\int DU \{\det D\}^{n_f}\, O[U,D^{-1}]\, e^{-S_g[U]}}{\int DU\, \{\det D\}^{n_f}\, e^{-S_g[U]}} = \prod_n \int \underbrace{dU_n \frac{1}{Z} \{\det D(U)\}^{n_f}\, e^{-S_g[U]}}\, O[U,D^{-1}]$$

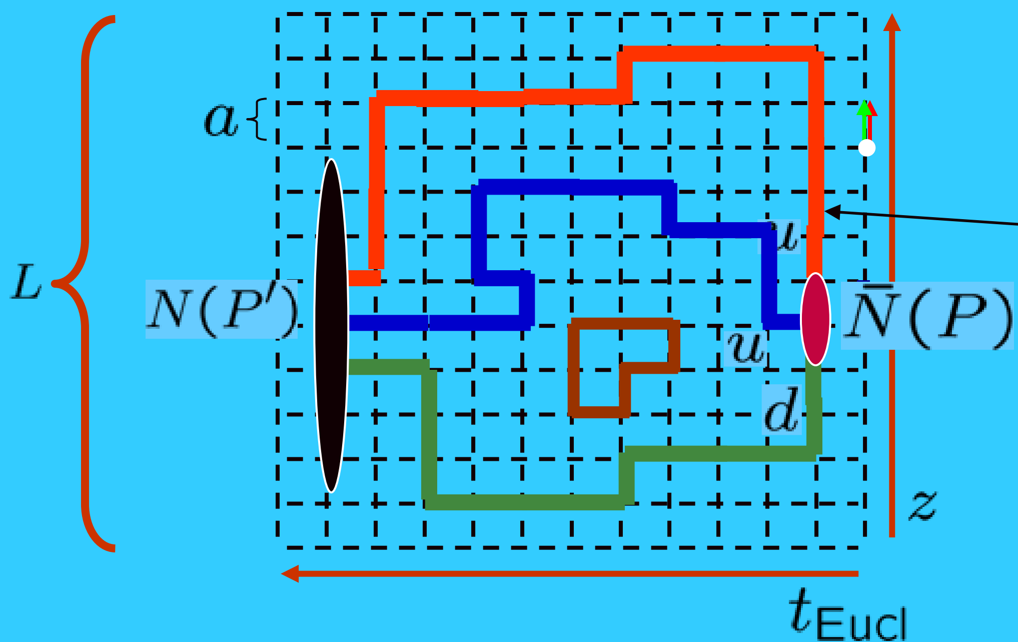
$$< \hat{O} > = \frac{1}{N} \sum_{U \in \frac{1}{Z} e^{-S_g[U] + \ln\{\det D\}^{n_f}}} O[U,D^{-1}]$$

$$\sum D^{-1}(U) D^{-1}(U) \ldots D^{-1}(U)$$

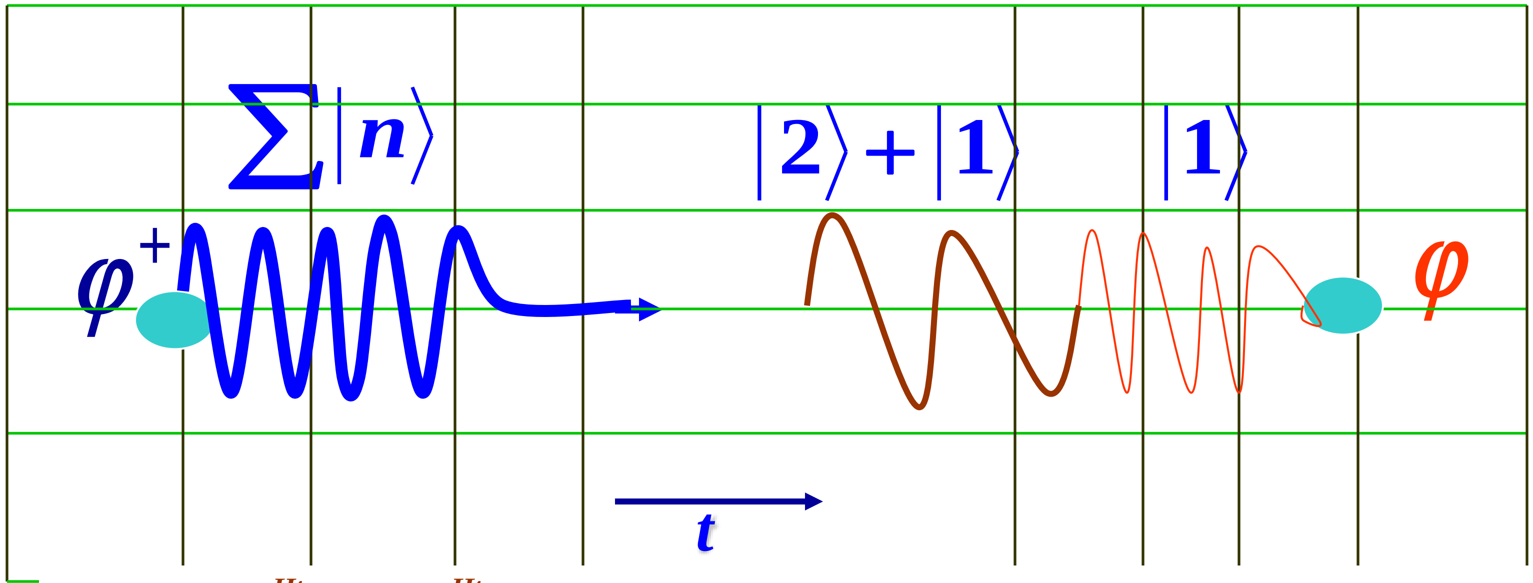


Parameters : gauge coupling and quark masses





quark propagators :
Inverse of very large matrix of space-time, spin and color



$$\varphi(t) = e^{Ht}\varphi(0)e^{-Ht}$$

$$G(t,\vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}.(\vec{x}-\vec{x}_0)} \sum_{n,\vec{q}} \langle 0|\varphi(x)|n,\vec{q}\rangle\langle n,\vec{q}|\varphi(x_0)|0\rangle$$

$$= \sum_{\vec{x}} e^{-i\vec{p}.(\vec{x}-\vec{x}_0)} \sum_{n,\vec{q}} \langle 0|e^{H(t-t_0)-i\vec{p}'.(\vec{x}-\vec{x}_0)}\varphi(x_0)e^{-H(t-t_0)+i\vec{p}'.(\vec{x}-\vec{x}_0)}|n,\vec{q}\rangle\langle n,\vec{q}|\varphi(x_0)|0\rangle$$

$$= \sum_{\vec{x}} e^{-i\vec{p}.(\vec{x}-\vec{x}_0)} \sum_{n,\vec{q}} e^{i\vec{q}.(\vec{x}-\vec{x}_0)-E_q^n(t-t_0)} \langle 0|\varphi(x_0)|n,\vec{q}\rangle\langle n,\vec{q}|\varphi(x_0)|0\rangle$$

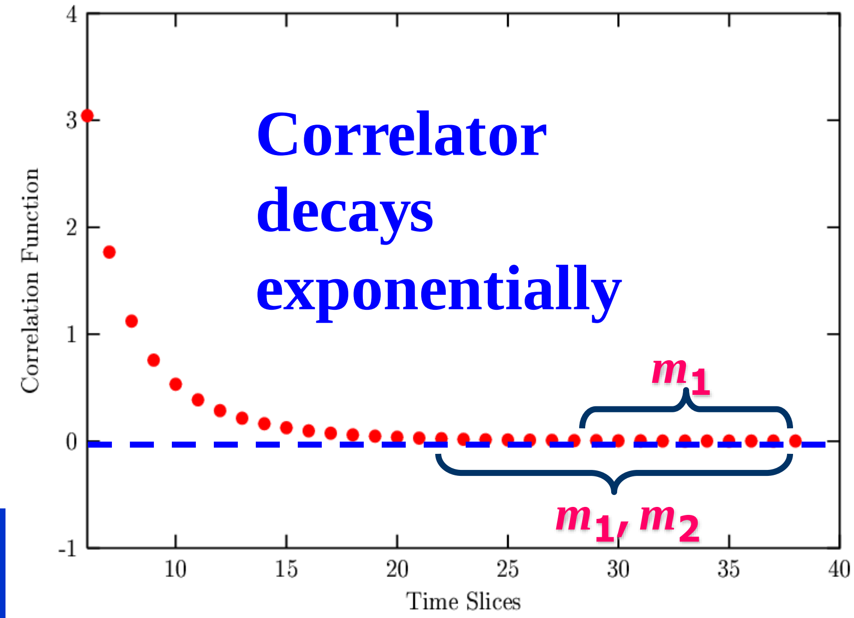
$$\approx \sum_{n,\vec{q}} \delta(\vec{p}-\vec{q}) e^{i(\vec{p}-\vec{q}).\vec{x}_0-E_q^n(t-t_0)} \langle 0|\varphi(x_0)|n,\vec{q}\rangle\langle n,\vec{q}|\varphi(x_0)|0\rangle$$

$$= \sum_{n} e^{-E_p^n(t-t_0)} \left|\langle 0|\varphi(x_0)|n,\vec{p}\rangle\right|^2$$

$$= \sum_{n} W_n e^{-E_p^n(t-t_0)} \xrightarrow[t\to\infty]{} W_1 e^{-E_1^n(t-t_0)}$$

Analysis (Extraction of Mass)

$$G(\tau) = \sum_{i=1}^{N} W_i \, e^{-m_i \tau} \underset{\tau\to\infty}{\approx} W_1 \, e^{-m_1 \tau}$$



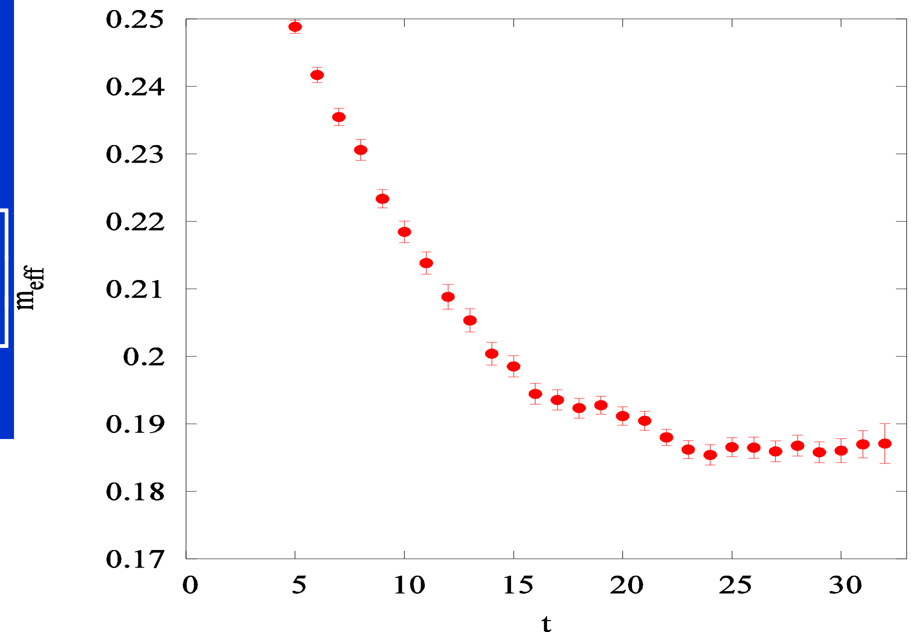
Effective mass :

$$\frac{G(\tau)}{G(\tau+1)} = e^{-m_1\tau + m_1(\tau+1)}$$

$$m(\tau) = \ln\left[\frac{G(\tau)}{G(\tau+1)}\right]$$

$$= \ln\left[\frac{|w_1|^2 \, e^{-E_1\tau} + |w_2|^2 \, e^{-E_2\tau} + \ldots}{|w_1|^2 \, e^{-E_1(\tau+a_\tau)} + |w_2|^2 \, e^{-E_2(\tau+a_\tau)} + \ldots}\right]$$

$$\approx a_\tau E_1 [1 + \vartheta(|w_2|^2 / |w_1|^2 \, e^{(E_2-E_1)\tau/a_\tau})$$



Hadron Spectrum Collaboration

Jefferson Lab, Univ. of Cambridge, Maryland, CMU, TIFR, Trinity College

Variational Analysis

ϕ_i : gauge invariant fields on a timeslice t that corresponds to Hilbert space operator ϕ_j whose quantum numbers are also carried by the states **|n>**.
Construct a matrix

$$C(t) = \begin{bmatrix} \langle 0|\phi_1(t)\phi_1^+(0)|0\rangle & \langle 0|\phi_1(t)\phi_2^+(0)|0\rangle & \dots & \dots \\ \langle 0|\phi_2(t)\phi_1^+(0)|0\rangle & \langle 0|\phi_2(t)\phi_2^+(0)|0\rangle & \dots & \dots \\ \dots & \dots & & \dots \\ \dots & \dots & & \dots \end{bmatrix}$$

- Need to find out variational coefficient $\{v_\alpha^{(m)}, \alpha = 1,2,\dots,n\}$ such that the overlap to a state is maximum

$$\Phi^{(m)}(t)|0\rangle = \sum_{\alpha}^{N} v_\alpha^{(m)}\phi_\alpha(t)|0\rangle$$

$$= (1-\varepsilon_m)e^{-\hat{H}t}|m\rangle + \sum_{n\neq m}\varepsilon_n e^{-\hat{H}t}|n\rangle \quad \text{with } \varepsilon_n << 1$$

- Variational solution → Generalized eigenvalue problem :

$$C(t)v^n(t,t_0) = \lambda_n(t,t_0)C(t_0)v^n(t,t_0)$$

"Rayleigh-Ritz method"
Diagonalize:
eigenvalues → spectrum
eigenvectors → spectral "overlaps" Z_i^n

- Eigenvalues give spectrum :

$$\lim_{t\to\infty}\lambda_n(t,t_0) = e^{-(t-t_0)E_n}(1+e^{-t\Delta E_n})$$

- Eigenvectors give the optimal operator :

$$\Phi^m(t) = v_1^m\phi_1(t) + v_2^m\phi_2(t) + \dots$$

Operators

Mesons: fermion bi-linears

$\bar{\psi}\Gamma\psi$	J = 0, 1	
$\bar{\psi}\Gamma\overleftrightarrow{D}\psi$	J = 0, 1, 2	gauge-covariant derivatives ~ 1^{--}
$\bar{\psi}\Gamma\overleftrightarrow{D}\overleftrightarrow{D}\psi$	J = 0, 1, 2, 3	coupling $\langle 1m_1; 1m_2 \vert L_{12}m_{12}\rangle \overleftrightarrow{D}_{m_1} \overleftrightarrow{D}_{m_2}$
$\bar{\psi}\Gamma\overleftrightarrow{D}\overleftrightarrow{D}\overleftrightarrow{D}\psi$	J = 0, 1, 2, 3, 4	2 derivatives can give chromo B field 1^{+-}

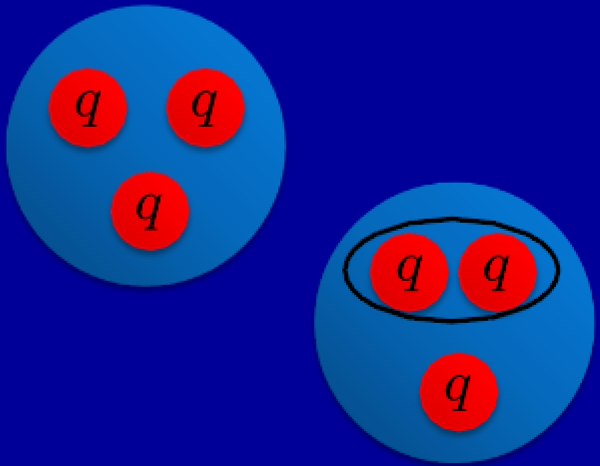
Baryons: three quarks

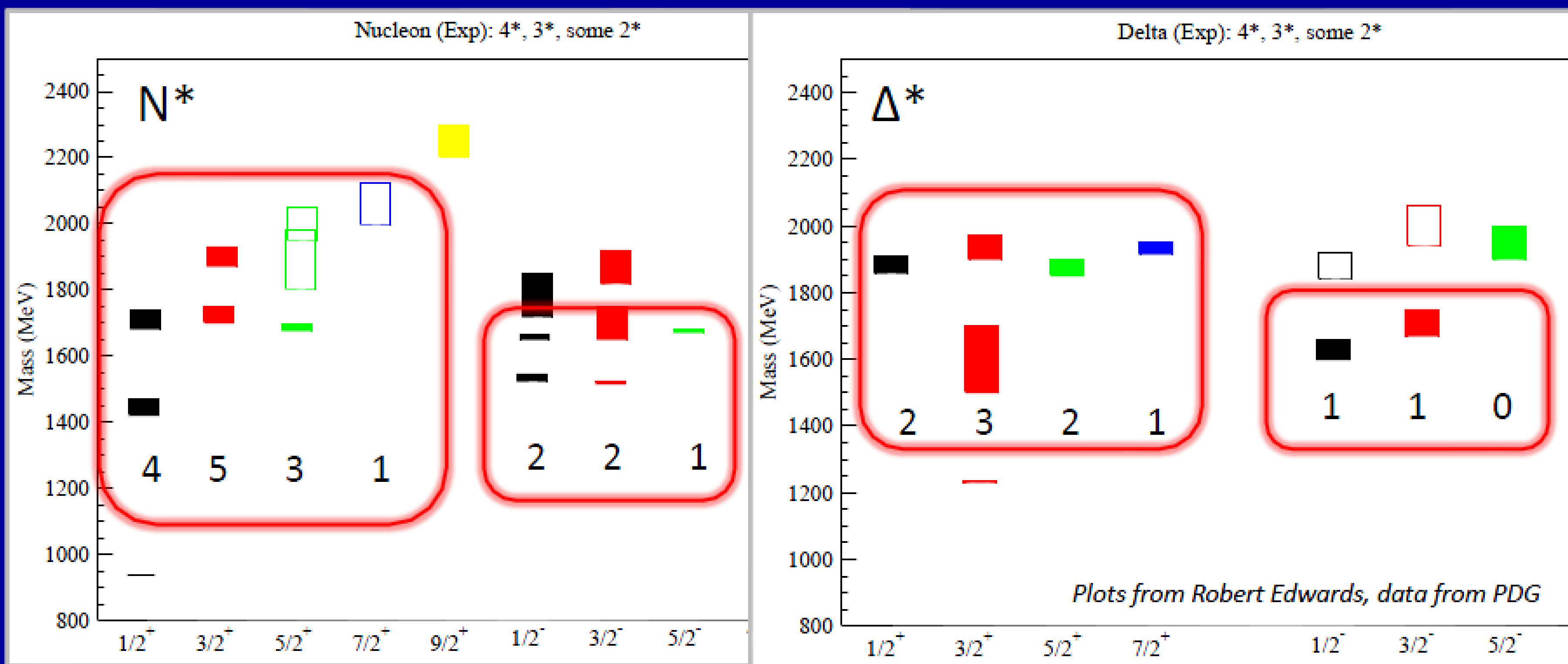
$\Phi^{J,j} = \langle 1l_1; 1l_2 \vert Ll\rangle\langle Ll; Ss \vert Jj\rangle \vec{D}_{l_1}\vec{D}_{l_2}\left[\psi\psi\psi\right]_s$

