Exploring the QCD critical region in the QCD-like two flavor models

Vivek Kumar Tiwari

University of Allahabad

ICMEC@Bose Institute Kolkata17 Jan 2014

Outline

- The QCD vacuum reveals itself through the spontaneously broken chiral symmetry and the color confinement phenomenon.
- QCD phase structure can be studied in effective model approach where both the chiral order parameter and the Polyakov loop which is a good indicator of the confinement-deconfinement transition, are coupled to the quarks.

[1] Schaefer et. al. PRD. 79, 014018 (2009)
[2] Schaefer et. al. PRD. 81, 074013 (2010)

- The QCD vacuum reveals itself through the spontaneously broken chiral symmetry and the color confinement phenomenon.
- QCD phase structure can be studied in effective model approach where both the chiral order parameter and the Polyakov loop which is a good indicator of the confinement-deconfinement transition, are coupled to the quarks.
- Polyakov loop extended Quark Meson (PQM) model is such an effective model[1] in which the bulk QCD thermodynamics has been computed and it shows agreement with the lattice QCD data[2] .
- We consider the modification of effective potential for the two quark flavor QM and PQM model after including the renormalized contribution of the ultraviolet divergent one loop fermionic vacuum fluctuation.

[1] Schaefer et. al. PRD. 79, 014018 (2009)
[2] Schaefer et. al. PRD. 81, 074013 (2010)

- We compute the phase diagrams for the modified QM and PQM models-renamed as QMVT and PQMVT in the presence of fermionic vacuum term-and locate the critical end point (CEP) in the μ and T plane.
- We will be plotting and comparing the contours of appropriately normalized constant quark number susceptibility and scalar susceptibility around the CEP in different model scenarios.

- We compute the phase diagrams for the modified QM and PQM models-renamed as QMVT and PQMVT in the presence of fermionic vacuum term-and locate the critical end point (CEP) in the μ and T plane.
- We will be plotting and comparing the contours of appropriately normalized constant quark number susceptibility and scalar susceptibility around the CEP in different model scenarios.
- Since the PQMVT and QMVT models have become QCD-like as they yield the second order transition at $\mu = 0$ on the temperature axis (as expected from the universality arguments) , we locate the tricritical point (TCP) also for the chiral limit of m_π=0.
- Finally we obtain the critical exponents from the power law scaling in the divergence of quark number susceptibility at the CEP and discuss the possible influence of TCP on the critical behavior around CEP.

Outline

- The model Lagrangian is $\mathcal{L}_{PQM} = \mathcal{L}_{QM} - \mathcal{U}\big(\Phi, \Phi^*, T\big)$

$$\mathcal{L}_{QM} = \bar{q}_f \big(i\gamma^\mu D_\mu - g\ \left(\sigma + i\gamma_5 \vec{\tau}.\vec{\pi}\right)\big) q_f + \mathcal{L}_m$$

Here $D_\mu = \partial_\mu - iA_\mu$ with $A_\mu = \delta_{\mu 0} A_0$ and $A_\mu = g_s A^a_\mu \lambda^a / 2$.

- The model Lagrangian is $\mathcal{L}_{PQM} = \mathcal{L}_{QM} - \mathcal{U}\big(\Phi, \Phi^*, T\big)$

$$\mathcal{L}_{QM} = \bar{q}_f\big(i\gamma^\mu D_\mu - g\ (\sigma + i\gamma_5\vec{\tau}.\vec{\pi})\big)q_f + \mathcal{L}_m$$

Here $D_\mu = \partial_\mu - iA_\mu$ with $A_\mu = \delta_{\mu 0}A_0$ and $A_\mu = g_s A_\mu^a \lambda^a/2$.