$\mathbf{1}\otimes\mathbf{1}\otimes\mathcal{S} \to \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$

Hadron Spectroscopy – Baryons

- Missing states?
- 'Freezing' of degrees of freedom?
- Gluonic excitations?
- Flavour structure

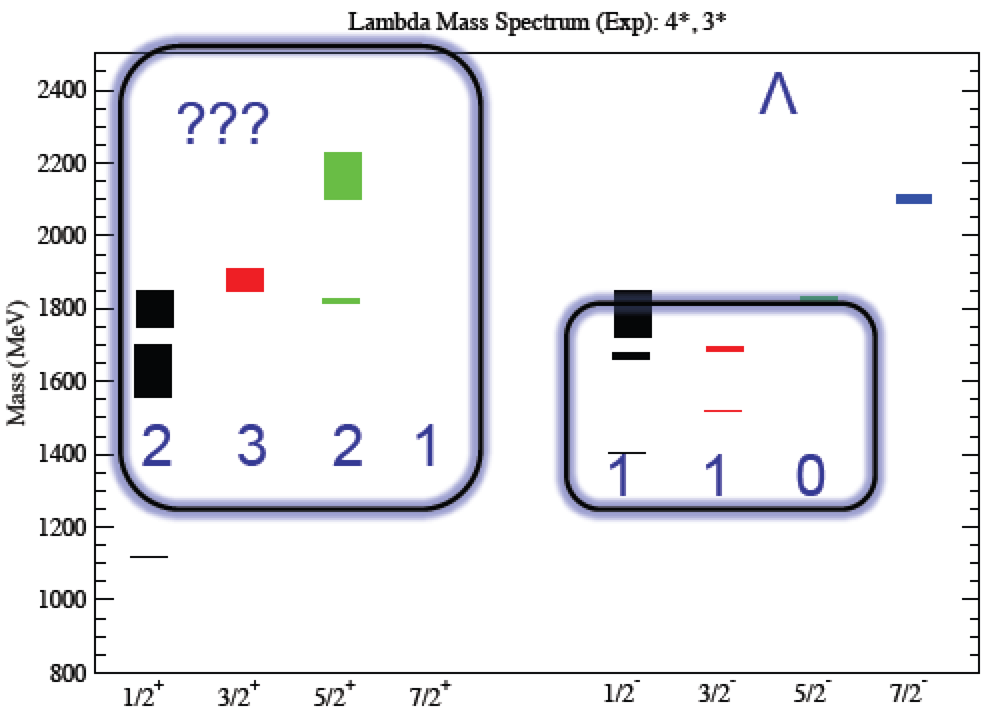


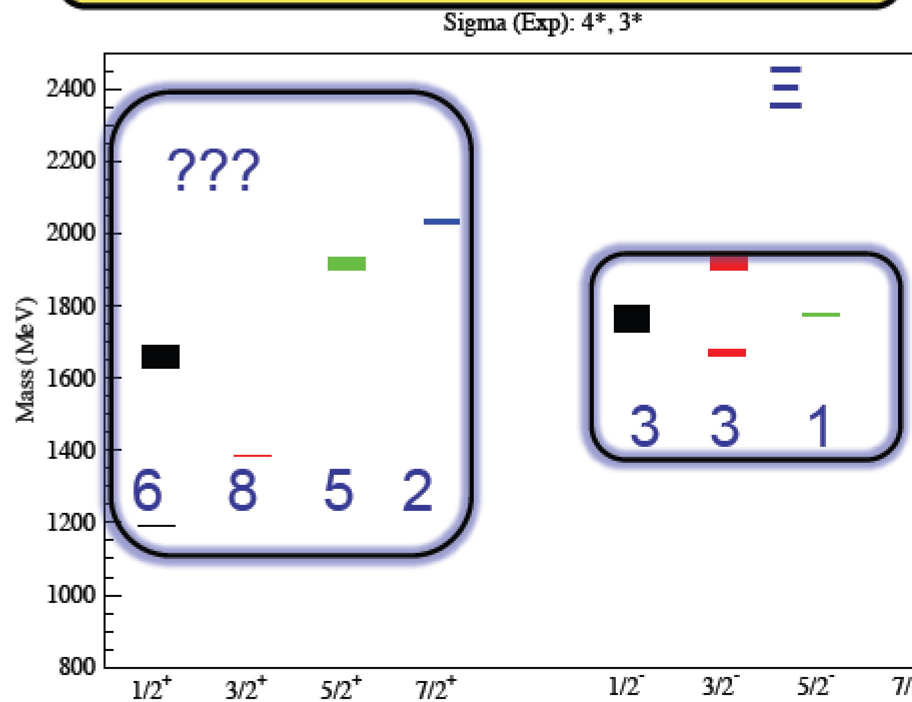


Strange Quark Baryon Spectrum

Strange quark baryon spectrum even sparser

Since SU(3) flavor symmetry broken, expect mixing of $\mathbf{8_F}$ & $\mathbf{10_F}$



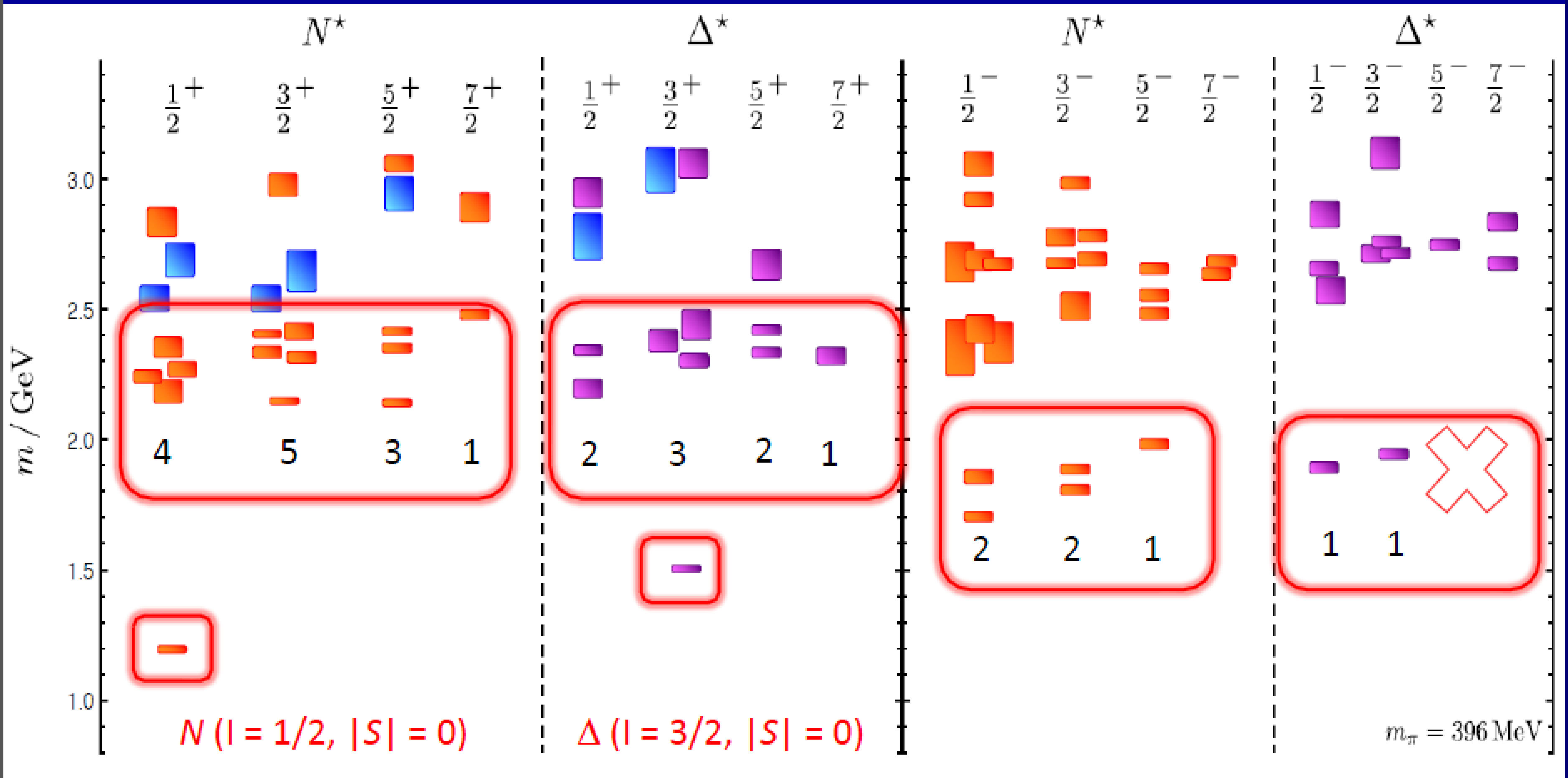


Even less known states in Ξ & Ω

@Edwar

N and Δ baryons

HSC : [PR D84 074508; D85 054016]



Counting expected in non. rel. quark model, SU(6) x O(3)

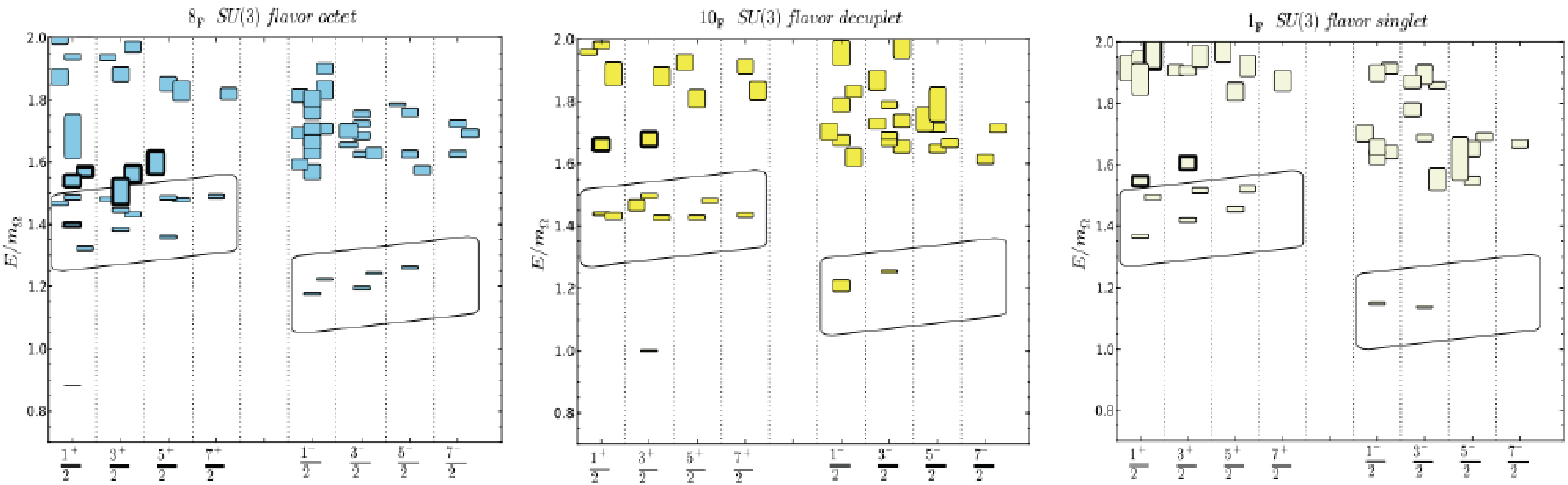
N_f = 2+1, $M_\pi \approx 400$ MeV

SU(3) flavor limit

In SU(3) flavor limit – have exact flavor Octet, Decuplet and Singlet representations

HSC : Phys.Rev. D87 (2013) 054506

8_F 10_F 1_F



$m_\pi \sim 700$ MeV

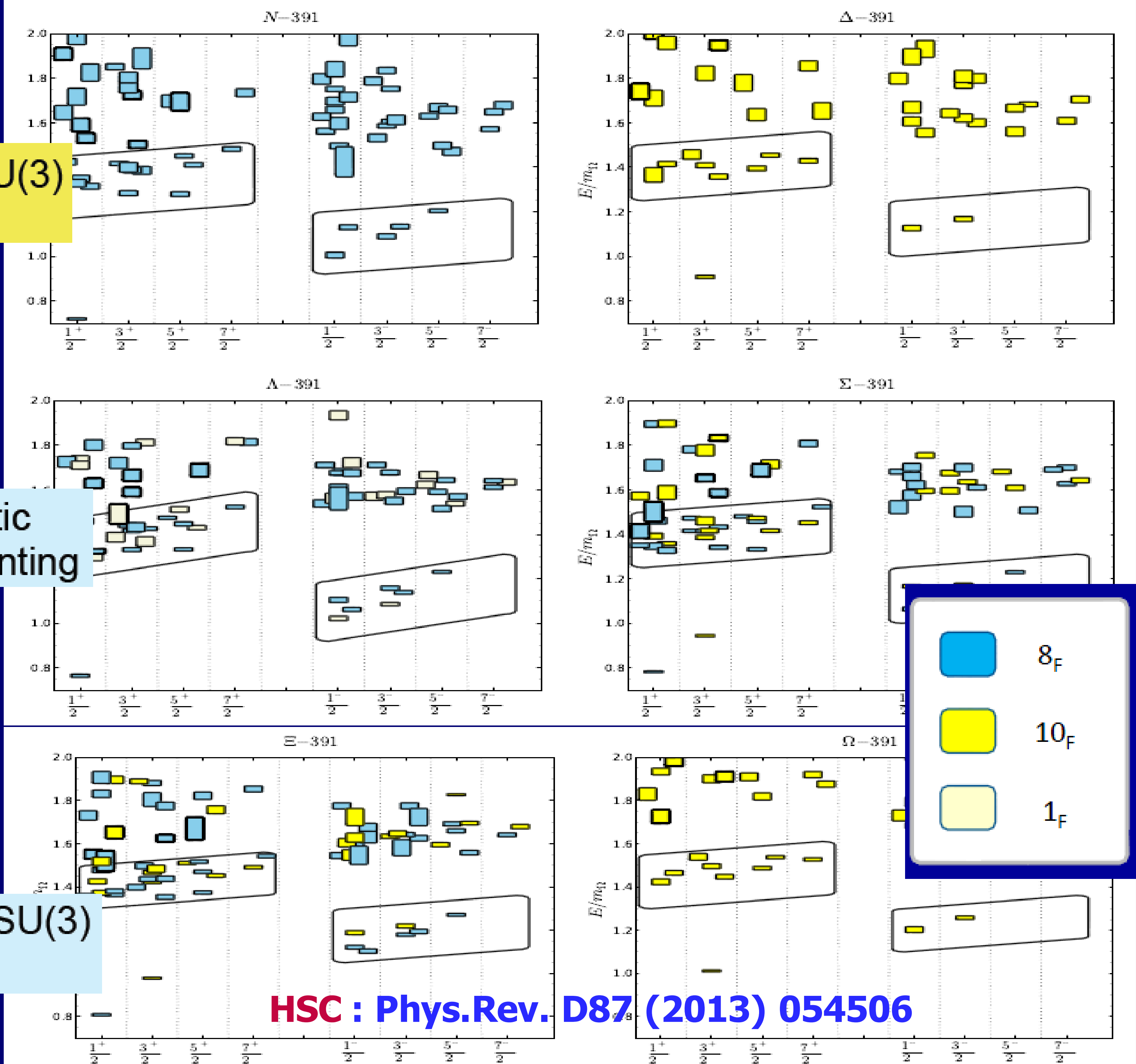
Full non-relativistic quark model counting

Additional levels with significant gluonic components

Light quarks – SU(3) flavor broken

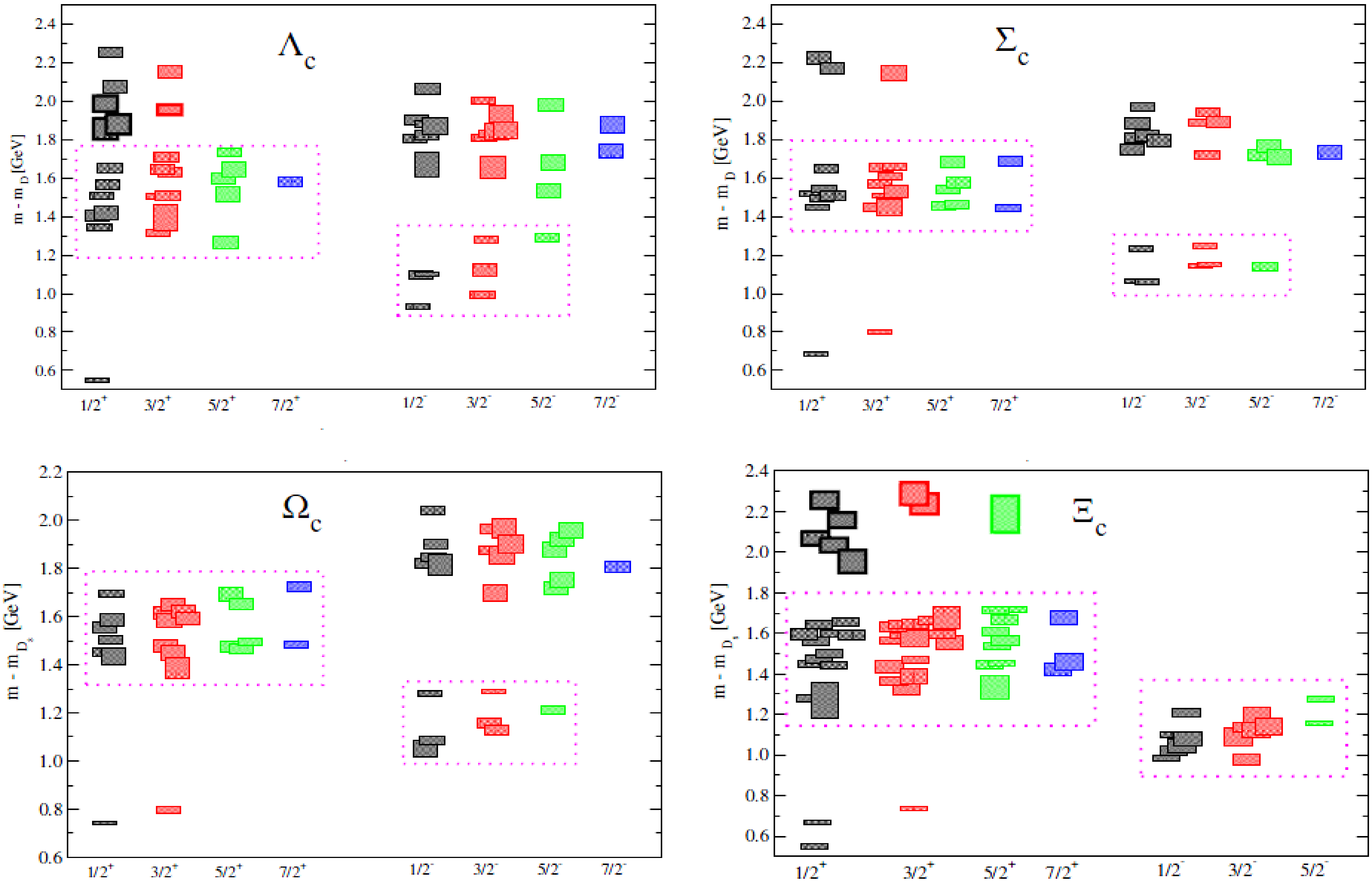
Full non-relativistic quark model counting

Some mixing of SU(3) flavor irreps



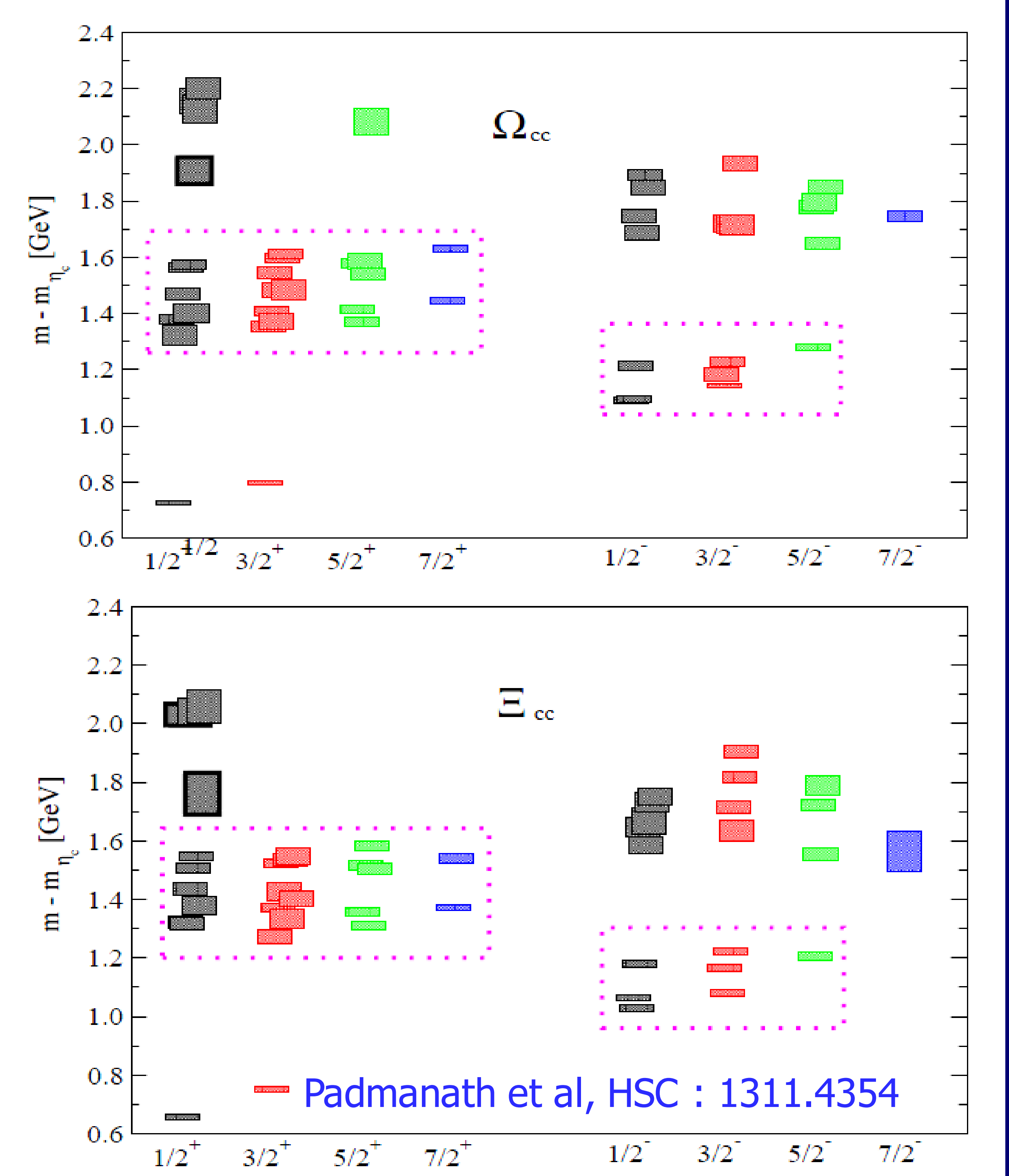
HSC : Phys.Rev. D87 (2013) 054506

Charm baryons

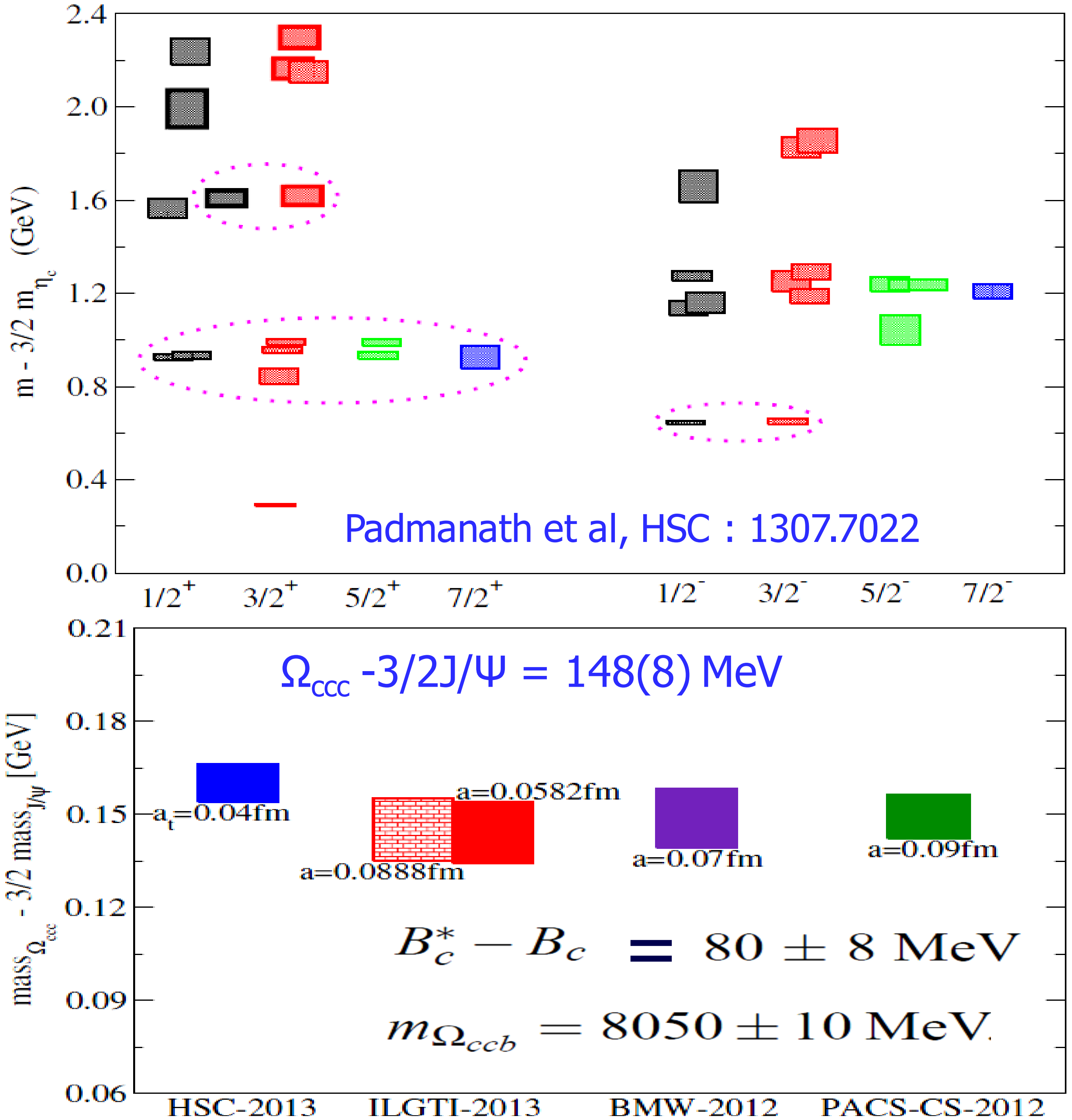


Padmanath et al, HSC : 1311.4806

CHARM BARYONS

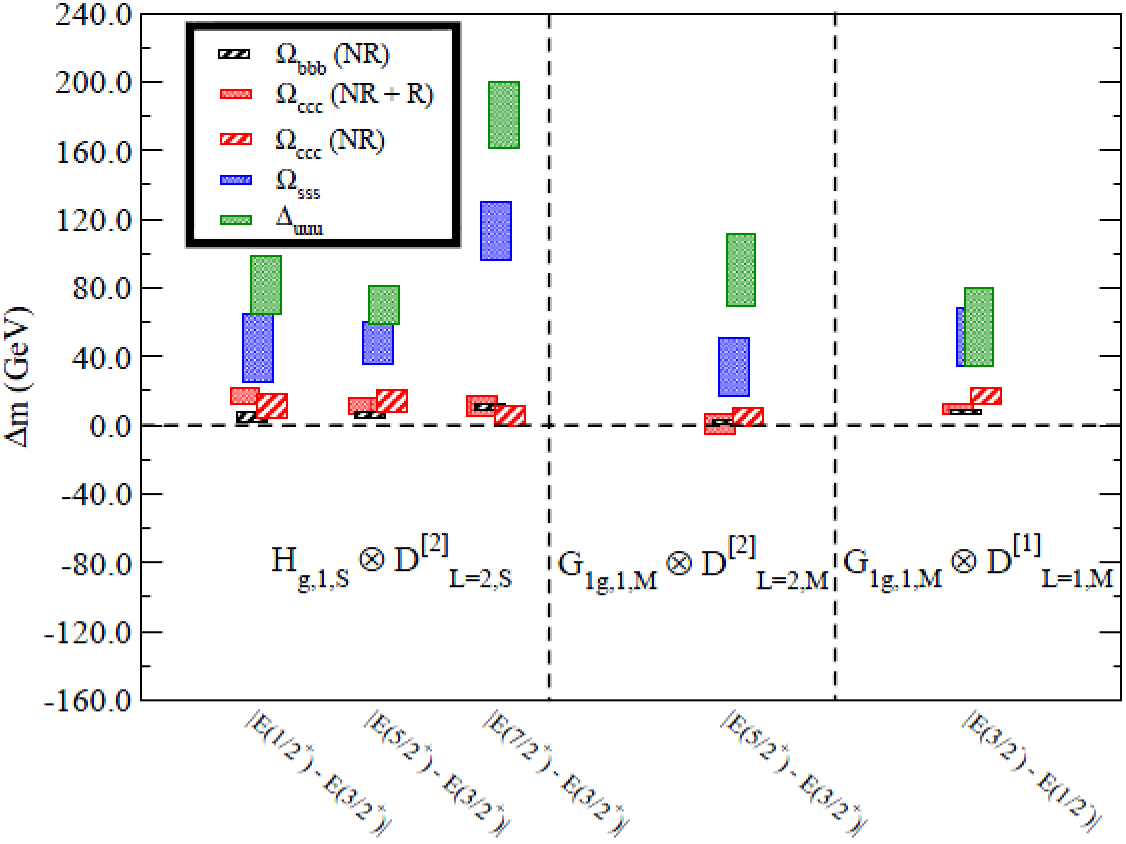


CHARM BARYONS



How heavy is charm?

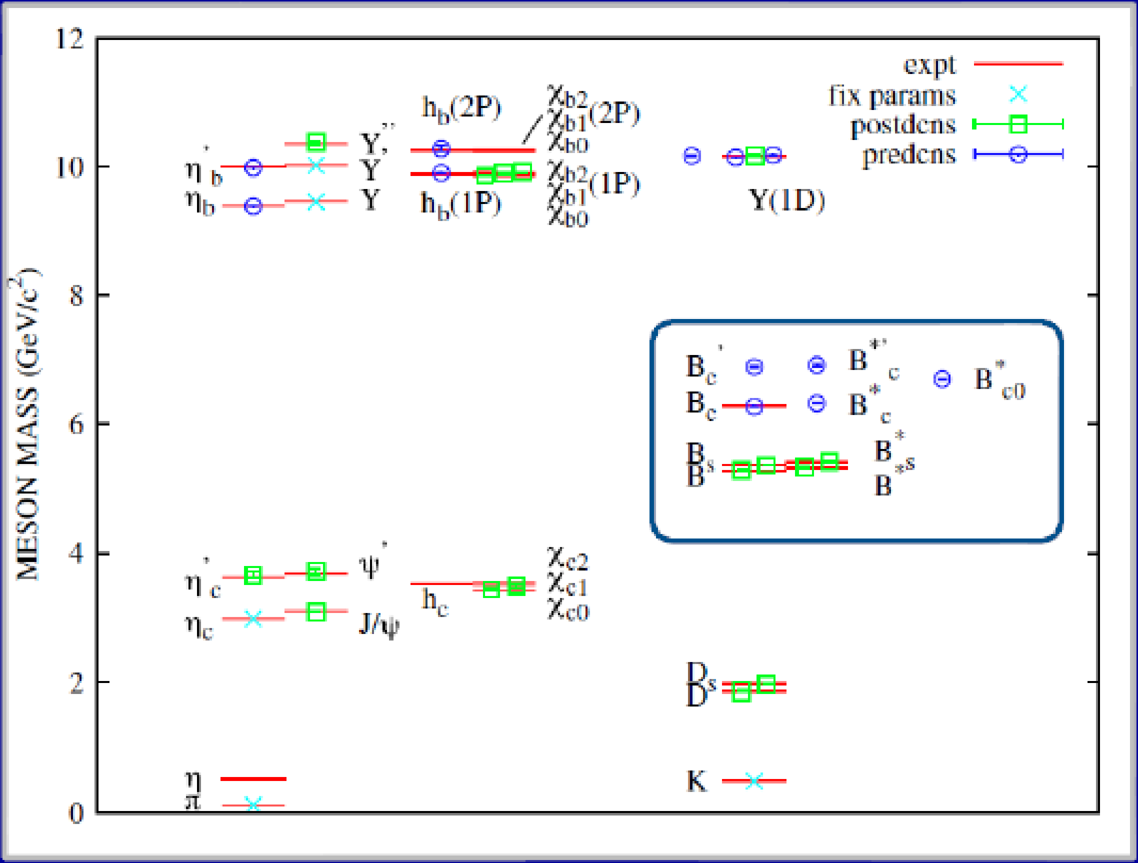
Can NRQCD sill work?