- The meson part

$$\mathcal{L}_m = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 - U(\sigma, \vec{\pi})$$

- The model Lagrangian is $\mathcal{L}_{PQM} = \mathcal{L}_{QM} - \mathcal{U}\big(\Phi, \Phi^*, T\big)$

$$\mathcal{L}_{QM} = \bar{q}_f \big(i\gamma^\mu D_\mu - g\ \big(\sigma + i\gamma_5 \vec{\tau}.\vec{\pi}\big)\big) q_f + \mathcal{L}_m$$

Here $D_\mu = \partial_\mu - iA_\mu$ with $A_\mu = \delta_{\mu 0} A_0$ and $A_\mu = g_s A_\mu^a \lambda^a / 2$.

- The meson part

$$\mathcal{L}_m = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 - U(\sigma, \vec{\pi})$$

- The pure mesonic potential is written as

$$U(\sigma, \vec{\pi}) = \frac{\lambda}{4}\left(\sigma^2 + \vec{\pi}^2 - v^2\right)^2 - h\sigma,$$

$h = f_\pi m_\pi^2$ for explicit chiral breaking, λ is quartic coupling of mesons and v is the vacuum expectation value of the σ.

Outline

- Thermal expectation value of color trace of Wilson loop in temporal direction defines the Polyakov Loop field as

$$\Phi = \frac{1}{N_c}\langle \mathrm{Tr}_c L(\vec{x})\rangle, \qquad \Phi^* = \frac{1}{N_c}\langle \mathrm{Tr}_c L^\dagger(\vec{x})\rangle$$

$$L(\vec{x}) = \mathcal{P}\exp\left[i\int_0^\beta d\tau A_0(\vec{x},\tau)\right]$$

Here $\mathcal{P}$ is the path ordering, A_0 is temporal Vector field, $\beta = \frac{1}{T}$

- Thermal expectation value of color trace of Wilson loop in temporal direction defines the Polyakov Loop field as

$$\Phi = \frac{1}{N_c}\langle \mathrm{Tr}_c L(\vec{x})\rangle, \qquad \Phi^* = \frac{1}{N_c}\langle \mathrm{Tr}_c L^\dagger(\vec{x})\rangle$$

$$L(\vec{x}) = \mathcal{P}\exp\left[i\int_0^\beta d\tau A_0(\vec{x},\tau)\right]$$

Here $\mathcal{P}$ is the path ordering, A_0 is temporal Vector field, $\beta = \frac{1}{T}$

- We have used the logarithmic form of Polyakov loop potential

[S. Rößner et al PRD 75,034007(2007)]

$$\frac{\mathcal{U}_{\log}(\Phi,\Phi^*,T)}{T^4} = -\frac{a(T)}{2}\Phi^*\Phi + b(T)\ln[1-6\Phi^*\Phi+4(\Phi^{*3}+\Phi^3)-3(\Phi^*\Phi)^2]$$

the temperature dependent coefficients are

$$a(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 \quad b(T) = b_3\left(\frac{T_0}{T}\right)^3$$

$$a_0 = 3.51\ , a_1 = -2.47 a_2 = 15.2, b_3 = -1.75, T_0 = 208 MeV$$

- Mean-field grand potential in presence of Polyakov loop

$$\Omega(T,\mu) = -\frac{T\ln Z}{V} = U(\sigma) + \Omega_{\bar{q}q}(T,\mu) + \mathcal{U}(\Phi,\Phi^*,T)$$

- Quark/antiquark contribution in presence of Polyakov loop

$$\Omega_{\bar{q}q}(T,\mu) = \Omega_{q\bar{q}}^{\mathrm{vac}} + \Omega_{q\bar{q}}^{\mathrm{T}}$$

- Mean-field grand potential in presence of Polyakov loop

$$\Omega(T,\mu) = -\frac{T\ln Z}{V} = U(\sigma) + \Omega_{\bar{q}q}(T,\mu) + \mathcal{U}(\Phi,\Phi^*,T)$$