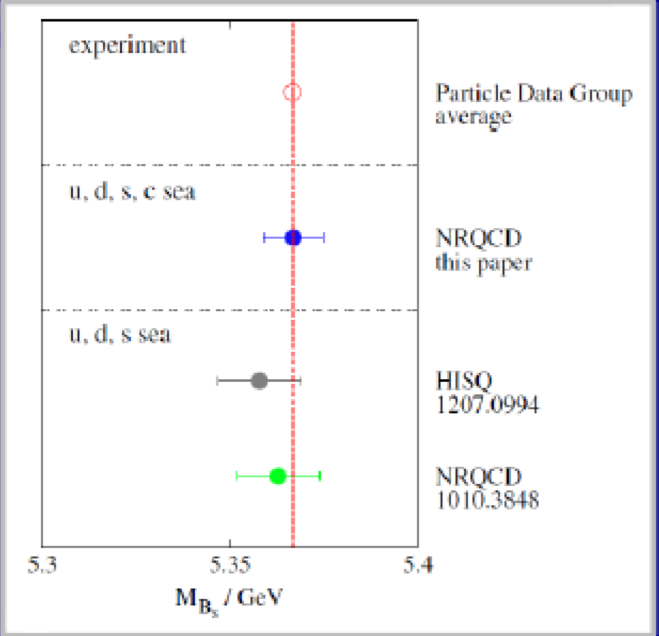


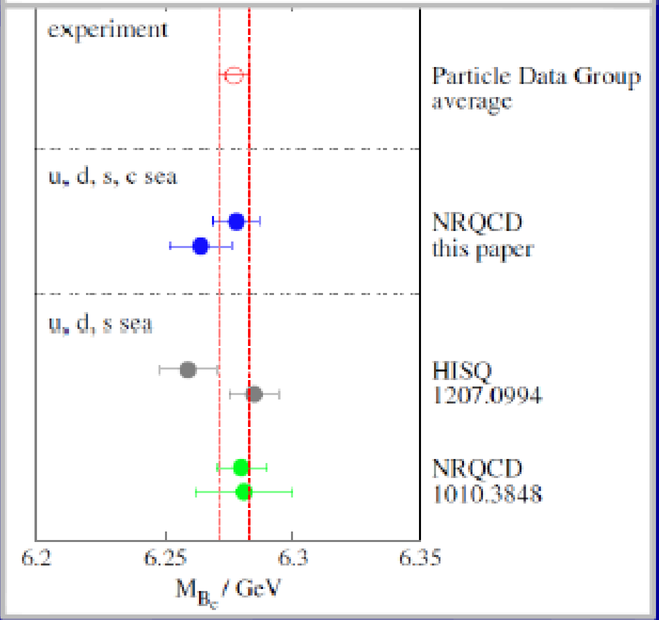
Padmanath et al, HSC : 1307.7022

Quarkonia and heavy-light mesons

Dowdall et al (HPQCD)
[PR D86, 094510 (2012)]

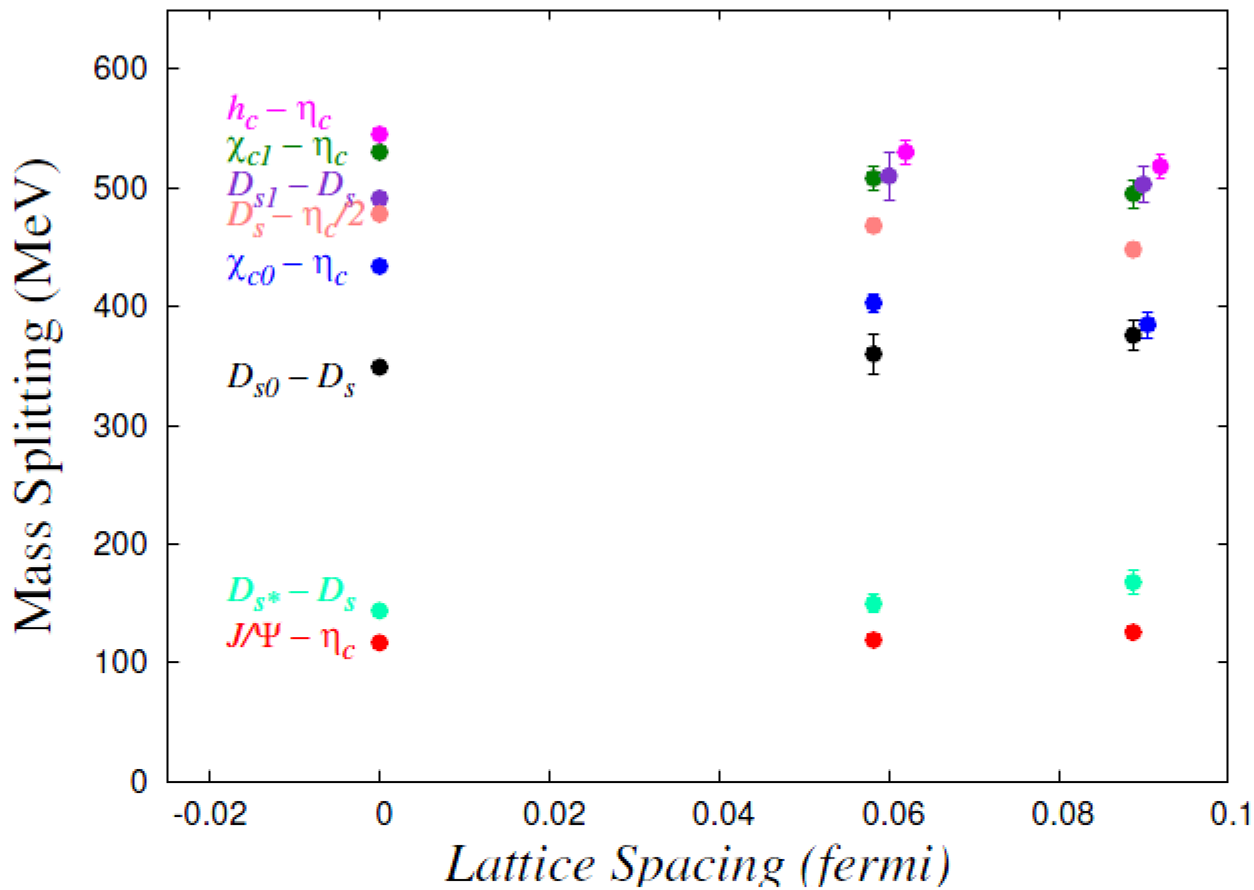






Dynamical (HISQ) $N_f = 2+1+1$ (u,d,s,c)
with non-rel b quark [O(α_s) corrections]

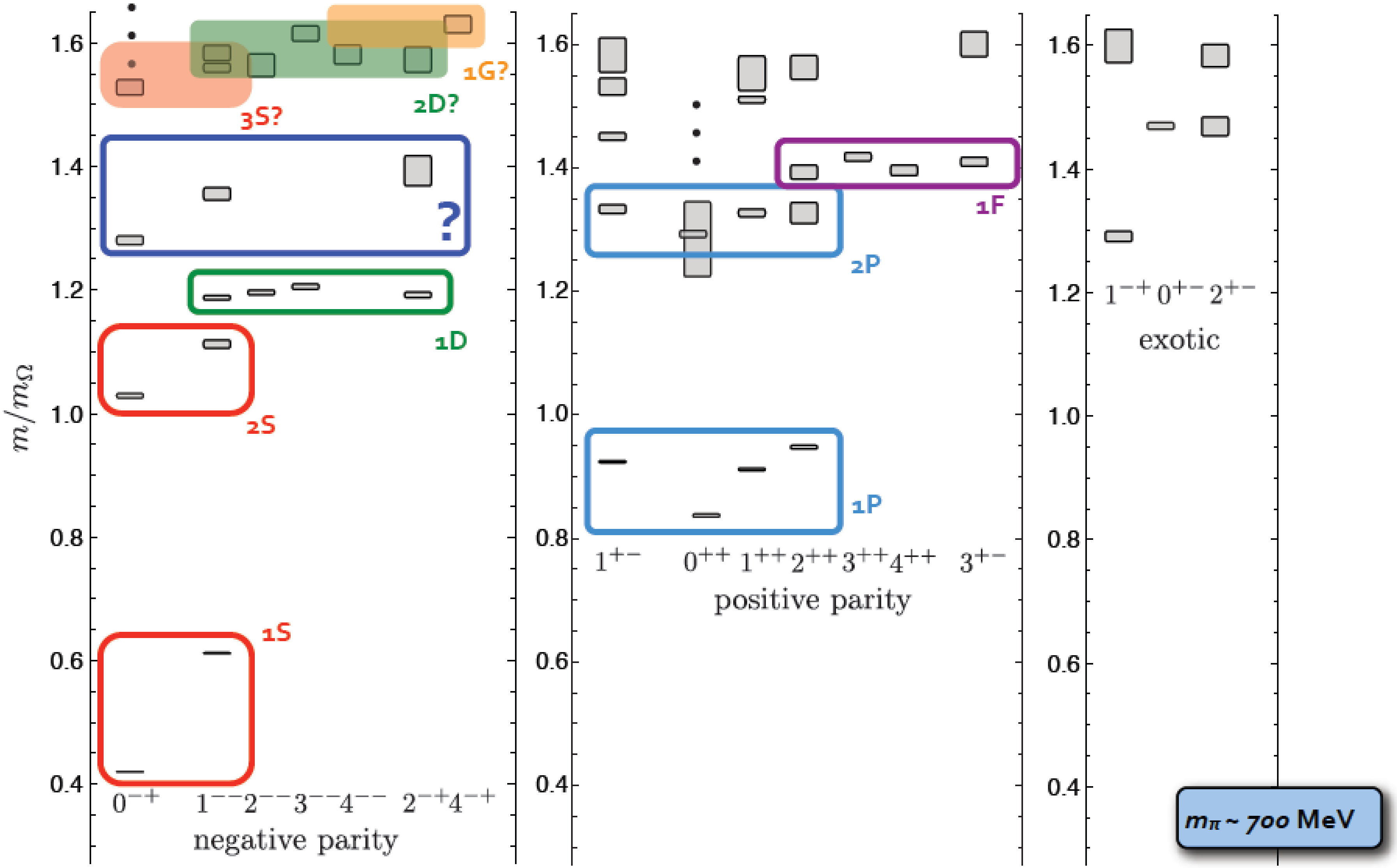
c.f. $N_f = 2+1$ (HISQ) with HISQ or non-rel b quark



Ω_{ccc} -3/2J/Ψ = 148(8) MeV

ILGTI@ arXiv:1312.3050, 1211.6277

Meson Spectra

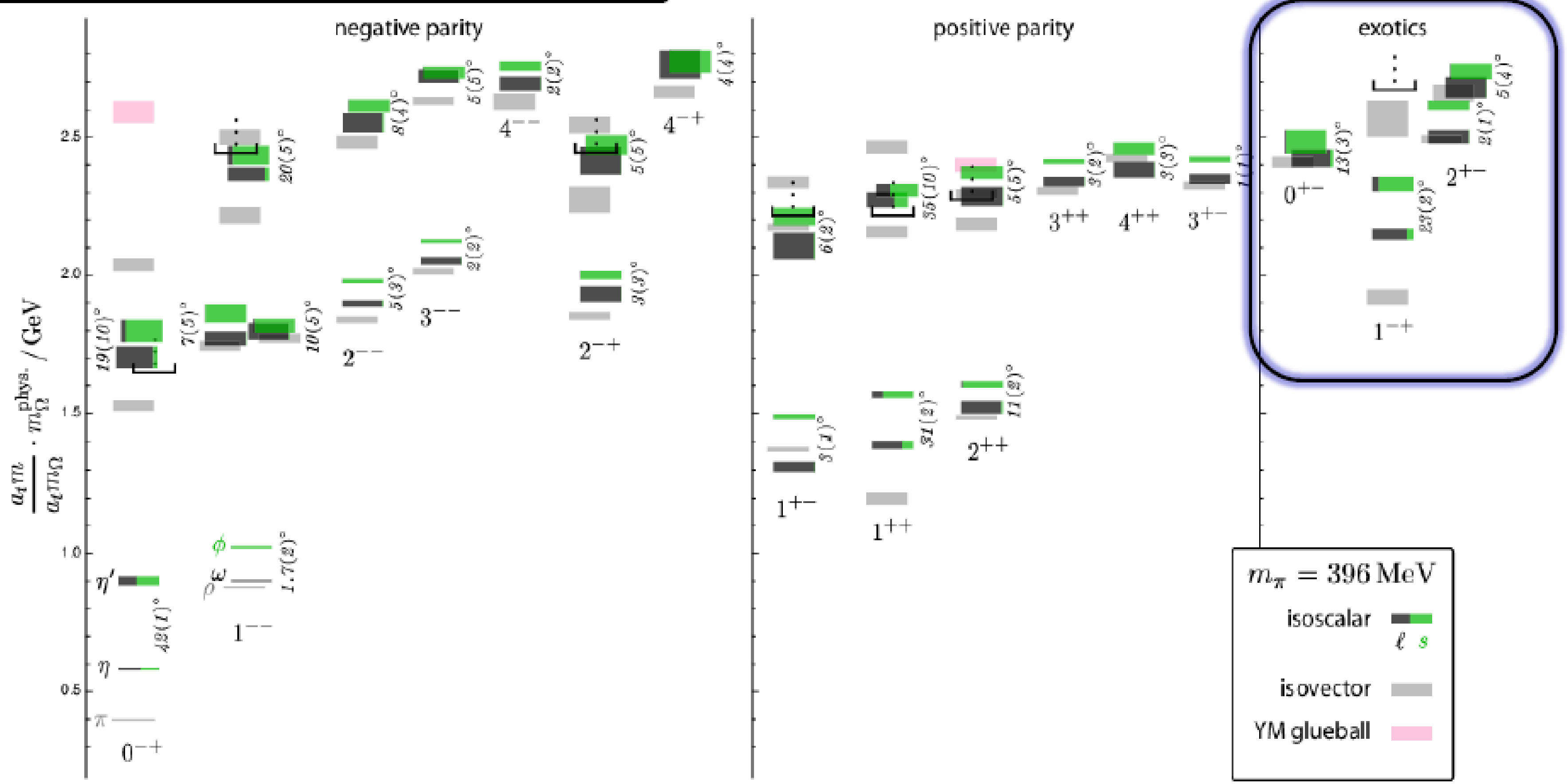


Dudek et al, HSC : Phys.Rev.Lett.103:262001,2009
Phys.Rev.D82:034508,2010

Isoscalar & isovector meson spectrum

Isoscalars: flavor mixing determined

Exotics

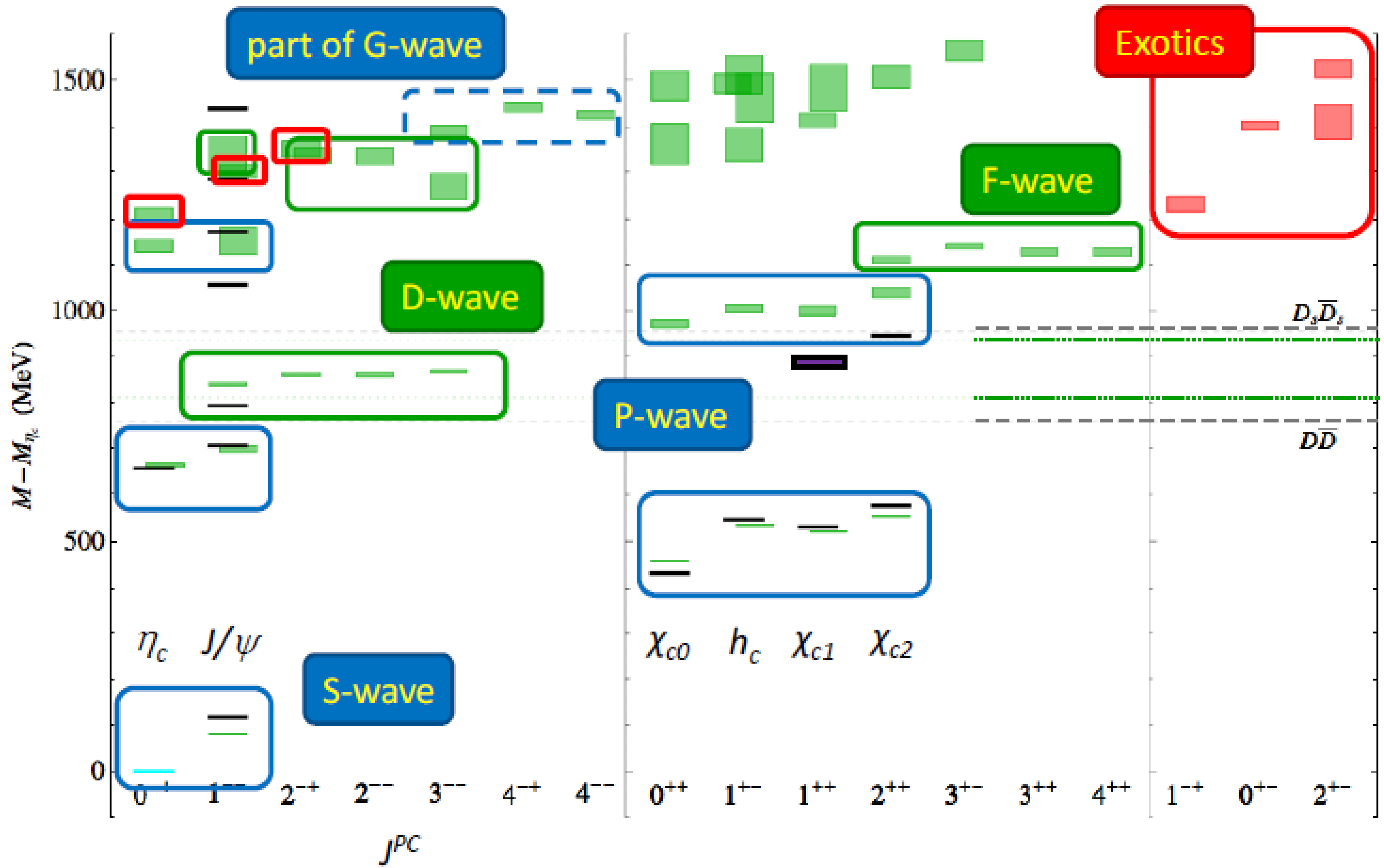


Will need to build PWA within mesons

Dudek et al, HSC: Phys.Rev.D83:111502,2011

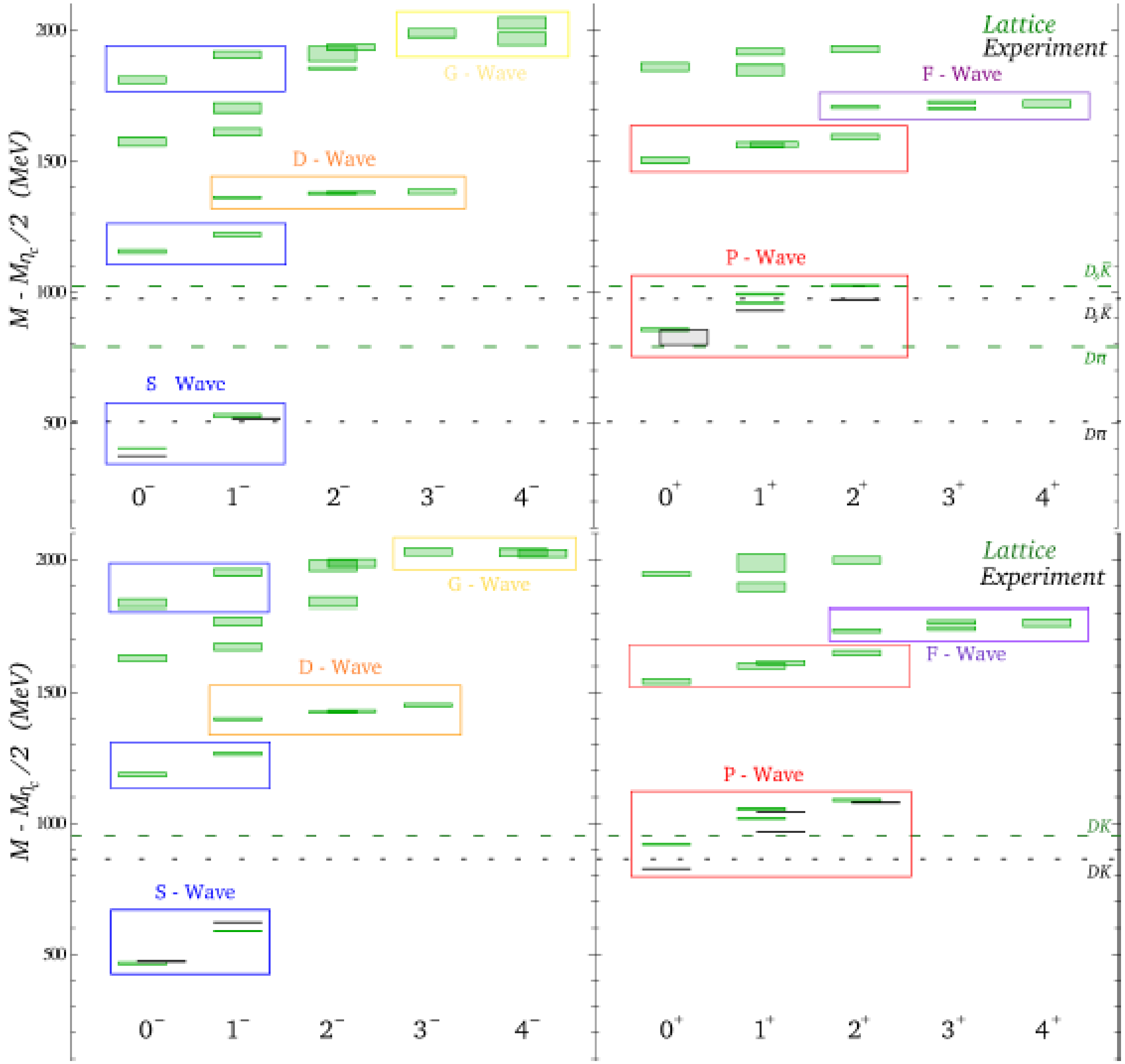
1102.4299

Charmonia spectra



Liu et al, HSC : JHEP 07 (2012) 126

D-D_s spectra



Moir et al, HSC : JHEP 05 (2013) 021

Identifying a Resonance State

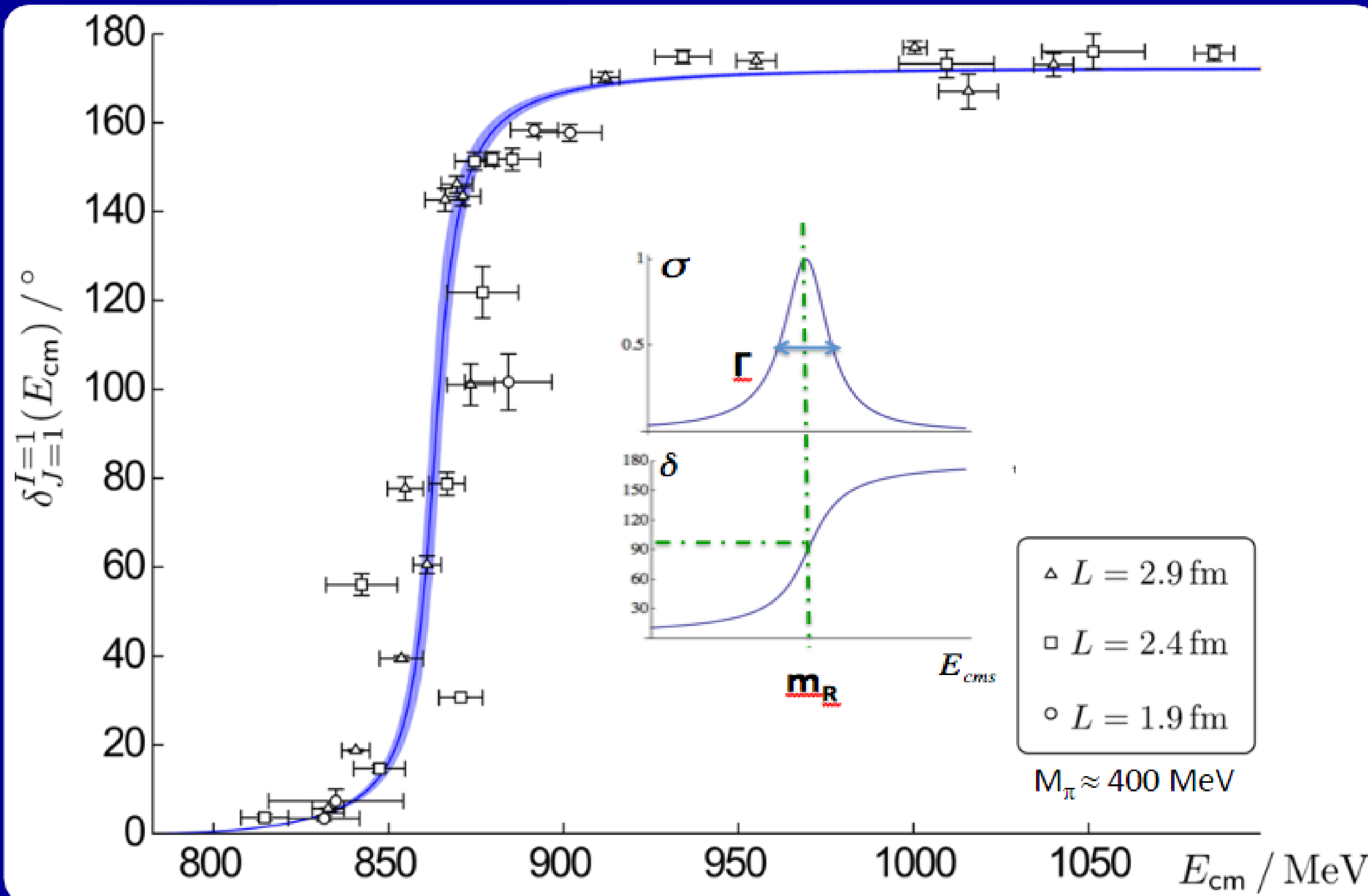
- Method 1 (quailitative) :
  - Study spectrum in a few volumes
  - Compare those with known multi-hadron decay channels
  - Resonance states will have no explicit volume dependence whereas scattering states will have inverse volume dependence.

- Method 2 (quantitative) :
  - Relate finite box energy to infinite volume phase shifts by Luscher formula
  - Calculate energy spectrum for several volumes to evaluate phase shifts for various volumes
  - Extract resonance parameters from phase shifts

The ρ resonance

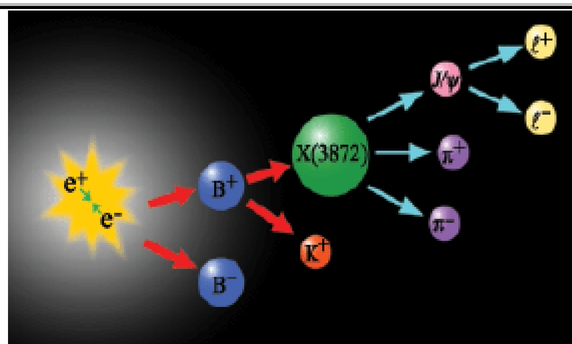
Rho decay

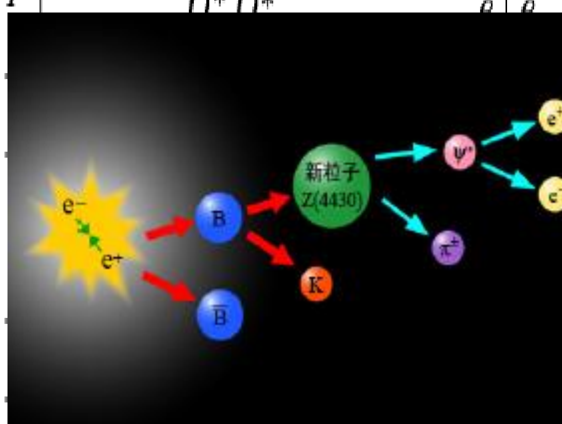
HSC : [PR D87, 034505]



Renaissance in Charmonia physics

state	M (MeV)	Γ (MeV)	J^{PC}	Decay Modes	Production Modes	Observed by:
$Y_s(2175)$	2175 ± 8				e^+e^- (ISR), $J/\psi \to \eta Y_s(2175)$	BaBar, BESII
$X(3872)$	3871.4 ± 0.6				$B \to KX(3872)$, $p\bar{p}$	Belle, CDF, D0, BaBar
$X(3875)$	3875.5 ± 1.5				$B \to KX(3875)$	Belle, BaBar
$Z(3940)$	3929 ± 5				$\gamma\gamma \to Z(3940)$	Belle
$X(3940)$	3942 ± 9	37 ± 17	J^{P+}	DD^*	$e^+e^- \to J/\psi X(3940)$	Belle
$Y(3940)$	3943 ± 17	87 ± 34	J^{P+}	$\omega J/\psi$	$B \to KY(3940)$	Belle, BaBar
$Y(4008)$	4008^{+82}_{-49}	226^{+97}_{-80}	1^{--}	$\pi^+\pi^- J/\psi$	e^+e^-(ISR)	Belle
$X(4160)$	4156 ± 29	139^{+113}_{-65}	J^{P+}	$D^*\bar{D}^*$	$e^+e^- \to J/\psi X(4160)$	Belle
$Y(4260)$	4264 ± 12	83 ± 22	1		e^-(ISR)	BaBar, CLEO, Belle
$Y(4350)$	4361 ± 13	74 ± 18	1		e^-(ISR)	BaBar, Belle
$Z(4430)$	4433 ± 5	45^{+35}_{-18}			$KZ^\pm(4430)$	Belle
$Y(4660)$	4664 ± 12	48 ± 15	1		e^-(ISR)	Belle
Y_b	$\sim 10,870$	?	1		e^- (ISR)	Belle

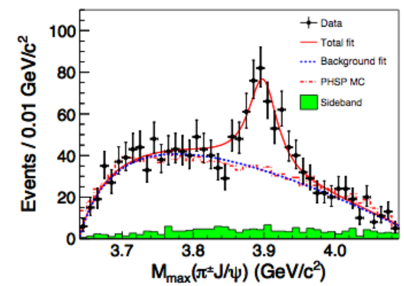




S. Olsen arXiv:0801.1153v1 (hep-ex)

Charged charmonia-like hadrons

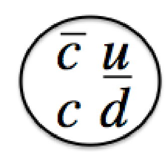
particle	decay	year	coll
$Z^+(4430)$	$\psi(2S)\ \pi^+$	2008	Belle, ~~BABAR~~
$Z^+(4050)$, $Z^+(4250)$	$\chi_{c1}\ \ \pi^+$	2008	Belle, unconfirmed
$Z_c^+(3900)$	$J/\psi\ \ \pi^+$	2013	BESIII, Belle, CLEOc
$Z_c^+(4020)$	$h_c(1P)\ \pi^+$	2013	BESIII preliminary
$Z_c^+(4025)$	$(D^*\ D^*)^+$	2013	BES III preliminary



[BESIII, 2013, arXiv:1303.5949]

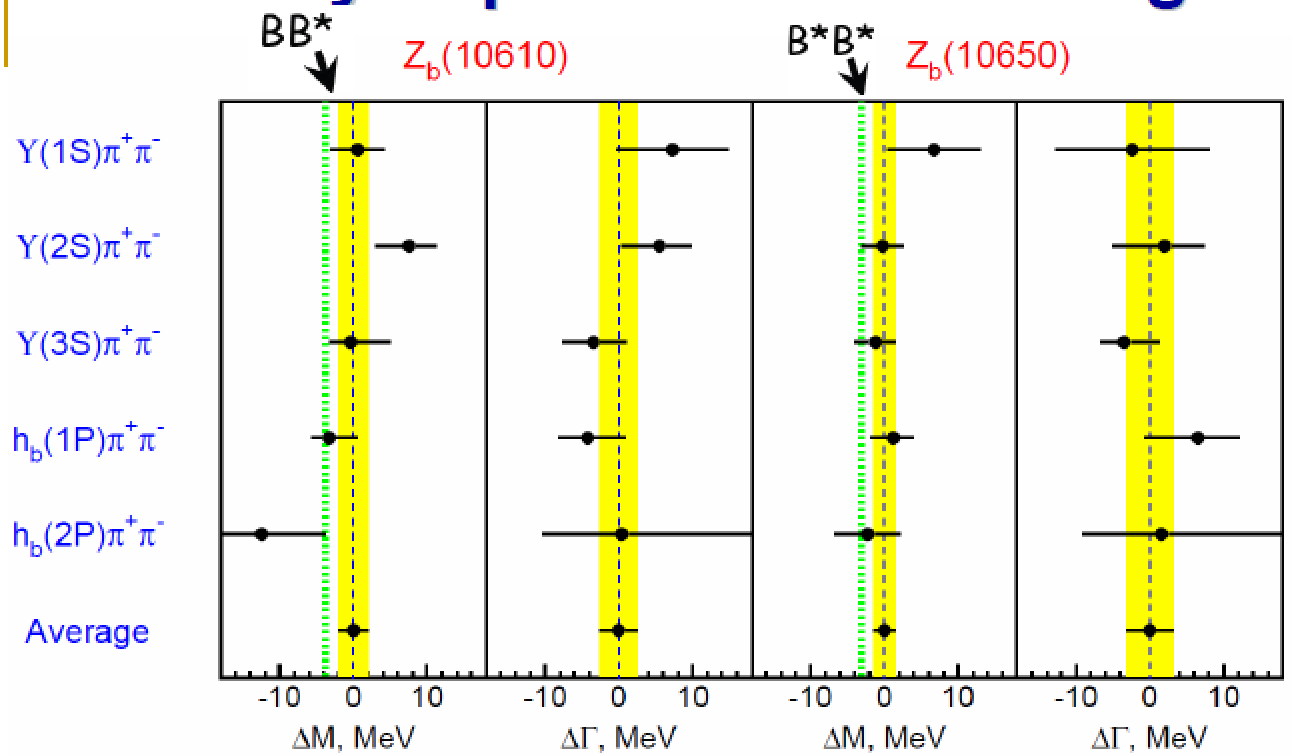
$Z_c^+(3900) \rightarrow\ \ J/\Psi\ \ \pi^+$

$\bar{c}c\ \ \bar{d}u$



Prelovsek@Charm13

Summary of parameters of charged Z_b states



$Z_{b1}(10610)$

M=10608.4±2.0 MeV

Γ=15.6±2.5 MeV

$Z_{b2}(10650)$

M=10653.2±1.5 MeV

Γ=14.4±3.2 MeV

- Relative phases: Υ(≈0°), h_b(≈180°)
- Mass just above B*B and B*B* thresholds
- Angular analysis favors $J^P=1^+$
  Indicates Z_b's could be molecules

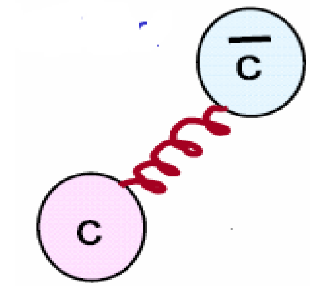
arXiv: 1105.4583

Many theoretical papers:
molecules interpretations:
arXiv:1106.2968 , arXiv:1105.5935
arXiv:1105.5829, arXiv:1107.0254
X. Liu, S.L.Zhu, G. Ding et. al
tetraquark states
arXiv:1108.2197 A. Ali (beauty11)
cusp effect:
arXiv:1105.5492 D. Bugg

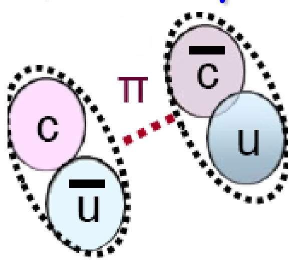
Exotics

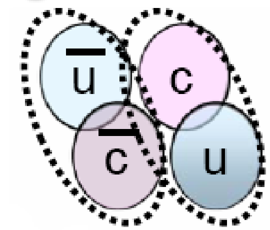
Exotics

- **States with excited gluon**
  - Glueballs (consitutent glue)
  - Hybrid mesons ($q\bar{q}$ meson + excited glue)
  - Hybrid baryons (qqq baryon + excited glue)



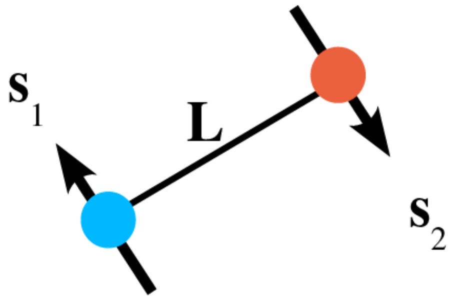
- **Multi-quark states**
  - Tetraquark, Pentaquark and higher number of quark states





- **These states are not well understood**
  - Quark model fails to explain these states

- Lack of understanding makes experimental identification difficult.