- Quark/antiquark contribution in presence of Polyakov loop

$$\Omega_{\bar{q}q}(T,\mu) = \Omega_{q\bar{q}}^{\mathrm{vac}} + \Omega_{q\bar{q}}^{\mathrm{T}}$$

$$= -2N_f \int \frac{d^3p}{(2\pi)^3}\left[N_c E_q \theta(\Lambda^2 - \vec{p}^{\,2}) + T\{\ln g_q^+ + \ln g_q^-\}\right]$$

where $g_q^+ = \left[1 + 3\Phi e^{-E_q^+/T} + 3\Phi^* e^{-2E_q^+/T} + e^{-3E_q^+/T}\right]$

$g_q^- = \left[1 + 3\Phi^* e^{-E_q^-/T} + 3\Phi e^{-2E_q^-/T} + e^{-3E_q^-/T}\right]$ $E_q^\pm = E_q \mp \mu$,

$E_q = \sqrt{p^2 + {m_q}^2}, \quad m_q = g\sigma$

Outline

- The vacuum term is replaced by the renormalization scale M dependent fermion vacuum loop correction as

$$\Omega_{q\bar{q}}^{\mathrm{vac}} = \frac{N_f N_c}{8\pi^2} m_f^4 \ln\left(\frac{m_f}{\mathrm{M}}\right)$$

- The grand potential in vacuum $T = 0, \mu = 0$ takes the form

$$\Omega(\sigma) = \Omega_{q\bar{q}}^{\mathrm{vac}} + U(\sigma) = -\frac{N_c N_f}{8\pi^2} g^4 \sigma^4 \ln\left(\frac{g\sigma}{M}\right) - \frac{\lambda v^2}{2}\sigma^2 + \frac{\lambda}{4}\sigma^4 - h\sigma$$

- The vacuum term is replaced by the renormalization scale M dependent fermion vacuum loop correction as

$$\Omega_{q\bar{q}}^{\mathrm{vac}} = \frac{N_f N_c}{8\pi^2} m_f^4 \ln\left(\frac{m_f}{\mathrm{M}}\right)$$

- The grand potential in vacuum $T = 0, \mu = 0$ takes the form

$$\Omega(\sigma) = \Omega_{q\bar{q}}^{\mathrm{vac}} + U(\sigma) = -\frac{N_c N_f}{8\pi^2} g^4 \sigma^4 \ln\left(\frac{g\sigma}{M}\right) - \frac{\lambda v^2}{2}\sigma^2 + \frac{\lambda}{4}\sigma^4 - h\sigma$$

- Using $\frac{\partial\Omega(\sigma)}{\partial\sigma}|_{\sigma=f_\pi} = 0$ and $\frac{\partial^2\Omega(\sigma)}{\partial\sigma^2}|_{\sigma=f_\pi} = m_\sigma^2$, we get

$$\Omega(\sigma) = -\frac{N_c N_f}{8\pi^2} g^4 \sigma^4 \ln\left(\frac{\sigma}{f_\pi}\right) - \frac{\lambda_r v_r^2}{2}\sigma^2 + \frac{\lambda_r}{4}\sigma^4 - h\sigma$$

Here $\lambda_r = (\frac{m_\sigma^2 - m_\pi^2}{2f_\pi^2}) + \frac{3N_c N_f}{8\pi^2} g^4$, $\lambda_r v_r^2 = (\frac{m_\sigma^2 - 3m_\pi^2}{2}) + \frac{N_c N_f}{4\pi^2} g^4\ f_\pi^2$

- Adding renormalized fermionic vacuum term to the PQM potential gives the grand potential in PQMVT model as

$$\Omega_{\mathrm{MF}}(T,\mu;\sigma,\Phi,\Phi^*) = \mathcal{U}(T;\Phi,\Phi^*) + \Omega(\sigma) + \Omega_{q\bar{q}}^{\mathrm{T}}(T,\mu;\sigma,\Phi,\Phi^*).$$

- Adding renormalized fermionic vacuum term to the PQM potential gives the grand potential in PQMVT model as