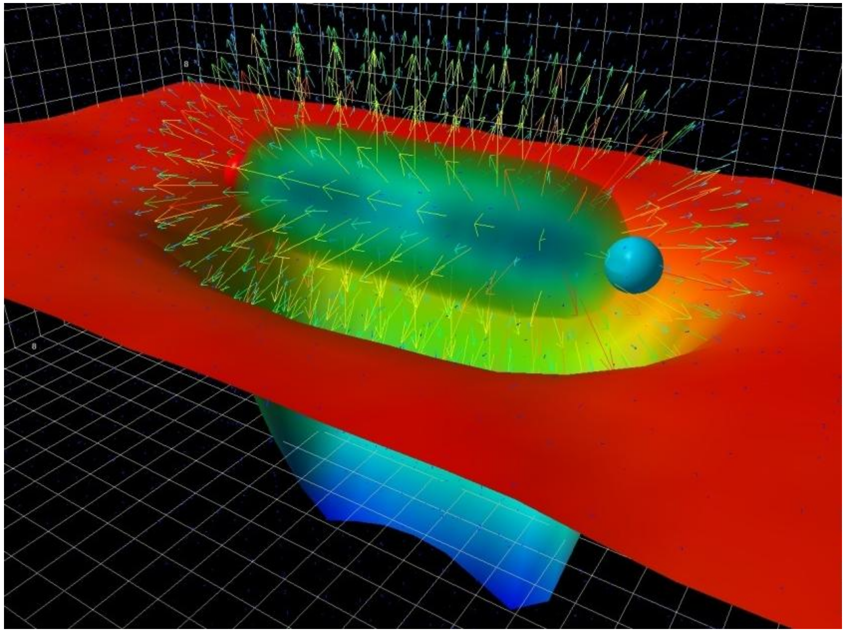


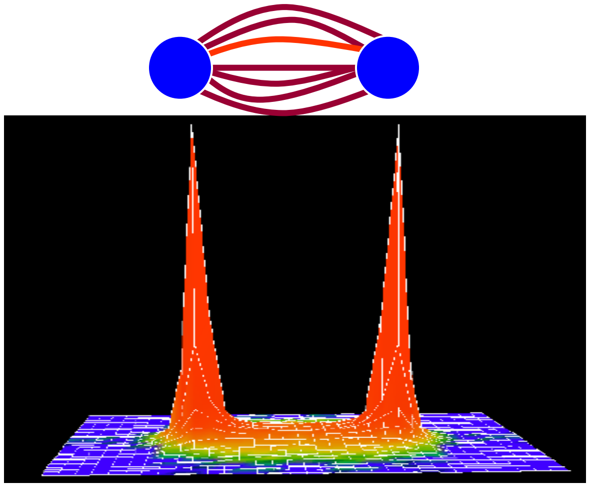
S = 0, 1
L = 0, 1, 2, 3…

$$\vec{J} = \vec{L} + \vec{S}, \quad P = (-1)^{L+1}, \quad C = (-1)^{L+S}$$

Allowed : $J^{PC} = 0^{-+}, 0^{++}, 1^{--}, 1^{+-}, 1^{++}, 2^{--}, 2^{-+}, 2^{++}, ...$

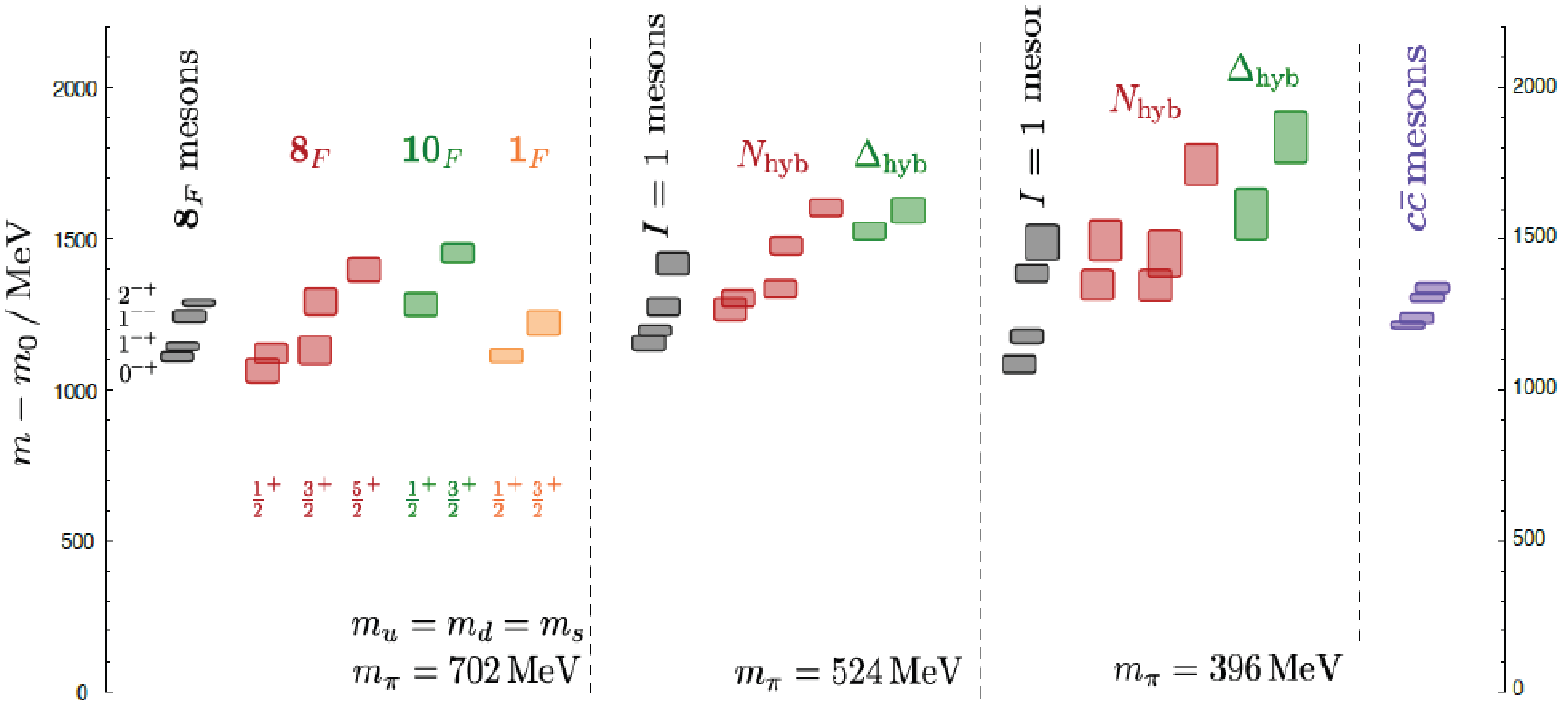
Forbidden (Exotics) : $J^{PC} = 0^{+-}, 0^{--}, 1^{-+}, 2^{+-}, 3^{-+}, 4^{+-}, ...$





Hybrid hadrons

"subtract off" the quark mass



$$m_0 = \begin{cases} m_\rho & \text{light mesons} \\ m_N & \text{baryons} \\ m_{\eta_c} & \text{charmonium} \end{cases}$$

Appears to be a single scale for gluonic excitations **~ 1.3 GeV**

Gluonic excitation transforming like a color **octet** with **$J^{PC} = 1^{+-}$**

arXiv:1201.2349

Prelovsek : arXiv:1307.5172v3 [hep-lat]

$X(3872)$	$m_X - \frac{1}{4}(m_{\eta_c} + 3m_{J/\psi})$	$m_X - (m_{D^0} + m_{D^{0*}})$
$\mathrm{lat}^{L\to\infty}$	815 ± 7 MeV	-11 ± 7 MeV
exp	804 ± 1 MeV	-0.14 ± 0.22 MeV

A state below 11+-7 MeV below DD* threshold

$$J^{PC} = 1^{++} \text{ and } I = 0$$

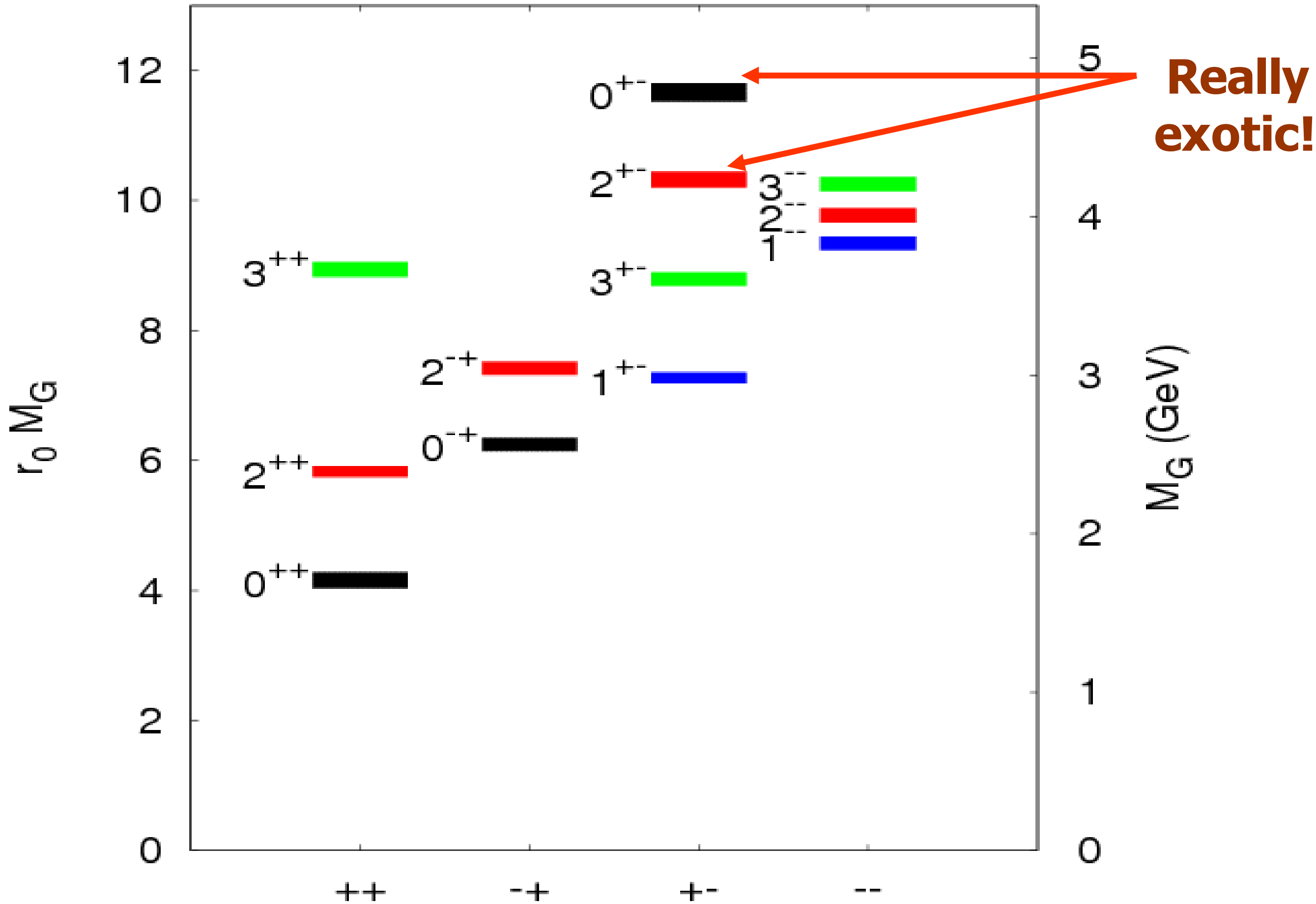
No signal for I = 1 channel

Glueball

SU(3) Spectra

A glueball is a purely gluonic bound state.

In the theory of QCD glueball self coupling Admits the existence of such a state.



Chen…Liu, Morningstar, Mathur, Peardon.. et al. Phys. Rev. D73, 014516 (2006)

Hadron Spectroscopy

Experiments

LHCb

ATLAS CMS

CLAS12 GlueX

+ others at 12 GeV JLab

BESIII KLOE2

COMPASS

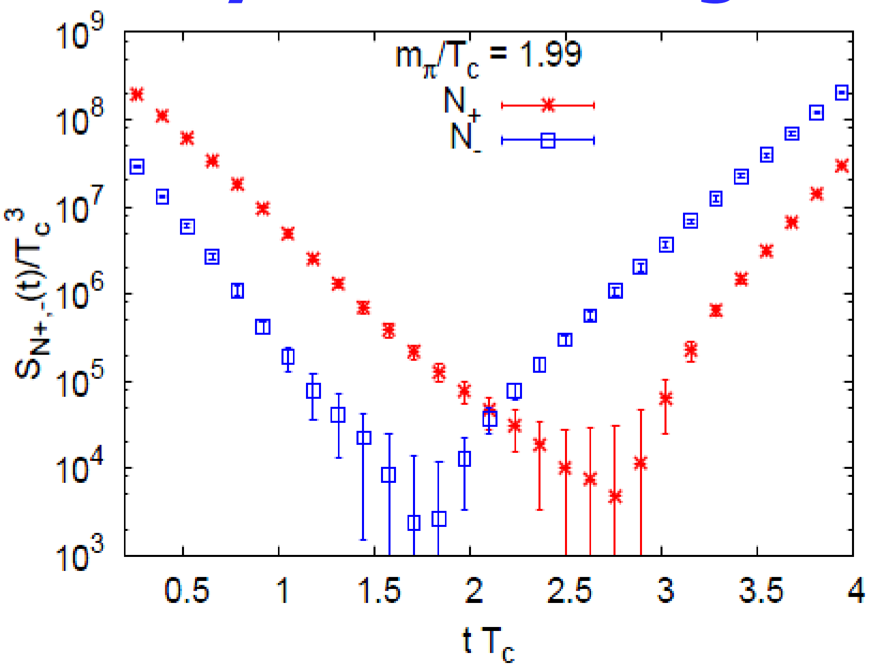
PANDA

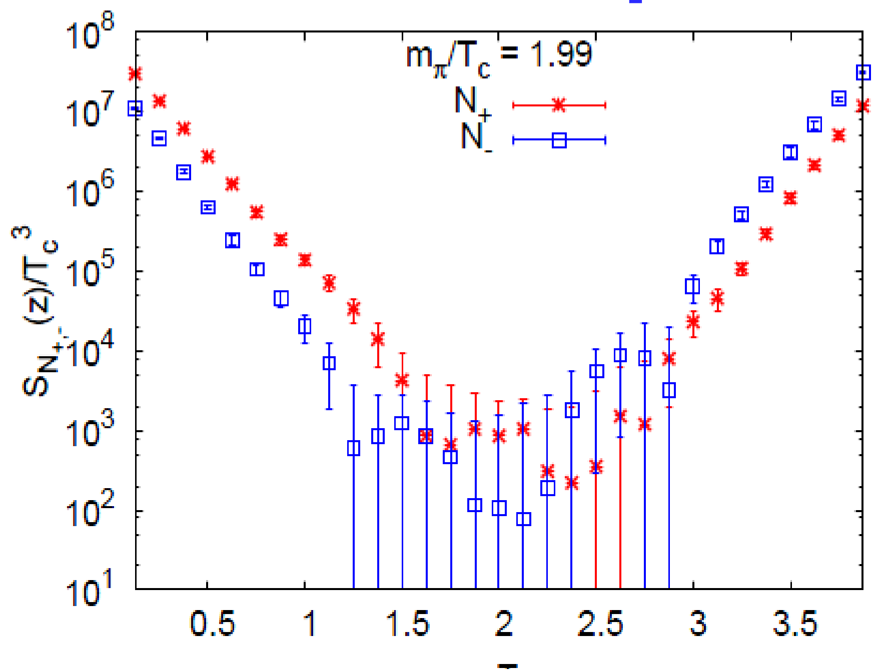
+ others at GSI

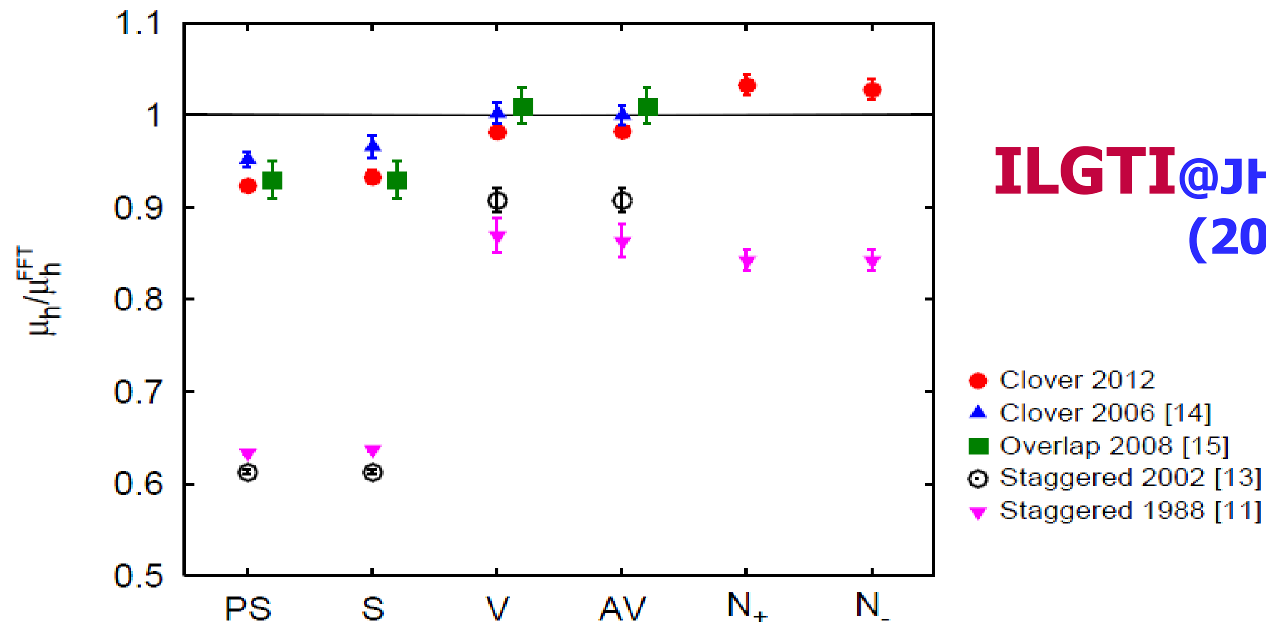
Belle II

ELSA MAMI J-PARC Spring-8

Baryon screening correlators at finite temp







ILGTI@JHEP 1302 (2013) 145

Nuclear physics from lattice

- **Use Luscher's formula on finite volume lattice to get phase shifts, scattering lengths** -- NPLQCD, Prog.Part.Nucl.Phys 66(2010)1

- **NBS wavefunction → NN potential** ....Ishii-Aoki-Hatsuda PRL99(2007)022001, PTP123(2010)89, arXiv:1206.5088

- **Binding energy for light nuclei on the lattice :** Yamazaki-Kuramashi-Ukawa (PACS-CS Coll.) PRD81(2010)111504, PRD84(2011)054506

- **Lattice effective field theory :** Rev. Mod. Phys. 81, 1773 (2009), Eur.Phys.J. A45 (2010) , Phys.Rev.Lett. 106 (2011) 192501

- **Strong coupling limit :** de Forcrand and Fromm,PRL104(2010)112005

NN Potential

Define potential from equal time Bethe-Salpeter amplitude of the two local interpolating operators separated by a distance r.

$$-\frac{1}{2\mu}\nabla^2\phi(\vec{r}) + \int d^3r' U(\vec{r},\vec{r}')\phi(\vec{r}') = E\phi(\vec{r})$$

$$\phi(\vec{r}) \equiv \frac{1}{24}\sum_{\mathcal{R}\in O}\frac{1}{L^3}\sum_{\vec{x}} P^{\tau}_{ij}P^{\sigma}_{\alpha\beta}\langle 0|N^i_{\alpha}(\mathcal{R}[\vec{r}]+\vec{x})N^j_{\beta}(\vec{x})|NN\rangle$$

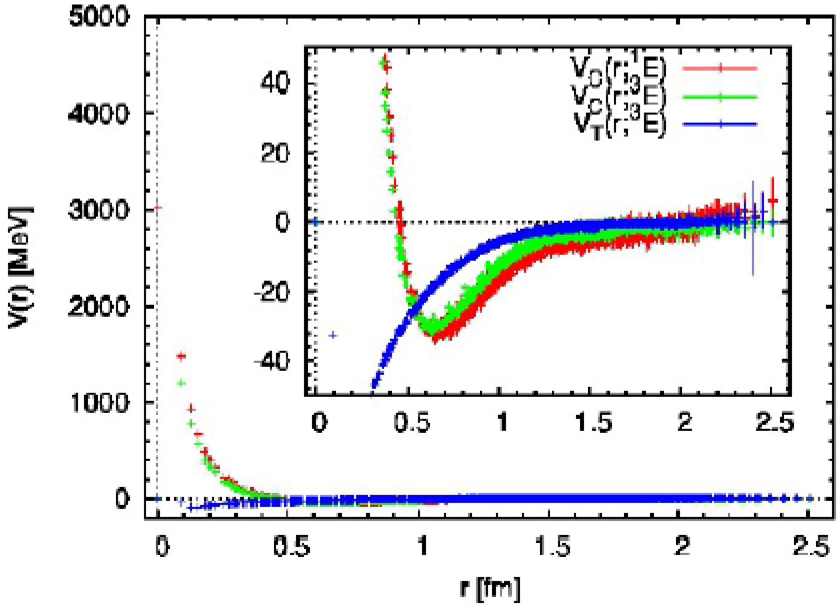
$$V_C(r) = E + \frac{1}{2\mu}\frac{\vec{\nabla}^2\phi(r)}{\phi(r)}$$

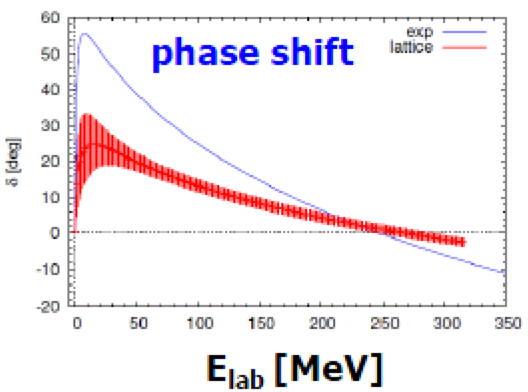
HALQCD : Phys. Rev. Lett. 99 022001 (2007)

NN potential on the lattice (positive parity)

$^{2S+1}L_J$

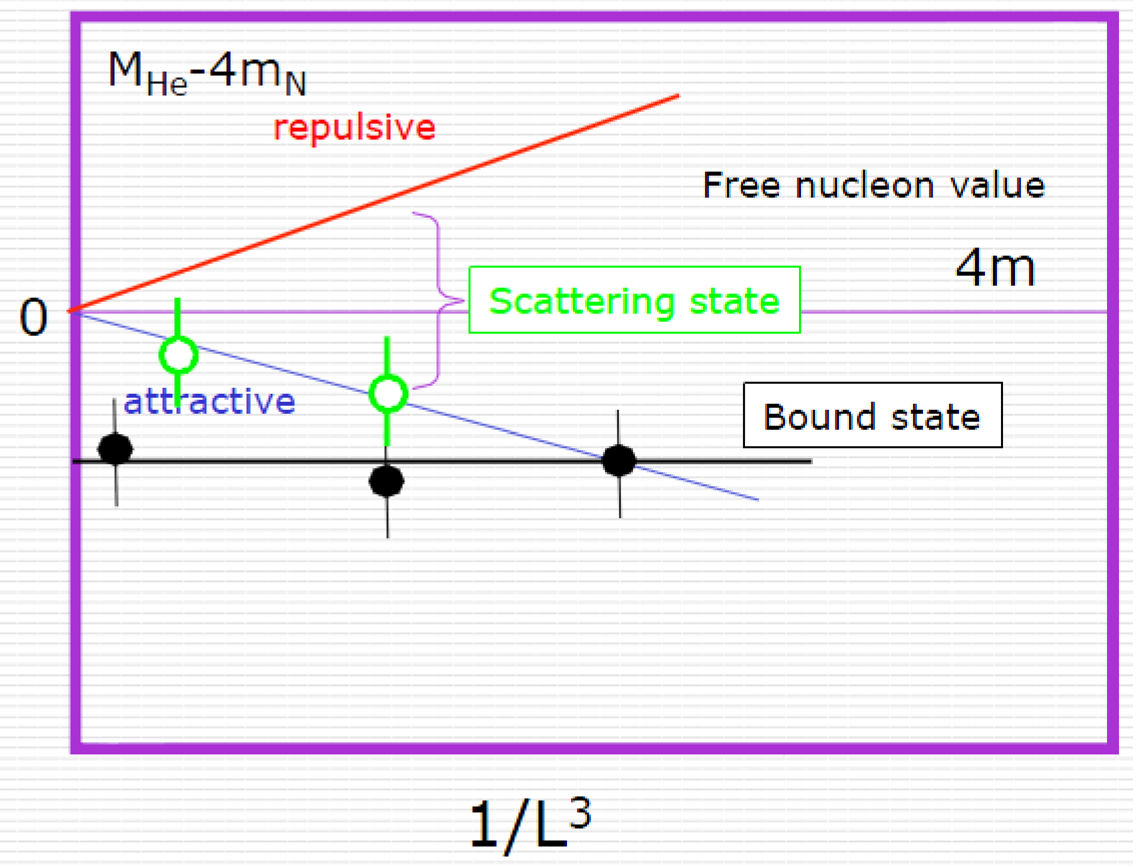
- "di-neutron" channel $^1S_0$ → central force
- "deuteron" channel $^3S_1 - ^3D_1$ → central & tensor force





Not Bound $a(^1S_0) = 1.6(1.1)$ fm

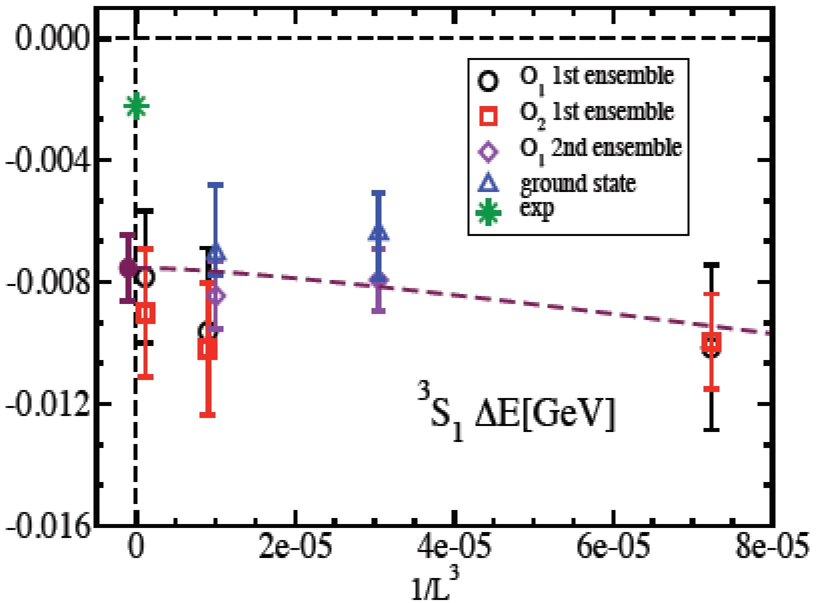
Nf=2+1 clover (PACS-CS), 1/a=2.2GeV,
L=2.9fm, m_π=0.7GeV, m_N=1.6GeV

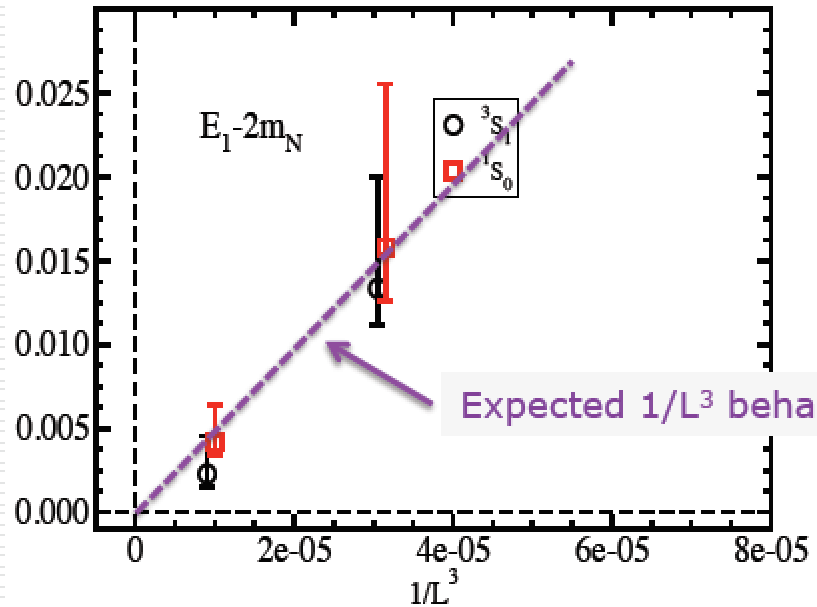


A Ukawa @Lattice13

Deuteron is a bound state (quenched QCD, m_π=0.8GeV)

1st excited state is a scattering state just above the threshold





$a_0 < 0$ evaluated from the 1st excited state energy consistent with bound state formation

a_0[fm](^{3}S$_1$)	a_0[fm](^{1}S$_0$)
$-1.05(24)(^{+0.05}_{-0.65})$	$-1.62(24)(^{+0.01}_{-0.75})$

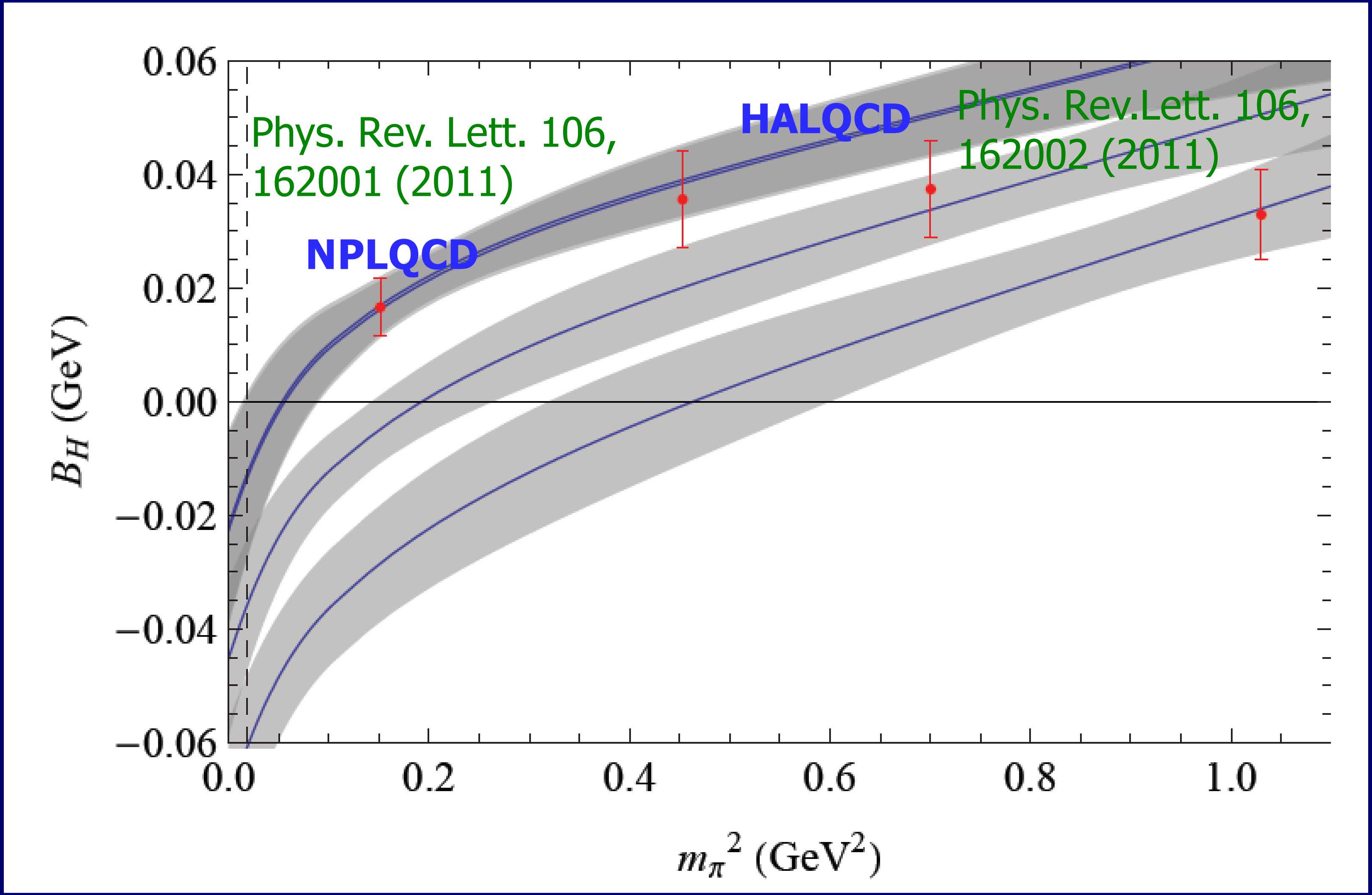
$$-\Delta E_\infty = \begin{cases} 43(12)(8) & \text{MeV for } ^4\text{He}, \\ 20.3(4.0)(2.0) & \text{MeV for } ^3\text{He}, \\ 11.5(1.1)(0.6) & \text{MeV for } ^3\text{S}_1, \\ 7.4(1.3)(0.6) & \text{MeV for } ^1\text{S}_0. \end{cases}$$

A Ukawa @Lattice13

Observables	PACS-CS $m_\pi = 0.8$ GeV L = 3,6,12 fm ($N_f = 0$)	PACS-CS $m_\pi = 0.5$ GeV L = 3−6fm ($N_f = 2+1$)	NPLQCD $m_\pi = 0.39$ GeV L = 2−4fm ($N_f = 2+1$)
Di-neutron($^1S_0$)	5.5(1.1)(1.0)	7.4(1.3)(0.6)	7.1(5.2)(7.3)
Deuteron ($^3S_1$-$^3D_1$)	9.1(1.1)(0.5)	11.5(1.1)(0.6)	11(5)(12)

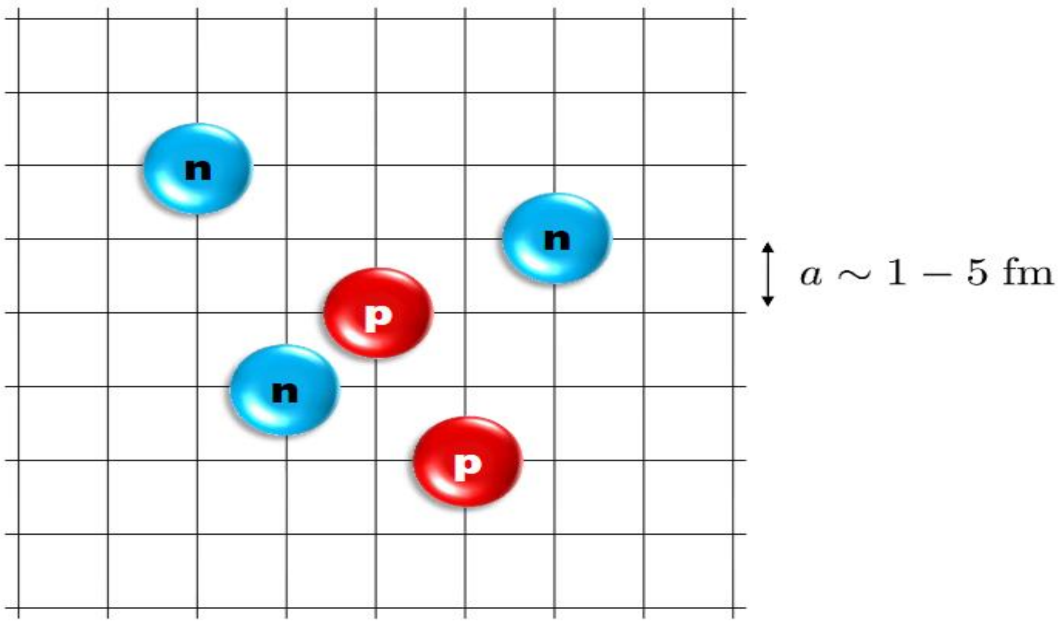
Observables	PACS-CS $m_\pi = 0.8$ GeV L = 3,6,12 fm ($N_f = 0$)	PACS-CS $m_\pi = 0.5$ GeV L = 3−6fm ($N_f = 2+1$)	NPLQCD $m_\pi = 0.81$ GeV L = 3.4,4.5,6.7fm ($N_f = 3$)
Di-neutron(^{3}He)	18.2(3.5)(2.9)	20.3(4.0)(2.0)	71(6)(5)
Deuteron (^{4}He)	27.7(7.8)(5.5)	43(12)(8)	110(20)(15)

H dibaryon (uuddss, I=0, $^1S_0$)



Shanahan et al, Phys. Rev. Lett. 107, 092004 (2011)

Lattice Effective Field Theory



Rev. Mod. Phys. 81, 1773 (2009)

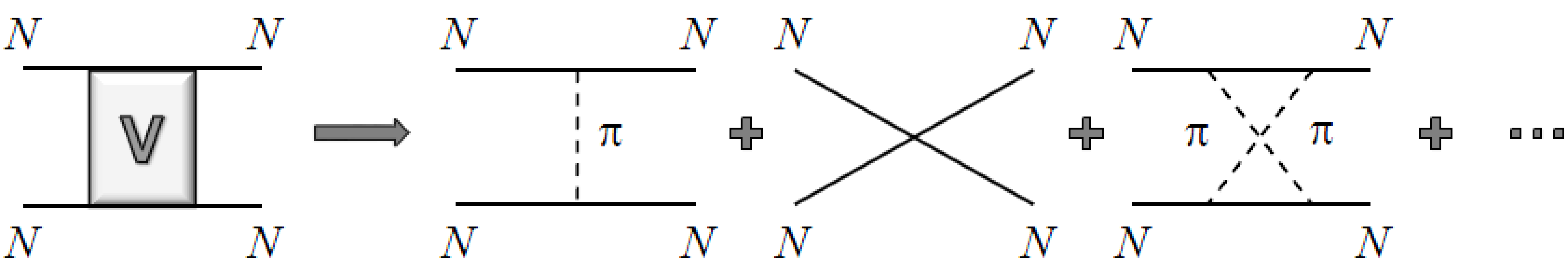
Eur.Phys.J. A45 (2010)