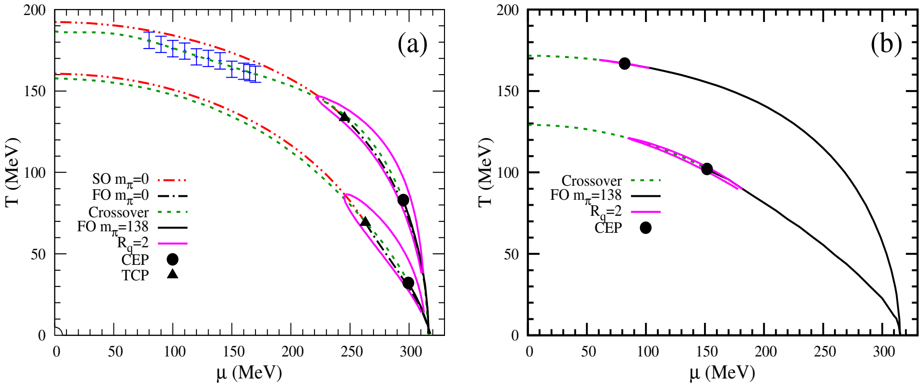
$$\Omega_{\mathrm{MF}}(T,\mu;\sigma,\Phi,\Phi^*) = \mathcal{U}(T;\Phi,\Phi^*) + \Omega(\sigma) + \Omega^{\mathrm{T}}_{q\bar{q}}(T,\mu;\sigma,\Phi,\Phi^*).$$

- We get the σ, Φ and Φ^* fields by searching the global minima of above equation for a given T and μ

$$\frac{\partial\Omega_{\mathrm{MF}}}{\partial\sigma} = \frac{\partial\Omega_{\mathrm{MF}}}{\partial\Phi} = \frac{\partial\Omega_{\mathrm{MF}}}{\partial\Phi^*} = 0$$

- We take $m_\pi = 138$ MeV, $m_\sigma = 500$ MeV, and $f_\pi = 93$ MeV in our computation. Constituent quark mass in vacuum $m_q^0 = 310$ MeV fixes the Yukawa coupling $g = 3.3$.

Outline

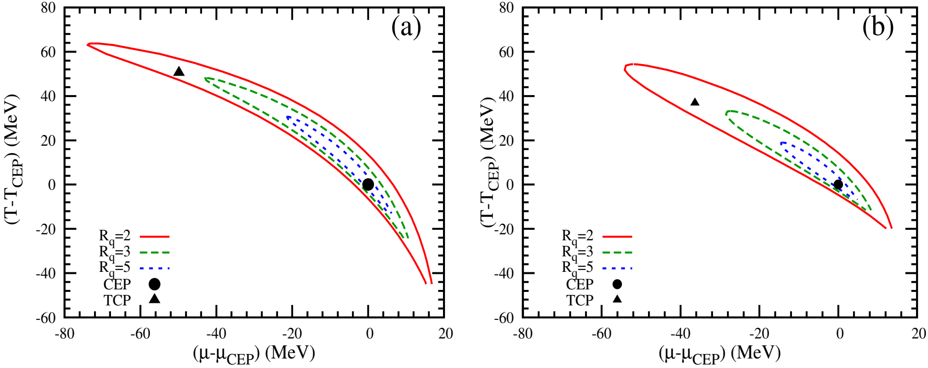


- CEP shifts to larger chemical potential $\mu_{CEP} = 295.2$ and lower $T_{CEP} = 83.0$ MeV in PQMVT model. PQM model $\mu_{CEP} = 81.0, T_{CEP} = 166.9$ MeV
- We get tricritical point (TCP) at $\mu_t = 245.3, T_t = 133.5$ MeV in the PQMVT where the second order line of O(4) critical points meets the first order line. QMVT model $\mu_t = 263.0, T_t = 69.1$ MeV. [3]