Phys.Rev.Lett. 106 (2011) 192501

Phys.Rev.Lett. 104 (2010) 142501

Low energy nucleons: Chiral effective field theory

Construct the effective potential order by order



V^{OPEP} Contact interactions V^{TPEP}

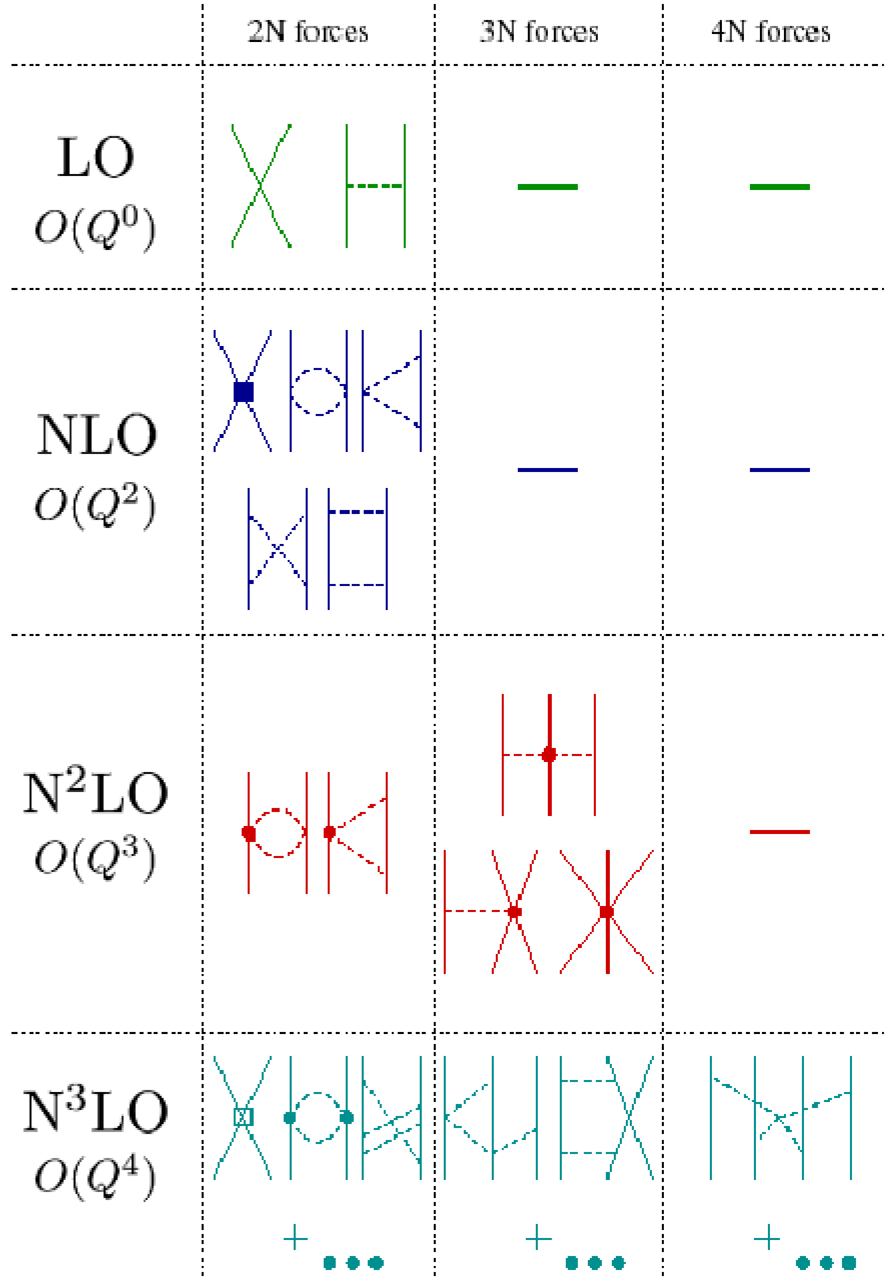
Leading order (LO) Next-to-leading order (NLO)

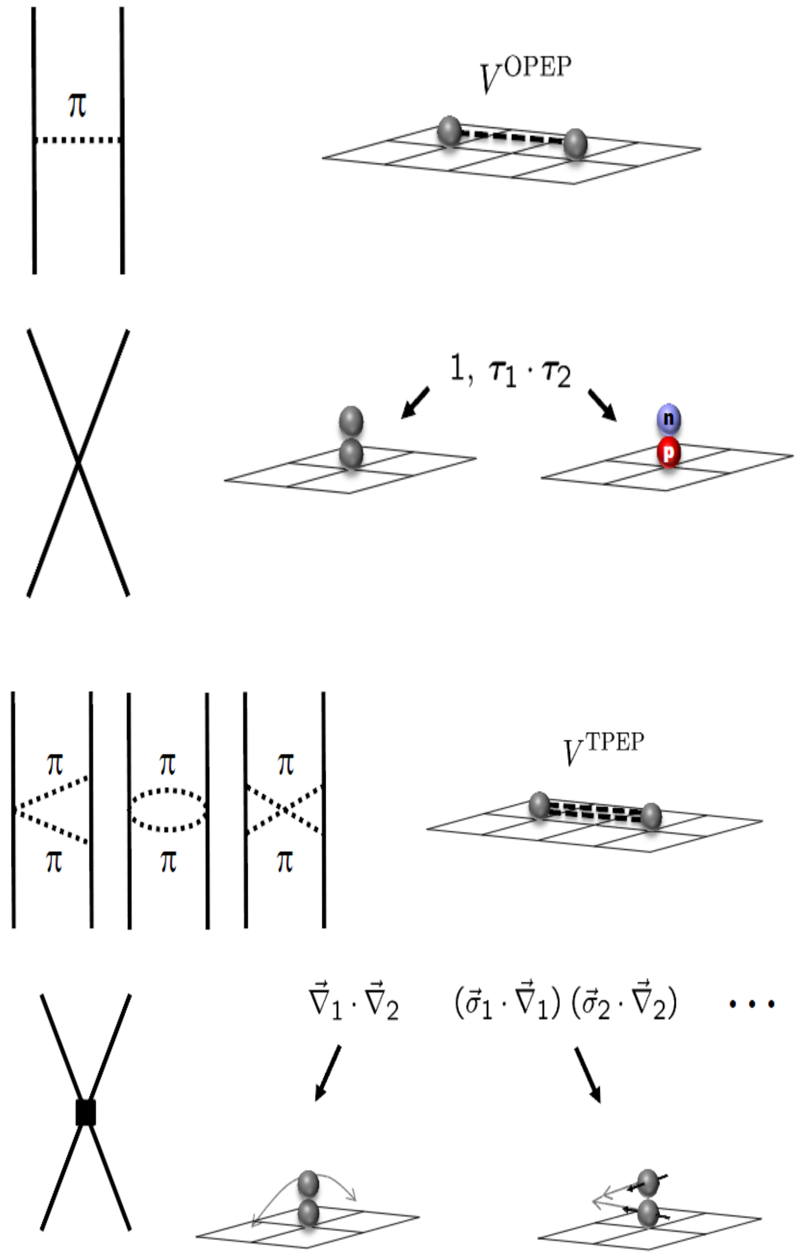
Nuclear Scattering Data

↓

Effective Field Theory

Ordonez et al. '94; Friar & Coon '94; Kaiser et al. '97; Epelbaum et al. '98, '03; Kaiser '99-'01; Higa et al. '03; ...




π
V^{OPEP}
$1,\ \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$
n
p
π
π
π
π
π
π
V^{TPEP}
$\vec{\nabla}_1 \cdot \vec{\nabla}_2$
$(\vec{\sigma}_1 \cdot \vec{\nabla}_1)\,(\vec{\sigma}_2 \cdot \vec{\nabla}_2)$
$\cdots$

LO* ($O(Q^0)$)	−24.8(2) MeV
NLO ($O(Q^2)$)	−23.8(2) MeV
NNLO ($O(Q^3)$)	−28.4(3) MeV
Experiment	−28.3 MeV

Helium-4

LO* ($O(Q^0)$)	−60.9(7) MeV
NLO ($O(Q^2)$)	−55(2) MeV
NNLO ($O(Q^3)$)	−58(2) MeV
Experiment	−56.5 MeV

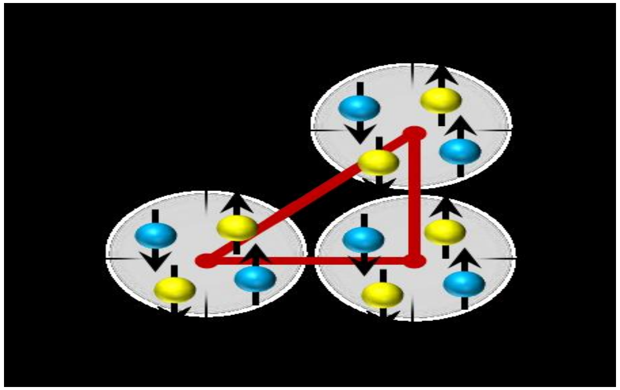
Beryllium-8

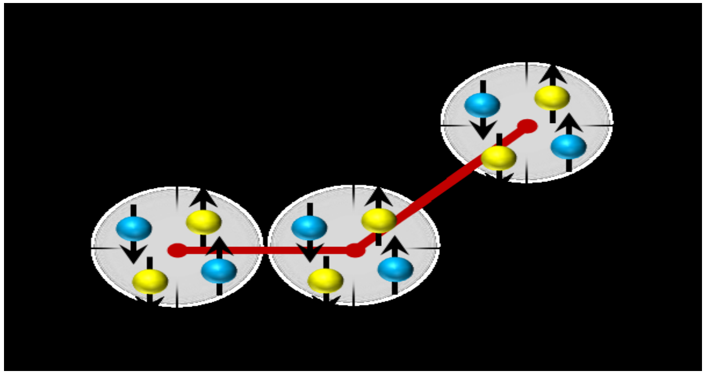
LO* ($O(Q^0)$)	−96(2) MeV
NLO ($O(Q^2)$)	−77(3) MeV
NNLO ($O(Q^3)$)	−92(3) MeV
Experiment	−92.2 MeV

Carbon-12

	2_1^+	0_2^+	2_2^+
LO* ($O(Q^0)$)	−94(2) MeV	−89(2) MeV	−88(2) MeV
NLO ($O(Q^2)$)	−74(3) MeV	−72(3) MeV	−70(3) MeV
NNLO ($O(Q^3)$)	−89(3) MeV	−85(3) MeV	−83(3) MeV
Experiment	−87.72 MeV	−84.51 MeV	−82.6(1) MeV (A,B) −81.1(3) MeV (C) −82.13(11) MeV (D)

Excited state : carbon-12 (even parity)



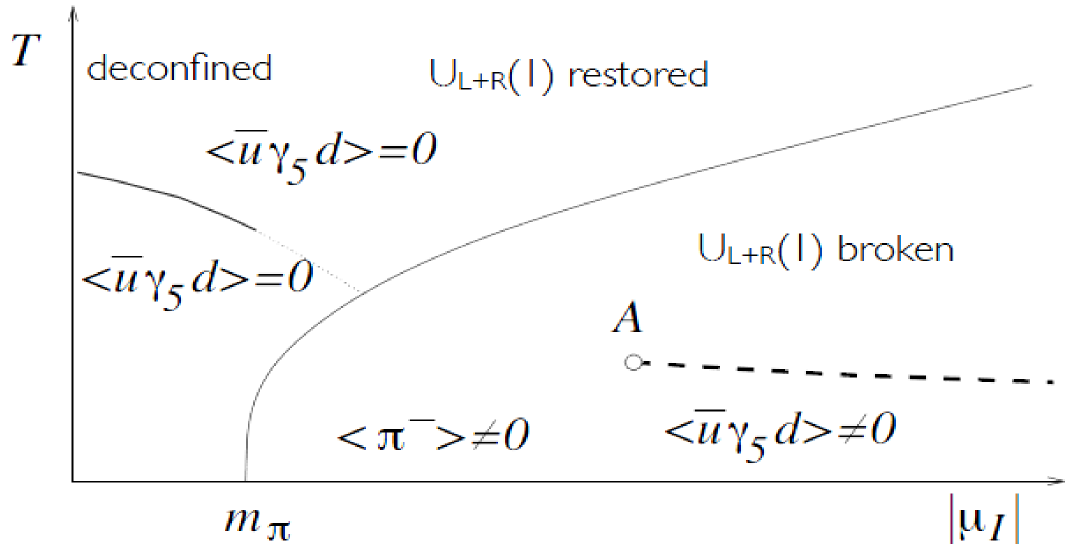


Epelbaum, Krebs, Lähde, D.L, Meißner, PRL 109 252501 (2012)

QCD with isospin chemical potential

Isospin chemical potential sets $\mu = \mu_u = - \mu_d$

Conjectured phase diagram [Son & Stephanov]



NB: equivalent to phase quenched QCD

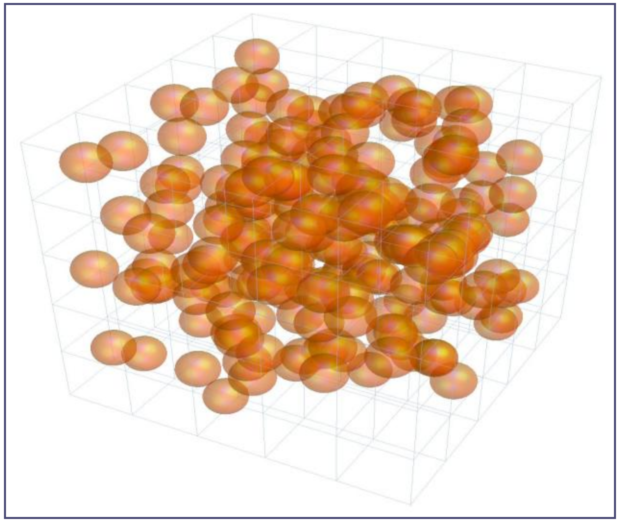
Kogut & Sinclair [PRD 66 (2002) 014508; PRD 66 (2002) 034505; PRD 70 (2004) 094501; PRD 77 (2008) 114503]

Cea, Cosmai, d'Elia, Papa & Sanfillipo Phys. Rev. D85 094512, 2012; PoS LATT12

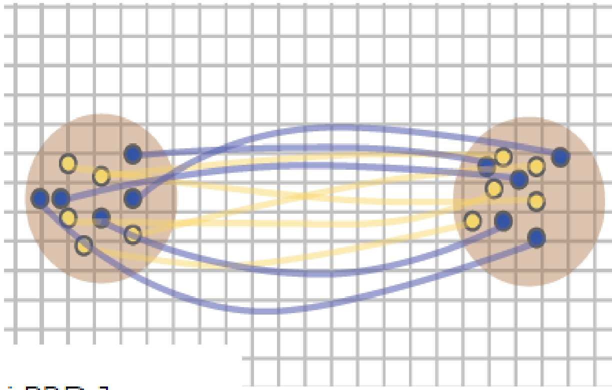
C Nonaka and M Kondo, Lattice2013

QCD with explicit isospin charge

Detmold *etal*



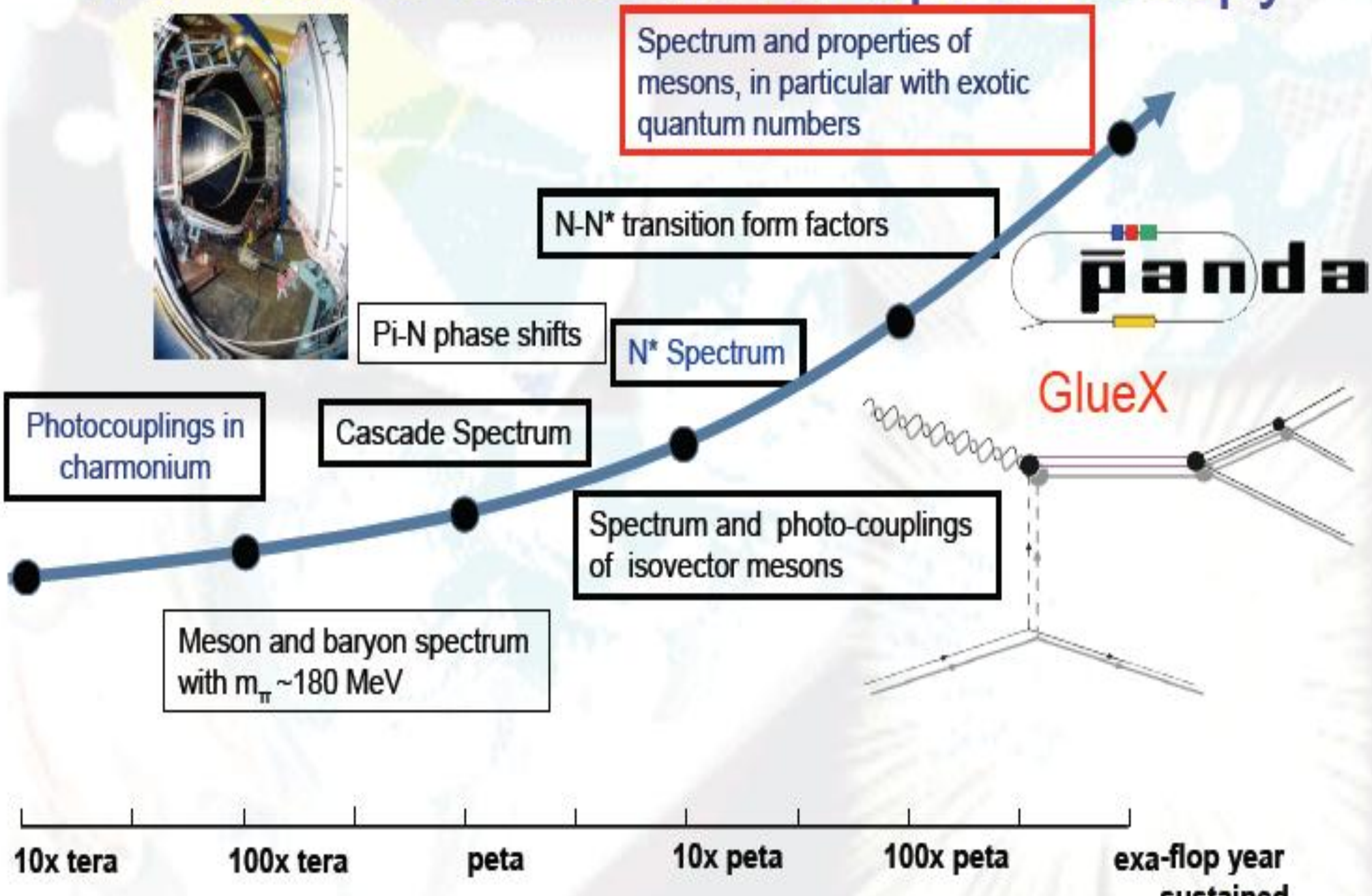
Another way of probing isospin density is by explicitly adding isospin density to the system



Construct correlation functions with "many pions"

- Quarkonium in medium [Detmold, Meinel & Shi PRD]
- Baryon masses in medium [Nicholson & Detmold, Latt13]
- Pion structure in medium [Detmold & H W Lin PoS Latt10]

The road to exascale for Spectroscopy



….D. Richards, Extreme scale computing