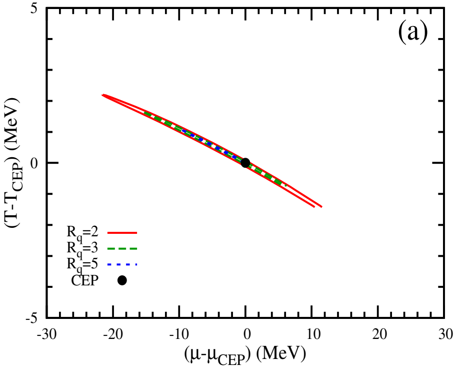
[3] V. K. Tiwari PRD 86 094032 (2012)

Outline

- Ratio $R_q = \frac{\chi_q}{\chi_q^{\text{free}}}$ and $\chi_q = -\frac{\partial^2 \Omega_{\text{MF}}}{\partial \mu^2}$, $\lim_{m_q \to 0} \chi_q(T,\mu) = 2\left[T^2 + \frac{3\mu^2}{\pi^2}\right] \equiv \chi_q^{\text{free}}$

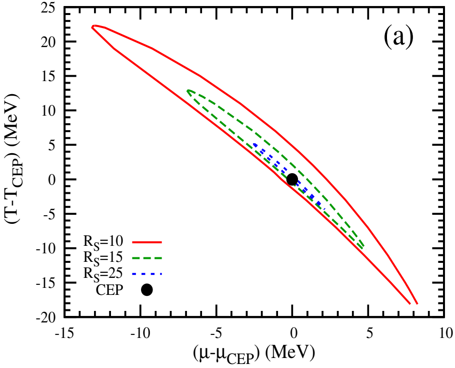


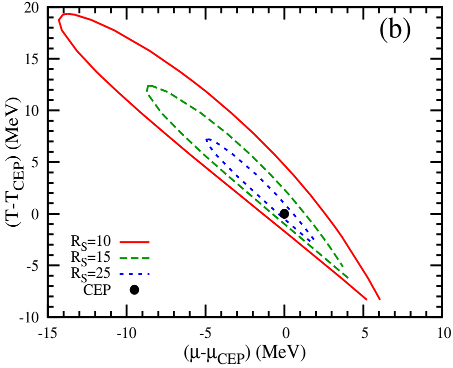
- The critical region size for PQMVT model is noticeably larger in both the directions μ and T. Width is smaller when compared to QMVT Model.
- Critical region size normal to the crossover line increases. Larger width for QMVT model only due to the fermionic vacuum fluctuation.

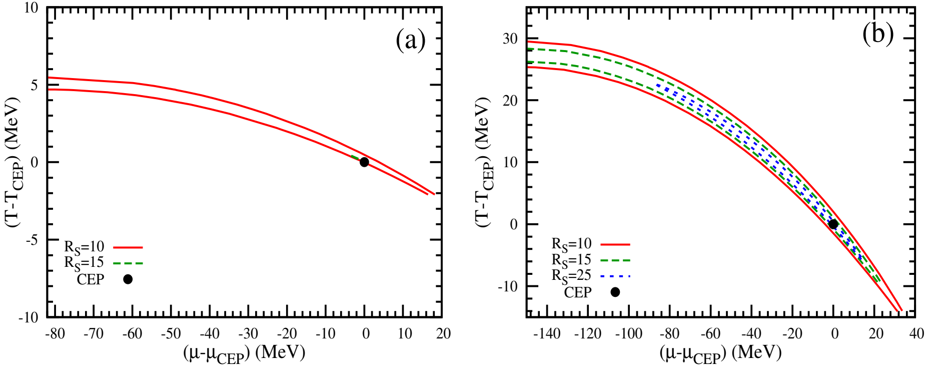


- Since the chiral crossover on the T axis at $\mu = 0$ is quite sharp and the first order line increases due to the effect of Polyakov loop potential, the critical region in the T direction gets significantly compressed in the PQM model contours when compared with the pure QM model plots.
- The transition at $\mu = 0$ is first order in the chiral limit for PQM/QM model. Hence we do not get TCP.

- All fluctuations of order parameter are encoded in the Scalar susceptibility $\chi_\sigma = -\frac{\partial^2 \Omega_{MF}}{\partial h^2} \sim m_\sigma^{-2}$. Normalized scalar susceptibility $R_s(T,\mu) = \frac{\chi_\sigma(T,\mu)}{\chi_\sigma(0,0)}$







- We get only the R_S=10 contour in the PQM model due to a sharp transition caused by the Polyakov loop. The σ mass decreases very sharply in a narrow temperature interval.

Outline

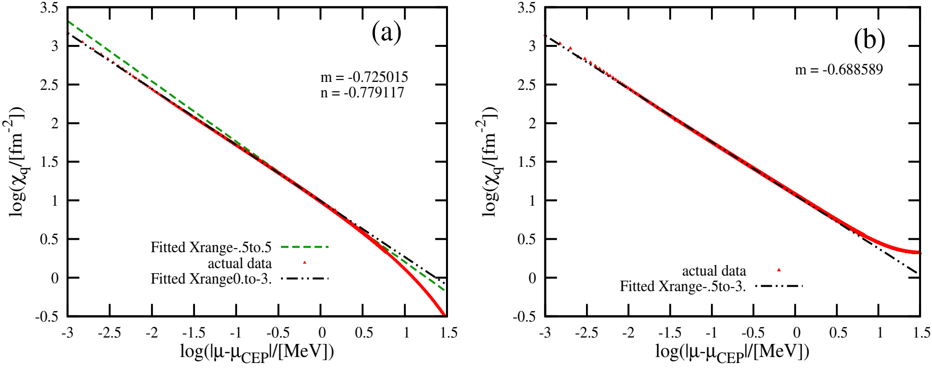
1. Introduction
   - PQM Model Lagrangian
   - Polyakov Loop Potential

2. Modified PQM Effective Potential
   - PQMVT Model

3. Phase Diagram and Critical Region
   - Proximity of the CEP to the TCP
   - Susceptibility Countours
   - Critical Exponents

4. Conclusion

- Fig(a) shows the PQMVT model plot of the $log(\chi_q)$ versus $log(\mu - \mu_{CEP})$ close to the CEP when the μ_{CEP} is approached from the lower μ side.
- Fig (b) shows the same plot as in Fig (a) when the μ_{CEP} is approached from the higher μ side.



Model	$\mu - \mu_{CEP} < 0$	$\mu - \mu_{CEP} > 0$
QM	0.6379 ± 0.0002	0.6648 ± 0.0001
PQM	0.6309 ± 0.0001	0.6668 ± 0.0001
QMVT	0.720 ± 0.00005	0.6938 ± 0.0002
PQMVT	0.725 ± 0.0002	0.6886 ± 0.0004

Table: Critical exponents of the quark-number susceptibility in the QM,PQM,QMVT and PQMVT models for two different paths parallel to the chemical potential axis approaching the μ_{CEP} from the lower $\mu < \mu_{CEP}$ and higher $\mu > \mu_{CEP}$ side.

Conclusion

- Fermionic vacuum correction leads to a significanly smoother transition at $\mu = 0$. CEP moves to higher chemical potential in consequence.
- In contrast Polyakov loop potential leades to quite a sharp transition and we get a very small and narrow critical region near the CEP.
- Critical region near the CEP for the PQMVT model is significantly large. It is stretched in the direction parallel to first order line.
- The width of the critical region increases in perpendicular direction to the crossover line due to the vacuum fluctuation.
- TCP gets located well within the enhanced quark number susceptibilty contour (R_q=2) in the PQM/QM model with vacuum fluctuation. TCP influences the critical behavior near CEP.

THANK YOU